Application of the Convolution theorem: image processing

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Convolution theorem indicates that Fourier transform of the convolution of two functions is the product of their Fourier transforms.

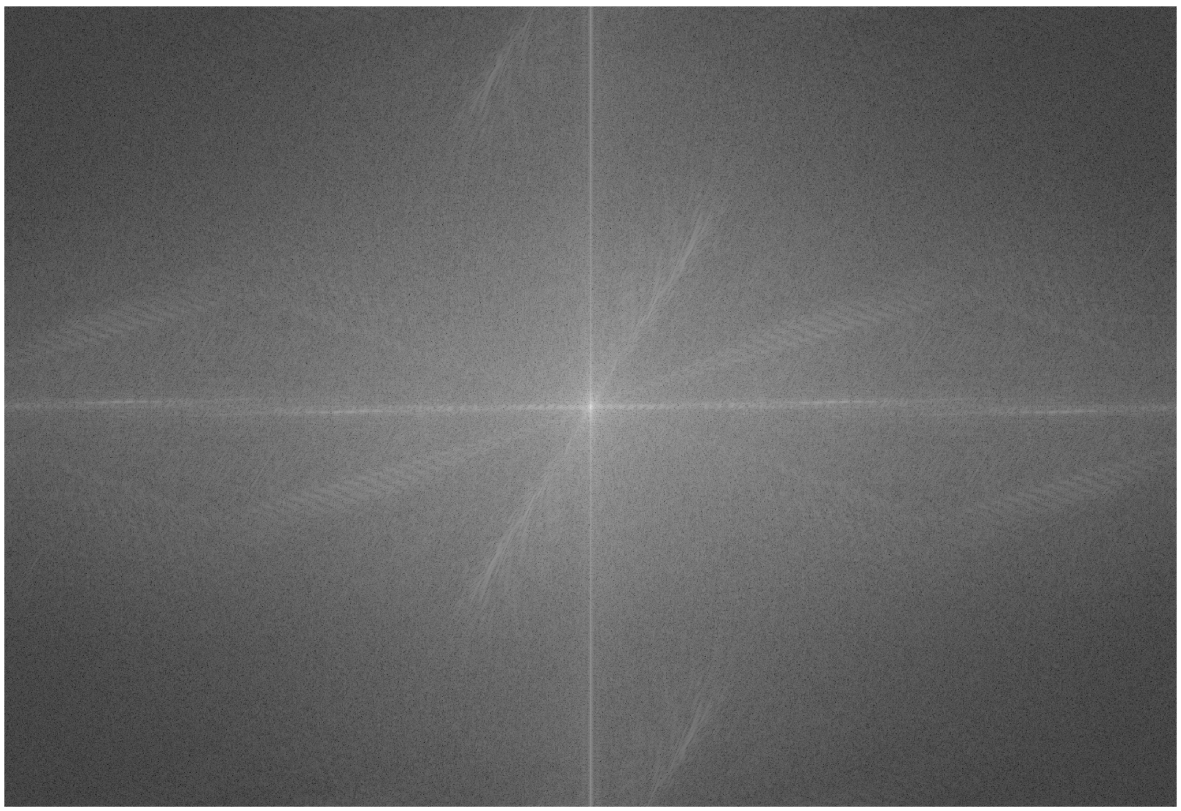
$$\mathcal{F}(x_n * y_n) = X_n Y_n$$

Import image and convert it into grayscale

```
img = imread('/Users/yue/Desktop/220C lecture slides/image_test.png');
if size(img, 3) == 3
    img_gray = rgb2gray(img);
else
    img_gray = img;
end
imshow(img_gray);
```

