

Noise Impact Analysis of Coupled Grid and Place Cells in Continuous Attractor Neural Networks

Yueying Luo
Yuanpei College
2300017819@stu.pku.edu.cn

January 19, 2025

Abstract

Spatial coding is a critical function of the brain, with place cells in the hippocampus (HPC) and grid cells in the medial entorhinal cortex (MEC) representing two primary neuronal types involved in spatial representation. Previous studies have shown that these two types of cells process different signals, employ different encoding mechanisms, and complement each other. However, how they collaborate to handle complex tasks under interference remains unclear. In this work, to address this challenge, we investigate the coupling dynamics of grid cells and place cells within a continuous attractor neural network. Specifically, we intend to model the effects of input noise and connection noise and decompose the system dynamics. We found that input noise and connection noise respectively induce distortions in the dynamics of coupled grid and place cell systems, revealing how noise disrupts the precision and stability of spatial representations within the network. Combined with recent research in spatial memory deficits in Alzheimer’s disease, we aim to provide mechanistic insights into the physiological basis of certain neuron-degenerative diseases.

Keywords: spatial coding, noise driven dynamics, continuous attractor neural network

1 Introduction

Understanding the dynamics of the brain is one of the ultimate goals in neuroscience. In the real world, the brain operates in an environment where signals are constantly subject to fluctuation. Specifically, noise can arise from various sources, including sensory inputs, neural circuitry, and environmental disturbances, and can significantly affect the stability and accuracy of spatial coding. Noise tolerance could play a significant role as a functional constraint in shaping the evolution of neuro-architectural designs across the animal kingdom.[CV10] Therefore, to investigate how brain cells interact with the environment as a whole, adapt to and mitigate noise is essential to uncover the resilience and robustness of the spatial representation system.

Two key types of neurons, place cells in the hippocampus (HPC)[OD71] and grid cells in the medial entorhinal cortex (MEC)[HFM⁺05], play a central role in the brain’s spatial representation system. Place cells are known to fire when an individual is in a specific location, while grid cells exhibit a regular, grid-like firing pattern that encodes self-motion cues.[CLK⁺19][MBJ⁺06] These two types of cells process different signals, adopt different encoding mechanisms, and complement each other to construct a coherent spatial representation.[MKM08]

Despite significant progress in understanding their individual properties, much remains unclear about how these neurons collaborate under complex and noisy conditions. There have been works to study cooperation of the two: for example, to extract grid cell characteristics from place cell inputs using non-negative principal component analysis, [DSMD16] or to explain grid-to-place-cell transformation account for the origin of place selectivity in the place cells[ASC14], yet they fail to conduct experiments in various noise conditions; and there exists attractor neural network optimization method under noisy condition like overlap training[WS90] and multiple bumps[WK22]; yet they focus on a single network instead of a system. We still lack explainable and computational model for solid analysis in collaboration of the two kinds of cells to cope with external disturbances such as sensory noise or internal perturbations within neural networks which potentially impacting navigation and

memory. Understanding how place cells and grid cells interact to maintain robust spatial coding under such conditions is crucial for advancing our knowledge of the brain’s navigation system.

In this work, we aim to address this challenge by investigating the coupling dynamics of grid cells and place cells within the framework of continuous attractor neural networks (CANNs). First we develop a model with reciprocally coupled continuous attractor neural networks based on previous work.[Ano24][WWF+16] We specifically model the effects of two types of noise respectively: input noise and connection noise to understand their impact on network dynamics. By employing wave function analysis,[FWW10][GS19] we can derive steady-state value of the synaptic current, decompose the perturbed dynamics into distortions of amplitude, position, width, and skewness. We also hope to examine system properties like diffusion and drift velocity to provide a detailed characterization of noise-driven effects on spatial representation.

Our main contribution is to reveal how different types and levels of noise affect the system’s activity dynamics and to identify differential sensitivities of grid and place cells to various noise sources, shedding light on their complementary roles in maintaining robust spatial representation. Such insights not only enhance our understanding of normal brain function but also provide a basis for understanding how this system fails in pathological conditions, providing a foundation for future experimental and therapeutic research.

