Using VGG19 for Neural Style Transfer Learning

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Abstract

In this project, we used the VGG19 neural network for neural transfer learning. We created our our own model with adjustable hyperparamters and with our choice of two content and two style images, we explore and discuss how changes in optimizer, target image initialization, seed, content loss (α), style loss (β), and training steps affect the target image. Among these, target image initialization, content loss (α), style loss (β), and training steps provide noticeable changes to the final rendered image. We also experimented with many different style images as well, and we find that styles with distinctive color palettes and uniform outlines work best in our model. However, styles with a variety or shapes, outlines, and characteristics do not perform as well. We end with a summary of main takeaways and a discussion on future work.

1 Introduction

One of the interesting applications of computer graphics is to perform style transfer from one image to another. Algorithms of image style transfer can be applied to photo and video editing softwares, enabling users to create artworks with various styles and contents without the need to know much about art. Yet style transfer is considered a difficult image processing task, due to lack of semantic information representation of images. The recent advance on studies in deep convolutional neural networks have provided some ideas on performing such a task. It is shown that convolutional neural networks trained for object recognition extract high-level semantic information of image in generic feature representations, which can be used for the style transfer task.

In this project, we implement a neural style transfer algorithm based on Gatys et al and various TensorFlow tutorials.

Gatys et al (2016) discussed how the VGG19 neural network can be used for neural transfer learning by specific layers used for the process and the hyperparameters that can be changed when rendering a target image. Several TensorFlow tutorials and Kaggle guides have outlined the steps to set this process up using Python 3–see TensorFlow's "Neural Style Transfer" and "Fast Style Transfer for Arbitrary Styles" as well as Basu's "Style Transfer Deep Learning Algorithm" on Kaggle for more details. We have taken elements and ideas from each of these guidelines to not only replicate the process as described by Gatys et al but also created a tool that allows us to easily switch between

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Figure 1: Our choice of content images (top row) and style images (bottom row). *Alma Mater* statue (top left). Columbia University's Morningside campus (top right). Portrait of Homer J. Simpson (bottom left). Jackson Pollock's *No. 5* (bottom right).

different options and hyperparameters, including the choice of seed, style loss, content loss, optimizer, and training steps.

In the spirit of Columbia, we have decided to use the *Alma Mater* statue and Morningside campus as our content images. For our style images, we chose two contrasting images: one is a more traditional notion of art–Jackson Pollock's abstract work *No. 5, 1948*–and one a more modern-day work–a portrait of Homer J. Simpson, from the popular television show *The Simpsons*. Please see Figure 1. We feel that this contrast of color, linework, and style help to better understand the general performance of the VGG19 neural network. We also utilize other style images–such as Caravaggio's *Supper at Emmaus* and Dali's *The Persistence of Memory* as examples that highlight the limitations of the VGG19 neural network.

These images serve as the data for our project, and we have done some preprocessing that resizes the images to 512 x 512 colored pixels.

1.1 Content Representation

Generally, images are represented in convolutional neural networks by the filter responses of each layer of the network, with feature representations of lower level focus more on the precise appearance or the exact pixel values while those of higher level capture more information of the objects and their arrangement.

For a layer l with N_l distinct filters, suppose the N_l feature maps are each of size M_l, and the responses in this layer are stored in a matrix $F_l \in \mathcal{R}^{N_l \times M_l}$, where M_l is the height times the width of the feature map and F^l_{ij} is the activation of the i^{th} filter at position j.

Denote the original content image and the generated image as $\vec{p}$ and $\vec{x}$ respectively, and their respective feature representations in layer l as P^l and F^l. The content loss is defined as

$$\mathcal{L}_{\text{content}}(\vec{p}, \vec{x}, l) = \frac{1}{2} \sum_{i,j} (F^l_{ij} - P^l_{ij})^2$$

Since convolutional neural networks extract more explicit information of the content of a image at higher layers, it is preferred to choose the content representation of the content from a higher layer feature representation.

