

References

- M. Ajtai and E. Szemerédi, *Sets of lattice points that form no squares*, *Studia Sci. Math. Hungar.* **9** (1974), 9–11 (1975). MR369299 Cited on page 78.
- M. Ajtai, V. Chvátal, M. M. Newborn, and E. Szemerédi, *Crossing-free subgraphs*, *Theory and practice of combinatorics*, North-Holland, 1982, pp. 9–12. MR806962 Cited on page 316.
- N. Alon and V. D. Milman, λ_1 , *isoperimetric inequalities for graphs, and superconcentrators*, *J. Combin. Theory Ser. B* **38** (1985), 73–88. MR782626 doi:10.1016/0095-8956(85)90092-9 Cited on page 121.
- Noga Alon, *Eigenvalues and expanders*, *Combinatorica* **6** (1986), 83–96. MR875835 doi:10.1007/BF02579166 Cited on pages 121 and 141.
- Noga Alon and Assaf Naor, *Approximating the cut-norm via Grothendieck’s inequality*, *SIAM J. Comput.* **35** (2006), 787–803. MR2203567 doi:10.1137/S0097539704441629 Cited on page 139.
- Noga Alon and Asaf Shapira, *A characterization of the (natural) graph properties testable with one-sided error*, *SIAM J. Comput.* **37** (2008), 1703–1727. MR2386211 doi:10.1137/06064888X Cited on page 94.
- Noga Alon and Joel H. Spencer, *The probabilistic method*, fourth ed., Wiley, 2016. MR3524748 Cited on pages 21, 165, 315, and 330.
- Noga Alon, Lajos Rónyai, and Tibor Szabó, *Norm-graphs: variations and applications*, *J. Combin. Theory Ser. B* **76** (1999), 280–290. MR1699238 doi:10.1006/jctb.1999.1906 Cited on page 88.
- Noga Alon, Eldar Fischer, Michael Krivelevich, and Mario Szegedy, *Efficient testing of large graphs*, *Combinatorica* **20** (2000), 451–476. MR1804820 doi:10.1007/s004930070001 Cited on page 88.
- Noga Alon, W. Fernandez de la Vega, Ravi Kannan, and Marek Karpinski, *Random sampling and approximation of MAX-CSPs*, vol. 67, 2003a, Special issue on STOC2002 (Montreal, QC), pp. 212–243. MR2022830 doi:10.1016/S0022-0000(03)00008-4 Cited on page 172.
- Noga Alon, Michael Krivelevich, and Benny Sudakov, *Turán numbers of bipartite graphs and related Ramsey-type questions*, vol. 12, 2003b, Special issue on Ramsey theory, pp. 477–494. MR2037065 doi:10.1017/S0963548303005741 Cited on page 39.
- Emil Artin, *Über die Zerlegung definiter Funktionen in Quadrate*, *Abh. Math. Sem. Univ. Hamburg* **5** (1927), 100–115. MR3069468 doi:10.1007/BF02952513 Cited on page 203.
- F. V. Atkinson, G. A. Watterson, and P. A. P. Moran, *A matrix inequality*, *Quart. J. Math. Oxford Ser.* **11** (1960), 137–140. MR118731 doi:10.1093/qmath/11.1.137 Cited on page 205.
- László Babai and Péter Frankl, *Linear algebra methods in combinatorics*, 2020, book draft <http://people.cs.uchicago.edu/~laci/CLASS/HANDOUTS-COMB/BaFrNew.pdf>. Cited on page 270.

- R. C. Baker, G. Harman, and J. Pintz, *The difference between consecutive primes. II*, Proc. Lond. Math. Soc. **83** (2001), 532–562. MR1851081 doi:10.1112/plms/83.3.532 Cited on page 47.
- Antal Balog and Endre Szemerédi, *A statistical theorem of set addition*, Combinatorica **14** (1994), 263–268. MR1305895 doi:10.1007/BF01212974 Cited on page 304.
- József Balogh, Robert Morris, and Wojciech Samotij, *Independent sets in hypergraphs*, J. Amer. Math. Soc. **28** (2015), 669–709. MR3327533 doi:10.1090/S0894-0347-2014-00816-X Cited on page 330.
- József Balogh, Ping Hu, Bernard Lidický, and Florian Pfender, *Maximum density of induced 5-cycle is achieved by an iterated blow-up of 5-cycle*, European J. Combin. **52** (2016), 47–58. MR3425964 doi:10.1016/j.ejc.2015.08.006 Cited on page 203.
- József Balogh, Robert Morris, and Wojciech Samotij, *The method of hypergraph containers*, Proceedings of the International Congress of Mathematicians—Rio de Janeiro 2018. Vol. IV. Invited lectures, World Scientific Publishing, 2018, pp. 3059–3092. MR3966523 Cited on page 330.
- Michael Bateman and Nets Hawk Katz, *New bounds on cap sets*, J. Amer. Math. Soc. **25** (2012), 585–613. MR2869028 doi:10.1090/S0894-0347-2011-00725-X Cited on page 244.
- F. A. Behrend, *On sets of integers which contain no three terms in arithmetical progression*, Proc. Natl. Acad. Sci. USA **32** (1946), 331–332. MR18694 doi:10.1073/pnas.32.12.331 Cited on pages 8 and 80.
- Clark T. Benson, *Minimal regular graphs of girths eight and twelve*, Canadian J. Math. **18** (1966), 1091–1094. MR197342 doi:10.4153/CJM-1966-109-8 Cited on page 51.
- V. Bergelson and A. Leibman, *Polynomial extensions of van der Waerden’s and Szemerédi’s theorems*, J. Amer. Math. Soc. **9** (1996), 725–753. MR1325795 doi:10.1090/S0894-0347-96-00194-4 Cited on page 9.
- Vitaly Bergelson, Bernard Host, and Bryna Kra, *Multiple recurrence and nilsequences*, Invent. Math. **160** (2005), 261–303, With an appendix by Imre Ruzsa. MR2138068 doi:10.1007/s00222-004-0428-6 Cited on page 269.
- Yonatan Bilu and Nathan Linial, *Lifts, discrepancy and nearly optimal spectral gap*, Combinatorica **26** (2006), 495–519. MR2279667 doi:10.1007/s00493-006-0029-7 Cited on page 121.
- G. R. Blakley and Prabir Roy, *A Hölder type inequality for symmetric matrices with nonnegative entries*, Proc. Amer. Math. Soc. **16** (1965), 1244–1245. MR184950 doi:10.2307/2035908 Cited on page 205.
- Jonah Blasiak, Thomas Church, Henry Cohn, Joshua A. Grochow, Eric Naslund, William F. Sawin, and Chris Umans, *On cap sets and the group-theoretic approach to matrix multiplication*, Discrete Anal. (2017), Paper No. 3, 27. MR3631613 doi:10.19086/da.1245 Cited on page 259.
- H. F. Blichfeldt, *A new principle in the geometry of numbers, with some applications*, Trans. Amer. Math. Soc. **15** (1914), 227–235. MR1500976 doi:10.2307/1988585 Cited on page 295.
- Thomas F. Bloom and Olof Sisask, *Breaking the logarithmic barrier in Roth’s theorem on arithmetic progressions*, 2020. arXiv:2007.03528 Cited on pages 7, 8, 80, 255, and 324.
- N. Bogolyubov, *Sur quelques propriétés arithmétiques des presque-périodes*, Ann. Chaire Phys. Math. Kiev **4** (1939), 185–205. MR20164 Cited on page 288.