2 Material and Method

2.1 A Coupled CANN Model

We adopted the one-dimensional CANN network model with coupled place and grid cells proposed in previous work,[Ano24] where a 1D CANN is used with location coordinate to model the place cell ensemble in HPC, referred to as P-CANN, in which neurons are aligned according to their preferred locations $x \in (-L/2, L/2)$, and multiple 1D CANNs with phase coordinate to model grid cell modules with different spacings in MEC, referred to as G-CANNs, and in each G-CANN, neurons are aligned according to their preferred phases $\theta \in (0, 2\pi]$. The P-CANN and each G-CANN are reciprocally connected, and no connection between G-CANNs exists.

The recurrent connections between neurons in the network are set as follows. For two neurons at locations x and x' in the P-CANN, their connection is $W_p(x, x') = \frac{J_p}{(\sqrt{2\pi}a_p)} \exp\left[-\frac{(x-x')^2}{2a_p^2}\right]$, with a_p controlling the range of recurrent connections and J_p the connection strength. For two neurons at phases θ and θ' in a G-CANN, their connection is $W_g(\theta, \theta') = \frac{J_g}{(\sqrt{2\pi}a_{gi})} \exp\left[-\frac{\|\theta-\theta'\|^2}{2a_{gi}^2}\right]$. For place cells at location x and grid cells at phase θ^i , the connection is set as,

$$W_{gi,p}(x, \theta^i) = \frac{J_{gi,p}}{\sqrt{2\pi}a_{gi,p}} \exp\left[-\frac{\|\theta^i - \varphi^i(x)\|^2}{2a_{gi,p}^2}\right] \quad (1)$$

where $\varphi^i(x) = \text{mod}\left(\frac{x}{\lambda^i}, 1\right) \times 2\pi$

Denote $U_p(x, t)$ the synaptic current to place cells at location x at time t , and $R_p(x, t)$ the corresponding firing rate. The dynamics of place cells are written as,

$$\begin{aligned} \tau_p \frac{dU_p(x, t)}{dt} = & -U_p(x, t) + \rho_p \int_{-L/2}^{L/2} W_p(x, x') R_p(x', t) dx' \\ & + \sum_{i=1}^M \rho_g \int_{-\pi}^{\pi} W_{gi,p}(x, \theta^i) R_g(\theta^i, t) d\theta^i + I_p(x), \end{aligned} \quad (2)$$

where τ_p denotes the time constant, ρ_p and ρ_g the densities of place and grid cells, respectively. $I_p(x)$ represents the external inputs conveying the location information from the environmental cue.

Denote $U_g(\theta^i, t)$ the synaptic current at time t to grid cells at phase θ^i in the i th module, and $R_g(\theta, t)$ the corresponding firing rate. The dynamics of grid cells are written as,

$$\begin{aligned} \tau_g \frac{dU_g(\theta^i, t)}{dt} = & -U_g(\theta^i, t) + \rho_g \int_{-\pi}^{\pi} W_g(\theta^i, \theta'^i) R_g(\theta'^i, t) d\theta'^i \\ & + \rho_p \int_{-L/2}^{L/2} W_{gi,p}(x, \theta^i) R_p(x, t) dx + I_g(\theta^i), \quad i = 1, \dots, M, \end{aligned} \quad (3)$$

where τ_g denotes the time constant, and $I_g(\theta^i)$ the external inputs conveying the location information from the motion cue.

The firing rate of neurons is given by,

$$R_s(s, t) = \frac{U_s(s, t)^2}{1 + k_s \rho_s \int U_s(s, t)^2 ds}, \quad s = x, \theta. \quad (4)$$

The external inputs to the networks convey the location information of the animal. We set them to be,

$$I_p(x) = \alpha_p \left\{ \exp \left[-\frac{(x - z_0)^2}{4a_p^2} \right] + \xi_p \right\}, \quad (5)$$

$$I_g(\theta^i) = \alpha_{gi} \left\{ \exp \left[-\frac{\|\theta^i - \psi^i(z_0)\|^2}{4a_{gi}^2} \right] + \xi_g \right\}, \quad i = 1, \dots, M, \quad (6)$$

where z_0 denotes the true location of the animal and $\psi^i(z_0)$ the matched phase. α_{gi} and α_p represent the strengths of the external inputs. ξ_g and ξ_p are Gaussian noises of zero mean and variances σ_{gi}^2 and σ_p^2 , reflecting the ambiguity of sensory cues.