1.2 Style representation

The style representation is a bit more complicated in that we need to extract information on the texture of the image from feature representations or filter responses that generally represents the objects. In this model, the Gram matrix, consisting of correlations between different filter responses, is used to capture the texture information and as a representation of the style of an image. Such style representation can be constructed from any layer in the network, since both higher and lower level features contain information that can contribute to our modeling of the style of an image.

The Gram matrix $G^l \in \mathcal{R}^{N_l \times M_l}$ is defined such that G^l_{ij} is the inner product between the vectorized feature maps i and j in layer l:

$$G^l_{ij} = \sum_k F^l_{ik} F^l_{jk}$$

Denote the original style image and the generated image as $\vec{a}$ and $\vec{x}$ respectively, and their respective style representations in layer l as A^l and G^l. The contribution of layer l to the style loss is defined as

$$E_l = \frac{1}{4N_l^2 M_l^2} \sum_{i,j} (G^l_{ij} - A^l_{ij})^2$$

The total style loss is a weighted sum of the contribution of each layers:

$$\mathcal{L}_{\text{style}}(\vec{a}, \vec{x}) = \sum_{l=0}^{L} w_l E_l$$

where w_l are the weighting factors.

1.3 Style transfer

The general idea of the style transfer algorithm is to generate an image that is close to the style image in their style representations and to the content image in their content representations. Therefore, the algorithm tries to minimize the weighted sum of content and style loss:

$$\mathcal{L}_{\text{total}}(\vec{p}, \vec{a}, \vec{x}) = \alpha \mathcal{L}_{\text{content}}(\vec{p}, \vec{x}) + \beta \mathcal{L}_{\text{style}}(\vec{a}, \vec{x})$$

2 Methods

In the project, we started from understanding the algorithm of the paper and implementing it. First, we referred to several tutorial resources and we started from picking the VGG network layers to code the loss function and the gradient descent optimizer. Then we train the model and visualize it. After that, we change the optimizer from gradient descent to Adam based on a better empirical result. And we also wrap all the process to a function so that we will be able to tune all the parameters and apply them to other images.

2.1 Data Preprocessing

For both the content image and the style image, they are by default RBG images. Using the TensorFlow, we preprocessed both images to create a three-channel tensor, with each channel representing one layer of color. Then we resize them as 512×512-pixel images.

2.2 Deep Image Representations Implementation

VGGNet was invented by VGG (Visual Geometry Group) from University of Oxford. In 2014, VGGNet beats the GoogleNet and secured the first and the second places in the localization and classification tracks in the ILSVRC(ImageNet Large Scale Visual Recognition Competition).

VGG19 is a model, showing a significant improvement on the prior-art configurations by pushing the depth to 19 weight layers. The main way for VGG19 to increase depth is by using an architecture with very small (3×3) convolution filters. It is pertained based on ImageNet, which is an image database organized according to the WordNet hierarchy with hundreds and thousands of images in each nodes, and over five hundred images per node. ILSVRC uses a subset of ImageNet, which includes 50,000 images with labels of the 1000 categories.

Although VGG19 is basically used for classification purpose, but the layers pertained in the VGG19 can grasp the main outline and the main content of an image. Thus, in this project, we do not need all the layers of VGG19 (16 convolution layers, 3 Fully connected layer, 5 MaxPool layers and 1 SoftMax layer), we have specially excluded those layers which are used for classification.

We used the feature space provided by a normalized version of the 16 convolution layers of the 19-layer VGG network. Specifically, we use one layer of Convolutional 2-Dimensional layers in VGG19 model as the representation of an image's content, and five other Convolutional 2-Dimensional layers to represent an image's style. In our model of transfer learning, we used the pertained layer weights of VGG19 and freeze the weight, so that we can apply the VGG19 model's weight to our images, extracting the content information and the style information from our input images.

2.3 Loss Function Implementation

In the beginning, we calculate the loss of the content based on the formula illustrated before. However, in the implementation, we change the loss function and the optimizer based on the empirical result and we believe that this is a better validation of the content loss that we change it from a mean square error loss function to another square error loss with a different scaling.