- Béla Bollobás, *Relations between sets of complete subgraphs*, Proceedings of the Fifth British Combinatorial Conference (Univ. Aberdeen, Aberdeen, 1975), 1976, pp. 79–84. MR0396327 Cited on page 216.
- Béla Bollobás, *Modern graph theory*, Springer-Verlag, 1998. MR1633290 doi:10.1007/978-1-4612-0619-4 Cited on page 59.
- J. A. Bondy and U. S. R. Murty, *Graph theory*, Springer, 2008. MR2368647 doi:10.1007/978-1-84628-970-5 Cited on page 59.
- J. A. Bondy and M. Simonovits, *Cycles of even length in graphs*, J. Combin. Theory Ser. B **16** (1974), 97–105. MR340095 doi:10.1016/0095-8956(74)90052-5 Cited on page 37.
- C. Borgs, J. T. Chayes, L. Lovász, V. T. Sós, and K. Vesztegombi, *Convergent sequences of dense graphs. I. Subgraph frequencies, metric properties and testing*, Adv. Math. **219** (2008), 1801–1851. MR2455626 doi:10.1016/j.aim.2008.07.008 Cited on pages 162 and 185.
- J. Bourgain, *On triples in arithmetic progression*, Geom. Funct. Anal. **9** (1999), 968–984. MR1726234 doi:10.1007/s000390050105 Cited on page 255.
- J. Bourgain, N. Katz, and T. Tao, *A sum-product estimate in finite fields, and applications*, Geom. Funct. Anal. **14** (2004), 27–57. MR2053599 doi:10.1007/s00039-004-0451-1 Cited on page 322.
- J. Bourgain, A. A. Glibichuk, and S. V. Konyagin, *Estimates for the number of sums and products and for exponential sums in fields of prime order*, J. Lond. Math. Soc. **73** (2006), 380–398. MR2225493 doi:10.1112/S0024610706022721 Cited on page 322.
- W. G. Brown, *On graphs that do not contain a Thomsen graph*, Canad. Math. Bull. **9** (1966), 281–285. MR200182 doi:10.4153/CMB-1966-036-2 Cited on pages 46 and 47.
- W. G. Brown, P. Erdős, and V. T. Sós, *Some extremal problems on r -graphs*, New directions in the theory of graphs (Proc. Third Ann Arbor Conf., Univ. Michigan, Ann Arbor, Mich, 1971), 1973, pp. 53–63. MR0351888 Cited on page 77.
- Boris Bukh, *Random algebraic construction of extremal graphs*, Bull. Lond. Math. Soc. **47** (2015), 939–945. MR3431574 doi:10.1112/blms/bdv062 Cited on pages 53 and 57.
- Boris Bukh, *Extremal graphs without exponentially-small bicliques*, 2021. arXiv:2107.04167 Cited on page 53.
- Mei-Chu Chang, *A polynomial bound in Freiman’s theorem*, Duke Math. J. **113** (2002), 399–419. MR1909605 doi:10.1215/S0012-7094-02-11331-3 Cited on page 274.
- Sourav Chatterjee, *An introduction to large deviations for random graphs*, Bull. Amer. Math. Soc. **53** (2016), 617–642. MR3544262 doi:10.1090/bull/1539 Cited on page 186.
- Sourav Chatterjee, *Large deviations for random graphs*, Springer, 2017, Lecture notes from the 45th Probability Summer School held in Saint-Flour, June 2015, École d’Été de Probabilités de Saint-Flour. [Saint-Flour Probability Summer School]. MR3700183 doi:10.1007/978-3-319-65816-2 Cited on page 186.

- Sourav Chatterjee and S. R. S. Varadhan, *The large deviation principle for the Erdős-Rényi random graph*, European J. Combin. **32** (2011), 1000–1017. MR2825532 doi:10.1016/j.ejc.2011.03.014 Cited on page 186.
- Jeff Cheeger, *A lower bound for the smallest eigenvalue of the Laplacian*, Problems in analysis (Papers dedicated to Salomon Bochner, 1969), 1970, pp. 195–199. MR0402831 Cited on page 121.
- F. R. K. Chung, R. L. Graham, P. Frankl, and J. B. Shearer, *Some intersection theorems for ordered sets and graphs*, J. Combin. Theory Ser. A **43** (1986), 23–37. MR859293 doi:10.1016/0097-3165(86)90019-1 Cited on page 227.
- F. R. K. Chung, R. L. Graham, and R. M. Wilson, *Quasi-random graphs*, Combinatorica **9** (1989), 345–362. MR1054011 doi:10.1007/BF02125347 Cited on pages 106, 115, and 328.
- Fan R. K. Chung, *Spectral graph theory*, American Mathematical Society, 1997. MR1421568 Cited on page 149.
- D. Conlon and W. T. Gowers, *Combinatorial theorems in sparse random sets*, Ann. of Math. **184** (2016), 367–454. MR3548529 doi:10.4007/annals.2016.184.2.2 Cited on page 330.
- David Conlon, *Extremal numbers of cycles revisited*, Amer. Math. Monthly **128** (2021), 464–466. MR4249723 doi:10.1080/00029890.2021.1886845 Cited on page 51.
- David Conlon and Jacob Fox, *Graph removal lemmas*, Surveys in combinatorics 2013, Cambridge University Press, 2013, pp. 1–49. MR3156927 Cited on page 103.
- David Conlon and Yufei Zhao, *Quasirandom Cayley graphs*, Discrete Anal. (2017), Paper No. 6, 14. MR3631610 doi:10.19086/da.1294 Cited on page 138.
- David Conlon, Jacob Fox, and Benny Sudakov, *An approximate version of Sidorenko’s conjecture*, Geom. Funct. Anal. **20** (2010), 1354–1366. MR2738996 doi:10.1007/s00039-010-0097-0 Cited on pages 115, 190, and 225.
- David Conlon, Jacob Fox, and Yufei Zhao, *The Green-Tao theorem: an exposition*, EMS Surv. Math. Sci. **1** (2014), 249–282. MR3285854 doi:10.4171/EMSS/6 Cited on pages 323, 325, and 350.
- David Conlon, Jacob Fox, and Yufei Zhao, *A relative Szemerédi theorem*, Geom. Funct. Anal. **25** (2015), 733–762. MR3361771 doi:10.1007/s00039-015-0324-9 Cited on pages 323, 329, 338, and 340.
- David Conlon, Jeong Han Kim, Choongbum Lee, and Joonkyung Lee, *Some advances on Sidorenko’s conjecture*, J. Lond. Math. Soc. **98** (2018), 593–608. MR3893193 doi:10.1112/jlms.12142 Cited on page 222.
- Don Coppersmith and Shmuel Winograd, *Matrix multiplication via arithmetic progressions*, J. Symbolic Comput. **9** (1990), 251–280. MR1056627 doi:10.1016/S0747-7171(08)80013-2 Cited on page 80.
- Ernie Croot, Vsevolod F. Lev, and Péter Pál Pach, *Progression-free sets in \mathbb{Z}_4^n are exponentially small*, Ann. of Math. **185** (2017), 331–337. MR3583357 doi:10.4007/annals.2017.185.1.7 Cited on pages 244 and 255.
- Giuliana Davidoff, Peter Sarnak, and Alain Valette, *Elementary number theory, group theory, and Ramanujan graphs*, Cambridge University Press, 2003. MR1989434 doi:10.1017/CBO9780511615825 Cited on pages 147 and 149.

