Quantum states and circuits

Austin Fowler

Classical states

- Bit strings: 0001100101
- Don't need to think about the physical implementation
- Just focus on the rules of manipulation
- Given two input registers A and B, and output C we can perform:
    - C=A+B
    - C=A*B
    - C=A AND B
    - C=A XOR B
    - C=A>>B
    - etc, etc, etc

Quantum states

$$|\Psi\rangle = \sum_{i=0}^{2^n-1} c_i|i\rangle, \quad c_i \in \mathbb{C}, \quad \sum_{i=0}^{2^n-1} |c_i|^2 = 1$$

quantum register or quantum state

Quantum states

$$|\Psi\rangle = \sum_{i=0}^{2^n-1} c_i|i\rangle, \quad c_i \in \mathbb{C}, \quad \sum_{i=0}^{2^n-1} |c_i|^2 = 1$$

amplitude

probability

Quantum gates

$$X|0\rangle = |1\rangle$$
$$X|1\rangle = |0\rangle$$

$$X_2(\alpha|000\rangle + \beta|111\rangle) = \alpha|100\rangle + \beta|011\rangle$$
$$X_1(\alpha|000\rangle + \beta|111\rangle) = \alpha|010\rangle + \beta|101\rangle$$
$$X_0(\alpha|000\rangle + \beta|111\rangle) = \alpha|001\rangle + \beta|110\rangle$$