We change the content loss function to the following:

For a layer l with N_l distinct filters, suppose the N_l feature maps are each of size M_l, and the responses in this layer are stored in a matrix $F_l \in \mathcal{R}^{N_l \times M_l}$, where M_l is the height times the width of the feature map and F_{ij}^l is the activation of the i^{th} filter at position j.

Denote the original content image and the generated image as $\vec{p}$ and $\vec{x}$ respectively, and their respective feature representations in layer l as P^l and F^l. The content loss is defined as

$$\mathcal{L}_{\text{content}}(\vec{p}, \vec{x}, l) = \frac{1}{N_l M_l} \sum_{i,j} (F_{ij}^l - P_{ij}^l)^2$$

We change the style loss function to:

The Gram matrix $G^l \in \mathcal{R}^{N_l \times M_l}$ is defined such that G_{ij}^l is the inner product between the vectorized feature maps i and j in layer l:

$$G_{ij}^l = \sum_k F_{ik}^l F_{jk}^l$$

Denote the original style image and the generated image as $\vec{a}$ and $\vec{x}$ respectively, and their respective style representations in layer l as A^l and G^l. The contribution of layer l to the style loss is defined as

$$E_l = \frac{1}{M_l} \sum_{i,j} (G_{ij}^l - A_{ij}^l)^2$$

The total style loss is the average contribution of each layers, denote the number of layers as L:

$$\mathcal{L}_{\text{style}}(\vec{a}, \vec{x}) = \frac{1}{L} \sum_{l=1}^{L} E_l$$

And we implement the total loss the same way as in the paper.

2.4 Optimizer Implementation

In the paper, the author suggests using L-BFGS while we are using Adam as the optimizer, with learning rate of 0.1. The reason for us to change from the L-BFGS to Adam is based on empirical result. We will provide more detailed proof in the final report.

2.5 Training The Model

By applying Adam to the both the content loss function and the style loss function to build the model, we wrap up the process to build a function, which help us with changing all the parameters we have to compare with, such as different images, different random seed, different number of iteration steps, different way to initialize the target images, different weights of content vs. style Loss. randomness. So that, we are able to train the model with different parameters and output the result with visualization.

3 Results

We find that the algorithm described above does indeed allow us to combine the content of a given image with the style of another image. To demonstrate this, Figure 2 displays each of the four combinations of the content and style images mentioned previously. These images were all generated using the content image to initialize the target, running 100 training steps with alpha = 1 and beta = 1e-14 – these parameters will be varied in later sections. Figure 2 (a) combines the content of the *Alma Mater* statue picture with the Homer Simpson style. With this combination, the image shows *Alma Mater* with the crisp black outlines and mostly white yellow color palette of the Homer Simpson image. Figure 2 (b) combines the content of the *Alma Mater* statue picture with Jackson Pollock's style. Here, *Alma Mater* is rendered with the darker colors and disorderly (lack of) pattern for which Pollock is known. Figure 2 (c) combines the content of the Columbia campus picture with the Homer Simpson style. Again, we see the crisp outlines and bright white yellow colors from this distinctive style. Figure 2 (d) combines the content of the Columbia Morningside campus picture with Jackson Pollock's style. With this style, the campus looks much darker and less orderly. The next few sections explore the effects of key parameters on the generated images–namely, the number of training iterations, the initialization of the target image, and the relative weighting between content loss and style loss. For illustrative purposes, we examine these effects on two of the four combinations: *Alma Mater* + Homer Simpson, and Columbia Campus + Jackson Pollock.