2.2 Noise Simulation

In the above section, we have considered noise of external input and simulate poisson spiking behavior into gaussian noise model according to central limit theorem. Here we further introduce noise in synaptic connectivity strengths to represent variability in biological networks.

$$W_\beta(x, y) \rightarrow W_\beta(x, y) + V_{\alpha\beta}(x, y).$$

where $V_{\alpha\beta}(x, y)$ is a connectivity noise matrix independently drawn from a zero-mean Gaussian distribution.

Correspondingly, the dynamics of place cells and grid cells should be written as,

$$\begin{aligned} \tau_p \frac{dU_p(x, t)}{dt} = & -U_p(x, t) + \rho_p \int_{-L/2}^{L/2} W'_p(x, x') R_p(x', t) dx' \\ & + \sum_{i=1}^M \rho_g \int_{-\pi}^{\pi} W'_{gi,p}(x, \theta^i) R_g(\theta^i, t) d\theta^i + I_p(x), \end{aligned} \quad (7)$$

$$\begin{aligned} \tau_g \frac{dU_g(\theta^i, t)}{dt} = & -U_g(\theta^i, t) + \rho_g \int_{-\pi}^{\pi} W'_g(\theta^i, \theta'^i) R_g(\theta'^i, t) d\theta'^i \\ & + \rho_p \int_{-L/2}^{L/2} W'_{gi,p}(x, \theta^i) R_p(x, t) dx + I_g(\theta^i), \quad i = 1, \dots, M, \end{aligned} \quad (8)$$

2.3 Evaluation Parameter

Then we proceed to analyze how different types of noise distort system dynamics. In the following section, we hope to vary noise level of input noise and connection noise respectively and test performance of our model. We will use pearson correlation coefficient r to measure the strength and direction of a linear relationship between the true position x and position decoded by our model y .

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}} \quad (9)$$

Furthermore, based on research on linear coordinates, [WK22] we know that input noise and spike noise primarily cause the diffusion of neuronal activity, while connectivity noise generates a drift velocity field, influencing the motion of neuronal populations. We can use diffusion coefficient D to evaluate bump structures undergoing diffusion relative to coherent motion. For input noise and spike noise, D is written as,

$$D_{input} = \frac{\sigma^2 \Delta t}{4\tau^2 \int dx \left(\frac{ds}{dx} \right)^2} \quad (10)$$

$$D_{\text{spike}} = \frac{\int dx s(x) \left(\frac{ds}{dx}\right)^2}{4\tau^2 \left[\int dx \left(\frac{ds}{dx}\right)^2\right]^2}. \quad (11)$$

where σ^2 represents the variance of the input noise, $s(x)$ is the baseline firing rate configuration under noise-free and drive-free conditions.

For connection noise, drift velocity can be derive as,

$$v_{\text{conn}}(\theta) = -\frac{\sum_{\alpha\beta} \int \int dx dy v_{\alpha\beta}(x, y) \frac{ds(x-\theta)}{dx} s(y-\theta)}{2\tau \int dx \left(\frac{ds}{dx}\right)^2}. \quad (12)$$

where $v_{\alpha\beta}(x, y)$ represents the perturbation of the connection weights, and $s(x)$ is the baseline firing rate configuration. The integral term in the denominator accumulates the rate of change of the local field, reflecting the drift velocity of the bump position θ caused by connection noise in the absence of external drive.

To adapt to the model we use, $s(x)$ here should be firing rate calculated on the steady-state value of the synaptic current $U_p(x, t)$ when there are no reciprocal connections ($J_{p,g} = 0$) and no external synaptic inputs ($I_p = I_g = 0$). For place cells,

$$\bar{U}_p(x) = A_p \exp\left(-\frac{(x-z)^2}{4a_p^2}\right) \quad (13)$$

where A_p represents the height of the activity bump, a_p controls the width of the activity bump, and z is a free parameter indicating the center position of the activity bump in place cells.

