Regression metrics: (R)MSPE, MAPE, (R)MSLE

Plan for the video

1) **Regression**
   - MSE, RMSE, R-squared
   - MAE
   - (R)MSPE, MAPE
   - (R)MSLE
2) **Classification:**
   - Accuracy, LogLoss, AUC
   - Cohen's (Quadratic weighted) Kappa

From MSE and MAE to MSPE and MAPE

- **Shop 1**: predicted 9, sold 10, MSE = 1
- **Shop 2**: predicted 999, sold 1000, MSE = 1

From MSE and MAE to MSPE and MAPE

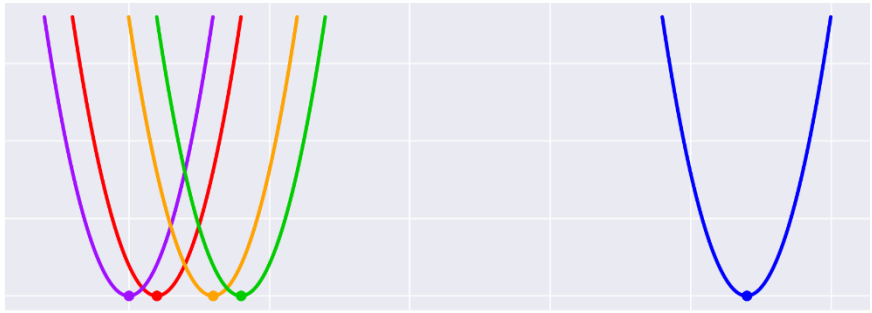
- **Shop 1**: predicted 9, sold 10, MSE = 1
- **Shop 2**: predicted 999, sold 1000, MSE = 1

- **Shop 1**: predicted 9, sold 10, MSE = 1
- **Shop 2**: predicted 900, sold 1000, MSE = 10000

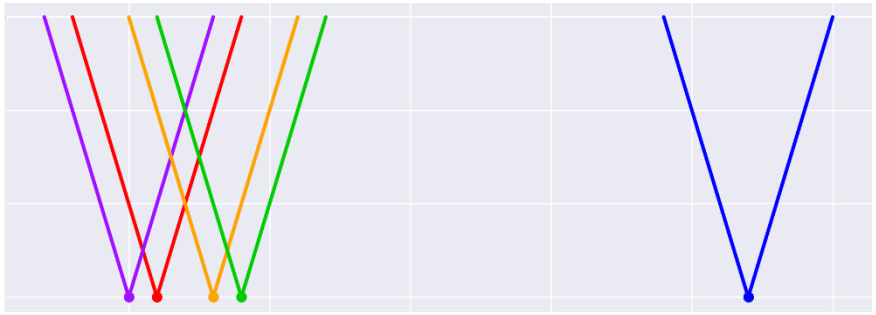
- **Shop 1**: predicted 9, sold 10, relative_metric = 1
- **Shop 2**: predicted 900, sold 1000, relative_metric = 1