3.1 Effect of Training Iterations

In these examples, since the target image is initialized using the content image, then each step in the training process serves to infuse more of the style into the picture. Figures 3 and 4 show the evolution of the learning process after different numbers of training iterations (10, 25, 50, 75, and 100 steps). For the *Alma Mater* + Homer Simpson combination (Figure 3), there is a clear progression of the style seeping into the content. After 10 steps (Figure 3 (a)), the color palette has barely begun to transform. After 25 steps (Figure 3 (b)), the black outlines are forming and some of the white spaces are getting whiter. This process continues, adding the distinctive yellow coloring as well, as we go through more training steps. Between the 75-step and 100-step images (Figure 3 (d) and (e)), the returns are diminishing, with the latter image having only marginally brighter yellows and clearer

Figure 2: Main output from our neural style transfer learning. Function settings: initialization from content image, seed $= 0$, steps $= 100$, alpha $= 1$, beta $= 1 \times 10^{-14}$. (a) Top left: *Alma Mater* and Homer Simpson. (b) Top right: *Alma Mater* and Pollock's *No. 5*. (c) Bottom left: Morningside Campus and Homer Simpson. (d) Bottom right: Morningside Campus and Pollock's *No. 5*

white spaces. For the Columbia Campus + Jackson Pollock combination (Figure 4), there is also a progression as the steps go by, though the target image seems to stabilize sooner. To the naked eye, there is not much difference between the 50-step, 75-step, and 100-step images (Figure 4 (c), (d), and (e)).

3.2 Effect of Target Image Initialization

As with any high-dimensional optimization problem, the initialization of the target has a notable impact on the final output. Figures 5 and 6 show the differences in the generated images using varying initializations of the target image. Specifically, for each of these content/style combinations, we initialize with the content image and with two different random white noise images. For the *Alma Mater* + Homer Simpson combination, initializing with the content image (Figure 5 (a)) renders the clearest picture of the *Alma Mater* statue. While initializing with random white noise (Figure 5 (b) and (c)) still gives us a recognizable pattern, the outline is not nearly as crisp. Using different random seeds generates different-looking images, though unsurprisingly neither is observably "better" than the other. For the Columbia Campus + Jackson Pollock combination, initializing with the content image (Figure 6 (a)) also gives the clearest picture of the campus. The difference is perhaps more apparent here, as the random initializations (Figure 6 (b) and (c)) only have very vague building and walkway shapes. This is likely because of the style itself; Pollock's method of creating the artwork via paint drip is itself a somewhat random process, so the random initialization is more similar to

Figure 3: Results from varying the number of training iterations. *Alma Mater* and Homer Simpson. Function settings: initialization from content image, seed $= 0$, alpha $= 1$, beta $= 1 \times 10^{-14}$. Steps = 10 (a, top left), 25, (b, top right) 50, (c, middle left), 75 (d, middle right), and 100 (e, bottom).