- L. E. Dickson, *On the congruence $x^n + y^n + z^n \equiv 0 \pmod{p}$* , J. Reine Angew. Math. **135** (1909), 134–141. MR1580764 doi:10.1515/crll.1909.135.134 Cited on page 1.
- Reinhard Diestel, *Graph theory*, fifth ed., Springer, 2017. MR3644391 doi:10.1007/978-3-662-53622-3 Cited on page 59.
- Jozef Dodziuk, *Difference equations, isoperimetric inequality and transience of certain random walks*, Trans. Amer. Math. Soc. **284** (1984), 787–794. MR743744 doi:10.2307/1999107 Cited on page 121.
- Zeev Dvir, *Incidence theorems and their applications*, Found. Trends Theor. Comput. Sci. **6** (2012), 257–393. MR3004132 doi:10.1561/04000000056 Cited on page 322.
- Yves Edel, *Extensions of generalized product caps*, Des. Codes Cryptogr. **31** (2004), 5–14. MR2031694 doi:10.1023/A:1027365901231 Cited on page 244.
- György Elekes, *On the number of sums and products*, Acta Arith. **81** (1997), 365–367. MR1472816 doi:10.4064/aa-81-4-365-367 Cited on pages 314 and 315.
- Michael Elkin, *An improved construction of progression-free sets*, Israel J. Math. **184** (2011), 93–128. MR2823971 doi:10.1007/s11856-011-0061-1 Cited on page 80.
- Jordan S. Ellenberg and Dion Gijswijt, *On large subsets of \mathbb{F}_q^n with no three-term arithmetic progression*, Ann. of Math. **185** (2017), 339–343. MR3583358 doi:10.4007/annals.2017.185.1.8 Cited on pages 244 and 255.
- P. Erdős, *On some extremal problems on r -graphs*, Discrete Math. **1** (1971), 1–6. MR297602 doi:10.1016/0012-365X(71)90002-1 Cited on page 32.
- P. Erdős and M. Simonovits, *A limit theorem in graph theory*, Studia Sci. Math. Hungar. **1** (1966), 51–57. MR205876 Cited on page 32.
- P. Erdős and E. Szemerédi, *On sums and products of integers*, Studies in pure mathematics, Birkhäuser, 1983, pp. 213–218. MR820223 Cited on page 313.
- P. Erdős, A. Rényi, and V. T. Sós, *On a problem of graph theory*, Studia Sci. Math. Hungar. **1** (1966), 215–235. MR223262 Cited on page 46.
- Paul Erdős, *On some problems in graph theory, combinatorial analysis and combinatorial number theory*, Graph theory and combinatorics (Cambridge, 1983), Academic Press, 1984, pp. 1–17. MR777160 Cited on page 202.
- P. Erdős, *On sets of distances of n points*, Amer. Math. Monthly **53** (1946), 248–250. MR15796 doi:10.2307/2305092 Cited on pages 28 and 30.
- P. Erdős and A. H. Stone, *On the structure of linear graphs*, Bull. Amer. Math. Soc. **52** (1946), 1087–1091. MR18807 doi:10.1090/S0002-9904-1946-08715-7 Cited on page 32.
- Paul Erdős, *Some remarks on number theory*, Riveon Lematematika **9** (1955), 45–48. MR73619 Cited on page 314.
- Paul Erdős and Paul Turán, *On Some Sequences of Integers*, J. Lond. Math. Soc. **11** (1936), 261–264. MR1574918 doi:10.1112/jlms/s1-11.4.261 Cited on page 6.

- Chaim Even-Zohar, *On sums of generating sets in \mathbb{Z}_2^n* , *Combin. Probab. Comput.* **21** (2012), 916–941. MR2981161 doi:10.1017/S0963548312000351 Cited on page 282.
- Helmut Finner, *A generalization of Hölder’s inequality and some probability inequalities*, *Ann. Probab.* **20** (1992), 1893–1901. MR1188047 Cited on pages 207 and 209.
- Kevin Ford, *The distribution of integers with a divisor in a given interval*, *Ann. of Math.* **168** (2008), 367–433. MR2434882 doi:10.4007/annals.2008.168.367 Cited on page 314.
- Jacob Fox, *A new proof of the graph removal lemma*, *Ann. of Math.* **174** (2011), 561–579. MR2811609 doi:10.4007/annals.2011.174.1.17 Cited on page 76.
- Jacob Fox and Huy Tuan Pham, *Popular progression differences in vector spaces II*, *Discrete Anal.* (2019), Paper No. 16, 39. MR4042159 doi:10.19086/da Cited on page 268.
- Jacob Fox and Benny Sudakov, *Dependent random choice*, *Random Structures Algorithms* **38** (2011), 68–99. MR2768884 doi:10.1002/rsa.20344 Cited on pages 39 and 59.
- Jacob Fox and Yufei Zhao, *A short proof of the multidimensional Szemerédi theorem in the primes*, *Amer. J. Math.* **137** (2015), 1139–1145. MR3372317 doi:10.1353/ajm.2015.0028 Cited on page 10.
- Jacob Fox, Huy Tuan Pham, and Yufei Zhao, *Tower-type bounds for Roth’s theorem with popular differences*, *J. Eur. Math. Soc. (JEMS)* (2022). Cited on page 269.
- Peter Frankl and Vojtěch Rödl, *Extremal problems on set systems*, *Random Structures Algorithms* **20** (2002), 131–164. MR1884430 doi:10.1002/rsa.10017.abs Cited on page 98.
- G. A. Freiman, *Foundations of a structural theory of set addition*, American Mathematical Society, Providence, R.I., 1973, Translated from the Russian. MR0360496 Cited on page 274.
- Ehud Friedgut, *Hypergraphs, entropy, and inequalities*, *Amer. Math. Monthly* **111** (2004), 749–760. MR2104047 doi:10.2307/4145187 Cited on page 227.
- Joel Friedman, *A proof of Alon’s second eigenvalue conjecture and related problems*, *Mem. Amer. Math. Soc.* **195** (2008), viii+100. MR2437174 doi:10.1090/memo/0910 Cited on page 146.
- Alan Frieze and Ravi Kannan, *Quick approximation to matrices and applications*, *Combinatorica* **19** (1999), 175–220. MR1723039 doi:10.1007/s004930050052 Cited on pages 170 and 172.
- William Fulton and Joe Harris, *Representation theory*, Springer-Verlag, 1991, A first course, Readings in Mathematics. MR1153249 doi:10.1007/978-1-4612-0979-9 Cited on page 133.
- Zoltán Füredi, *On a Turán type problem of Erdős*, *Combinatorica* **11** (1991), 75–79. MR1112277 doi:10.1007/BF01375476 Cited on page 39.
- Zoltan Füredi and David S. Gunderson, *Extremal numbers for odd cycles*, *Combin. Probab. Comput.* **24** (2015), 641–645. MR3350026 doi:10.1017/S0963548314000601 Cited on page 36.
- Zoltán Füredi and Miklós Simonovits, *The history of degenerate (bipartite) extremal graph problems*, *Erdős centennial*, János Bolyai Mathematical Society, 2013, pp. 169–264. MR3203598 doi:10.1007/978-3-642-39286-3_7 Cited on page 59.