From MSE and MAE to MSPE and MAPE

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

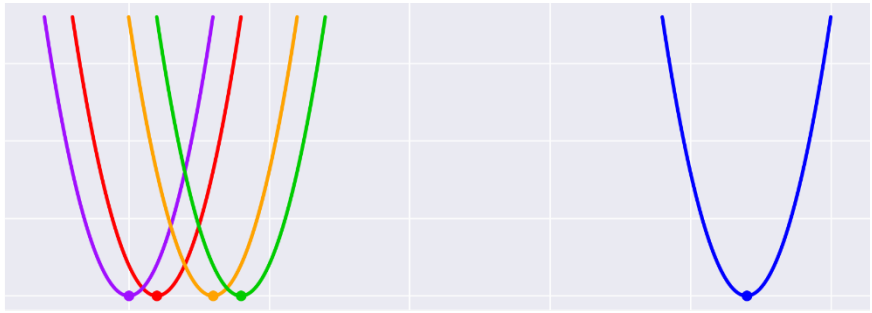


$$\text{MAE} = \frac{1}{N}\sum_{i=1}^{N}|y_i - \hat{y}_i|$$

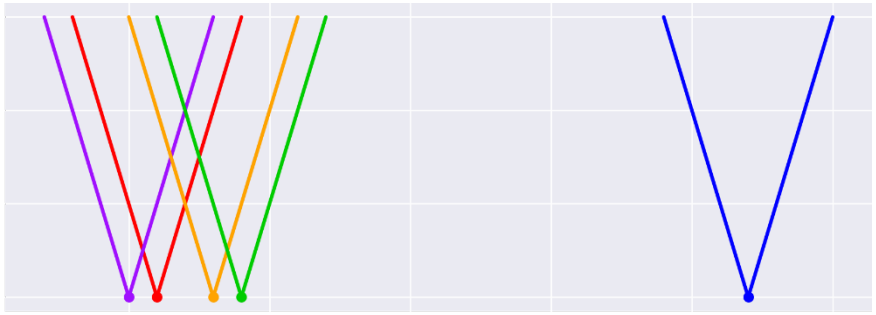


From MSE and MAE to MSPE and MAPE

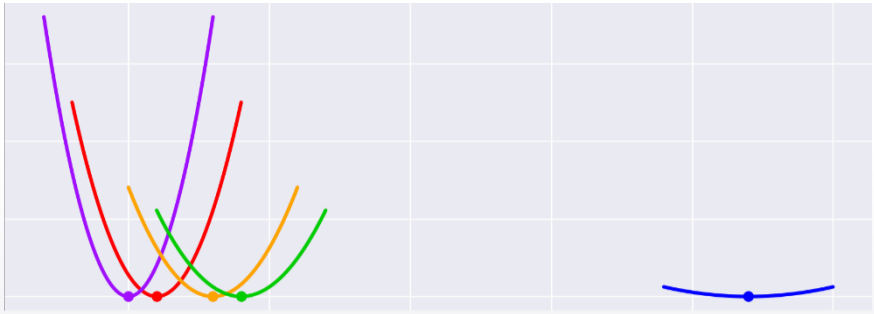
$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$



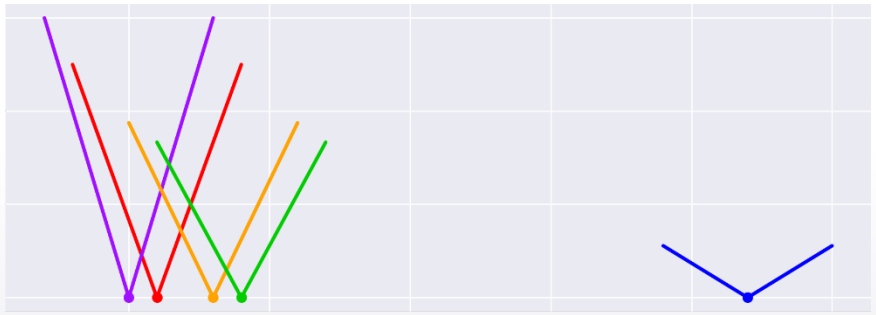
$$\text{MAE} = \frac{1}{N}\sum_{i=1}^{N}|y_i - \hat{y}_i|$$



$$\text{MSPE} = \frac{100\%}{N}\sum_{i=1}^{N}\left(\frac{y_i - \hat{y}_i}{y_i}\right)^2$$



$$\text{MAPE} = \frac{100\%}{N}\sum_{i=1}^{N}\left|\frac{y_i - \hat{y}_i}{y_i}\right|$$



