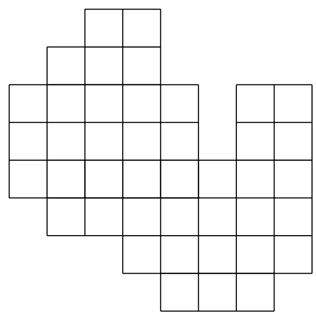
Problem Set 9. Due 11/20

Reminder: You must acknowledge your sources and collaborators (even if it is "none", you must write so). Failure to do so on this problem set will result in an automatic 2-point deduction.

1. Show that in any graph G (not necessarily bipartite), the size of any *maximal* matching M (i.e. a matching that cannot be enlarged by including more edges of G) is at least half the size of a *maximum* matching. Also, give an example where it is exactly one-half.

2. Let G be a connected graph with at least 4 vertices such that every edge is contained in some perfect matching of G. Show that G is 2-edge-connected.

3. Can the following figure be tiled by dominoes? Give a tiling or a short proof that no tiling exists. (What does this have to do with bipartite matchings?)



4. Twenty-four rooks are placed on a standard 8×8 chessboard so that every row and every column contains exactly three rooks. Prove that one can select eight non-attacking rooks (i.e., no two rooks on the same row or column).

5. Take a standard deck of cards, and deal them out into 13 piles of 4 cards each. Show that it is always possible to select exactly 1 card from each pile, such that the 13 selected cards contain exactly one card of each rank (A, 2, 3, . . . , Q, K).

6. Construct a 5-regular graph without a perfect matching (you should prove that your construction indeed has no perfect matching).

7. Prove that the edge set of a 2-edge-connected 3-regular graph can be partitioned into paths of 3 edges each.