For grid cells,

$$\bar{U}_g(\theta_i) = A_g \exp\left(-\frac{\|\theta_i - \phi_i\|^2}{4a_{gi}^2}\right) \quad (14)$$

where A_g and a_{gi} represent the height and width of the activity bump for grid cells, respectively. θ_i denotes the phase of grid cells in the i -th module, while ϕ_i indicates the center phase of the activity bump.

3 Results

We conducted a simulation to evaluate the model's performance under the influence of noise, specifically in the input and connection.

3.1 Input Noise

We first set non-zero values for I_g and I_p , indicating the presence of both environmental cues and self-motion cues. The strength of the noise was varied from zero to one, and the corresponding decoding results are presented in Figure 1. The blue curve represents the true position, while the orange curve depicts the position predicted by our model. The results show that the decoding accuracy remains high, with a Pearson correlation coefficient exceeding 0.9, when the noise level is below 0.3, demonstrating the model's robust resistance to noise. As the noise level increases, the decoded position curve becomes more irregular, oscillating around the true position. Despite this, the model still effectively captures the approximate direction of positional changes.

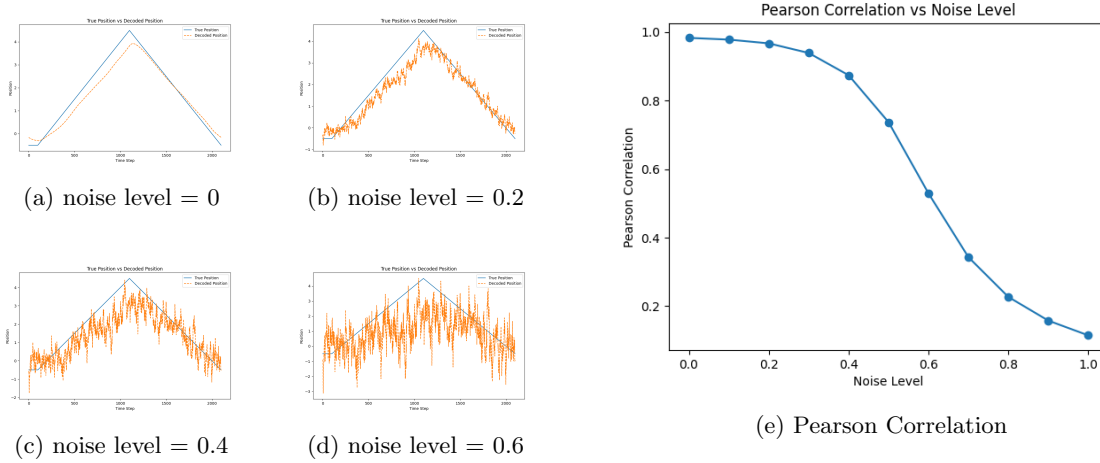


Figure 1: Decode Performance with Environmental Cue

We then simulated a scenario in which self-motion cues were the sole source of spatial information, akin to navigating in the dark. In this case, place cells decoded positional information exclusively based on input from grid cells. Unsurprisingly, the model’s performance decreased compared to conditions where additional environmental cues were available. For instance, the Pearson correlation coefficient dropped from 0.983 to 0.868 under zero noise conditions. Moreover, we observed that even minimal interference (noise level less than 0.1) significantly impaired decoding performance. This deterioration can be attributed to the phase-space coding strategy employed by grid cells, which exhibits heightened sensitivity to phase changes.