- H. Furstenberg, *Ergodic behavior of diagonal measures and a theorem of Szemerédi on arithmetic progressions*, J. Analyse Math. **31** (1977), 204–256. MR0498471 Cited on pages 7 and 9.
- H. Furstenberg and Y. Katznelson, *An ergodic Szemerédi theorem for commuting transformations*, J. Analyse Math. **34** (1978), 275–291. MR531279 doi:10.1007/BF02790016 Cited on pages 7 and 9.
- David Galvin, *Three tutorial lectures on entropy and counting*, 2014. arXiv:1406.7872 Cited on page 231.
- David Galvin and Prasad Tetali, *On weighted graph homomorphisms*, Graphs, morphisms and statistical physics, American Mathematical Society, 2004, pp. 97–104. MR2056231 doi:10.1090/dimacs/063/07 Cited on pages 210, 212, and 228.
- Michel X. Goemans and David P. Williamson, *Improved approximation algorithms for maximum cut and satisfiability problems using semidefinite programming*, J. Assoc. Comput. Mach. **42** (1995), 1115–1145. MR1412228 doi:10.1145/227683.227684 Cited on page 172.
- A. W. Goodman, *On sets of acquaintances and strangers at any party*, Amer. Math. Monthly **66** (1959), 778–783. MR107610 doi:10.2307/2310464 Cited on pages 198 and 199.
- W. T. Gowers, *Lower bounds of tower type for Szemerédi’s uniformity lemma*, Geom. Funct. Anal. **7** (1997), 322–337. MR1445389 doi:10.1007/PL00001621 Cited on pages 69, 263, and 269.
- W. T. Gowers, *A new proof of Szemerédi’s theorem for arithmetic progressions of length four*, Geom. Funct. Anal. **8** (1998), 529–551. MR1631259 doi:10.1007/s000390050065 Cited on page 311.
- W. T. Gowers, *Additive and combinatorial number theory*, 1998b, online lecture notes written by Jacques Verstraëte based on a course given by W. T. Gowers, <https://www.dpmms.cam.ac.uk/~wtg10/>. Cited on page 311.
- W. T. Gowers, *A new proof of Szemerédi’s theorem*, Geom. Funct. Anal. **11** (2001), 465–588. MR1844079 doi:10.1007/s00039-001-0332-9 Cited on pages 7, 8, 245, 271, 275, 304, and 324.
- W. T. Gowers, *Quasirandomness, counting and regularity for 3-uniform hypergraphs*, Combin. Probab. Comput. **15** (2006), 143–184. MR2195580 doi:10.1017/S0963548305007236 Cited on pages 102 and 103.
- W. T. Gowers, *Hypergraph regularity and the multidimensional Szemerédi theorem*, Ann. of Math. **166** (2007), 897–946. MR2373376 doi:10.4007/annals.2007.166.897 Cited on pages 98 and 102.
- W. T. Gowers, *Quasirandom groups*, Combin. Probab. Comput. **17** (2008), 363–387. MR2410393 doi:10.1017/S0963548307008826 Cited on pages 128, 129, 133, 134, and 135.
- W. T. Gowers, *Decompositions, approximate structure, transference, and the Hahn-Banach theorem*, Bull. Lond. Math. Soc. **42** (2010), 573–606. MR2669681 doi:10.1112/blms/bdq018 Cited on page 338.
- W. T. Gowers, *A new way of proving sumset estimates*, 2011, blog post <https://gowers.wordpress.com/2011/02/10/>. Cited on page 277.
- Ronald L. Graham, Bruce L. Rothschild, and Joel H. Spencer, *Ramsey theory*, second ed., Wiley, 1990, A Wiley-Interscience Publication. MR1044995 Cited on page 12.
- B. Green, *A Szemerédi-type regularity lemma in abelian groups, with applications*, Geom. Funct. Anal. **15** (2005), 340–376. MR2153903 doi:10.1007/s00039-005-0509-8 Cited on pages 261, 263, 266, and 268.

- Ben Green, *Roth's theorem in the primes*, Ann. of Math. (2) **161** (2005), 1609–1636. MR2180408 doi:10.4007/annals.2005.161.1609 Cited on page 330.
- Ben Green, *Finite field models in additive combinatorics*, Surveys in combinatorics 2005, Cambridge University Press, 2005c, pp. 1–27. MR2187732 doi:10.1017/CBO9780511734885.002 Cited on pages 270 and 300.
- Ben Green, *Long arithmetic progressions of primes*, Analytic Number Theory: A Tribute to Gauss and Dirichlet, American Mathematical Society, 2007, pp. 149–167. MR2362199 Cited on pages 323 and 350.
- Ben Green, *Additive combinatorics (book review)*, Bull. Amer. Math. Soc. **46** (2009), 489–497. MR2507281 doi:10.1090/S0273-0979-09-01231-2 Cited on page 12.
- Ben Green, *Additive combinatorics*, 2009b, lecture notes <http://people.maths.ox.ac.uk/greenbj/notes.html>. Cited on pages 270 and 311.
- Ben Green, *Approximate algebraic structure*, Proceedings of the International Congress of Mathematicians—Seoul 2014. Vol. 1, Kyung Moon Sa, 2014, pp. 341–367. MR3728475 Cited on page 350.
- Ben Green and Imre Z. Ruzsa, *Freiman's theorem in an arbitrary abelian group*, J. Lond. Math. Soc. **75** (2007), 163–175. MR2302736 doi:10.1112/jlms/jdl021 Cited on page 275.
- Ben Green and Terence Tao, *The primes contain arbitrarily long arithmetic progressions*, Ann. of Math. **167** (2008), 481–547. MR2415379 doi:10.4007/annals.2008.167.481 Cited on pages 9, 323, 329, 338, and 350.
- Ben Green and Terence Tao, *Linear equations in primes*, Ann. of Math. **171** (2010), 1753–1850. MR2680398 doi:10.4007/annals.2010.171.1753 Cited on page 350.
- Ben Green and Terence Tao, *An equivalence between inverse sumset theorems and inverse conjectures for the U^3 norm*, Math. Proc. Cambridge Philos. Soc. **149** (2010), 1–19. MR2651575 doi:10.1017/S0305004110000186 Cited on page 301.
- Ben Green and Terence Tao, *An arithmetic regularity lemma, an associated counting lemma, and applications*, An irregular mind, János Bolyai Mathematical Society, 2010c, pp. 261–334. MR2815606 doi:10.1007/978-3-642-14444-8_7 Cited on page 269.
- Ben Green and Terence Tao, *New bounds for Szemerédi's theorem, III: a polylogarithmic bound for $r_4(N)$* , Mathematika **63** (2017), 944–1040. MR3731312 doi:10.1112/S0025579317000316 Cited on page 7.
- Ben Green and Julia Wolf, *A note on Elkin's improvement of Behrend's construction*, Additive number theory, Springer, 2010, pp. 141–144. MR2744752 doi:10.1007/978-0-387-68361-4_9 Cited on page 80.
- Ben Green, Terence Tao, and Tamar Ziegler, *An inverse theorem for the Gowers $U^{s+1}[N]$ -norm*, Ann. of Math. **176** (2012), 1231–1372. MR2950773 doi:10.4007/annals.2012.176.2.11 Cited on page 350.
- A. Grothendieck, *Résumé de la théorie métrique des produits tensoriels topologiques*, Bol. Soc. Mat. São Paulo **8** (1953), 1–79. MR94682 Cited on page 138.
- Andrzej Grzesik, *On the maximum number of five-cycles in a triangle-free graph*, J. Combin. Theory Ser. B **102** (2012), 1061–1066. MR2959390 doi:10.1016/j.jctb.2012.04.001 Cited on page 202.

