ISE 533 LEO_Wyndor

Group 5: Yueru Zhang, Xuchen Shao, Zhaoqi Xiao, Yongjun Kim

Problem Statement

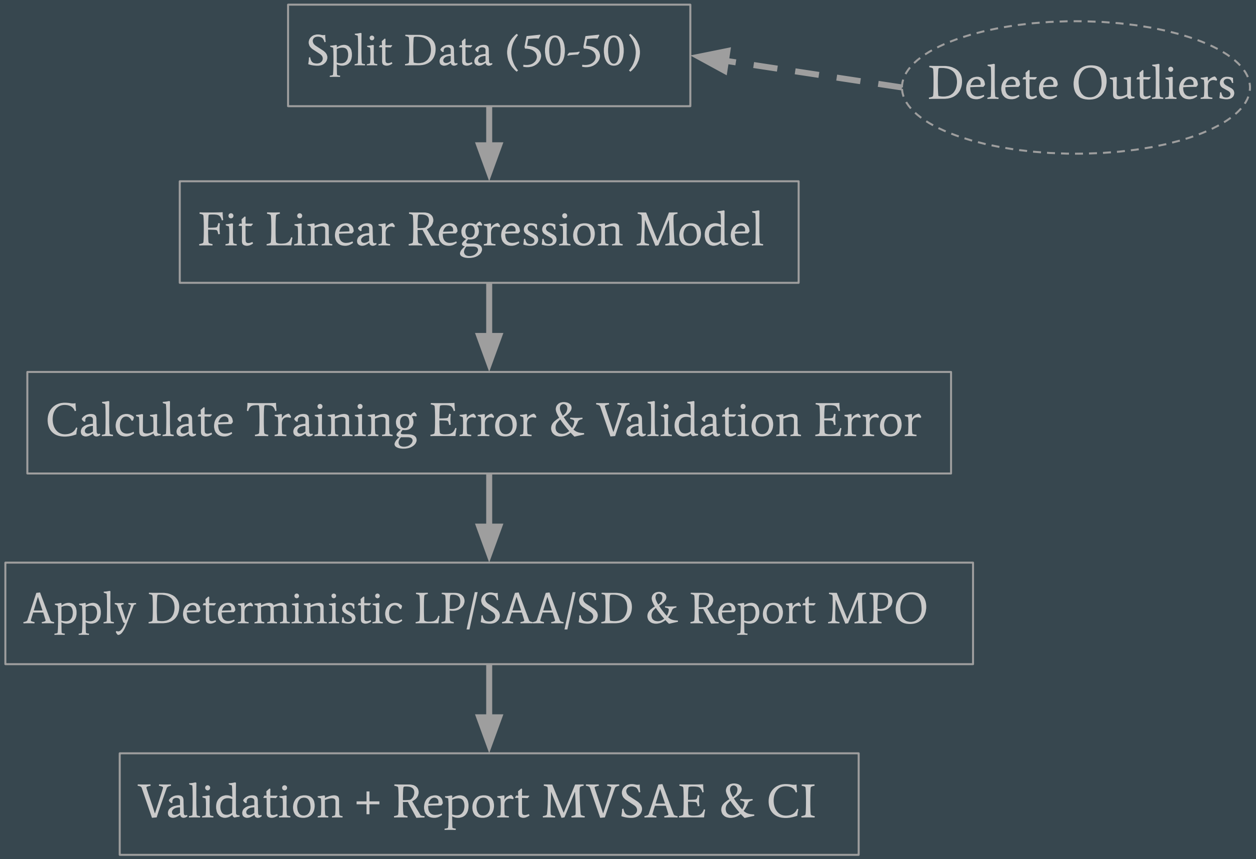
The Wyndor Glass Co

- Produce 2 high quality glass doors (y_A, y_B)
- Resource provided by 3 companies
- Advertising: TV & Radio → Total budget: $200,000
- Potential door sales is related to marketing strategy (Advertising)

Goal

- Determine the optimal expenditure of TV and Radio (x_1, x_2)
- Maximize Wyndor's expected profit

Workflow


Split Data (50-50)
Delete Outliers
Fit Linear Regression Model
Calculate Training Error & Validation Error
Apply Deterministic LP/SAA/SD & Report MPO
Validation + Report MVSAE & CI

Advertising-Operations Integrated (AOI)

