ECON 204: Microeconomics III	**Spring 2017**
Lecture 1 — April 3rd	
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1.1 Walrasian Equilibrium

- $N = \{1, ..., n\}$ agents $\quad l$ goods $\quad$ no production, exchange economy
- R^l_+ consumption set $\quad + :$ non-negative $\quad ++ :$ positive
- $x_i \in R^l_+ \quad x_i = (x_{i1}, ..., x_{il})$
- $e_i \in R^l_+ \rightarrow$ endowment
- $u_i : R^l_+ \rightarrow R$ utility function

Definition 1. Walrasian Equilibrim: Given economy $E = (e_1, u_1, ..., e_n, u_n)$, a Walrasian Equilibirum is a vector (p, x) with $p \in R^l_+ \quad x = (x_1, ..., x_n) \in R^{nl}_+$ such that:

1. Agents are maximizing their utilities: for all $i \in N$

$$x_i \in arg \max_{x_i'} \; u_i(x_i') \quad \forall i$$

$$s.t. \;\; p \cdot x_i' \leq p \cdot e_i$$

2. Markets clears:

$$\sum x_i = \sum e_i$$

Note 1. Here we assumed that agents are price takers.

Note 2. 1. $\leq$ can be $=$ if utility is non-decreasing 2. In class, we had $\leq$ instead, if utility is non-decreasing.

1.2 Welfare Theorems

Definition 2. An allocation x is **feasible** if $\sum x_i \leq \sum e_i$.

Definition 3. Given an economy E, a feasible allocation x is **weakly Pareto optimal** if there is no other feasible allocation $x' = (x_1', ..., x_n') \in R^{nl}_+$ such that

$$u_i(x_i') > u_i(x_i) \quad \forall i$$

Definition 4. Given an economy E, a feasible allocation x is **Pareto optimal** if $\nexists x'$ feasible such that

$$u_i(x_i') \geq u_i(x_i) \quad \forall i$$
$$u_j(x_j') > u_i(x_j) \ for\ some\ j$$

Definition 5. Local non-satiation: Utility function $u : D \to R$ is **locally non-satiated** if $\forall x \in D, \epsilon > 0, \quad \exists x' \in D$ s.t.

$$u(x_i') > u(x_i)$$
$$\|x' - x\| < \epsilon$$

First Welfare Theorem: Given economy E, if $p, (x_1, ...x_n)$ is WE, then it is weakly Pareto Efficient; If $\forall i,\ u_i$ is locally non-satiated, then it is Pareto Efficient.
1. Simpler Version:

Proof. Suppose $\exists x' = (x_1', ...x_n')$ s.t.

$$u_i(x_i') > u_i(x_i) \quad \forall i$$

implies that it's not affordable (otherwise (p, x) wouldn't be WE,) i.e.

$$p \cdot x_i' \ > \ p \cdot e_i \ \ \forall i$$
$$\Rightarrow p \cdot \sum x_i' \ > \ p \cdot \sum e_i$$
$$\Rightarrow \sum x_i' \nleq \sum e_i$$

□

2. Strict Pareto Efficiency:

Proof. Suppose $\exists x' = (x_1', ...x_n')$ s.t.

$$u_i(x_i') \geq u_i(x_i) \quad \forall i$$
$$u_j(x_j') > u_j(x_j)$$

Then, applying local-non-satiation*, we have

$$p \cdot x_i' \ \geq \ p \cdot e_i \ \ \forall i$$
$$p \cdot x_j' \ > \ p \cdot e_j$$

following the same argument,

$$\Rightarrow p \cdot \sum x_i' \ > \ p \cdot \sum e_i$$
$$\Rightarrow \sum x_i' \nleq \sum e_i$$

□

Claim 1. Local non-satiation $\Rightarrow$ If $u_i(x_i') = u_i(x_i)$, then $p \cdot x_i' \geq p \cdot e_i$.

Proof. Suppose on the contrary, $p \cdot x_i' < p \cdot e_i$. Since $p \geq 0$, find $\epsilon > 0$ s.t.

$$p \cdot (x_i' + (\epsilon, ..., \epsilon)) < p \cdot e_i$$

And thus $\exists$ y_i s.t.

$$u_i(y_i) > u_i(x_i') = u_i(x_i)$$

$$\|yi - x_i'\| \leq \epsilon \Rightarrow p \cdot y_i < p \cdot e_i$$

by non-satiation. This contradicts that x_i is the demand at p (feasible allocation that maximizes utility.)

□

1.3 Arrow Debreu's Existence Theorem

See ECON 202

Example 1.

- $n = 2,\ l = 2,\ e_1 = (1, 0),\ e_2 = (0, 1)$
- $u_i = [\min(x_{i1}, x_{i2})]^\alpha$
- $p = (p_1, p_2)$

We solve for Walrasian Equilibirum: Given price $p = (1,\ p)$ (we can almost always normalize price to (p, 1) unless price of 2nd good is 0). For person 1, we have $x_{11} + px_{12} = 1$ and $x_{11} = x_{12}$ to maximize it's utility

$$x_1 = (\frac{1}{1+p}, \frac{1}{1+p})$$

and similarly for person two:

$$x_2 = (\frac{p}{1+p}, \frac{p}{1+p})$$

Market clears! So we found WE!
1st person $\rightarrow$ wealth 1, 2nd person $\rightarrow$ wealth of p. There are multiple $WE's$, the higher p, the better off 2nd person. But all equilibriums are Pareto Efficient.

