

18.S997 (FALL 2017) PROBLEM SET 2

1. Let the *half-graph* H_n be the bipartite graph on $2n$ vertices $\{a_1, \dots, a_n, b_1, \dots, b_n\}$ with edges $\{a_i b_j : i \leq j\}$.
 - (a) For every $\epsilon > 0$, explicitly construct an ϵ -regular partition of H_n into $O(1/\epsilon)$ parts.
 - (b) Show that there is some $c > 0$ such that for every sufficiently small $\epsilon > 0$, every integer k and sufficiently large multiple n of k , every partition of the vertices of H_n into k equal-sized parts contains at least ck pairs of parts which are not ϵ -regular.
2. Show that there is some absolute constant $C > 0$ such that for every $\epsilon > 0$, every graph on n vertices contains an ϵ -regular pair of vertex subsets each with size at least δn , where $\delta = 2^{-\epsilon^{-C}}$.
3. Show that for every $\epsilon > 0$ there exists $\delta > 0$ such that every graph on n vertices contains an ϵ -regular subset of vertices of size at least δn . (Here a vertex subset X is called an *ϵ -regular set* if the pair (X, X) is ϵ -regular, i.e., for all $A, B \subset X$ with $|A|, |B| \geq \epsilon|X|$, one has $|d(A, B) - d(X, X)| \leq \epsilon$.)
4. Show that for every $\epsilon > 0$ there exists $\delta > 0$ such that if $A \subset [n]$ has fewer than δn^2 many triples $(x, y, z) \in A^3$ with $x + y = z$, then there is some $B \subset A$ with $|A \setminus B| \leq \epsilon n$ such that B is sum-free, i.e., there do not exist $x, y, z \in B$ with $x + y = z$.

... to be continued ... check back later (last updated: September 28, 2017)