1st Stage

$$\begin{aligned} \text{Max} \quad & -0.1x_1 - 0.5x_2 + \mathbb{E}[\text{Profit}(\tilde{\omega})] \\ \text{s.t.} \quad & x_1 + x_2 \le 200 \\ & x_1 - 0.5x_2 \ge 0 \\ & L_1 \le x_1 \le U_1, L_2 \le x_2 \le U_2 \end{aligned}$$

x_1: Expenditures of TV

x_2: Expenditures of Radio

$\tilde{\omega}$: Total sales

[L1, U1] = [0.7, 293.6], [L2, U2] = [0.3, 49.4]

2nd Stage

$$\begin{aligned} \text{Profit}(\omega) \quad = \quad \text{Max} \quad & 3y_A + 5y_B \\ \text{s.t.} \quad & y_A \le 8 \\ & 2y_B \le 24 \\ & 3y_A + 2y_B \le 36 \\ & y_A + y_B \le \omega \\ & y_A, y_B \ge 0 \end{aligned}$$

y_A: # of production of door A

y_B: # of production of door B

ω: Total sales profit

The total production should not be more than the estimated demand

Deterministic LP - Description Analytics

Linear Regression

- Split data into training and validation set (100 rows each)
- Build a linear regression model using the training set

```
Sales ~ 1 + TV + Radio

Coefficients:
```

	Coef.	Std. Error	t	Pr(>|t|)	Lower 95%	Upper 95%
(Intercept)	2.32059	0.469254	4.95	<1e-05	1.38925	3.25193
TV	0.0495754	0.00215066	23.05	<1e-40	0.0453069	0.0538438
Radio	0.186071	0.0119735	15.54	<1e-27	0.162307	0.209835

- $\beta_0 = 2.32059, \beta_1 = 0.04958, \beta_2 = 0.186071$

Deterministic LP - Description Analytics

- Calculate Epsilon (error) of training and validation sets

$$\varepsilon_{ti} := \omega_i - \beta_0 - \beta_1 x_{1i} - \beta_2 x_{2i}$$

```
100-element Vector{Float64}:
  1.3386365562274456
 -1.4392817063207648
 -4.650764526221992
 -2.1095329560725684
  1.923266640554342
  1.1316724323736
  0.0080037605393137731
  0.760492542312619
 -0.302015218744657
 -0.33351286837195904
 -1.2832452700154349
  1.9463706869123811
 -4.005202461760707
  ⋮
 -2.4787929999435843
  1.8531220115729141
 -0.5096709441454266
 -0.0678770963885249
  2.532669868428602
  1.6204840918008792
 -1.2661292396212112
  1.7246604554741367
  0.6728971482197483
 -1.1263325512492113
  1.9638226800122744
  1.304856493923488
```

$$\varepsilon_{vi} := \omega_i - \beta_0 - \beta_1 x_{1i} - \beta_2 x_{2i}$$

```
100-element Vector{Float64}:
 -2.413942345861635
  0.9840121443247511
 -0.39338263654201455
  0.5256995897058871
  1.2734603950968655
  1.6623127045308839
 -0.0301239059487024
 -0.8315736856287721
  0.4393436762110561
  1.5167995524016646
  1.7343402780649608
  0.529862931458716
 -2.5387440939550103
  ⋮
  0.2090196906238635
 -1.957590935433247
  0.16532137006122127
 -3.185531152073013
  1.2008921602620237
  1.8269035214378384
  1.1952593738099786
  0.9338521268549975
  2.6971683438969807
  1.7976626646872802
 -0.02588984102719394
 -2.02724291592345
```