- Larry Guth, *Polynomial methods in combinatorics*, American Mathematical Society, 2016. MR3495952 doi:10.1090/ulect/064 Cited on pages 270 and 322.
- Larry Guth and Nets Hawk Katz, *On the Erdős distinct distances problem in the plane*, Ann. of Math. **181** (2015), 155–190. MR3272924 doi:10.4007/annals.2015.181.1.2 Cited on pages 30 and 322.
- G. H. Hardy and S. Ramanujan, *The normal number of prime factors of a number n* [*Quart. J. Math.* **48** (1917), 76–92], Collected papers of Srinivasa Ramanujan, AMS Chelsea Publishing, 2000, pp. 262–275. MR2280878 Cited on page 315.
- Johan Håstad, *Some optimal inapproximability results*, J. ACM **48** (2001), 798–859. MR2144931 doi:10.1145/502090.502098 Cited on page 172.
- Hamed Hatami and Serguei Norine, *Undecidability of linear inequalities in graph homomorphism densities*, J. Amer. Math. Soc. **24** (2011), 547–565. MR2748400 doi:10.1090/S0894-0347-2010-00687-X Cited on pages 187 and 204.
- Hamed Hatami, Jan Hladký, Daniel Král', Serguei Norine, and Alexander Razborov, *On the number of pentagons in triangle-free graphs*, J. Combin. Theory Ser. A **120** (2013), 722–732. MR3007147 doi:10.1016/j.jcta.2012.12.008 Cited on page 202.
- David Hilbert, *Ueber die Darstellung definiter Formen als Summe von Formenquadraten*, Math. Ann. **32** (1888), 342–350. MR1510517 doi:10.1007/BF01443605 Cited on page 203.
- David Hilbert, *Über ternäre definite Formen*, Acta Math. **17** (1893), 169–197. MR1554835 doi:10.1007/BF02391990 Cited on page 203.
- Shlomo Hoory, Nathan Linial, and Avi Wigderson, *Expander graphs and their applications*, Bull. Amer. Math. Soc. **43** (2006), 439–561. MR2247919 doi:10.1090/S0273-0979-06-01126-8 Cited on page 149.
- Kaave Hosseini, Shachar Lovett, Guy Moshkovitz, and Asaf Shapira, *An improved lower bound for arithmetic regularity*, Math. Proc. Cambridge Philos. Soc. **161** (2016), 193–197. MR3530502 doi:10.1017/S030500411600013X Cited on page 263.
- Kenneth Ireland and Michael Rosen, *A classical introduction to modern number theory*, second ed., Springer-Verlag, 1990. MR1070716 doi:10.1007/978-1-4757-2103-4 Cited on page 128.
- Herbert E. Jordan, *Group-Characters of Various Types of Linear Groups*, Amer. J. Math. **29** (1907), 387–405. MR1506021 doi:10.2307/2370015 Cited on page 133.
- Jeff Kahn, *An entropy approach to the hard-core model on bipartite graphs*, Combin. Probab. Comput. **10** (2001), 219–237. MR1841642 doi:10.1017/S0963548301004631 Cited on pages 210, 212, and 228.
- G. Katona, *A theorem of finite sets*, Theory of graphs (Proc. Colloq., Tihany, 1966), 1968, pp. 187–207. MR0290982 Cited on page 193.
- Kiran S. Kedlaya, *Large product-free subsets of finite groups*, J. Combin. Theory Ser. A **77** (1997), 339–343. MR1429085 doi:10.1006/jcta.1997.2715 Cited on page 134.
- Kiran S. Kedlaya, *Product-free subsets of groups*, Amer. Math. Monthly **105** (1998), 900–906. MR1656927 doi:10.2307/2589282 Cited on page 134.

- Peter Keevash, *Hypergraph Turán problems*, Surveys in combinatorics 2011, Cambridge University Press, 2011, pp. 83–139. MR2866732 Cited on page 59.
- Subhash Khot, Guy Kindler, Elchanan Mossel, and Ryan O'Donnell, *Optimal inapproximability results for MAX-CUT and other 2-variable CSPs?*, SIAM J. Comput. **37** (2007), 319–357. MR2306295 doi:10.1137/S0097539705447372 Cited on page 172.
- Robert Kleinberg, David E. Speyer, and Will Sawin, *The growth of tri-colored sum-free sets*, Discrete Anal. (2018), Paper No. 12, 10. MR3827120 doi:10.19086/da.3734 Cited on page 259.
- János Kollár, Lajos Rónyai, and Tibor Szabó, *Norm-graphs and bipartite Turán numbers*, Combinatorica **16** (1996), 399–406. MR1417348 doi:10.1007/BF01261323 Cited on pages 48 and 49.
- J. Komlós and M. Simonovits, *Szemerédi's regularity lemma and its applications in graph theory*, Combinatorics, Paul Erdős is eighty, Vol. 2 (Keszthely, 1993), János Bolyai Mathematical Society, 1996, pp. 295–352. MR1395865 Cited on page 102.
- János Komlós, Ali Shokoufandeh, Miklós Simonovits, and Endre Szemerédi, *The regularity lemma and its applications in graph theory*, Theoretical aspects of computer science (Tehran, 2000), Springer, 2002, pp. 84–112. MR1966181 doi:10.1007/3-540-45878-6_3 Cited on page 102.
- S. V. Konyagin and I. D. Shkredov, *On sum sets of sets having small product set*, Proc. Steklov Inst. Math. **290** (2015), 288–299, Published in Russian in Tr. Mat. Inst. Steklova **2** (2015), 304–316. MR3488800 doi:10.1134/S0081543815060255 Cited on page 321.
- T. Kővári, V. T. Sós, and P. Turán, *On a problem of K. Zarankiewicz*, Colloq. Math. **3** (1954), 50–57. MR65617 doi:10.4064/cm-3-1-50-57 Cited on page 26.
- Bryna Kra, *The Green-Tao theorem on arithmetic progressions in the primes: an ergodic point of view*, Bull. Amer. Math. Soc. **43** (2006), 3–23. MR2188173 doi:10.1090/S0273-0979-05-01086-4 Cited on page 350.
- M. Krivelevich and B. Sudakov, *Pseudo-random graphs*, More sets, graphs and numbers, Springer, 2006, pp. 199–262. MR2223394 doi:10.1007/978-3-540-32439-3_10 Cited on page 149.
- Joseph B. Kruskal, *The number of simplices in a complex*, Mathematical optimization techniques, University of California Press, 1963, pp. 251–278. MR0154827 Cited on page 193.
- Serge Lang and André Weil, *Number of points of varieties in finite fields*, Amer. J. Math. **76** (1954), 819–827. MR65218 doi:10.2307/2372655 Cited on page 57.
- Joonkyung Lee, MathOverflow post, 2019, <https://mathoverflow.net/q/189222/>. Cited on page 206.
- Frank Thomson Leighton, *New lower bound techniques for VLSI*, Math. Systems Theory **17** (1984), 47–70. MR738751 doi:10.1007/BF01744433 Cited on page 316.
- J.L. Xiang Li and Balazs Szegedy, *On the logarithmic calculus and Sidorenko's conjecture*, 2011. arXiv:1107.1153 Cited on pages 222 and 225.
- L. H. Loomis and H. Whitney, *An inequality related to the isoperimetric inequality*, Bull. Amer. Math. Soc. **55** (1949), 961–962. MR0031538 doi:10.1090/S0002-9904-1949-09320-5 Cited on page 208.