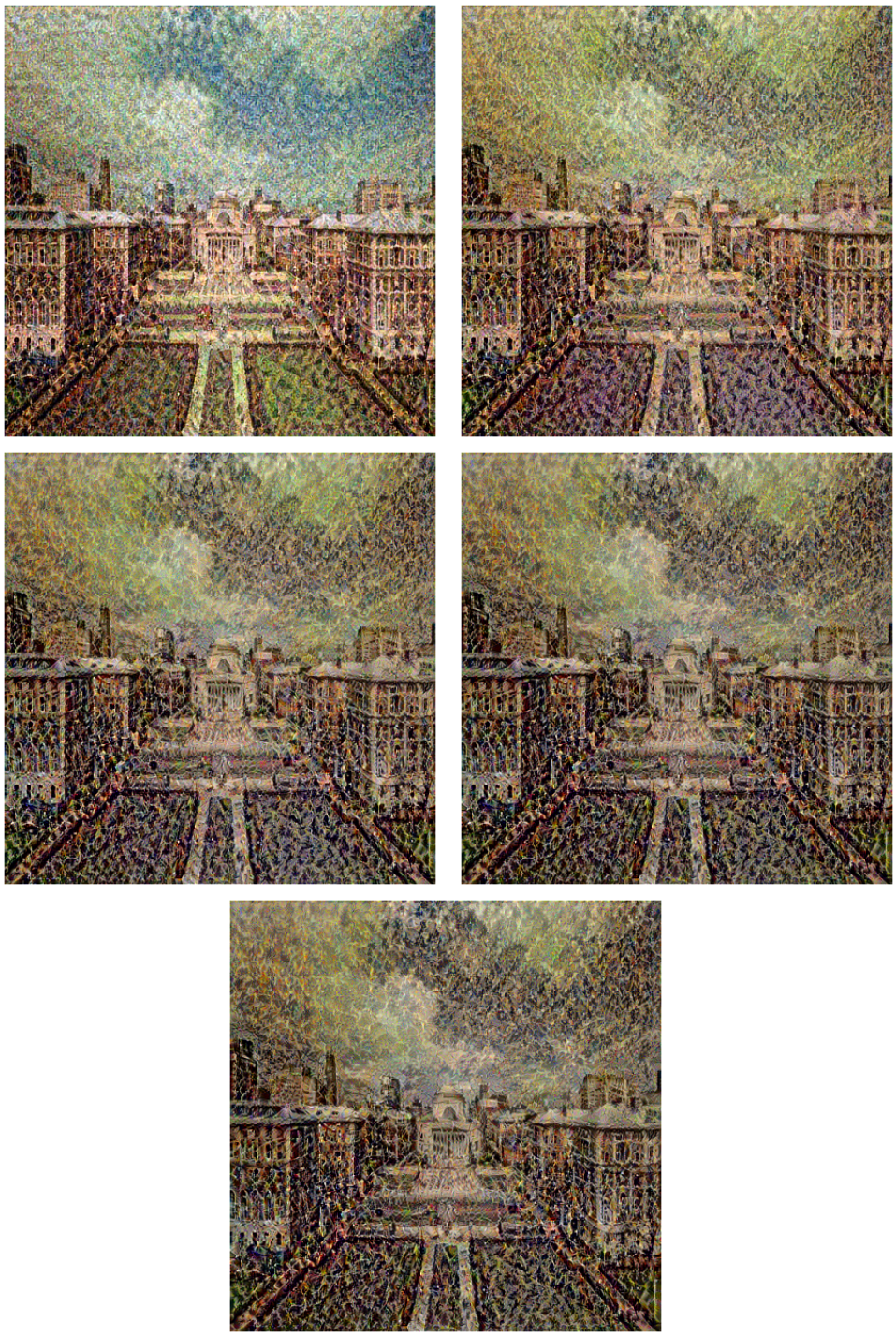

Figure 4: Results from varying the number of training iterations. Morningside campus and Pollock's *No. 5*. Function settings: initialization from content image, seed $= 0$, alpha $= 1$, beta $= 1 \times 10^{-14}$. Steps = 10 (a, top left), 25, (b, top right) 50, (c, middle left), 75 (d, middle right), and 100 (e, bottom).

Figure 5: Results from varying the target image initialization. *Alma Mater* and Homer Simpson. Function settings: steps = 100, alpha $= 1$, beta $= 1 \times 10^{-14}$. (a) Left: initialization from content image, seed $= 0$. (b) Middle: initialization from random white noise image, seed $= 0$. (c) Right: initialization from random white noise image, seed $= 1$.

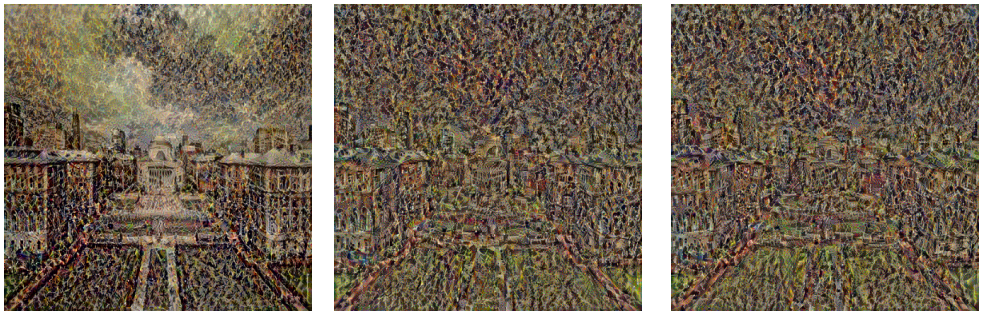

Figure 6: Results from varying the target image initialization. Morningside campus and Pollock's *No. 5*. Function settings: steps = 100, alpha $= 1$, beta $= 1 \times 10^{-14}$. (a) Left: initialization from content image, seed $= 0$. (b) Middle: initialization from random white noise image, seed $= 0$. (c) Right: initialization from random white noise image, seed $= 1$.

