ECON 204: Microeconomics III	**Spring 2017**
Lecture 10 — June 5th	
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10.1 Inefficiency $\to$ Transfers

Example 1. (continue'd) If we distribute cost unevenly after $x > 3$, so that $a_i \to$ proportion to i-th person, with $\sum_i a_i = 1$. Then marginal utility at public good level $x = 3$:

$$mu_1 = 1 - 6a_1$$

$$mu_2 = 2 - 6a_2$$

$$mu_3 = 4 - 6a_3$$

Set $a_1 = \frac{1}{8}, a_2 = \frac{1}{4}, a_3 = \frac{5}{8}$, all $mu_i > 0$ and balanced. So the median mechanism is inefficient :)

Mechanism with Transfers:

- $A =$ decision/ alternatives, $t \in R^n$
- $\Theta \to$ type space
- $f : \Theta \to A \times R^n$ social choice function
  $f(\theta) = (d(\theta), t_1(\theta), ..., t_n(\theta)$

Definition 1. A **feasible** mechanism gives $\sum t_i(\theta) \leq 0, \ \forall \theta \in \Theta$

Definition 2. A **balanced** mechanism gives $\sum t_i(\theta) = 0, \ \forall \theta \in \Theta$

- utility $\to u_i(d(\theta), \theta_i) + t_i(\theta)$ quasi-linear?

Definition 3. $d(\cdot)$ is **efficient** if it maximizes sum of u_i

$$d(\theta) \in arg \max_d \sum u_i(d, \theta_i)$$

d efficient meaning d maximizes sum of utilities assuming announced θ_i are true types

Claim 1. $(d(\cdot), t(\cdot))$ is PE $\Rightarrow$ d is efficient and t is balanced.

Proof. Need t to be feasible; this is why we want efficient d and balanced t. $\square$

10.2 Vickrey-Clarke-Groves

Example 2. **Q:** How to make people's interest align with maximizing total utilities?
A: Add the other people's utilities u_{-i}.

Q: How to assure $DSIC$?
A: The same; as long as d is efficient.

Q: How to get feasibility?
A: $-H$ where $H \geq 0$, and make $t \leq 0$. Note that there are a couple different ways to make it feasible.

General Mechanism:

$$t_i(\hat{\theta}) = \sum_{j \neq i} u_j(d(\hat{\theta}), \hat{\theta}_i) + x_i(\hat{\theta}_{-i})$$

Claim 2. Mechanism is $DSIC$ if d is efficient.

Proof. truth telling gives

$$u_i(d(\theta_i, \hat{\theta}_{-i}), \theta_i) + \sum_{j \neq i} u_j(d(\theta_i, \hat{\theta}_{-i}), \hat{\theta}_j) + x_i(\hat{\theta}_{-i})$$

$$\geq u_i(d(\tilde{\theta}_i, \hat{\theta}_{-i}), \theta_i) + \sum_{j \neq i} u_j(d(\tilde{\theta}_i, \hat{\theta}_{-i}), \hat{\theta}_j) + x_i(\hat{\theta}_{-i})$$

since d is efficient as $d(\theta) \in arg \max_d \sum u_i(d, \theta_i)$

□

10.3 Clarke's Pivotal Mechanism

$$x_i(\hat{\theta}_{-i}) = -\max_d \sum_{j \neq i} u_j(d, \hat{\theta}_j)$$

so transfer is

$$t_i(\hat{\theta}) = \sum_{j \neq i} u_j(d(\theta_i, \hat{\theta}_{-i}), \hat{\theta}_j) - \max_d \sum_{j \neq i} u_j(d, \hat{\theta}_j)$$

$$= \sum_{j \neq i} u_j(d(\theta_i, \hat{\theta}_{-i}), \hat{\theta}_j) - \sum_{j \neq i} u_j(d(\hat{\theta}_{-i}), \hat{\theta}_j)$$

i.e. how much i increases other people's utilities over announced $\hat{\theta}_{-i}$

Example 3. Median Mechanism:

- $n = 3$
- $\theta_1 = 0 \quad \theta_2 = 6 \quad \theta_3 = 12$

Take median, $\bar{\theta} = 6, \bar{\theta}_{-2} = 6$ so player 2's not pivotal transfer is 0.
Look at $t_1 : \bar{\theta}_{-1} = 9$

$$t_1 = -(6-6)^2 - (12-6)^2 - [-(6-9)^2 - (12-9)^2]$$

Example 4. 2nd price auction is a pivot mechanism.
Social surplus doesn't change $\rightarrow t = 0$
Social surplus changes $\rightarrow t =$ the increase of social surplus

Note 1. non-pivotal type $\rightarrow$ gets 0 transfer