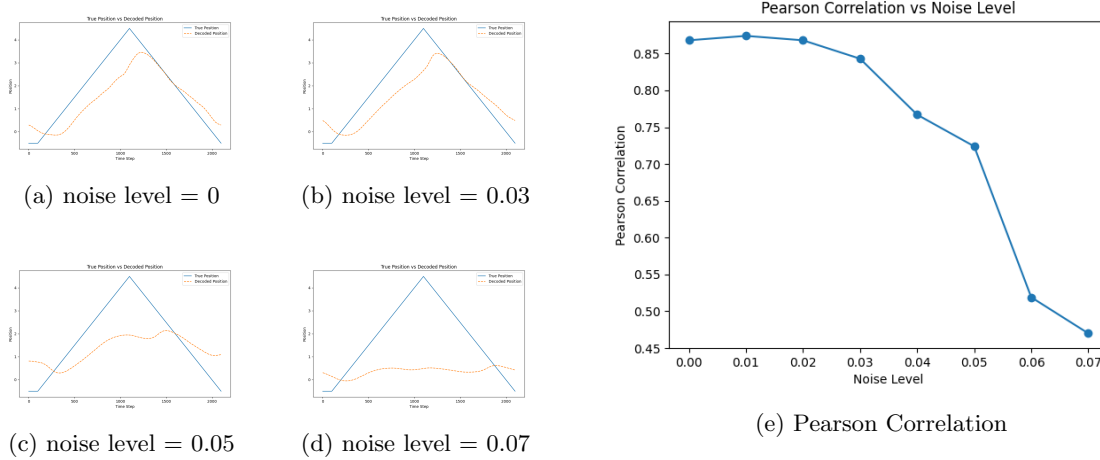


Figure 2: Decode Performance with only Self-Motion Cue

3.2 Connection Noise

We simulated connection noise between the P-CANN (Place Cell Attractor Neural Network) and G-CANNs (Grid Cell Attractor Neural Networks) to assess its impact on model performance. Remarkably, the model maintained perfect decoding performance regardless of the noise intensity. While the noise introduced minor fluctuations in the decoded position curve, applying smoothing techniques allowed the model to accurately reflect positional changes. This robustness can be attributed to the averaging effect of Gaussian noise across the network, which tends to converge toward zero, minimizing its overall impact. Additionally, slight variations in synaptic weights among neurons do not significantly disrupt the network’s attractor dynamics, allowing the system to maintain stable and accurate decoding. This result highlights the biological plausibility of the model, as neural networks in the brain are also known

to tolerate moderate noise without significant loss of functionality, leveraging redundancy and stability inherent in their architecture.

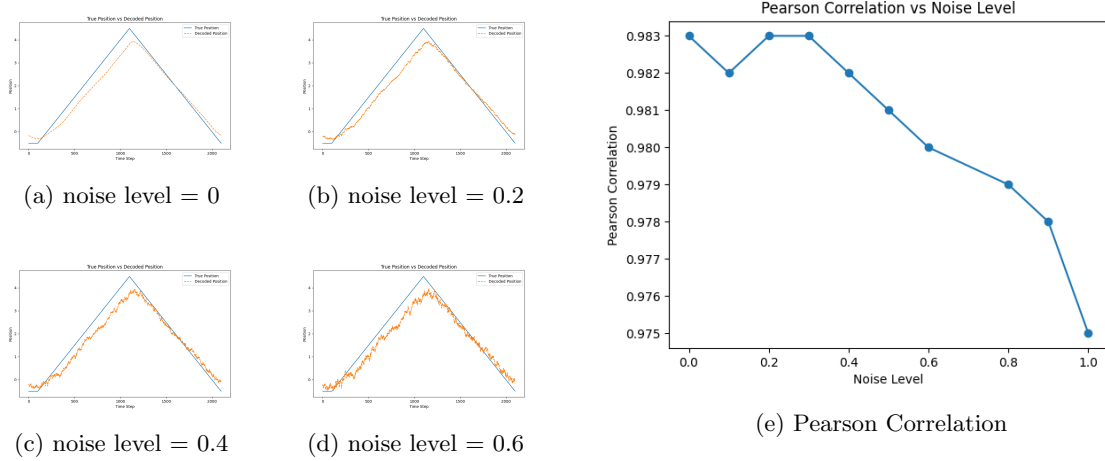


Figure 3: Decode Performance with Environmental Cue

4 Discussion

In this study, we have employed several types of noise in a computational model where grid cell and place cell complement each other for spatial coding. We first examine grid cell and place cell for their noise resistance properties. Then we distort connection to see how our model react to fluctuation in the network. Using simulations, we further demonstrate how noise disrupts the precision and stability of spatial representations.