the style image than to the content image. Therefore, the optimization process approaches a local minimum for the loss function that minimizes the style loss more than the content loss. The takeaway from this experimentation is that initializing the target image with the content image renders an output that more closely adheres to the shapes in the content image. Initializing with random white noise, on the other hand, gives the output a more amorphous shape, which can vary with the use of different random seeds.

Figure 7: Results from varying the content vs. style loss weights. *Alma Mater* and Homer Simpson. Function settings: initialization from content image, seed $= 0$, steps = 100, alpha $= 1$. (a) Left: beta $= 1 \times 10^{-13}$. (b) Middle: beta $= 1 \times 10^{-14}$. (c) Right: beta $= 1 \times 10^{-15}$.

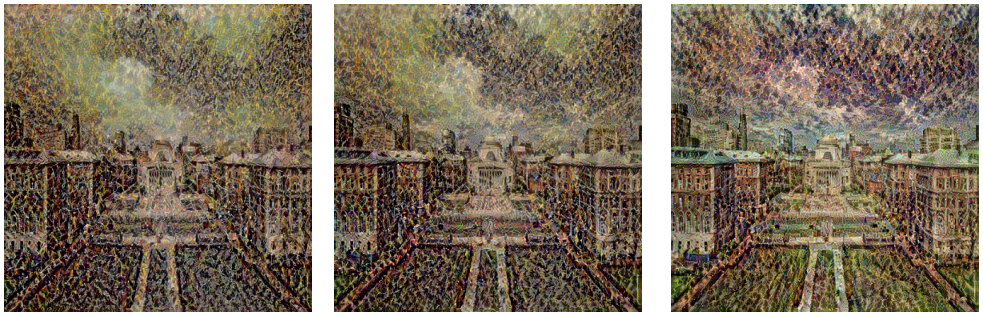

Figure 8: Results from varying the content vs. style loss weights. Morningside campus and Pollock's *No. 5*. Function settings: initialization from content image, seed = 0, steps = 100, alpha = 1. (a) Left: beta = 1×10^{-13}. (b) Middle: beta = 1×10^{-14}. (c) Right: beta = 1×10^{-15}.

3.3 Effect of Content vs. Style Loss Tradeoff

The algorithm's loss function is, in actuality, a linear combination of two distinct loss functions: the content loss and the style loss. We must manage the tradeoff between content and style by adjusting the parameters that define the relative weights between content loss (alpha) and style loss (beta). As mentioned previously, our content loss and style loss functions are scaled differently than those in the Gatys et al paper, so the ratios of alpha to beta may be different. While we experimented with many different combinations, for illustrative purposes we show alpha = 1 with beta = 1e-13, 1e-14, and 1e-15. For the *Alma Mater* + Homer Simpson combination (Figure 7), there are clear differences between these three parameter choices. With beta = 1e-13 (Figure 7 (a)), many of the statue's features have been washed away, with only the basic outlines remaining. Since this is the example where the style loss receives the highest weight, it is unsurprising that this image looks the most "cartoony". With beta = 1e-14 (Figure 7 (b)), more of the statue's finer features appear, and the yellow white color palette is not as sharp. With beta = 1e-15 (Figure 7 (c)), there is very little of the Homer Simpson style. This weighting seems to tilt too heavily toward the content loss. For the Columbia Campus + Jackson Pollock combination (Figure 8), the parameter choices still have an impact, but it is not quite as obvious as it was in the previous example. All three images are reminiscent of Pollock's chaotic splashes of paint, but the color palette is darker when the style loss has greater weight (beta = 1e-13, Figure 8 (a)) and more colorful (like the content image) when the style loss has less weight (beta = 1e-15, Figure 8 (c)). Based on this analysis, it is clear that giving more relative weight to the style loss injects more of the style (and less of the content) into the final image, and vice versa. From our experimentation, we empirically observed that a beta-to-alpha ratio on the order of 1e-14 seems to strike the best balance, while 1e-13 may be appropriate if the goal is to recreate the style more closely.