Deterministic LP - Description Analytics

- F-test

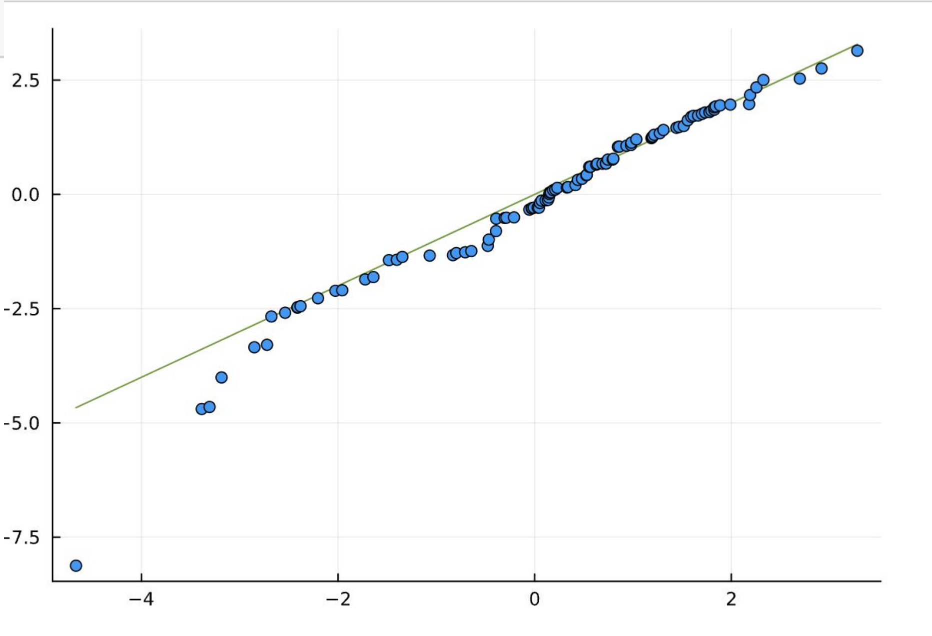
```
VarianceFTest(epsilon_vi_saa, epsilon_ti_saa)
```

```
Variance F-test
---------------
Population details:
    parameter of interest:   variance ratio
    value under h_0:         1.0
    point estimate:          0.717883

Test summary:
    outcome with 95% confidence: fail to reject h_0
    two-sided p-value:           0.1008

Details:
    number of observations: [100, 100]
    F statistic:            0.7178832775527605
    degrees of freedom:     [99, 99]
```

- QQ Plot



Deterministic LP - Prescriptive Analytics

Deterministic LP Model

$$\mathbb{E}[\tilde{\omega}] = \mathbb{E}[\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \tilde{\varepsilon}] = \beta_0 + \beta_1 x_1 + \beta_2 x_2.$$

$$\begin{aligned}
\text{Max} \quad & -0.1x_1 - 0.5x_2 + 3y_A + 5y_B \\
\text{s.t.} \quad & x_1 + x_2 \le 200 \\
& x_1 - 0.5x_2 \ge 0 \\
& y_A \le 8 \\
& 2y_B \le 24 \\
& 3y_A + 2y_B \le 36 \\
& -\beta_1 x_1 - \beta_2 x_2 + y_A + y_B \le \beta_0 \\
& y_A, y_B \ge 0 \\
& L_1 \le x_1 \le U_1, \quad L_2 \le x_2 \le U_2
\end{aligned}$$

Result

Methodology	x_1	x_2	MPO (in $)
Deterministic LP	172.422	27.578	$40,969

Deterministic LP - Model Validation Sample Average Estimate

- Using ε_{vi} and calculate $\omega_i := \beta_0 + \beta_1\hat{x}_1 + \beta_2\hat{x}_2 + \varepsilon_{vi}$
- Calculate $Profit(\omega_i)$ in each scenario with $\hat{x}_1, \hat{x}_2$ fixed by solving LPs
- Calculate $-0.1\hat{x}_1 - 0.5\hat{x}_2 + \frac{1}{N}\sum_{i=1}^{N} Profit(\omega_i)$ (average) & standard deviation
- Get 95% CI $\rightarrow$ MVSAE

```
-0.1x1 -0.5x2 + mean(profit) + 1.96 * std(profit)/10
44.428601180974155

-0.1x1 -0.5x2 + mean(profit) - 1.96 * std(profit)/10
42.58674099707976
```

Methodology	x_1	x_2	MPO (in $)	MVSAE (in $)
Deterministic LP	172.422	27.578	$40,969	$[38,758, 39,971]

SLP with SAA-Prescriptive Analytics

All-in-one model

Use the same linear regression model and ε_{ti}

$$
\begin{aligned}
\text{Max} \quad & -0.1x_1 - 0.5x_2 + \frac{1}{N}\sum_{i=1}^{N}(3y_{Ai} + 5y_{Bi}) \\
\text{s.t.} \quad & x_1 + x_2 \le 200 \\
& x_1 - 0.5x_2 \ge 0 \\
& y_{Ai} \le 8 \quad i = 1, \ldots, N \\
& 2y_{Bi} \le 24 \quad i = 1, \ldots, N \\
& 3y_{Ai} + 2y_{Bi} \le 36 \quad i = 1, \ldots, N \\
& -\beta_1 x_1 - \beta_2 x_2 + y_{Ai} + y_{Bi} \le \beta_0 + \varepsilon_{ti} \quad i = 1, \ldots, N \\
& L_1 \le x_1 \le U_1, \quad L_2 \le x_2 \le U_2, y_{Ai}, y_{Bi} \ge 0.
\end{aligned}
$$