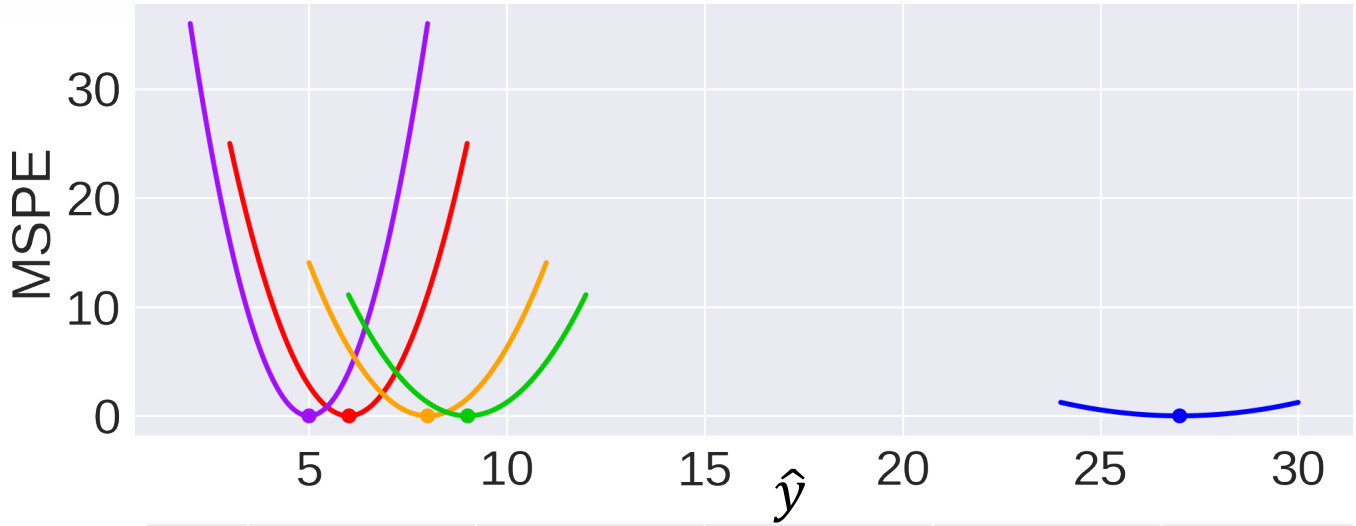
MSPE: constant

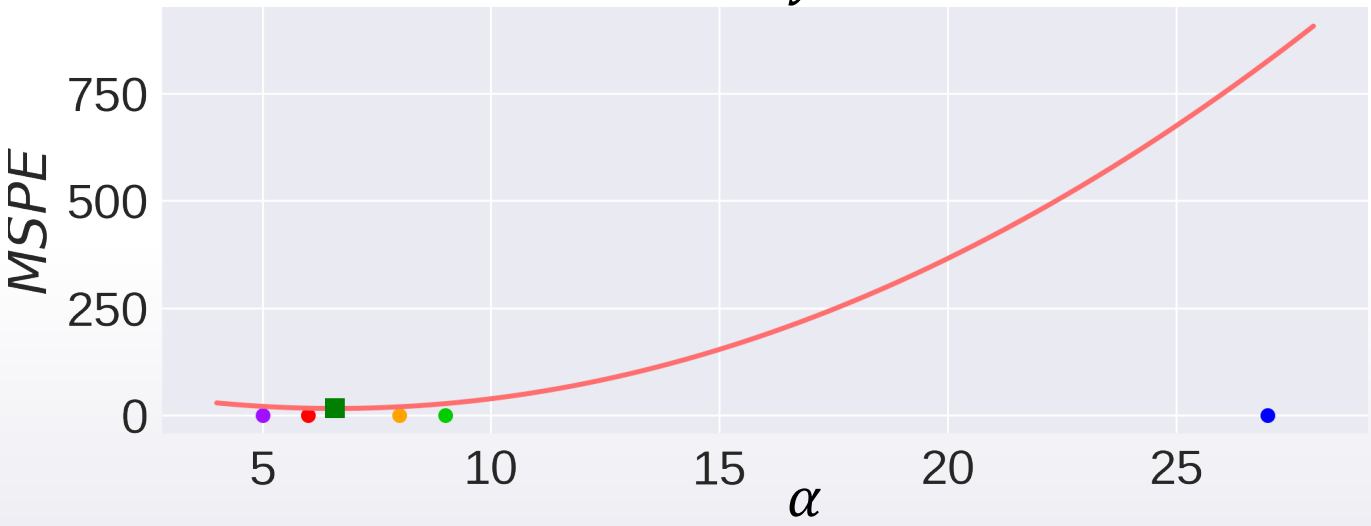
$$\text{MSPE} = \frac{100\%}{N}\sum_{i=1}^{N}\left(\frac{y_i - \alpha}{y_i}\right)^2$$

