18.S997 (FALL 2017) PROBLEM SET 2

1. Let the *half-graph* H_n be the bipartite graph on $2n$ vertices $\{a_1, \ldots, a_n, b_1, \ldots, b_n\}$ with edges $\{a_i b_j : i \leq j\}$.
   (a) For every $\epsilon > 0$, explicitly construct an ϵ-regular partition of H_n into $O(1/\epsilon)$ parts.
   (b) Show that there is some $c > 0$ such that for every sufficiently small $\epsilon > 0$, every integer k and sufficiently large multiple n of k, every partition of the vertices of H_n into k equal-sized parts contains at least ck pairs of parts which are not ϵ-regular.
2. Show that there is some absolute constant $C > 0$ such that for every $\epsilon > 0$, every graph on n vertices contains an ϵ-regular pair of vertex subsets each with size at least δn, where $\delta = 2^{-\epsilon^{-C}}$.

. . . to be continued . . . check back later (last updated: September 25, 2017)