SLP with SAA-Prescriptive Analytics

All-in-one model

```
saa_model = Model(GLPK.Optimizer)

@variable(saa_model, x1 >= 0)
@variable(saa_model, x2 >= 0)
@variable(saa_model, ya[1:N] >= 0)
@variable(saa_model, yb[1:N] >= 0)

@objective(saa_model, Max, -0.1 * x1 - 0.5 * x2 + 1 / N * sum(3 * ya[i] + 5 * yb[i] for i = 1:N))
@constraint(saa_model, c1, x1 + x2 <= 200)
@constraint(saa_model, c2, x1 - 0.5 * x2 >= 0)
@constraint(saa_model, c3[i = 1:N], ya[i] <= 8)
@constraint(saa_model, c4[i = 1:N], 2 * yb[i] <= 24)
@constraint(saa_model, c5[i = 1:N], 3 * ya[i] + 2 * yb[i] <= 36)
@constraint(saa_model, c6[i = 1:N], -1 * beta_1_saa * x1 - beta_2_saa * x2 + ya[i] + yb[i] <= beta_0_saa + epsilon_ti_saa[i])
@constraint(saa_model, c7, x1 >= L1_saa)
@constraint(saa_model, c8, x1 <= U1_saa)
@constraint(saa_model, c9, x2 >= L2_saa)
@constraint(saa_model, c10, x2 <= U2_saa)

optimize!(saa_model)
@assert(termination_status(saa_model) == OPTIMAL)
@show objective_value(saa_model)
x1 = value.(x1)
@show value.(x1)
x2 = value.(x2)
@show value.(x2)
@show value.(ya)
@show value.(yb)
```

```
objective_value(saa_model) = 40.35176166537608
value.(x1) = 186.35494835621043
value.(x2) = 13.645051643789573
value.(ya) = [3.4367892124691117, 0.65887094992
554285, 1.7961374374970092, 1.764639787869708,
28892629830047767, 4.0, 0.0, 3.795990100567806
298988631, 0.8574231785272417, 3.36248816022703
67810563083, 4.0, 0.7300776549674453, 3.178472
611089000324952, 4.0, 1.1086018298759779, 3.301
63717694975, 2.8780112923047554, 3.555248754103
635247665545, 2.2035874914380145, 2.51581491120
0035, 2.5245007671235804, 2.301678097310065, 2.
8953448715061, 0.0, 4.0, 4.0, 0.0, 0.0, 2.24752
66, 2.768639031556738, 3.931374755950186, 2.768
341677, 2.1412118334199643, 0.0, 2.257927751291
79670264597496, 0.0, 3.9512746678145803, 1.5884
620455, 3.822813111715803, 2.7710498044614145,
value.(yb) = [12.0, 12.0, 9.447388130019675, 11
```

SLP with SAA-Validation

3. Calculate $\omega_i := \beta_0 + \beta_1 \hat{x}_1 + \beta_2 \hat{x}_2 + \varepsilon_{vi}$ with ε_{vi} for i = 1...N.

For each scenario, calculate Profit(ω_i) with $\hat{x}_1, \hat{x}_2$ fixed by solving the second stage LPs.

$$
\begin{aligned}
\text{Profit}(\omega) \quad = \quad \text{Max} \quad & 3y_A + 5y_B \\
\text{s.t.} \quad & y_A \leq 8 \\
& 2y_B \leq 24 \\
& 3y_A + 2y_B \leq 36 \\
& y_A + y_B \leq \omega \\
& y_A, y_B \geq 0
\end{aligned}
$$

```
100-element Vector{Any}:
 58.421051551900156
 69.24649440169925
 65.11431005909895
 67.87155673784267
 70.1148391540156
 71.28139608231766
 66.20408625087889
 63.79973691183868
 67.61248899735817
 70.84485662592999
 71.49747880291989
 67.88404676310114
 57.79704281143328
  ⋮
 66.92151704059658
 60.421685162425256
 66.79042207890866
 54.56310752084327
 69.89713444951107
 71.77516853303851
 69.88023609015494
 69.09601434928999
 72.0
 71.68744596278684
 66.21678844564342
 60.21272922095465
```