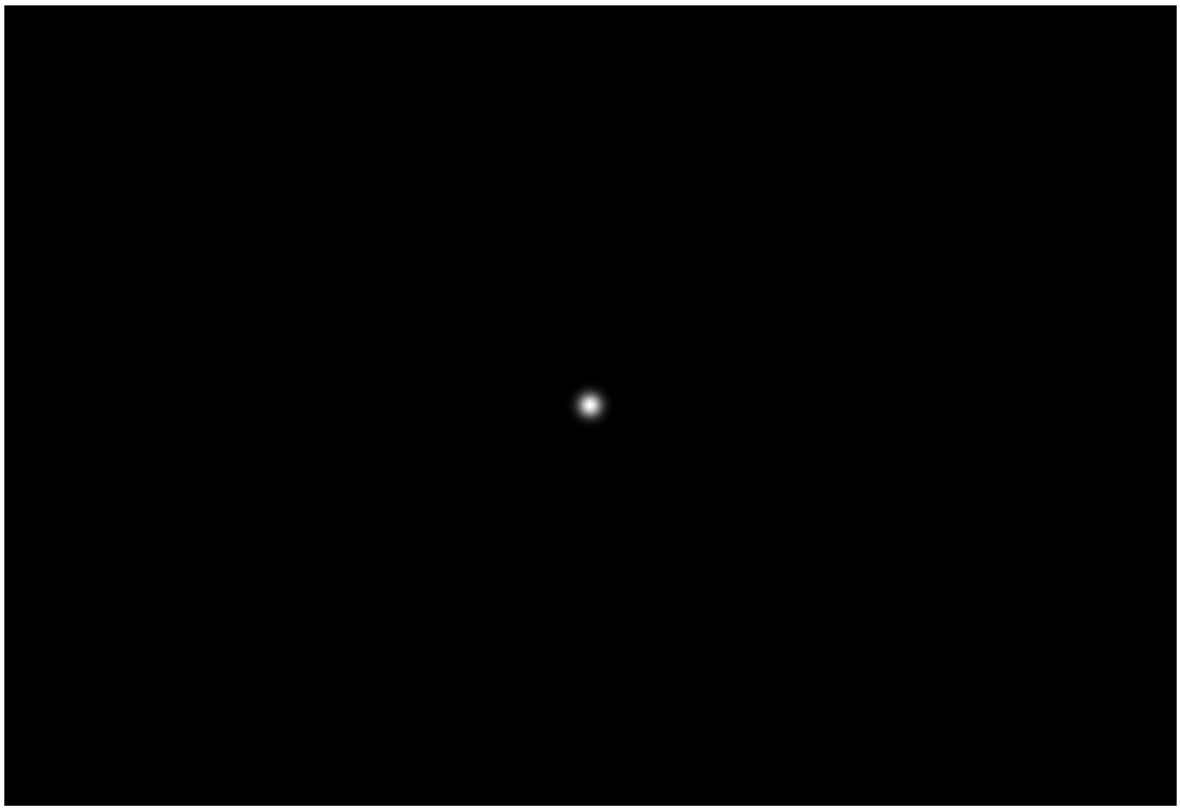
```
F_original = fft2(img_gray);% 2D fourier transform
F_shifted= fftshift(F_original); % Shift 0 frequency to center
magnitude_spectrum = log(1 + abs(F_shifted));  % Using log scale to enhance
visibility
```

```
imshow(magnitude_spectrum, []);
```

