ECON 204: Microeconomics III	**Spring 2017**

Lecture 6 — May 8th

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6.1 Akerlof Example

Example 1. Akerlof: $r(\theta) = 2/3\theta$, $Funif(0,1)$

$$w = E[\theta | r(\theta) \leq w]$$

$$= E[\theta | \frac{2}{3}\theta \leq w] = \begin{cases} \frac{3}{4}w & if w < \frac{2}{3} \\ \frac{1}{2} & otherwise \end{cases}$$

$$i.e. \quad w = min\{\frac{3w}{4}, \frac{1}{2}\}$$

$w = 0$ is the only fixed point. So $\nexists$ equilibrium.
competitive equilibrium doesn't even exist.

6.2 Example of Insurance: Full Insurance

- buyer type θ, seller uninformed
- initial wealth is 1, π is prob of wealth $\downarrow 0$
- $\frac{1}{2}$ of the population is of type θ_H with $\pi = \pi_H$, and the other $\frac{1}{2}$ is θ_L with $\pi_L < \pi_H$

Lowest price insurance company's willing to charge to insure the whole market is:

$$p \geq \frac{\pi_H}{2} + \frac{\pi_L}{2} = \frac{\pi_H + \pi_L}{2} \qquad (1)$$

Outside option is self-insuring. Incentive constraint (IC) for buyer is:

$$\pi u(0) + (1-\pi)u(1) \leq u(1-p) \qquad (*)$$

u is non-decreasing utility function; insurer is risk-neutral. Observe that if $(*)$ is satisfied for the low's, it is satisfied for the high's. So we only need to check on the low's. Assume $u(x) = \sqrt{x}$. Then we need

$$\pi_L\sqrt{0} + (1-\pi_L)\sqrt{1} \leq \sqrt{1-p}$$

$$(1-\pi_L)^2 \leq 1-p$$

$$p \leq 1 - (1-\pi_L)^2 \qquad (2)$$

to sell insurance to the low's (and to the high's follows.)

So we need (1) and (2) to insure the whole market: (1) for insurers to break-even and (2) for low types to buy, i.e.

$$\frac{\pi_H + \pi_L}{2} \leq p \leq 1 - (1 - \pi_L)^2 \qquad (3)$$

Example 2. $\pi_L = \frac{1}{4}, \pi_H = \frac{3}{4}$ then

$$\frac{1}{2} \not\leq \frac{7}{16}$$

$\Rightarrow$ cannot serve the whole market on one contract! Can set $p = \pi_H$ and insure the H type only

6.3 Modified Insurance Model: Partial Insurance

- $p \rightarrow$ price
- $e \rightarrow$ deductable

Insuring the whole market, need "break-even for insurer"

$$(\frac{\pi_L}{2} + \frac{\pi_H}{2})c + p \geq \frac{\pi_H + \pi_L}{2} \qquad (1)$$

and IC for both types

$$\pi u(0) + (1 - \pi)u(1) \leq (1 - \pi)u(1 - p) + \pi u(1 - p - c) \;\; \forall \; type \; L, H \quad (2)$$

Now we solve for highest coverage with lowest price (price assured by "break-even" for insurer).

$$(1) \Rightarrow c \geq 1 - \frac{2p}{\pi_H + \pi_L}$$

We want the highest coverage, so let

$$c = 1 - \frac{2p}{\pi_H + \pi_L}$$

W.O.L.G. let $u(0) = 0$; low type's IC costraint is binding in the following example (need proof) though not in general. Plug in c to (2) for type L gets

$$\frac{\pi}{1 - \pi} u(p(\frac{2}{\pi_L + \pi_H} - 1)) \geq u(1) - u(1 - p) \qquad (3)$$

Example 3. $\pi_H = \frac{3}{4}, \pi_L = \frac{1}{4}, u(x) = \sqrt{x}$, (2) becomes

$$\frac{1}{3} u(p) \geq u(1) - u(1 - p)$$
$$\Rightarrow \frac{1}{3}\sqrt{p} \geq 1 - \sqrt{1 - p}$$
$$\Rightarrow 0 \leq p \leq 0.36$$

So insurance contracts ranges from $p = 0, c = 1$ to $p = 0.36, c = 0.28$, i.e. no insurance to maximum possible partial coverage.