1.4 Paradox With Uncertainty

- $S = \{s, s'\} \rightarrow$ states, equally likely
- Trading is ex-post

Equilibrium 1 In both states

$$p = 1$$

$$x_1 = x_2 = (\frac{1}{2}, \frac{1}{2})$$

$$u_1 = u_2 = (\frac{1}{2})^\alpha$$

Equilibrium 2 Different ex-post WE's in different states:

$$p = \frac{1}{3}\ in\ s \quad p = 3\ in\ s'$$

$$s \to x_1 = (\frac{3}{4}, \frac{3}{4}) \quad x_2 = (\frac{1}{4}, \frac{1}{4})$$

$$u_1 = (\frac{3}{4})^\alpha \quad u_2 = (\frac{1}{4})^\alpha$$

$$s' \to x_1 = (\frac{1}{4}, \frac{1}{4}) \quad x_2 = (\frac{3}{4}, \frac{3}{4})$$

$$u_1 = (\frac{1}{4})^\alpha \quad u_2 = (\frac{3}{4})^\alpha$$

Expected Utility For equilibrium 1,

$$EU_i = (\frac{1}{2})^\alpha \quad i = 1, 2$$

For equilibrium 2,

$$EU_i = \frac{1}{2} \cdot (\frac{3}{4})^\alpha + \frac{1}{2} \cdot (\frac{1}{4})^\alpha \quad i = 1, 2$$

If $\alpha = 1$, two are the same
$\alpha > 1$, 2nd equilibrium is better
$\alpha < 1$, 1st equilibrium is better

So the two equilibriums are both ex-post Pareto Efficient, but at least one of them is ex-ante Pareto inefficient depending on value of α. First Welfare Theorem breaks down when there's uncertainty involved.

Note 3. Ex-post Pareto Efficiency: maximizing utility in each state (after state is revealed) & Ex-ante efficiency: maximizing expected utility (over different states).

1.5 Information Set

Definition 6. S is the state space. An **information set** is $I \subseteq 2^S \setminus \{\emptyset\}$ s.t.

$$\forall \pi \in I, \pi' \in I, \pi \neq \pi'$$

$$\pi \cap \pi' = \emptyset$$

and

$$\bigcup_{\pi \in I} \pi = S$$

i.e. an information set is a partition of S.

Example 2.
- $S = \{s_1, s_2, s_3, s_4, s_5\}$
- $I_i = \{\{s_1, s_2\}, \{s_3, s_4\}, \{s_5\}\}$
- $I_i(s_2) = \{s_1, s_2\}$

Definition 7. Information set I is **coarser** than I' or I' is **finer** than I if

$$bad\ \ definition: \quad \forall \pi \in I, \exists \pi' \in I'\ s.t.\ \pi' \subseteq \pi$$

$$\forall \pi' \in I', \exists \pi \in I\ s.t.\ \pi' \subseteq \pi$$

Definition 8. $I \vee I'$ is **join** defined as the coarsest partition finer than I, I'.
$I \wedge I'$ is **meet** defined as the finest partition coarser than both.

Join and meet are unique.

1.6 Rational Expectation Equilibrium

- $N = \{1, ..., n\}, S =$ states, l goods
- $e_i : S \to R^l_+$ endowment $e_i(s) \quad i \in N$, measurable w.r.t. I_i (not I_i meet I_p)
- I_i : information set, partition of S
- $u_i : R^{\#S \times l}_+ \to R \quad (x \in R^{\#S \times l}_+)$

Definition 9. Price $p : S \to \{(p_1, ..., p_l) \in R^l_+\ s.t.\ \sum\limits_{k=1}^{l} p_k = 1\}$. Price is normalized, so that no extra information is given from price. Most other normalization works; $(p, 1)$ has a risk of price of good 2 being 0.

Definition 10. I_p is defined as a partition of S so that

$$I_p(s) = I_p(s') \Leftrightarrow p(s) = p(s')$$

Definition 11. $E = (e_i, u_i, I_i)_{i \in N}$, price and consumption bundle $(p, x_1, ..., x_n)$ is a **Rationalized Expectation Equilibrium (REE)** if
1. $\forall i, x_i$ maximizes u_i over all feasible x'_i measurable w.r.t. $I_i \vee I_p$:

$$x_i \in arg\ \max u_i(x'_i)$$

$$s.t.\ p(s) \cdot x'_i(s) \leq p(s) \cdot e_i(s) \ \ \forall s$$

$$x'_i\ measurable\ w.r.t.\ I_i \vee I_p$$

2. Market clears:

$$\sum x_i = \sum e_i$$