ECON 204: Microeconomics III	**Spring 2017**
Lecture 2 — April 10th	
Lecturer: Matt Jackson	*Scribe: Y*

2.1 Non-existence Example

David Kreps (1997)

- $I_1 = \{\{s\}, \{s'\}\} \quad I_2 = \{\{s, s'\}\}$
- $e_1(s) = e_1(s') = e_2(s) = e_2(s') = (1, 10)$
- $u_i(x_i(s), s) = a_i(s) log(x_{i1}(s)) + x_{i2}(s)$ where
  $a_1(s) = 1 \quad a_1(s') = 2$
  $a_2(s) = 2 \quad a_2(s') = 1$

Note 1. U_i ought to be function of x_i.... So in this particular case, U_i is defined as weighted sum of $u_i(s)$ in each states; and to maximize U_i over all possible x_i', only need to maximize $u_i(s)$ over all possible $x(s)$, $\forall s$, since U_i is separable.

Claim 1. $\nexists$ REE

Proof. Normalize price to $(p(s), 1) \rightarrow$ (price of good 1, price of good 2)

Case 1: $p(s) = p(s')$
Since utility function are not the same*,

$$x_{11}(s) \neq x_{11}(s')$$

but since x_2 needs to be measurable w.r.t. $\{\{s, s'\}\}$

$$x_{21}(s) = x_{21}(s')$$

$\Rightarrow$ Market doesn't clear.
*utility function at $s : log(x_{11}) + x_{12}$, at $s' : 2log(x_{11}) + x_{12}$

Case 2: $p(s) \neq p(s')$
We calculate the demand of good 1 by 1 and 2 in different states*:
$x_{11} \rightarrow \frac{1}{p(s)}, \frac{2}{p(s')} \qquad x_{21} \rightarrow \frac{2}{p(s)}, \frac{1}{p(s')}$

$$\Rightarrow \frac{3}{p(s)} = \frac{3}{p_(s')}$$

contradicts the assumption that $p(s) \neq p(s')$

*calculation at s for person 1: maximize $log(x_1) + x_2$

$$s.t.\ px_1 + x_2 = p + 10$$

$$\Rightarrow x_2 = p + 10 - px_1$$

from constraint. Plug into objective and take FOC:

$$\frac{1}{x_1} - p = 0$$

□

2.2 Grossman-Stiglitz Paradox

2.3 Common Knowledge

- $S = \{s_1, ...s_k\} \quad P(s) \in [0,1]$ with $\sum_{s \in S} P(s) = 1$ prior
- $n = 2,\ I_1,\ I_2$

Definition 1. An **"event"** is $E \subseteq S$

Definition 2. $I^m = I_1 \wedge I_2$

Definition 3. E is **common knowledge at** $s \in S$ if $I^m(s) \subseteq E$

Definition 4. **Conditional probability** is defined as

$$q_i(E|s) = \frac{P(E \cap I_i(s))}{P(I_i(s))}$$

Example 1. $E = \{1, 2\},\ s = 3,\ I_i = \{\{1\}, \{2, 3\}\},\ P(s) = \frac{1}{3}$

$$q_i(E|s) = \frac{1}{2}$$

2.4 Aumman's Theorem; No Trade Theorem

Aumann's Theorem. Priors are the same. If $E = \{s | q_1(E'|s) = q_1\ \&\ q_2(E'|s) = q_2\}$ is c.k. at some s, then $q_1 = q_2$

Proof.

$$E = \{s|q_1(E'|s) = q_1,\ q_2(E'|s) = q_2\} \qquad (*)$$

$$I^m(s) \subseteq E \qquad (**)$$

Let $B = I^m(s) = \bigcup_{k\in K} B_k,\ B_k \in I_1 \quad \Rightarrow \ q_1(E'|s) = q_1,\ \forall s \in B$

$$\frac{P(E'\cap B_k)}{P(B_k)} = q_1,\ \forall B_k$$

$$\sum_{k\in K} P(E'\cap B_k) = q_1 \sum_{k\in K} P(B_k)$$

$$\Rightarrow P(E'\cap B) = q_1 P(B)$$

$$\Rightarrow q_1 = \frac{P(E'\cap B)}{P(B)} \Rightarrow q_1 = q_2 = \frac{P(E'\cap B)}{P(B)}$$

□

No Trade Theorem. Milgrom Stokey
1 pays 2 x if E' occurs; 2 pays 1 y if E' not occur. Then 1 will take bet if

$$q_1(E'|s)(-x) + (1 - q_1(E'|s))y > 0$$

$$\Leftrightarrow \ q_1(E'|s) < \frac{y}{x+y}$$

2 will take bet if

$$q_2(E'|s)x + (1 - q_2(E'|s))(-y) > 0$$

$$\Leftrightarrow \ q_2(E'|s) > \frac{y}{x+y}$$

Assumptions:
1. state s is realized
2. "Both will take bet" $E = \{s|q_1(E'|s) < \frac{y}{x+y} < q_2(E'|s)\}$ is c.k. at s via a similar line of argument can prove $P(E'\cap B) < \frac{x}{x+y}$ from 1 and a contradicting result from person 2

Rubinstein's Email Game: No investment.