SLP with SAA-Validation

Calculate $-0.1\hat{x}_1 - 0.5\hat{x}_2 + \frac{1}{N}\sum_{i=1}^{N} Profit(\omega_i)$ MVSAE

```
-0.1*x1 - 0.5*x2 + mean(profit) - 1.96 * std(profit)/10
```

```
40.03372588032506
```

```
-0.1*x1 - 0.5*x2 + mean(profit) + 1.96 * std(profit)/10
```

```
41.980139692802645
```

Methodology	x_1	x_2	MPO (in $)	MVSAE (in $)
SLP with SAA-Validation	186.35	13.65	$40351.8	$[40033.7, 41980.1]

SLP with SD

```
@variable(model_sd, tv_min <= x1 <= tv_max)
@variable(model_sd, radio_min <= x2 <= radio_max)
@variable(model_sd, ya >= 0)
@variable(model_sd, yb >= 0)

@objective(model_sd, Min, -1*(-0.1x1 -0.5x2 + 3ya + 5*yb))

@constraint(model_sd, c1, x1 + x2 <= 200)
@constraint(model_sd, c2, x1 - 0.5x2 >= 0)
@constraint(model_sd, c3, ya <= 8)
@constraint(model_sd, c4, 2yb <= 24)
@constraint(model_sd, c5, 3ya + 2yb <= 36)
@constraint(model_sd, c6, -b1 * x1 - b2 * x2 + ya + yb <= 0)

split_position = Position(c3, ya)

using Random
rng = Random.MersenneTwister(1234)


function mystoc()
    d = rand(rng, performance_traindf.error)
    binding = [Position(c6, "RHS") => b0 + d]
    return OneRealization(binding)
end

user_mean = [b0]

result = solve_sd(model_sd, split_position, user_mean, mystoc)
```

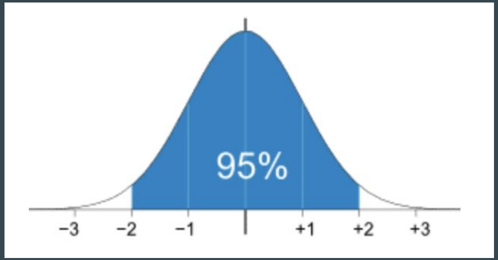
```
Starting evaluation.
...............
Upper bound estimate                    : -35.039221
Error in estimation                     : -0.015828
Confidence interval at 95%              : [-35.316521, -34.761920]
Number of observations                  : 1528
```

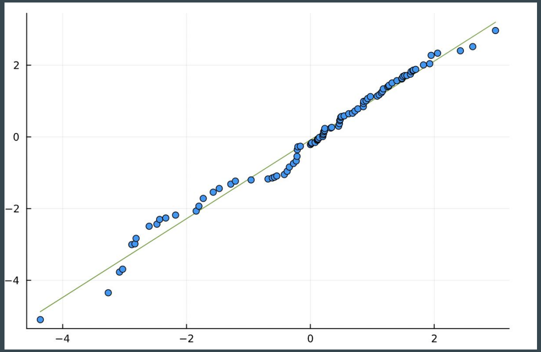
```
In  [21]: decision.(x1, Ref(result), CompromiseSolution)
Out[21]: 172.385773

In  [22]: decision.(x2, Ref(result), CompromiseSolution)
Out[22]: 27.614227
```

Outlier Removal

- Use Z-Score between -2 and 2






```
Variance F-test
---------------
Population details:
    parameter of interest:   variance ratio
    value under h_0:         1.0
    point estimate:          0.816557

Test summary:
    outcome with 95% confidence: fail to reject h_0
    two-sided p-value:           0.3256

Details:
    number of observations: [95, 97]
    F statistic:            0.8165571322709405
    degrees of freedom:     [94, 96]
```

Coefficients:

	Coef.	Std. Error	t	Pr(>|t|)	Lower 95%	Upper 95%
(Intercept)	2.67955	0.431113	6.22	<1e-07	1.82357	3.53554
TV	0.0471585	0.00199724	23.61	<1e-40	0.043193	0.0511241
Radio	0.188295	0.0109726	17.16	<1e-29	0.166509	0.210082

Results without Outliers

Methodology	x_1	x_2	MPO (in $)	MVSAE (in $)
Deterministic LP	171.519	28.481	$40,979	$[38,764, 39,956]
SLP with SAA	183.486	16.514	$40,012	$[39,179, 41,372]
SLP with SD	175.211	24.789	$36,276	$[35,999, 36,554]

Thank you!