q2 q1 q0

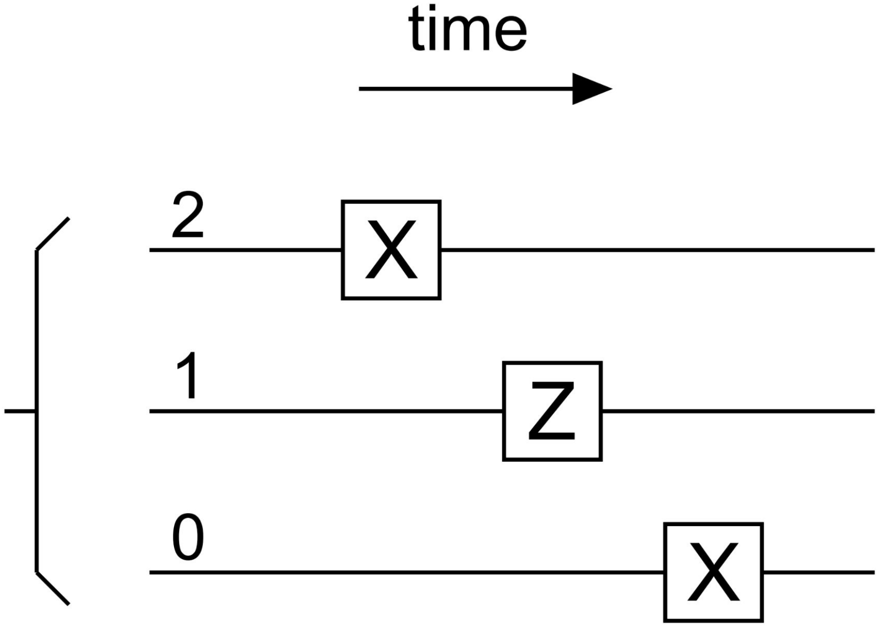
Quantum gates

$$Z|0\rangle = |0\rangle$$
$$Z|1\rangle = -|1\rangle$$

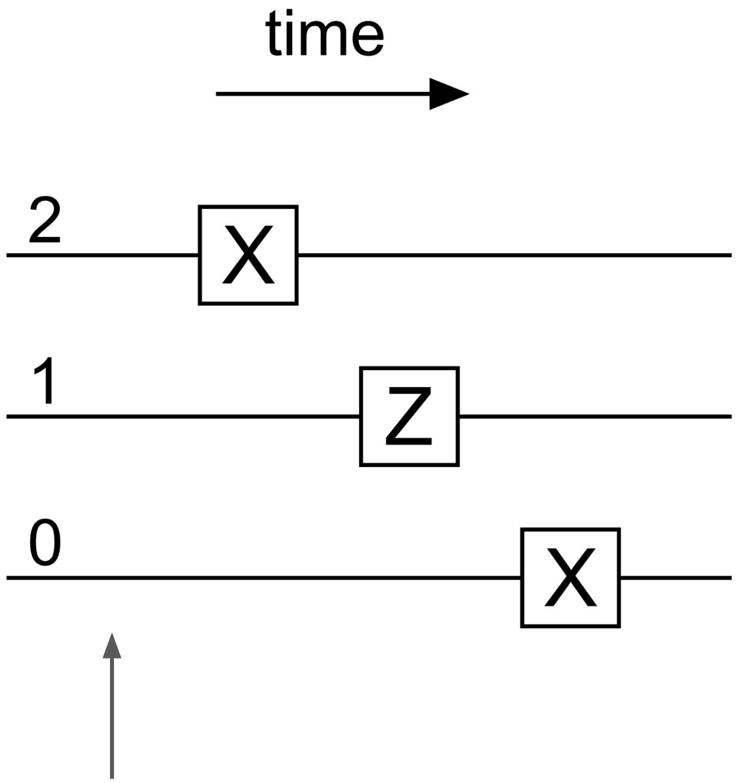
$$Z_2(\alpha|000\rangle + \beta|111\rangle) = \alpha|000\rangle - \beta|111\rangle$$
$$Z_1(\alpha|000\rangle + \beta|111\rangle) = \alpha|000\rangle - \beta|111\rangle$$
$$Z_0(\alpha|000\rangle + \beta|111\rangle) = \alpha|000\rangle - \beta|111\rangle$$

