Project Week 04

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Fintech 545

Problem 1

Calculate and compare the expected value and standard deviation of price at time t, each of the 3 types of price returns, assuming $r \sim N(0, \sigma^2)$*. Simulate each return equation using* $r \sim N(0, \sigma^2)$ *and show the mean and standard deviation match your expectations.*

To calculate the expected value and standard deviation of the price at time t for each type of price return given $r \sim N(0, \sigma^2)$, I used the following formulas:

Classical Brownian Motion: E(Pt) = Pt; Sigma(Pt) = Sigma

Arithmetic Return System: E(Pt) = Pt; Sigma(Pt) = Pt * Sigma

Geometric Brownian Motion: E(Pt) = Pt * e ^ ((sigma^2) / 2); Sigma(Pt) = Pt * sqrt(e ^ (sigma^2) * (e ^ (sigma^2) -1)). By assuming p0 = 100 and sigma = 0.3, I simulate each return equation 10000 times and get the calculated expected value and standard deviation. The plotted distribution of each simulation is shown in Figure 1 below and the result is shown in Table 1 below:

Figure 1

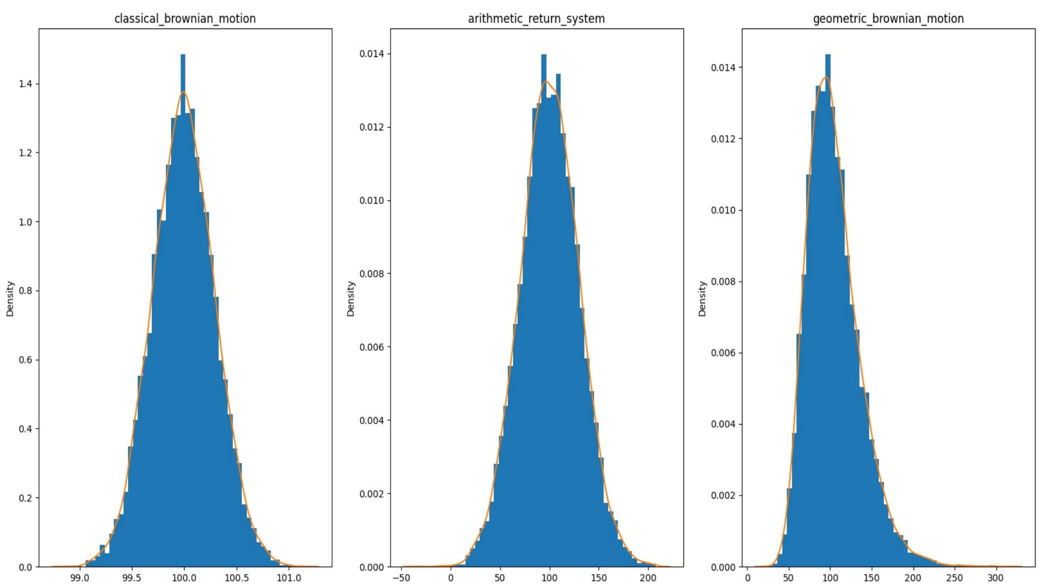


Table 1

	Calculated Mean	Actual Mean	Calculated Std	Actual Std
Classical Brownian Motion	100	99.99446988395	0.3	0.296266970453
Arithmetic Return System	100	100.32964017619	30.0	29.79164042902
Geometric Brownian Motion	104.6027859909	104.3834085052	32.10032389503	31.795683037018

We can see that the difference between the calculated mean and standard deviation using the formulas and the actual mean and standard deviation of the simulation is relatively small. This shows that the mean and standard deviation match my expectations.

Problem 2

Implement a function similar to the "return_calculate()" in this week's code. Allow the user to specify the method of return calculation.

Use DailyPrices.csv. Calculate the arithmetic returns for all prices.

Remove the mean from the series so that the mean(META)=0.

Calculate VaR

Using a normal distribution.

Using a normal distribution with an Exponentially Weighted variance (λ = 0. 94).

Using a MLE fitted T distribution.

Using a fitted AR(1) model.

Using a Historic Simulation.

Compare the 5 values.

