

### 18.S997 (FALL 2017) PROBLEM SET 1

1. (a) Let  $s$  and  $r$  be positive integers. Show that there is some integer  $n = n(s, r)$  so that if every edge of the complete graph  $K_n$  on  $n$  vertices is colored with one of  $r$  colors, then there is a monochromatic copy of  $K_s$ .  
(b) Let  $s \geq 3$  be a positive integer. Show that if the edges of the complete graph on  $\binom{2s-2}{s-1}$  vertices are colored with 2 colors, then there is a monochromatic copy of  $K_s$ .
2. Show that a graph with  $n$  vertices and  $m$  edges has at least

$$\frac{4m}{3n} \left( m - \frac{n^2}{4} \right)$$

many triangles.

3. Let  $S$  be a set of  $n$  points in the plane, with the property that no two points are at distance greater than 1. Show that  $S$  has at most  $\lfloor n^2/3 \rfloor$  pairs of points at distance greater than  $1/\sqrt{2}$ . Also, show that the bound  $\lfloor n^2/3 \rfloor$  is tight (i.e., cannot be improved).
4. Show that for every  $r \geq 1$  and  $\epsilon > 0$ , there is some  $c > 0$  so that any graph with at least  $(1 - \frac{1}{r} + \epsilon) \frac{n^2}{2}$  edges contains at least  $cn^{r+1}$  copies of  $K_{r+1}$ .
5. Show that for every  $\epsilon > 0$ , there exists  $\delta > 0$  such that for all sufficiently large  $n$ , every  $K_4$ -free with  $n$  vertices and at least  $(\frac{1}{3} - \delta)n^2$  edges contains 3 disjoint independent sets each of size at least  $(1 - \epsilon)n/3$ .

*... to be continued ... check back later (last updated: September 14, 2017)*