Computer-Linguistische Anwendungen

CLA | B.Sc. | LMU

Wiederholung: Lineare Algebra

Slides recycled from B. Roth / H. Schuetze

Wiederholung: Lineare Algebra

Punkt Produkt / Skalarprodukt

Beispiel:

$$\vec{w} \cdot \vec{c} = \sum_i w_i c_i$$

$$\begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = w_1 c_1 + w_2 c_2 + w_3 c_3$$

Linear Algebra: C = AB

			B	
			1	-2
			-1	2
A	1	1	0	0
	2	1	1	-2

$$C_{11} = A_{11}B_{11} + A_{12}B_{21}$$
$$C_{12} = A_{11}B_{12} + A_{12}B_{22}$$
$$C_{21} = A_{21}B_{11} + A_{22}B_{21}$$
$$C_{22} = A_{21}B_{12} + A_{22}B_{22}$$

Linear Algebra: C = AB

			B	
			0.7	0.3
			0.2	0.8
	0	1	0.2	0.8
A	0.2	0.8	0.3	0.7
	0.3	0.7	0.35	0.65

$$
\begin{aligned}
C_{11} &= A_{11}B_{11} + A_{12}B_{21} \\
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C_{21} &= A_{21}B_{11} + A_{22}B_{21} \\
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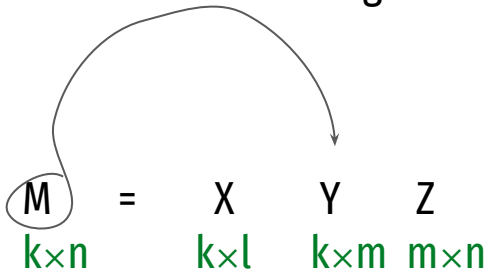
C_{31} &= A_{31}B_{11} + A_{32}B_{21} \\
C_{32} &= A_{31}B_{12} + A_{32}B_{22}
\end{aligned}
$$

Linear Algebra: C = AB

Matrix Faktorisierung



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$$

Wichtige Konzepte:

Euklidische Länge des Vektors $\vec{d}$

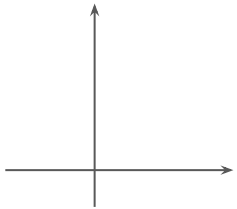
$$|\vec{d}| = \sqrt{\sum_{i=1}^{n} d_i^2}$$

Äquivalent ist: die Quadratwurzel des *Dot Products* von *d*

$\vec{c}$ und $\vec{d}$ sind orthogonal nur wenn

$$\sum_{i=1}^{n} c_i \cdot d_i = 0$$

Genau dann, wenn das dot product null ist.



Aufgabe:

V^T	d_1	d_2	d_3	d_4	d_5	d_6
1	−0.75	−0.28	−0.20	−0.45	−0.33	−0.12
2	−0.29	−0.53	−0.19	0.63	0.22	0.41
3	0.28	−0.75	0.45	−0.20	0.12	−0.33
4	0.00	0.00	0.58	0.00	−0.58	0.58
5	−0.53	0.29	0.63	0.19	0.41	−0.22

Show: column d_1 has unit length: $\sqrt{\sum_i d_{i1}^2} = 1$

Show: columns d_1, d_2 are orthogonal: $\sum_i d_{i1} \cdot d_{i2} = 0$

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Show: columns d_1, d_2 are orthogonal: $\sum_i d_{i1} \cdot d_{i2} = 0$

$$0.75^2 + 0.29^2 + 0.28^2 + 0.00^2 + 0.53^2 = 1.0059$$

$$-0.75 * -0.28 + -0.29 * -0.53 + 0.28 * -0.75 + 0.00 * 0.00 + -0.53 * 0.29 = 0$$