Quantum circuits

$$\alpha|000\rangle + \beta|111\rangle$$

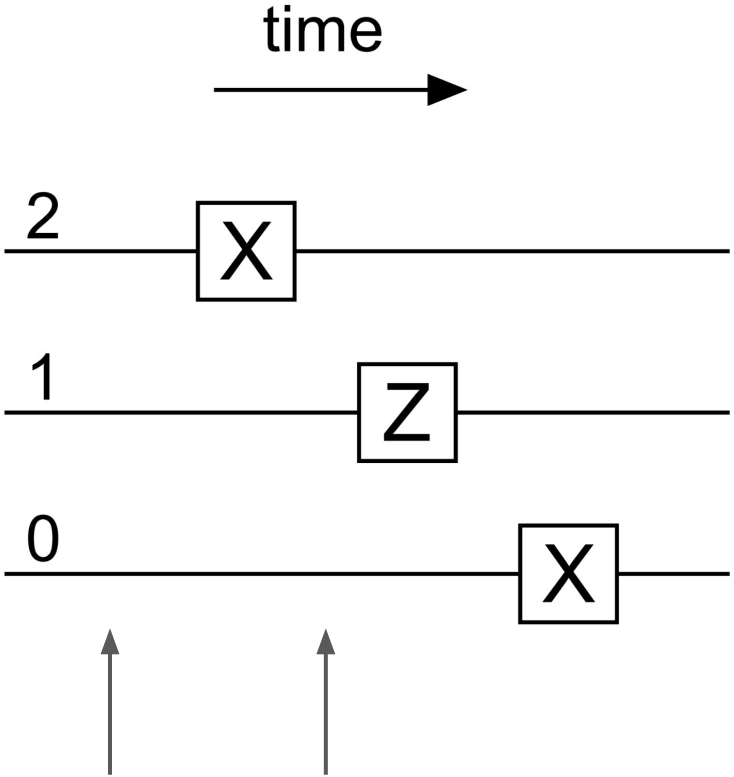


Quantum circuits



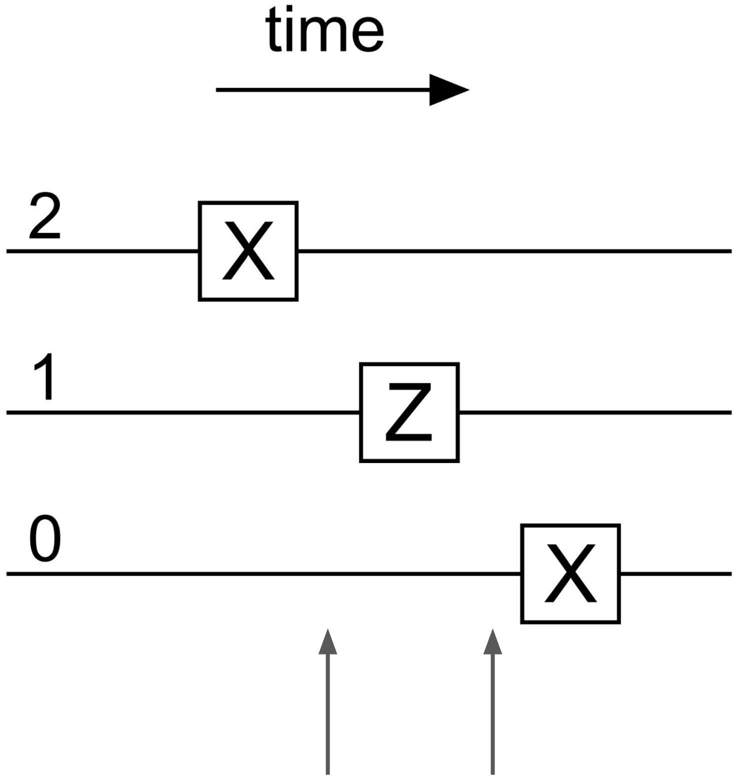
$$\alpha|000\rangle + \beta|111\rangle$$

Quantum circuits



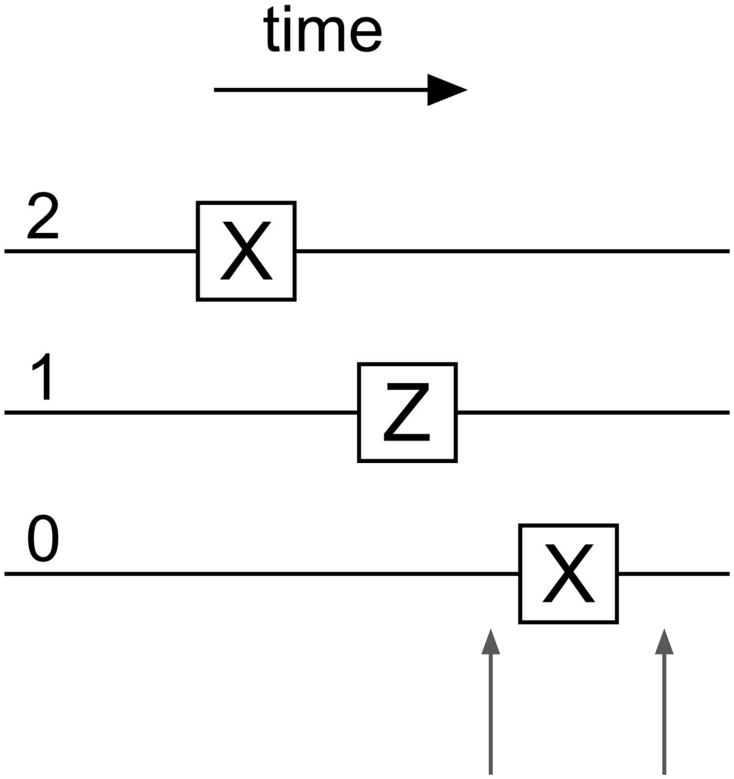
$$\alpha|000\rangle + \beta|111\rangle \qquad \alpha|100\rangle + \beta|011\rangle$$

Quantum circuits



$$\alpha|100\rangle + \beta|011\rangle \qquad \alpha|100\rangle - \beta|011\rangle$$

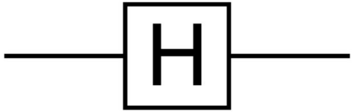
Quantum circuits



$$\alpha|100\rangle - \beta|011\rangle \qquad \alpha|101\rangle - \beta|010\rangle$$

More quantum gates

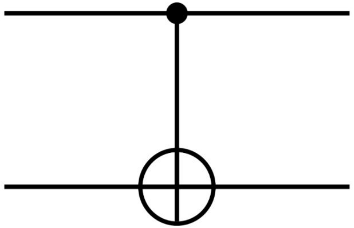
Hadamard



$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |-\rangle$$

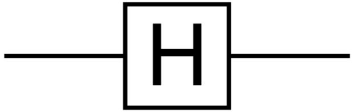
Controlled-NOT



$$CNOT(1,0)|00\rangle = |00\rangle$$
$$CNOT(1,0)|01\rangle = |01\rangle$$
$$CNOT(1,0)|10\rangle = |11\rangle$$
$$CNOT(1,0)|11\rangle = |10\rangle$$