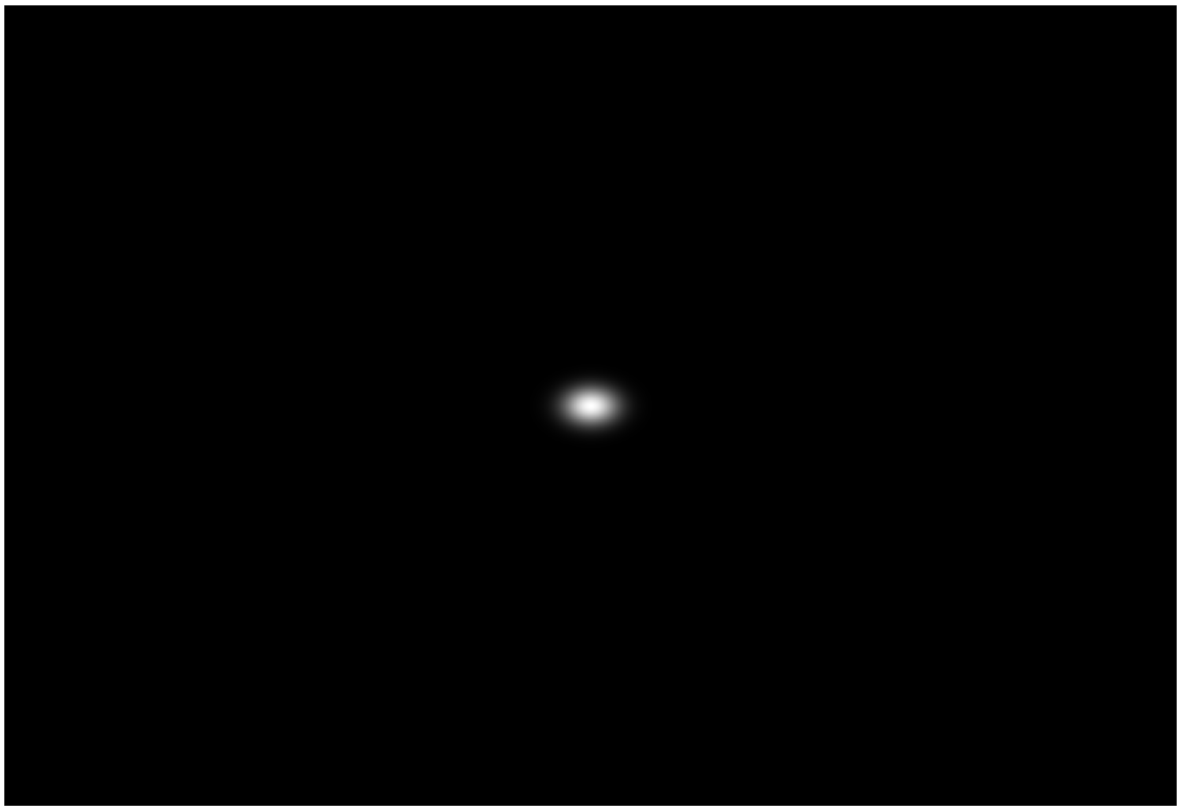


Low-pass filter

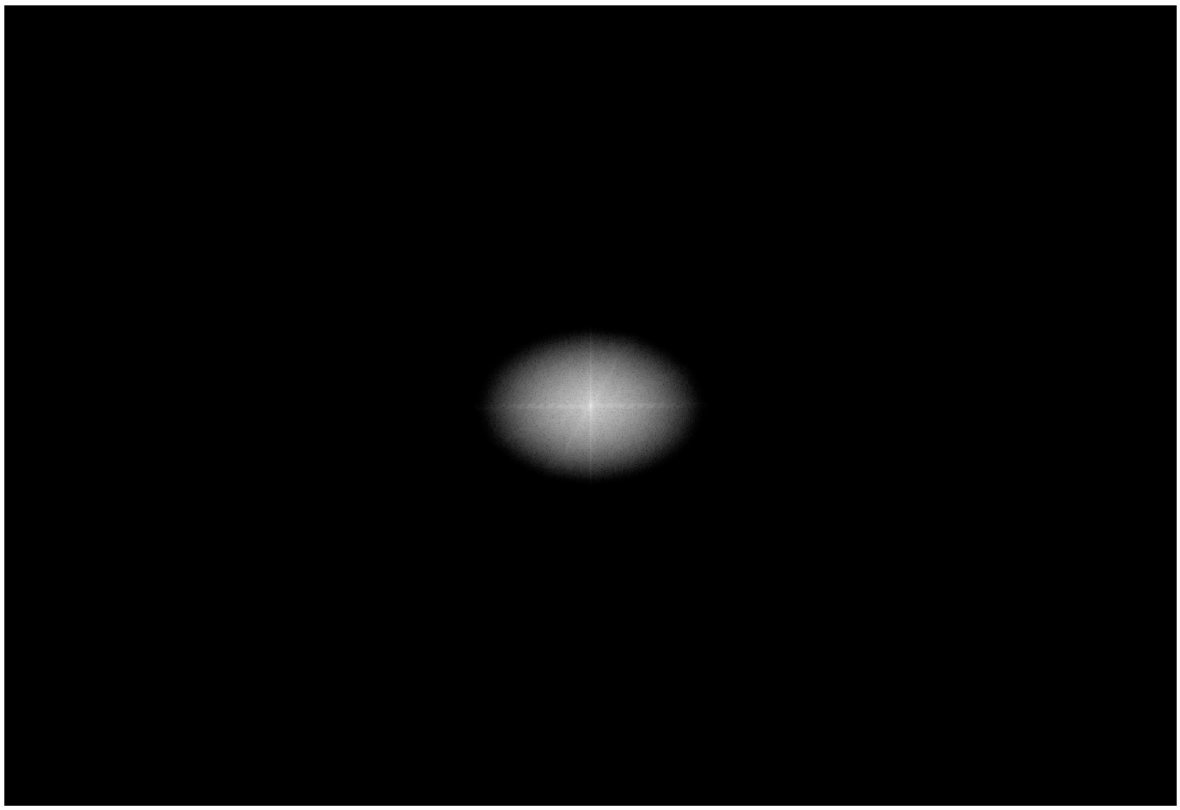
```
[n, m] = size(img_gray);
sigma = 10;
[X, Y] = meshgrid(1:m, 1:n);
centerX = ceil(m/2);
centerY = ceil(n/2);
gaussianFilter = exp(-((X-centerX).^2 + (Y-centerY).^2) / (2*sigma^2));
imshow(gaussianFilter,[]);
```



```
F_lowPass = fft2(gaussianFilter, n, m);
imshow(abs(fftshift(F_lowPass)) , []);
```



```
F_filtered = F_original .* F_lowPass;
imshow(log(1 + abs(fftshift(F_filtered))),[]);
```