- László Lovász, *Very large graphs*, Current developments in mathematics, 2008, International Press, 2009, pp. 67–128. MR2555927 Cited on page 186.
- László Lovász, *Large networks and graph limits*, American Mathematical Society, 2012. MR3012035 doi:10.1090/coll/060 Cited on pages 186, 195, and 230.
- László Lovász and Balázs Szegedy, *Limits of dense graph sequences*, J. Combin. Theory Ser. B **96** (2006), 933–957. MR2274085 doi:10.1016/j.jctb.2006.05.002 Cited on page 163.
- László Lovász and Balázs Szegedy, *Szemerédi's lemma for the analyst*, Geom. Funct. Anal. **17** (2007), 252–270. MR2306658 doi:10.1007/s00039-007-0599-6 Cited on page 159.
- Shachar Lovett, *Equivalence of polynomial conjectures in additive combinatorics*, Combinatorica **32** (2012), 607–618. MR3004811 doi:10.1007/s00493-012-2714-z Cited on page 301.
- Shachar Lovett, *An exposition of Sanders' quasi-polynomial Freiman-Ruzsa theorem*, Theory of Computing Library Graduate Surveys, vol. 6, 2015, pp. 1–14. Cited on page 311.
- Shachar Lovett and Oded Regev, *A counterexample to a strong variant of the polynomial Freiman-Ruzsa conjecture in Euclidean space*, Discrete Anal. (2017), Paper No. 8, 6. MR3651924 doi:10.19086/da.1640 Cited on page 302.
- Eyal Lubetzky and Yufei Zhao, *On the variational problem for upper tails in sparse random graphs*, Random Structures Algorithms **50** (2017), 420–436. MR3632418 doi:10.1002/rsa.20658 Cited on pages 209 and 212.
- A. Lubotzky, R. Phillips, and P. Sarnak, *Ramanujan graphs*, Combinatorica **8** (1988), 261–277. MR963118 doi:10.1007/BF02126799 Cited on page 146.
- Alexander Lubotzky, *Expander graphs in pure and applied mathematics*, Bull. Amer. Math. Soc. **49** (2012), 113–162. MR2869010 doi:10.1090/S0273-0979-2011-01359-3 Cited on page 149.
- W. Mantel, *Problem 28*, Wiskundige Opgaven **10** (1907), 60–61. Cited on page 14.
- Adam W. Marcus, Daniel A. Spielman, and Nikhil Srivastava, *Interlacing families I: Bipartite Ramanujan graphs of all degrees*, Ann. of Math. **182** (2015), 307–325. MR3374962 doi:10.4007/annals.2015.182.1.7 Cited on pages 147 and 149.
- G. A. Margulis, *Explicit group-theoretic constructions of combinatorial schemes and their applications in the construction of expanders and concentrators*, Problemy Peredachi Informatsii **24** (1988), 51–60. MR939574 Cited on page 146.
- Ju. V. Matiyasevich, *The Diophantineness of enumerable sets*, Dokl. Akad. Nauk. SSSR. **191** (1970), 279–282. MR0258744 Cited on page 188.
- Jiří Matoušek, *Thirty-three miniatures*, American Mathematical Society, 2010, Mathematical and algorithmic applications of linear algebra. MR2656313 doi:10.1090/stml/053 Cited on page 270.
- Roy Meshulam, *On subsets of finite abelian groups with no 3-term arithmetic progressions*, J. Combin. Theory Ser. A **71** (1995), 168–172. MR1335785 doi:10.1016/0097-3165(95)90024-1 Cited on page 239.
- Hermann Minkowski, *Geometrie der Zahlen*, Teubner, 1896. MR249269 Cited on page 294.

- Moshe Morgenstern, *Existence and explicit constructions of $q + 1$ regular Ramanujan graphs for every prime power q* , J. Combin. Theory Ser. B **62** (1994), 44–62. MR1290630 doi:10.1006/jctb.1994.1054 Cited on page 146.
- Guy Moshkovitz and Asaf Shapira, *A short proof of Gowers' lower bound for the regularity lemma*, Combinatorica **36** (2016), 187–194. MR3516883 doi:10.1007/s00493-014-3166-4 Cited on page 69.
- Guy Moshkovitz and Asaf Shapira, *A tight bound for hypergraph regularity*, Geom. Funct. Anal. **29** (2019), 1531–1578. MR4025519 doi:10.1007/s00039-019-00512-5 Cited on page 102.
- T. S. Motzkin, *The arithmetic-geometric inequality*, Inequalities (Proc. Sympos. Wright-Patterson Air Force Base, Ohio, 1965), Academic Press, 1967, pp. 205–224. MR0223521 Cited on page 203.
- T. S. Motzkin and E. G. Straus, *Maxima for graphs and a new proof of a theorem of Turán*, Canadian J. Math. **17** (1965), 533–540. MR175813 doi:10.4153/CJM-1965-053-6 Cited on page 215.
- H. P. Mulholland and C. A. B. Smith, *An inequality arising in genetical theory*, Amer. Math. Monthly **66** (1959), 673–683. MR110721 doi:10.2307/2309342 Cited on page 205.
- Jaroslav Nešetřil and Moshe Rosenfeld, *I. Schur, C. E. Shannon and Ramsey numbers, a short story*, vol. 229, 2001, Combinatorics, graph theory, algorithms and applications, pp. 185–195. MR1815606 doi:10.1016/S0012-365X(00)00208-9 Cited on page 4.
- V. Nikiforov, *The number of cliques in graphs of given order and size*, Trans. Amer. Math. Soc. **363** (2011), 1599–1618. MR2737279 doi:10.1090/S0002-9947-2010-05189-X Cited on page 196.
- N. Nikolov and L. Pyber, *Product decompositions of quasirandom groups and a Jordan type theorem*, J. Eur. Math. Soc. (JEMS) **13** (2011), 1063–1077. MR2800484 doi:10.4171/JEMS/275 Cited on page 135.
- A. Nilli, *On the second eigenvalue of a graph*, Discrete Math. **91** (1991), 207–210. MR1124768 doi:10.1016/0012-365X(91)90112-F Cited on page 141.
- Giuseppe Pellegrino, *Sul massimo ordine delle calotte in $S_{4,3}$* , Matematiche (Catania) **25** (1970), 149–157 (1971). MR363952 Cited on page 241.
- Sarah Peluse, *Bounds for sets with no polynomial progressions*, Forum Math. Pi **8** (2020), e16, 55. MR4199235 doi:10.1017/fmp.2020.11 Cited on page 9.
- Giorgis Petridis, *New proofs of Plünnecke-type estimates for product sets in groups*, Combinatorica **32** (2012), 721–733. MR3063158 doi:10.1007/s00493-012-2818-5 Cited on page 277.
- Nicholas Pippenger and Martin Charles Golumbic, *The inducibility of graphs*, J. Combin. Theory Ser. B **19** (1975), 189–203. MR401552 doi:10.1016/0095-8956(75)90084-2 Cited on page 202.
- Helmut Plünnecke, *Eine zahlentheoretische Anwendung der Graphentheorie*, J. Reine Angew. Math. **243** (1970), 171–183. MR266892 doi:10.1515/crll.1970.243.171 Cited on page 277.
- D. H. J. Polymath, *A new proof of the density Hales-Jewett theorem*, Ann. of Math. **175** (2012), 1283–1327. MR2912706 doi:10.4007/annals.2012.175.3.6 Cited on page 7.
- Jaikumar Radhakrishnan, *Entropy and counting*, Computational Mathematics, Modelling and Algorithms (J. C. Misra, ed.), Narosa, 2003. Cited on page 231.

