ECON 204: Microeconomics III **Spring 2017**

Lecture 5 — May 1st

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5.1 Moral Hazard and Adverse Selection

- Moral Hazard: Hidden Action
- Adverse Selection: Hidden Type

5.2 Principle Agent Problem

- e: "effort"
- π: principal's profit $\sim F_e(\pi)$.
- $w(\pi)$: wage as a function of profits

The principle agent problem is formulated as:

$$\max_{w(\pi),e} \int (\pi - w(\pi))\ dF_e(\pi)$$

$$s.t. \int v(w(\pi))\ dF_e(\pi) - g(e) \geq \bar{u} \tag{1}$$

$$\int v(w(\pi))\ dF_e(\pi) - g(e) \geq \int v(w(\pi))\ dF'_e(\pi) - g(e'),\ \forall e' \neq e \tag{2}$$

$$w(\pi) \geq 0 \tag{3}$$

Note 1. $g(e)$ is cost of effort, and $\bar{u}$ is reservation utility;
constraint (2) is equivalent to a maximization problem;
(3) means no threat, and e is non-oberservable to make the problem non-trivial.

5.3 Risk Sharing

If e is observable, then principal can contract on e and pays agent contingent on a specific effort level being observed. $\forall e$, agent solves the minimization problem:

$$min \int w(\pi) dF_e(\pi)$$

$$s.t.(1)$$

Solve by taking Lagrangian,

$$L = \int w(\pi) dF_e(\pi) + \lambda[\bar{u} - \int v(w(\pi))\ dF_e(\pi) - g(e)]$$

To maximize the Lagrangian = the integral over all possible function $w(\pi)$, we maximized the integrant at each point π of $w(\pi)$.
Take derivative with respect to w, and get

$$dF_e(\pi) - \lambda v'(w(\pi)) dF_e(\pi) = 0$$

$$\Rightarrow v'(w(\pi)) = \frac{1}{\lambda} \quad \forall \pi$$

$$\Rightarrow v(\bar{w}) = g(e) + \bar{u}$$

$$\Rightarrow \bar{w} = v^{-1}(g(e) + \bar{u})$$

So w is constant wage, and we pay exactly $g(e) + \bar{u}$

Note 2. Observe that $w \geq 0$ so (3) is automatically satisfied.

Note 3. In the case where e is observable, next step would be to choose e such that principal's profit is maximized. Pick e s.t.

$$max \int \pi dF_e(\pi) - \bar{w}(e)$$

This is called "first best, " i.e. optimal solution w/o IC constraint. This is the same as maximizing total utility.

5.4 More on Risk Sharing

Example 1.

- e_L, e_H, $\pi \in \{0, 100\}$
- $e_L : \begin{cases} \frac{3}{4} \rightarrow 0 \\ \frac{1}{4} \rightarrow 100 \end{cases}$
- $e_H : \begin{cases} \frac{1}{4} \rightarrow 0 \\ \frac{3}{4} \rightarrow 100 \end{cases}$
- $v(w) = \sqrt{(w)}$
- $g(e_L) = 0,\ g(e_H) = 5,\ \bar{u} = 0$

e **observable:** (i) Suppose we want to get $e = e_L$, then just set $\bar{w}(\pi) = (g(e_L) + \bar{u})^2 = 0$ for $\forall \pi$, and it satisfies all constraints.
(ii) If we want $e = e_H$, then need

$$\sqrt{\bar{w}} - g(e_H) \geq \bar{u}$$

$$\sqrt{25} - 5 = 0$$

$$\bar{w} = 25$$

(iii) profits: $e_L : \frac{1}{4}100 - 0 = 25 \qquad e_H : \frac{3}{4}100 - 25 = 50$
Will pick e_H. Profit is $(50, 0)$
Note: If $g(e_H) = 10$, principal will want to contract on e_L.

e **unobservable:** $wage = w(0), w(100)$
Work with IC constraint:
(i) In order to induce low effort e_L, need to pay $w(0) = w(100) = 0$.
(ii) In order to induce high effort e_H, these are the wages we need to pay:

$$\frac{1}{4}\sqrt{w(0)} + \frac{3}{4}\sqrt{w(100)} - g(e_H) \geq \frac{3}{4}\sqrt{w(0)} + \frac{1}{4}\sqrt{w(100)} - 0$$

$$g(e_H) = 5$$

$$\frac{1}{2}\sqrt{w(100)} - \frac{1}{2}\sqrt{w(0)} \geq 5$$

$$\sqrt{w(100)} \geq \sqrt{w(0)} + 10$$

To minimize wage paid, we let $w(0) = 0 \Rightarrow w(100) = 100$
So $E[\pi - w(\pi)] = 0$
(iii) Contracting on e_L is better since it yields expected payoff $= \frac{1}{4}x100 - 0 = 25$
Payoff is (25, 0)

In summary: e observable: contract is $e_H \to \bar{w} = 25$ payoff $= (50, 0)$
e unobservable: contract is to ensure $e_L \to w = 0$ payoff $= (25, 0)$
If agent commits to be observed, both players can be better off —- there is real inefficiency here.
a variation: if $g(e_H) = 1$, then ....

5.5 Solving the Original Problem

Assume there are two effort levels: e_L and e_H, $g(e_H) > g(e_L)$, and $F_{eH}\ FOSD\ F_{eL}$

Claim 1. (1) and (2) are binding.

5.6 Principal-Agent Problem: Hidden Type

- Agent has hidden type = productivity $\theta \in [0, 1]$, with distribution $F(\theta)$
- $r(\theta)$ is reservation wage/ outside option

Assume that principal pays constant wage, then competitive REE. Wage is a fixed point:

$$w^* = E[\theta | w^* \geq r(\theta)]$$

We show that there is no efficient equilibrium in the following two examples.

Example 2. Assume $r(\theta) = r \in [0, 1], \forall\theta$ and that $0 < F(r) < 1$.
Case I. $w < r$, nobody works, then it's not efficient because of under-hiring. There exists θ *s.t.* $\theta > r$

$$0 < \theta - (r + \epsilon)$$

$$r < r + \epsilon$$

Case II. $w \geq r$, everybody works, then it's not efficient since there's over-hiring. $E[\theta] = w$. There exists θ *s.t.* $\theta < r$

$$-(w - \theta) < -(w - r + \epsilon)$$

$$w < r + (w - r + \epsilon)$$

There's a better contract that both principal and agent prefers. If $E[\theta] >= r$, there could be equilibrium, but not efficient. otherwise there's no equilibrium.