Best constant:
weighted target mean

Data:

X	Y
-1	4
1	3
-2	6
3	7
3	25





MAPE: constant

$$\mathrm{MAPE} = \frac{100\%}{N} \sum_{i=1}^{N} \left| \frac{y_i - \alpha}{y_i} \right|$$

Best constant:
?

Data:

X	Y
-1	4
1	3
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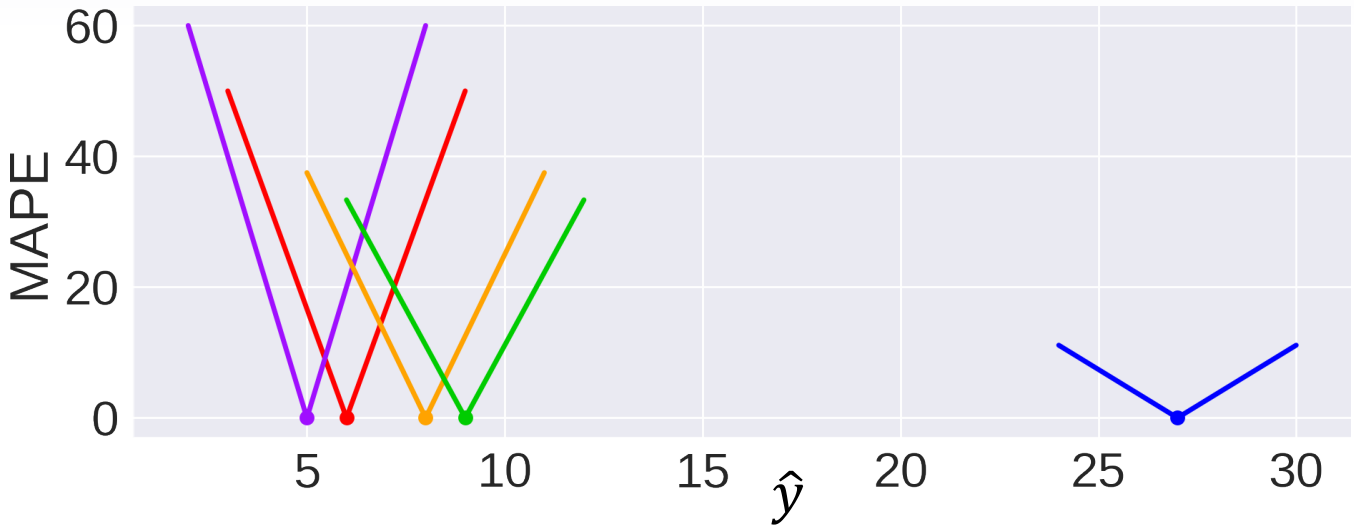
3	7
3	25



MAPE: constant

$$\text{MAPE} = \frac{100\%}{N} \sum_{i=1}^{N} \left| \frac{y_i - \alpha}{y_i} \right|$$

Best constant:
weighted target median

Data:

X	Y
-1	4
1	3
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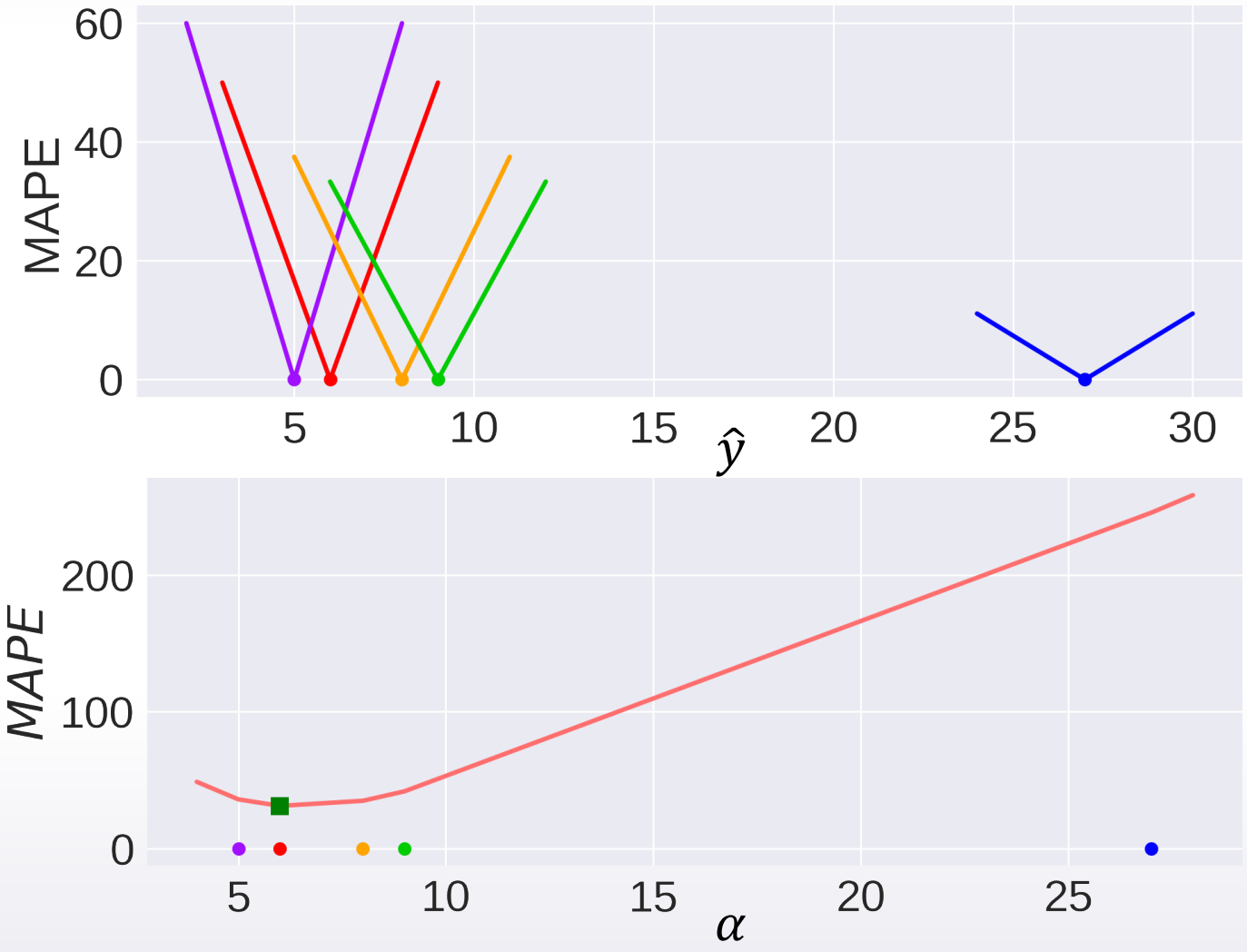
3	25



(R)MSLE: Root Mean Square Logarithmic Error

$$\begin{aligned}\text{RMSLE} &= \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\log(y_i+1)-\log(\hat{y}_i+1)\right)^2} = \\ &= RMSE\left(\log(y_i+1),\log(\hat{y}_i+1)\right) = \\ &= \sqrt{MSE\left(\log(y_i+1),\log(\hat{y}_i+1)\right)}\end{aligned}$$

(R)MSLE: Root Mean Square Logarithmic Error

$$\text{RMSLE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\log(y_i+1) - \log(\hat{y}_i+1)\right)^2}$$

Data:

X	Y
-1	4
1	3
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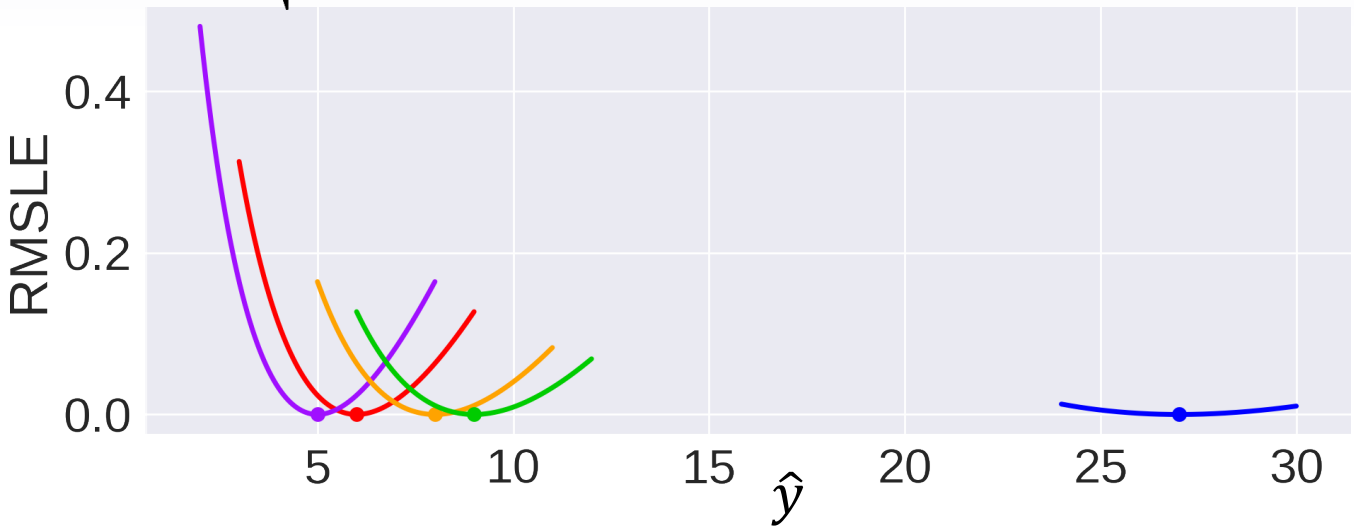
3	7
3	25



(R)MSLE: constant

$$\text{RMSLE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\log(y_i+1)-\log(\alpha+1)\right)^2} =$$

$$= RMSE\left(\log(y_i+1), \log(\alpha+1)\right) =$$

$$= \sqrt{MSE\left(\log(y_i+1), \log(\alpha+1)\right)}$$

(R)MSLE: constant

$$\text{RMSLE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\log(y_i+1) - \log(\alpha+1)\right)^2} =$$
$$= RMSE\left(\log(y_i+1), \log(\alpha+1)\right) =$$
$$= \sqrt{MSE\left(\log(y_i+1), \log(\alpha+1)\right)}$$

- Best constant in log space is a mean target value
- We need to exponentiate it to get an answer

(R)MSLE: constant

$$\text{RMSLE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\log(y_i+1) - \log(\alpha+1)\right)^2}$$

Data:

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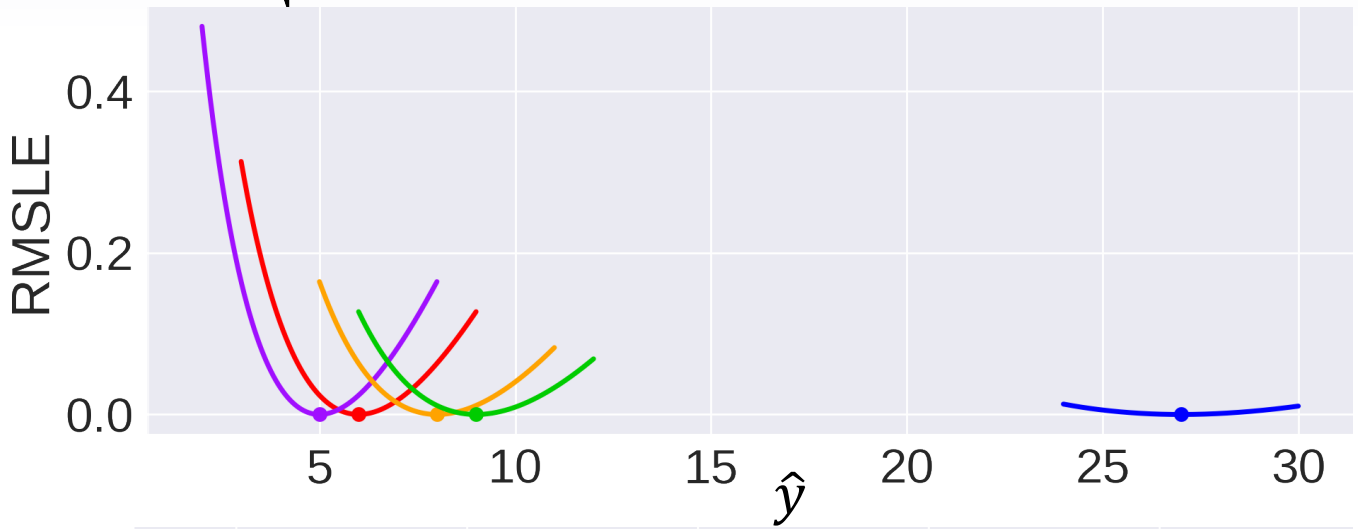

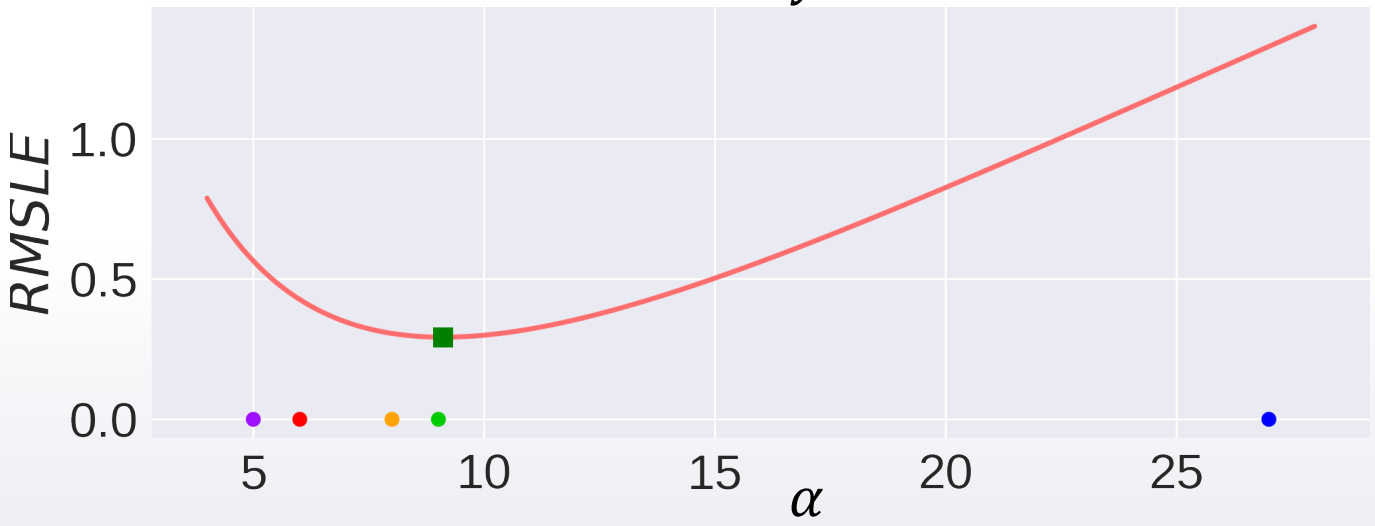

Compare the constants

Metric	Constant
MSE	11
RMSLE	9.11
MAE	8
MSPE	6.6
MAPE	6

Conclusion

Discussed the metrics, sensitive to relative errors:

- **(R)MSPE**
  - Weighted version of MSE
- **MAPE**
  - Weighted version of MAE
- **(R)MSLE**
  - MSE in log space