Our model demonstrated that phase-space coding is highly susceptible to noise, as even small perturbations can significantly alter the phase. In contrast, localized space coding proved to be more robust, relying on discrete, localized fields to effectively represent spatial information.[\[SF11\]](#)[\[MKM08\]](#) Additionally, we verified the integration of information between grid cells and place cells, which substantially increased the system’s resilience to noise compared to their individual performances. This finding aligns with previous studies suggesting that cooperative interactions between these neural populations enhance robustness and reliability in spatial navigation.[\[BBB14\]](#)[\[IBJD17\]](#) We also observed that connection noise did not notably affect decoding accuracy. This robustness can be attributed to the properties of Gaussian noise, which has a mean of zero, allowing its effects to be mitigated in large-scale networks through compensatory mechanisms and redundancy.

Research on Alzheimer’s disease (AD) has shown that fast gamma oscillations are disrupted in aged APP-KI mice, both in the medial entorhinal cortex (MEC) and the hippocampal CA1 region. Furthermore, the coupling of fast gamma oscillations between the MEC and CA1 is diminished, indicating impaired signal transfer within the MEC→CA1 circuit.[\[JBS+20\]](#)[\[MGJC17\]](#) Abnormal neuronal activity in AD may arise from an increase in internal or external sources of “noise”. For instance, the accumulation of β -amyloid plaques ($A\beta$ plaques) can disrupt normal inter-neuronal communication, producing effects similar to the artificial noise introduced in experimental settings. Additionally, tau protein pathology and other neurodegenerative factors may exacerbate this noise effect, further impairing the functionality of neural networks. These disruptions in network communication and oscillatory dynamics are thought to contribute to the cognitive deficits observed in AD.

For future work, we aim to enhance our model’s ability to emulate the biological processes underlying spatial coding in the brain and to investigate the system’s dynamics using advanced analytical techniques. Regarding cell spiking behavior, Poisson noise would be a biologically relevant alternative, introducing variability that could slightly alter calculations. Furthermore, neural noise is not independent across neurons, as demonstrated in previous research.[\[AD99\]](#) Additionally, the assumption of a unimodal Gaussian tuning curve as an approximation for place cells is a simplification that does not fully capture their complex dynamics.[\[FKN+08\]](#)

To deepen our understanding, advanced techniques such as wave function decomposition or Fourier analysis could be employed to explore the system’s dynamics more thoroughly. Beyond the Pearson correlation coefficient, metrics such as diffusion coefficients and drift velocities as mentioned above could offer further insights into the effects of noise on spatial representation.[WK22] Moreover, the relationship between positional velocity and decoding accuracy presents a compelling avenue for future exploration, as an abrupt change in the stimulus may influence grid-place cell interactions during navigation.[FWW10]

Recent studies suggest that the hippocampal-entorhinal loop (HPC-MEC) plays a collaborative role in representing abstract spaces and other cognitive functions.[SG21][PBNE21][KW21] Our goal is to develop a more detailed and biologically plausible model to investigate these processes, bridging our findings with experimental data to provide a comprehensive understanding of spatial and cognitive coding in the brain.

Acknowledgments

In my paper, I used an LLM for language polishing and code debugging but did not use it to generate the core research content. Additionally, I have carefully reviewed the content.