- Alexander A. Razborov, *Flag algebras*, J. Symbolic Logic **72** (2007), 1239–1282. MR2371204 doi:10.2178/jsl/1203350785 Cited on page 201.
- Alexander A. Razborov, *On the minimal density of triangles in graphs*, Combin. Probab. Comput. **17** (2008), 603–618. MR2433944 doi:10.1017/S0963548308009085 Cited on pages 195 and 201.
- Alexander A. Razborov, *Flag algebras: an interim report*, The mathematics of Paul Erdős. II, Springer, 2013, pp. 207–232. MR3186665 doi:10.1007/978-1-4614-7254-4_16 Cited on page 231.
- Christian Reiher, *The clique density theorem*, Ann. of Math. **184** (2016), 683–707. MR3549620 doi:10.4007/annals.2016.184.3.1 Cited on page 196.
- Omer Reingold, Luca Trevisan, Madhur Tulsiani, and Salil Vadhan, *New proofs of the Green-Tao-Ziegler dense model theorem: an exposition*, 2008. arXiv:0806.0381 Cited on page 338.
- V. Rödl, B. Nagle, J. Skokan, M. Schacht, and Y. Kohayakawa, *The hypergraph regularity method and its applications*, Proc. Natl. Acad. Sci. USA **102** (2005), 8109–8113. MR2167756 doi:10.1073/pnas.0502771102 Cited on pages 7, 98, and 102.
- K. F. Roth, *On certain sets of integers*, J. Lond. Math. Soc. **28** (1953), 104–109. MR51853 doi:10.1112/jlms/s1-28.1.104 Cited on pages 6, 61, 233, and 249.
- I. Z. Ruzsa, *Generalized arithmetical progressions and sumsets*, Acta Math. Hungar. **65** (1994), 379–388. MR1281447 doi:10.1007/BF01876039 Cited on page 274.
- Imre Z. Ruzsa, *An application of graph theory to additive number theory*, Sci. Ser. A Math. Sci. **3** (1989), 97–109, with Addendum in **4** (1990/91), 93–94. MR2314377 Cited on page 277.
- Imre Z. Ruzsa, *An analog of Freiman’s theorem in groups*, no. 258, 1999, Structure theory of set addition, pp. xv, 323–326. MR1701207 Cited on pages 280, 282, and 300.
- Imre Z. Ruzsa, *Sumsets and structure*, Combinatorial number theory and additive group theory, Birkhäuser Verlag, 2009, pp. 87–210. MR2522038 doi:10.1007/978-3-7643-8962-8 Cited on pages 277 and 311.
- Imre Z. Ruzsa and Endre Szemerédi, *Triple systems with no six points carrying three triangles*, Combinatorics (Proc. Fifth Hungarian Colloq., Keszthely, 1976), Vol. II, 1978, pp. 939–945. MR519318 Cited on pages 10, 61, 74, and 77.
- Bruce E. Sagan, *The symmetric group*, second ed., Springer-Verlag, 2001, Representations, combinatorial algorithms, and symmetric functions. MR1824028 doi:10.1007/978-1-4757-6804-6 Cited on page 133.
- Ashwin Sah, Mehtaab Sawhney, David Stoner, and Yufei Zhao, *The number of independent sets in an irregular graph*, J. Combin. Theory Ser. B **138** (2019), 172–195. MR3979229 doi:10.1016/j.jctb.2019.01.007 Cited on page 214.
- Ashwin Sah, Mehtaab Sawhney, David Stoner, and Yufei Zhao, *A reverse Sidorenko inequality*, Invent. Math. **221** (2020), 665–711. MR4121160 doi:10.1007/s00222-020-00956-9 Cited on page 214.
- Ashwin Sah, Mehtaab Sawhney, and Yufei Zhao, *Patterns without a popular difference*, Discrete Anal. (2021), Paper No. 8, 30. MR4293329 doi:10.19086/da Cited on page 269.
- R. Salem and D. C. Spencer, *On sets of integers which contain no three terms in arithmetical progression*, Proc. Natl. Acad. Sci. USA **28** (1942), 561–563. MR7405 doi:10.1073/pnas.28.12.561 Cited on page 80.

- Tom Sanders, *On the Bogolyubov-Ruzsa lemma*, Anal. PDE **5** (2012), 627–655. MR2994508 doi:10.2140/apde.2012.5.627 Cited on pages 274, 300, and 303.
- Tom Sanders, *The structure theory of set addition revisited*, Bull. Amer. Math. Soc. **50** (2013), 93–127. MR2994996 doi:10.1090/S0273-0979-2012-01392-7 Cited on pages 274, 303, and 311.
- A. Sárközy, *On difference sets of sequences of integers. I*, Acta Math. Acad. Sci. Hungar. **31** (1978), 125–149. MR466059 doi:10.1007/BF01896079 Cited on page 9.
- David Saxton and Andrew Thomason, *Hypergraph containers*, Invent. Math. **201** (2015), 925–992. MR3385638 doi:10.1007/s00222-014-0562-8 Cited on page 330.
- Mathias Schacht, *Extremal results for random discrete structures*, Ann. of Math. **184** (2016), 333–365. MR3548528 doi:10.4007/annals.2016.184.2.1 Cited on page 330.
- Richard H. Schelp and Andrew Thomason, *A remark on the number of complete and empty subgraphs*, Combin. Probab. Comput. **7** (1998), 217–219. MR1617934 doi:10.1017/S0963548397003234 Cited on page 216.
- Tomasz Schoen, *Near optimal bounds in Freiman’s theorem*, Duke Math. J. **158** (2011), 1–12. MR2794366 doi:10.1215/00127094-1276283 Cited on page 274.
- Tomasz Schoen and Ilya D. Shkredov, *Roth’s theorem in many variables*, Israel J. Math. **199** (2014), 287–308. MR3219538 doi:10.1007/s11856-013-0049-0 Cited on page 8.
- Tomasz Schoen and Olof Sisask, *Roth’s theorem for four variables and additive structures in sums of sparse sets*, Forum Math. Sigma **4** (2016), e5, 28 pp. MR3482282 doi:10.1017/fms.2016.2 Cited on page 8.
- Alexander Schrijver, *Combinatorial optimization. Polyhedra and efficiency.*, Springer-Verlag, 2003. MR1956924 Cited on page 59.
- I. Schur, *Über die kongruenz $x^m + y^m \equiv z^m \pmod{p}$* , Jber. Deutsch. Math.-Verein **25** (1916). Cited on pages 1 and 4.
- J. Schur, *Untersuchungen über die Darstellung der endlichen Gruppen durch gebrochene lineare Substitutionen*, J. Reine Angew. Math. **132** (1907), 85–137. MR1580715 doi:10.1515/crll.1907.132.85 Cited on page 133.
- Jean-Pierre Serre, *Linear representations of finite groups*, Springer-Verlag, 1977. MR0450380 Cited on page 129.
- Adam Sheffer, *Polynomial methods and incidence theory*, Cambridge University Press, 2022. Cited on page 322.
- I. D. Shkredov, *On a generalization of Szemerédi’s theorem*, Proc. Lond. Math. Soc. **93** (2006), 723–760. MR2266965 doi:10.1017/S0024611506015991 Cited on page 80.
- A. F. Sidorenko, *Inequalities for functionals generated by bipartite graphs*, Diskret. Mat. **3** (1991), 50–65. MR1138091 doi:10.1515/dma.1992.2.5.489 Cited on page 206.
- Alexander Sidorenko, *A correlation inequality for bipartite graphs*, Graphs Combin. **9** (1993), 201–204. MR1225933 doi:10.1007/BF02988307 Cited on page 188.