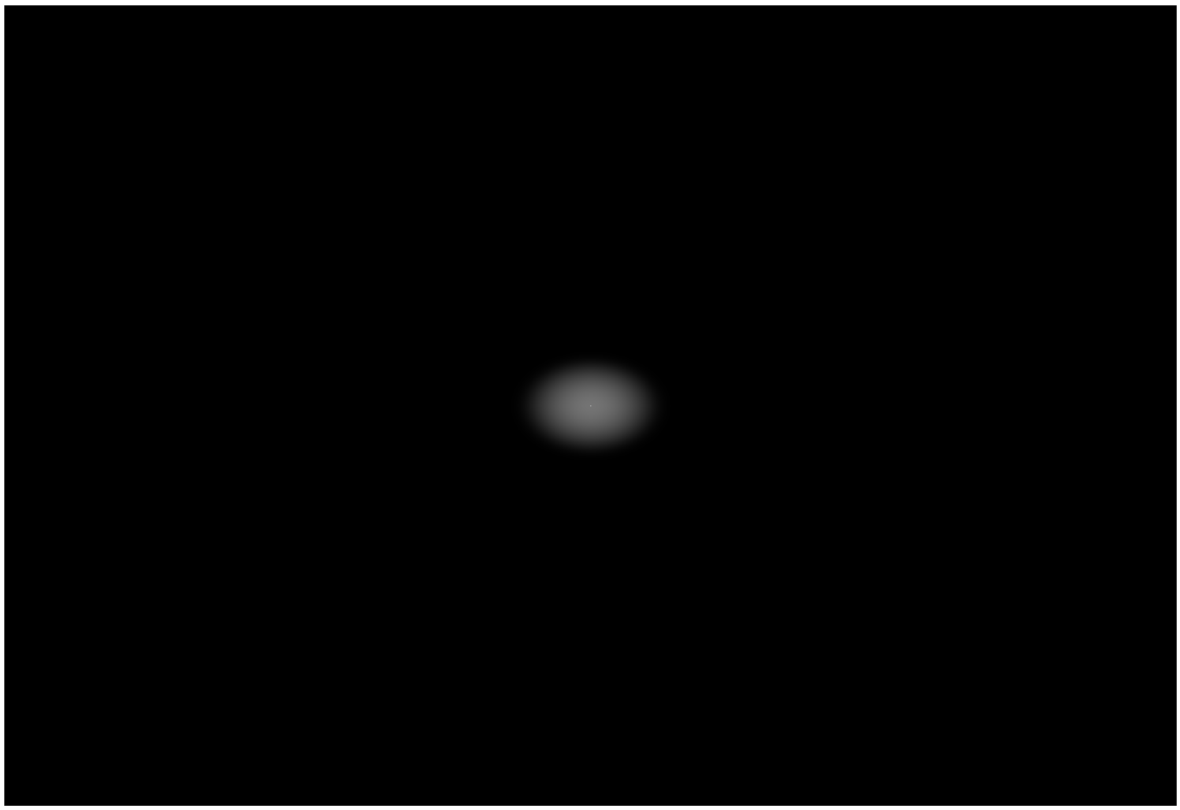


```
filtered_img = ifft2(F_filtered);
F_shifted2 = fftshift(filtered_img);
imshow(abs(F_shifted2), []);
```