To implement a function similar to the "return_calculate()" in this week's code, I converted the code into python code and used it to calculate the arithmetic returns for all prices using the DailyPrices.csv file. Then I removed the mean from the series so that the mean(META)=0. I used META to calculate VaR using five methods: normal distribution, normal distribution with an Exponentially Weighted variance (λ = 0. 94), MLE fitted T distribution, fitted AR(1) model, and historic simulation. The plotted distribution of each simulation is shown in Figure 2 below and the VaR result is shown in Table 2 below:

Figure 2

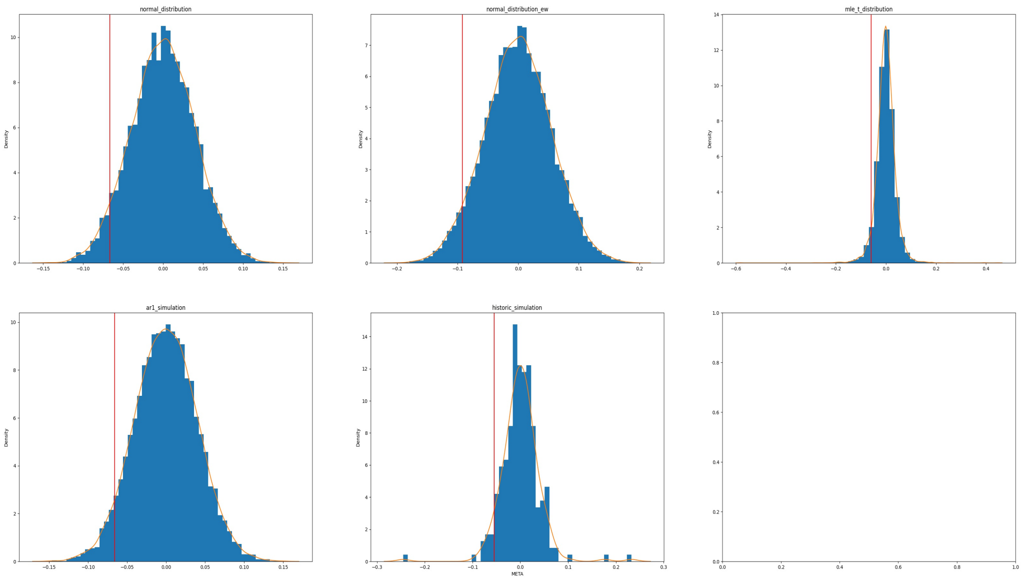


Table 2

	Normal Distribution	Normal Distribution with EW Var	MLE fitted T distribution	Fitted AR(1) model	Historic Simulation
VaR	0.0664955577	0.09240237401	0.05858878708	0.06599187488	0.05462007908

From the graph and table, we can see that the MLE fitted T distribution model has the closest VaR to the Historic Simulation. Their distribution histogram also shows a similar shape, which causes the similarity in their VaRs. Among the five VaRs, the normal distribution with an Exponentially Weighted variance and λ = 0. 94 has the highest VaR and is a lot greater than that of other models. The graph of AR(1) shows that it is very similar to a normal distribution, therefore its VaR is also close to that of the normal distribution.

Problem 3

Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0.

This file contains the stock holdings of 3 portfolios. You own each of these portfolios. Using an exponentially weighted covariance with lambda = 0.94, calculate the VaR of each portfolio as well as your total VaR (VaR of the total holdings). Express VaR as a $.

Discuss your methods and your results.

Choose a different model for returns and calculate VaR again. Why did you choose that model? How did the model change affect the results?

To calculate the VaR of each portfolio as well as the total VaR (VaR of the total holdings), I first used the Delta Normal method, which can be calculated using the formula $VaR(\alpha) =- PV * F^{-1}(\alpha) * \nabla R^{T} \Sigma \nabla R$. The VaR result is shown in Table 3 below:

Table 3

	Portfolio A	Portfolio B	Portfolio C	Portfolio ALL
VaR	5670.20292014734	4494.59841077826	3786.58901080905	13577.0754189771

Then I chose different models for returns and calculate VaR again by using Monte Carlo Simulation and Historic Simulation. The plotted distribution of each simulation is shown in Figure 3 and Figure 4 below and the VaR result is shown in Table 4 below:

Figure 3

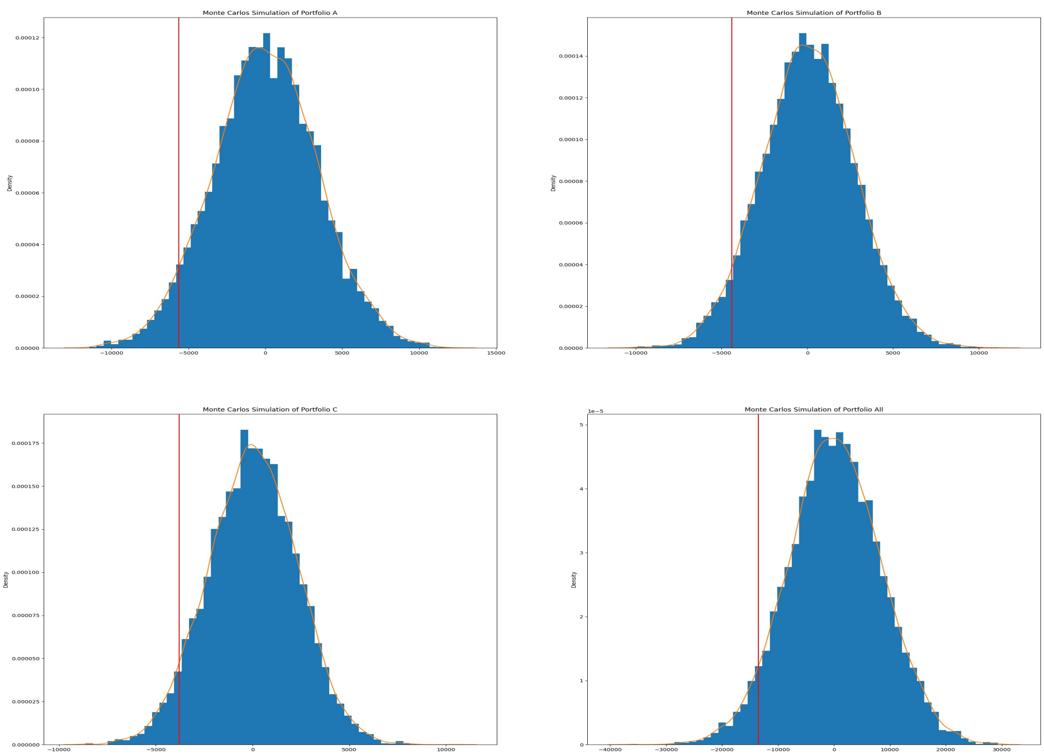


Figure 4

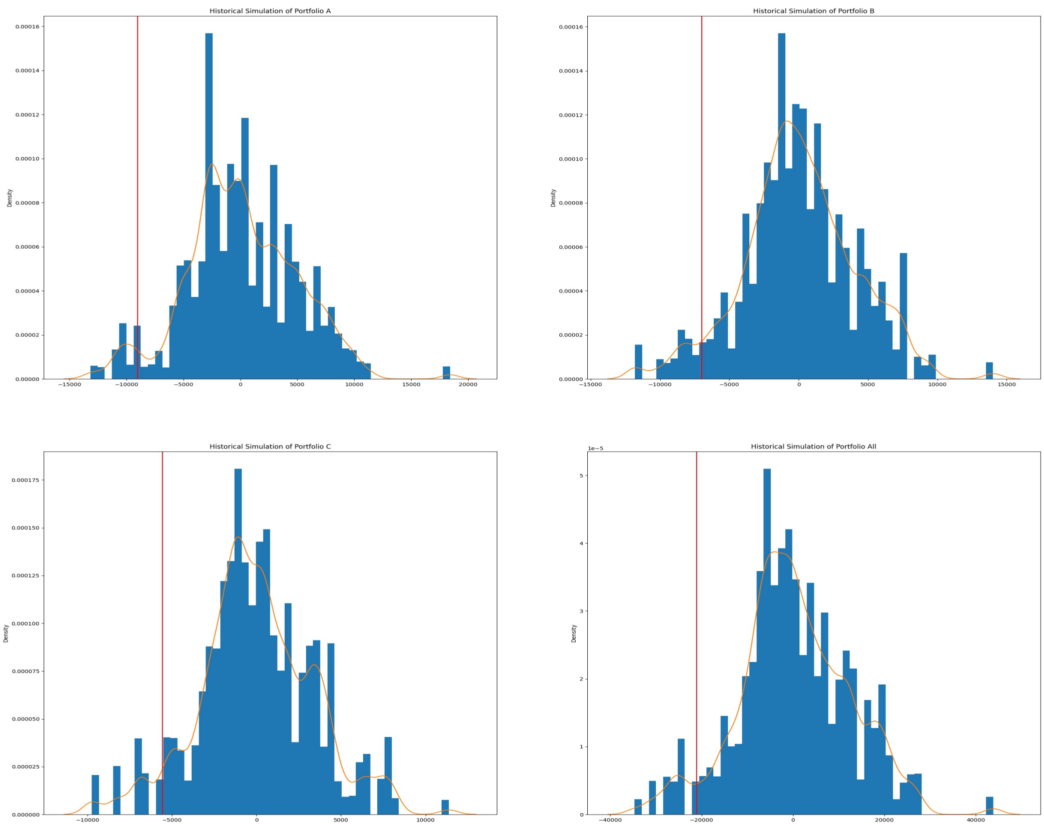
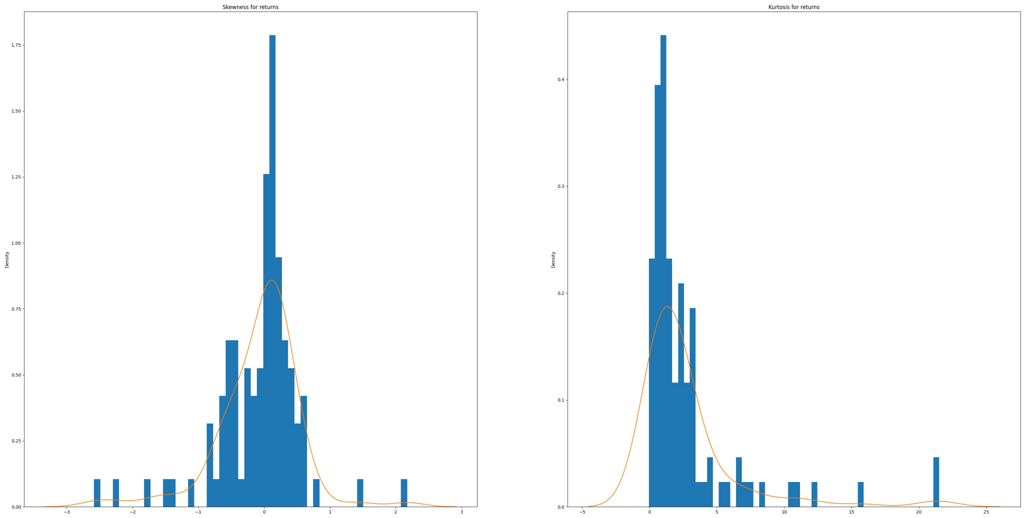


Table 4

	Portfolio A	Portfolio B	Portfolio C	Portfolio All
VaR for Monte Carlo	5614.9589525	4396.2733605337	3800.17150354744	13502.654936811
VaR for Historic	9005.0672162	7001.11745441394	5558.7244034558	21103.3980107681

From the graph and table, we can see that the VaR for Monte Carlo is the smallest and the VaR for Historic is the largest. The difference between Delta Normal and Monte Carlo is due to the additional assumption of linearity for Delta Normal. This causes the model to estimate more risk to the portfolio. We can also see that the while the difference between Monte Carlo VaR and Delta Normal VaR is relatively small, the VaR for Historic Simulation is a lot greater. This is because the distribution of returns is not normal. To show this, I calculated the skewness and kurtosis for the returns, which are shown in Figure 5 below.

Figure 5



The violation of normal distribution for Monte Carlo and Delta Normal will make them underestimate the risk for returns since there are more outliers with extreme values in reality. And the Historic Simulation can provide a better illustration of these outliers.