More quantum gates

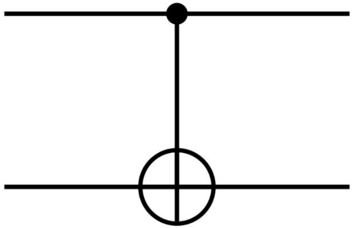
Hadamard



$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |-\rangle$$

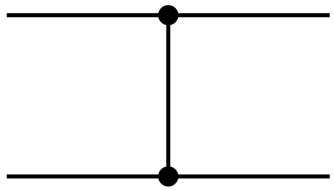
Controlled-X



$$CX(1,0)|00\rangle = |00\rangle$$
$$CX(1,0)|01\rangle = |01\rangle$$
$$CX(1,0)|10\rangle = |11\rangle$$
$$CX(1,0)|11\rangle = |10\rangle$$

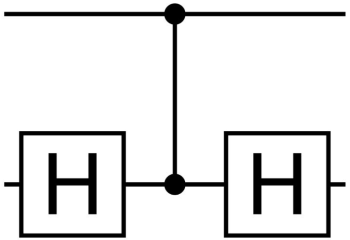
More quantum gates

Controlled-Z



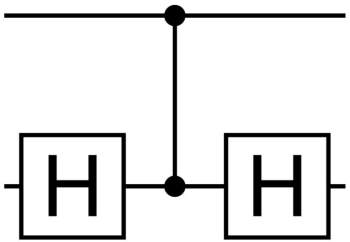
$$CZ(1,0)|00\rangle = |00\rangle$$
$$CZ(1,0)|01\rangle = |01\rangle$$
$$CZ(1,0)|10\rangle = |10\rangle$$
$$CZ(1,0)|11\rangle = -|11\rangle$$

Test your understanding



$$|00\rangle \rightarrow \ ?$$
$$|01\rangle \rightarrow \ ?$$
$$|10\rangle \rightarrow \ ?$$
$$|11\rangle \rightarrow \ ?$$

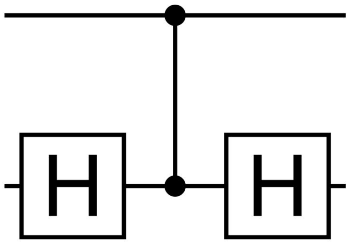
Test your understanding



$$|00\rangle \to \ ?$$
$$|01\rangle \to \ ?$$
$$|10\rangle \to \ ?$$
$$|11\rangle \to \ ?$$

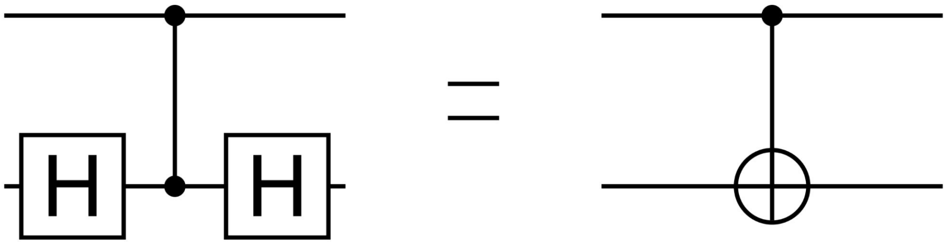
$$
\begin{aligned}
|10\rangle &\to |1\rangle\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\
&= \frac{1}{\sqrt{2}}(|1\rangle|0\rangle + |1\rangle|1\rangle) \\
&= \frac{1}{\sqrt{2}}(|10\rangle + |11\rangle) \\
&\to \frac{1}{\sqrt{2}}(|10\rangle - |11\rangle) \\
&\to \frac{1}{\sqrt{2}}\left(|1\rangle\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) - |1\rangle\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right) \\
&= |11\rangle
\end{aligned}
$$