4 Discussion

4.1 Main Takeaways

In summary, we have utilized the VGG19 neural network to combine Columbia-themed content images–the *Alma Mater* statue and Morningside campus–with traditional and non-traditional style images–Jackson Pollock's *No. 5* and a portrait of Homer J. Simpson from *The Simpsons*, respectively–to create different renderings of the university in those artistic styles. Using the methods outlined in Gatys et al (2016) and various TensorFlow tutorials, we developed our own tool that allow us to adjust many different hyperparameters create these renderings.

This project is a visual example of the power and limitations that neural transfer learning can provide. Being able to successfully train and select the right hyperparameters for a convolutional neural network gives rise to a unique approach to art and design.

Graphic designers could potentially find a new rival or ally in neural networks. Social media giants such as Facebook and Snapchat can utilize this technology to expand their filter options. The world of digital art–or art in general–could find its next major movement.

Figure 9: Other style images and results with other style images. (a) Top left: Salvador Dali's *The Persistence of Memory*. (b) Top right: *Alma Mater* and Dali's *The Persistence of Memory*. Function settings: initialization from content image, seed $= 0$, steps $= 100$, alpha $= 1$, beta $= 1 \times 10^{-14}$. (c) Bottom left: Michelangelo Merisi Caravaggio's *Supper at Emmaus* (d) Bottom right: *Alma Mater* and Caravaggio's *Supper at Emmaus*. Function settings: initialization from content image, seed $= 0$, steps $= 100$, alpha $= 1$, beta $= 1 \times 10^{-14}$.

4.2 Commentary on Other Style Images

Beyond the examples shown above, we explored many other artistic styles, some of which were more successful than others in blending with the content images. Based on this experimentation, we noticed an interesting pattern: The style images that worked best for these combinations tended to be those with distinctive color palettes (e.g. Homer Simpson's bright white and yellow, or Jackson Pollock's dreary browns) and recognizable characteristics (e.g. Homer Simpson's sharp borders, or Jackson Pollock's random splatter). On the other hand, the style images that did not work as well tended to be those with a greater variety of shades and more diversity of shapes within the composition. One example of a style image that did not work as well is Salvador Dali's *The Persistence of Memory* (Figure 9 (a)). This painting is known for its surreal depiction of melting clocks against a rocky background. However, these qualities do not translate well when run through the algorithm (Figure 9 (b)), perhaps because the shapes in the painting all have somewhat different characteristics (e.g. sharp edges in the landscape but flaccidity and roundness in the clocks). Another example of a style image that did not work as well is Michelangelo Merisi da Caravaggio's *Supper at Emmaus* (Figure 9 (c)). Caravaggio is known for using shading and shadow to elicit an almost photorealistic appearance. However, this technique is difficult for the algorithm to learn (Figure 9 (d)), as it requires a very specific way of blending of many shades.

4.3 Further Research

Although our work builds on the the work of Gatys et al (2016) and various TensorFlow tutorials, we feel that there are many more changes worth considering in future work. One challenge we have seen– as discussed above–is how certain style images do not work well with our current implementation. There are many possible reasons for this: (1) we need to train for a greater number of steps that goes beyond the memory and computing power we currently have access to (2) we have yet to discover the right combination of current hyperparameters within the range of training steps we are currently capable of (3) the choice of layers of in the VGG19 neutral network should be different from our current choice, and (4) we need an entirely different neural network different from VGG19 altogether.

Additionally, we also find difficulty in quantifying exactly which rendered image is arguably better based on changing the hyperparameters alone. However, this may not even be necessary as the issue is more a matter of opinion and artistic preference. If this were to be used in application, the "best" result would often be dictated by one's intentions, the context in which the output is used, and the audience that is viewing the output.

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