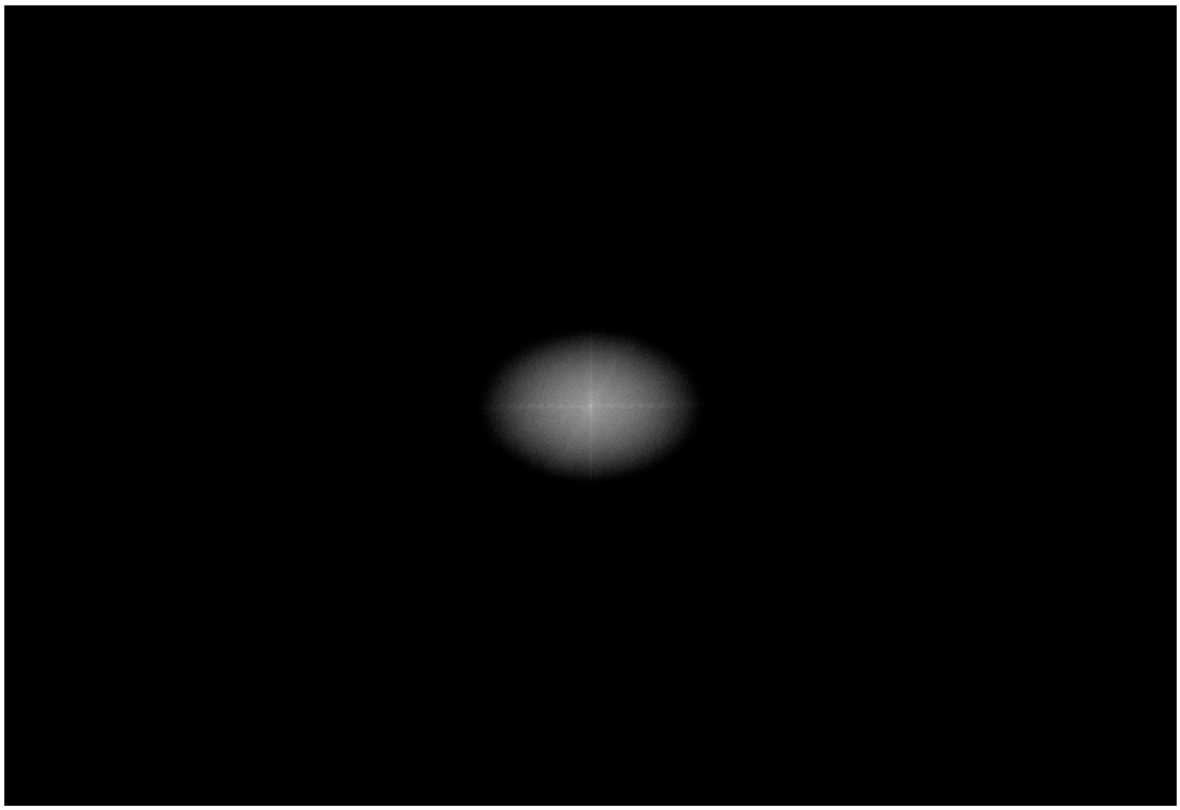
High-pass filter

```
highPassFilter = 1 - gaussianFilter;
imshow(highPassFilter,[]);
```

```
F_img = fft2(img_gray);
F_highPass = fft2(highPassFilter, n, m);
imshow(log(1 + abs(fftshift(F_highPass))),[]);
```



```
F_filtered = F_original .* F_highPass;
imshow(log(1 + abs(fftshift(F_filtered))),[]);
```



```
filtered_img = ifft2(F_filtered);
F_shifted3 = fftshift(filtered_img);
imshow(abs(F_shifted3), []);
```