Test your understanding



$$|00\rangle \to |00\rangle$$
$$|01\rangle \to |01\rangle$$
$$|10\rangle \to |11\rangle$$
$$|11\rangle \to |10\rangle$$

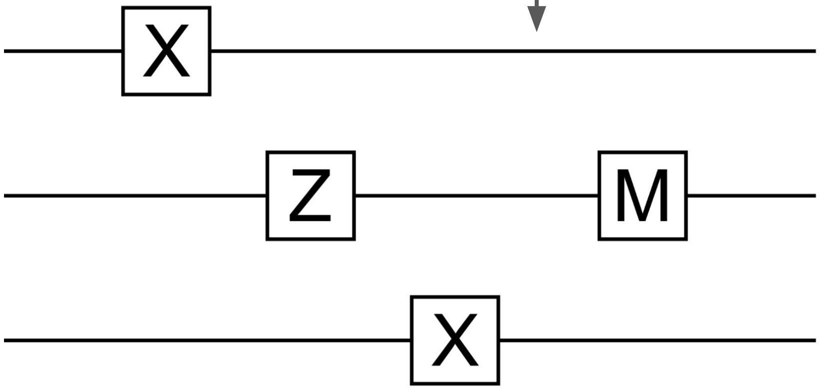
Test your understanding



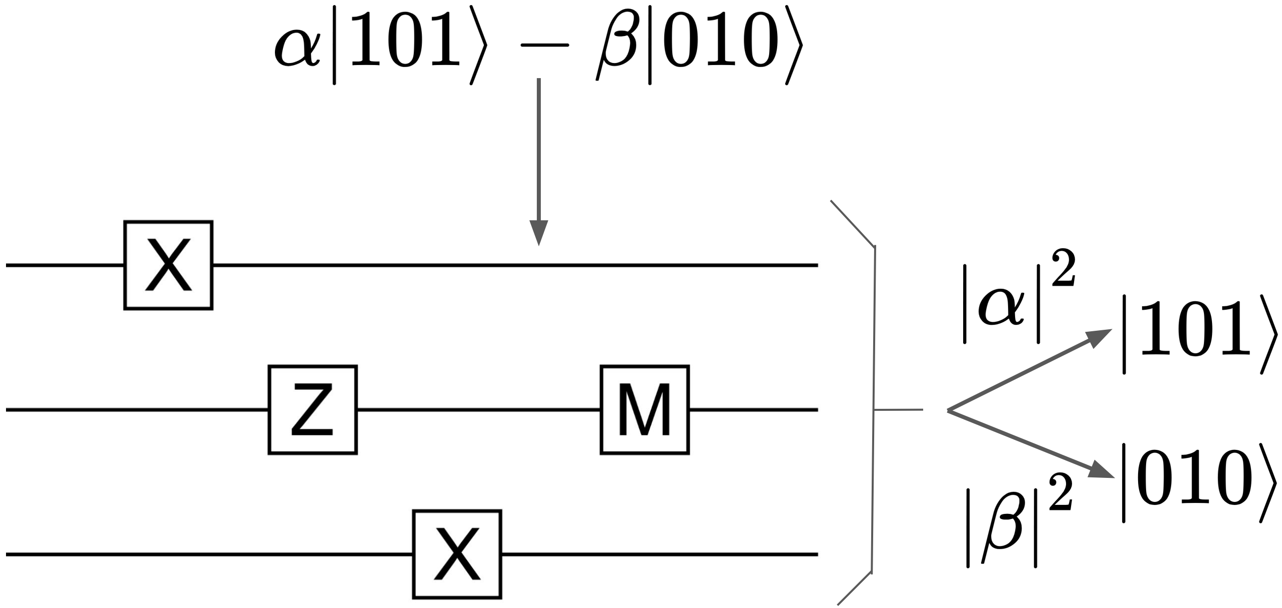
$$|00\rangle \rightarrow |00\rangle$$
$$|01\rangle \rightarrow |01\rangle$$
$$|10\rangle \rightarrow |11\rangle$$
$$|11\rangle \rightarrow |10\rangle$$

Measurement

$$\alpha|101\rangle - \beta|010\rangle$$



Measurement



Entire system collapses to one state or the other with probabilities $|\alpha|^2$, $|\beta|^2$