6.4 Social Ranking

Arrow's thought: people with different preferences arrive at company's shareholder meeting, then what?

- $A =$ alternatives finte
- $N = \{1, ..., n\}$ people finite
- $L_i(A)$ is a strict preference linear ordering: complete, transitive, asymmetric. $i \in N$
- $\#A = 2 \Rightarrow$ May's theorem; $\#A \geq 3 \Rightarrow$ Arrow's theorem

Note 1. People have strict preferences but society have weak preferences.

Example 4. $A = \{a, b\}$. Preferences $\succ_i, i \in N$. abuse of notation: i prefers a to b

$$a \succ_i b$$

Definition 1. Individual rankings $\succ = (\succ_1, \succ_2, ... \succ_n) \Rightarrow$ social ranking $R(\succ)$.
If $\#A = 2, W.O.L.G.$ say $A = \{a, b\}$, and $\succ_i = a$ if i prefers a to b.

Definition 2. A social welfare ordering rule satisfy neutrality if:

$$Given \succ, \succ', s.t. \succ_i \neq \succ'_i \; \forall i$$

then

$$a \; R(\succ) \; b \Rightarrow b \; R(\succ') \; a$$
$$b \; R(\succ) \; a \Rightarrow a \; R(\succ') \; b$$

Definition 3. Anonimity: symmetrically treat all voters

$$permutation \; \pi : N \to N$$

$$\succ_{\pi(i)} = \succ_i$$
$$R(\succ_\pi) = R(\succ)$$

Definition 4. Monotonicity (weak):

$$\succ, \succ', s.t. \;\; \succ_i = \succ'_i, \;\; \forall i \neq j$$

$$a \succ_j b, \;\; b \succ'_j a$$

then

$$b \; R(\succ) \; a \Rightarrow b \; R(\succ') \; a$$
$$a \; R(\succ') \; b \Rightarrow a \; R(\succ) \; b$$

Theorem 1. If R (complete) satisfies neutrality, anonymity, monotonicity, then $\exists\ q,\ 0 \leq q \leq \frac{n}{2}$, such that

$$a\ R(\succ)\ b\ if N_a(\succ) \geq q$$

$$bR(\succ)a\ if N_b(\succ) \geq q$$

"quota rule", where

$$N_a(\succ) = \#\{i|\succ_i = a\},\ N_b(\succ) = \#\{\succ_i = b\}$$

Note 2. Since preferences are strict, $N_a + N_b = n$.

Theorem 2. May's Theorem. substituting monotonicity with strict monotonicity, we have $q = \frac{n}{2}$ "majority rule."

Proof. Anonymity $\Rightarrow R(\succ) = R(\succ')$ when

$$N_a(\succ) = N_a(\succ')\ \ (N_b(\succ) = N_b(\succ'))$$

Adding monotonicity $\Rightarrow$ if $N_a(\succ) < N_a(\succ')$, then

$$a\ R(\succ)\ b \Rightarrow a\ R(\succ')\ b$$

thus,

$$\exists q_a\ s.t.\ a\ R(\succ)\ b \Longleftrightarrow N_a(\succ) \geq q_a$$

Neutrality $\Rightarrow q_a = q_b = q$
Complete $\Rightarrow q \leq \frac{n}{2}$
Strict Monotonicity $\Rightarrow q = \frac{n}{2}$ $\square$

Example 5. Ordinal preferences may lead to inefficiency.

Example 6. When there is cycle in preferences, voting order matters.

Example 7. Borda Count: pros: break cycles. cons: irrelevant alternatives matter.