- M. Simonovits, *External graph problems with symmetrical extremal graphs. Additional chromatic conditions*, Discrete Math. **7** (1974), 349–376. MR337690 doi:10.1016/0012-365X(74)90044-2 Cited on page 36.
- Robert Singleton, *On minimal graphs of maximum even girth*, J. Combinatorial Theory **1** (1966), 306–332. MR201347 Cited on page 51.
- Jozef Skokan and Lubos Thoma, *Bipartite subgraphs and quasi-randomness*, Graphs Combin. **20** (2004), 255–262. MR2080111 doi:10.1007/s00373-004-0556-1 Cited on pages 115 and 190.
- József Solymosi, *Note on a generalization of Roth's theorem*, Discrete and computational geometry, Springer, 2003, pp. 825–827. MR2038505 doi:10.1007/978-3-642-55566-4_39 Cited on page 78.
- József Solymosi, *Bounding multiplicative energy by the sumset*, Adv. Math. **222** (2009), 402–408. MR2538014 doi:10.1016/j.aim.2009.04.006 Cited on pages 314 and 320.
- K. Soundararajan, *Additive combinatorics*, 2007, online lecture notes, <http://math.stanford.edu/~ksound/Notes.pdf>. Cited on page 311.
- Daniel A. Spielman, *Spectral and algebraic graph theory*, 2019, textbook draft <http://cs-www.cs.yale.edu/homes/spielman/sagt/>. Cited on page 149.
- Elias M. Stein and Rami Shakarchi, *Fourier analysis*, Princeton University Press, 2003, An introduction. MR1970295 Cited on page 270.
- B. Sudakov, E. Szemerédi, and V. H. Vu, *On a question of Erdős and Moser*, Duke Math. J. **129** (2005), 129–155. MR2155059 doi:10.1215/S0012-7094-04-12915-X Cited on page 304.
- Balázs Szegedy, *An information theoretic approach to sidorenko's conjecture*, 2015. arXiv:1406.6738 Cited on page 222.
- László A. Székely, *Crossing numbers and hard Erdős problems in discrete geometry*, Combin. Probab. Comput. **6** (1997), 353–358. MR1464571 doi:10.1017/S0963548397002976 Cited on page 318.
- E. Szemerédi, *On sets of integers containing no k elements in arithmetic progression*, Acta Arith. **27** (1975), 199–245. MR369312 doi:10.4064/aa-27-1-199-245 Cited on page 6.
- Endre Szemerédi and William T. Trotter, Jr., *Extremal problems in discrete geometry*, Combinatorica **3** (1983), 381–392. MR729791 doi:10.1007/BF02579194 Cited on pages 314 and 317.
- Terence Tao, *A variant of the hypergraph removal lemma*, J. Combin. Theory Ser. A **113** (2006), 1257–1280. MR2259060 doi:10.1016/j.jcta.2005.11.006 Cited on page 102.
- Terence Tao, *Structure and randomness in combinatorics*, 48th Annual IEEE Symposium on Foundations of Computer Science (FOCS'07), 2007a, pp. 3–15. doi:10.1109/FOCS.2007.17 Cited on pages 266 and 270.
- Terence Tao, *The dichotomy between structure and randomness, arithmetic progressions, and the primes*, International Congress of Mathematicians. Vol. I, European Mathematical Society, 2007b, pp. 581–608. MR2334204 doi:10.4171/022-1/22 Cited on pages 7 and 350.
- Terence Tao, *The spectral proof of the szemerédi regularity lemma*, 2012, blog post <https://terrytao.wordpress.com/2012/12/03/>. Cited on page 266.

- Terence Tao, *A proof of Roth's theorem*, 2014, blog post <https://terrytao.wordpress.com/2014/04/24/>. Cited on page 268.
- Terence Tao and Van Vu, *Additive combinatorics*, Cambridge University Press, 2006. MR2289012 doi:10.1017/CBO9780511755149 Cited on pages 12, 274, and 277.
- Terence Tao and Tamar Ziegler, *The primes contain arbitrarily long polynomial progressions*, Acta Math. **201** (2008), 213–305. MR2461509 doi:10.1007/s11511-008-0032-5 Cited on page 10.
- Terence Tao and Tamar Ziegler, *A multi-dimensional Szemerédi theorem for the primes via a correspondence principle*, Israel J. Math. **207** (2015), 203–228. MR3358045 doi:10.1007/s11856-015-1157-9 Cited on page 10.
- Alfred Tarski, *A decision method for elementary algebra and geometry*, RAND Corporation, 1948. MR0028796 Cited on page 188.
- Andrew Thomason, *Pseudorandom graphs*, Random graphs '85 (Poznań, 1985), North-Holland, 1987, pp. 307–331. MR930498 Cited on page 106.
- Andrew Thomason, *A disproof of a conjecture of Erdős in Ramsey theory*, J. Lond. Math. Soc. **39** (1989), 246–255. MR991659 doi:10.1112/jlms/s2-39.2.246 Cited on page 199.
- Paul Turán, *On a Theorem of Hardy and Ramanujan*, J. Lond. Math. Soc. **9** (1934), 274–276. MR1574877 doi:10.1112/jlms/s1-9.4.274 Cited on page 315.
- Paul Turán, *Eine Extremalaufgabe aus der Graphentheorie*, Mat. Fiz. Lapok **48** (1941), 436–452 (Hungarian, with German summary). Cited on page 17.
- B. L. van der Waerden, *Beweis einer baudetschen vermutung*, Nieuw Arch. Wisk. **15** (1927), 212–216. Cited on page 6.
- P. Varnavides, *On certain sets of positive density*, J. Lond. Math. Soc. **34** (1959), 358–360. MR106865 doi:10.1112/jlms/s1-34.3.358 Cited on page 331.
- I. M. Vinogradov, *The representation of an odd number as a sum of three primes.*, Dokl. Akad. Nauk. SSSR. **16** (1937), 139–142. Cited on page 292.
- R. Wenger, *Extremal graphs with no C^4 's, C^6 's, or C^{10} 's*, J. Combin. Theory Ser. B **52** (1991), 113–116. MR1109426 doi:10.1016/0095-8956(91)90097-4 Cited on page 51.
- Douglas B. West, *Introduction to graph theory*, Prentice Hall, 1996. MR1367739 Cited on page 59.
- Avi Wigderson, *Representation theory of finite groups, and applications*, Lecture notes for the 22nd McGill invitational workshop on computational complexity, 2012, https://www.math.ias.edu/~avi/TALKS/Green_Wigderson_lecture.pdf. Cited on pages 129 and 136.
- David Williams, *Probability with martingales*, Cambridge University Press, 1991. MR1155402 doi:10.1017/CBO9780511813658 Cited on pages 174 and 175.
- J. Wolf, *Finite field models in arithmetic combinatorics—ten years on*, Finite Fields Appl. **32** (2015), 233–274. MR3293412 doi:10.1016/j.ffa.2014.11.003 Cited on page 270.

- Julia Wolf, *Arithmetic and polynomial progressions in the primes [after Gowers, Green, Tao and Ziegler]*, no. 352, 2013, Séminaire Bourbaki. Vol. 2011/2012. Exposés 1043–1058, pp. Exp. No. 1054, ix–x, 389–427. MR3087352 Cited on page 350.
- K. Zarankiewicz, *Problem 101*, Colloq. Math. **2** (1951), 201. Cited on page 25.
- Yufei Zhao, *The number of independent sets in a regular graph*, Combin. Probab. Comput. **19** (2010), 315–320. MR2593625 doi:10.1017/S0963548309990538 Cited on pages 210 and 212.
- Yufei Zhao, *An arithmetic transference proof of a relative Szemerédi theorem*, Math. Proc. Cambridge Philos. Soc. **156** (2014), 255–261. MR3177868 doi:10.1017/S0305004113000662 Cited on page 338.
- Yufei Zhao, *Extremal regular graphs: independent sets and graph homomorphisms*, Amer. Math. Monthly **124** (2017), 827–843. MR3722040 doi:10.4169/amer.math.monthly.124.9.827 Cited on page 214.