References

- [AD99] Larry F Abbott and Peter Dayan. The effect of correlated variability on the accuracy of a population code. *Neural computation*, 11(1):91–101, 1999.
- [Ano24] Anonymous. Complementary coding of space with coupled place cells and grid cells. In *Submitted to The Thirteenth International Conference on Learning Representations*, 2024. under review.
- [ASC14] Amir H Azizi, Natalie Schieferstein, and Sen Cheng. The transformation from grid cells to place cells is robust to noise in the grid pattern. *Hippocampus*, 24(8):912–919, 2014.
- [BBB14] Daniel Bush, Caswell Barry, and Neil Burgess. What do grid cells contribute to place cell firing? *Trends in neurosciences*, 37(3):136–145, 2014.
- [CLK⁺19] Guifen Chen, Yi Lu, John A King, Francesca Cacucci, and Neil Burgess. Differential influences of environment and self-motion on place and grid cell firing. *Nature communications*, 10(1):630, 2019.
- [CV10] Allen Cheung and Robert Vickerstaff. Finding the way with a noisy brain. *PLoS computational biology*, 6(11):e1000992, 2010.
- [DSMD16] Yedidyah Dordek, Daniel Soudry, Ron Meir, and Dori Derdikman. Extracting grid cell characteristics from place cell inputs using non-negative principal component analysis. *eLife*, 5:e10094, mar 2016.
- [FKN⁺08] André A Fenton, Hsin-Yi Kao, Samuel A Neymotin, Andrey Olypher, Yevgeniy Vayntrub, William W Lytton, and Nandor Ludvig. Unmasking the cal ensemble place code by exposures to small and large environments: more place cells and multiple, irregularly arranged, and expanded place fields in the larger space. *Journal of Neuroscience*, 28(44):11250–11262, 2008.
- [FWW10] CC Alan Fung, KY Michael Wong, and Si Wu. A moving bump in a continuous manifold: a comprehensive study of the tracking dynamics of continuous attractor neural networks. *Neural Computation*, 22(3):752–792, 2010.
- [GS19] David J Griffiths and Darrell F Schroeter. *Introduction to quantum mechanics*. Cambridge university press, 2019.
- [HFM⁺05] Torkel Hafting, Marianne Fyhn, Sturla Molden, May-Britt Moser, and Edvard I Moser. Microstructure of a spatial map in the entorhinal cortex. *Nature*, 436(7052):801–806, 2005.

- [IBJD17] Revekka Ismakov, Omri Barak, Kate Jeffery, and Dori Derdikman. Grid cells encode local positional information. *Current Biology*, 27(15):2337–2343, 2017.
- [JBS⁺20] Heechul Jun, Allen Bramian, Shogo Soma, Takashi Saito, Takaomi C Saido, and Kei M Igarashi. Disrupted place cell remapping and impaired grid cells in a knockin model of alzheimer’s disease. *Neuron*, 107(6):1095–1112, 2020.
- [KW21] Eric B Knudsen and Joni D Wallis. Hippocampal neurons construct a map of an abstract value space. *Cell*, 184(18):4640–4650, 2021.
- [MBJ⁺06] Bruce L McNaughton, Francesco P Battaglia, Ole Jensen, Edvard I Moser, and May-Britt Moser. Path integration and the neural basis of the ‘cognitive map’. *Nature Reviews Neuroscience*, 7(8):663–678, 2006.
- [MGJC17] Alexandra J Mably, Brian J Gereke, Dylan T Jones, and Laura Lee Colgin. Impairments in spatial representations and rhythmic coordination of place cells in the 3xtg mouse model of alzheimer’s disease. *Hippocampus*, 27(4):378–392, 2017.
- [MKM08] Edvard I. Moser, Emilio Kropff, and May-Britt Moser. Place cells, grid cells, and the brain apos;s spatial representation system. *Annual Review of Neuroscience*, 31(Volume 31, 2008):69–89, 2008.
- [OD71] John O’Keefe and Jonathan Dostrovsky. The hippocampus as a spatial map: preliminary evidence from unit activity in the freely-moving rat. *Brain research*, 1971.
- [PBNE21] Michael Peer, Iva K Brunec, Nora S Newcombe, and Russell A Epstein. Structuring knowledge with cognitive maps and cognitive graphs. *Trends in cognitive sciences*, 25(1):37–54, 2021.
- [SF11] Sameet Sreenivasan and Ila Fiete. Grid cells generate an analog error-correcting code for singularly precise neural computation. *Nature neuroscience*, 14(10):1330–1337, 2011.
- [SG21] Marielena Sosa and Lisa M Giocomo. Navigating for reward. *Nature Reviews Neuroscience*, 22(8):472–487, 2021.
- [WK22] Raymond Wang and Louis Kang. Multiple bumps can enhance robustness to noise in continuous attractor networks. *PLOS Computational Biology*, 18(10):e1010547, 2022.
- [WS90] KYM Wong and D Sherrington. Optimally adapted attractor neural networks in the presence of noise. *Journal of Physics A: Mathematical and General*, 23(20):4659, 1990.
- [WWF⁺16] Si Wu, KY Michael Wong, CC Alan Fung, Yuanyuan Mi, and Wenhao Zhang. Continuous attractor neural networks: candidate of a canonical model for neural information representation. *F1000Research*, 5, 2016.