Problem Set 1. Due 9/11

1. How many subsets does $[n]$ have that contain exactly one of the elements 1 and 2?
2. In how many ways can the elements of $[n]$ be permuted so that the sum of every two consecutive elements in the permutation is odd?
3. A host invites n couples to a party. She wants to ask a subset of the $2n$ guests to give a speech, but she does not want to ask both members of any couple to give speeches. In how many ways can she proceed?
4. We want to select an ordered pair (A, B) of subsets of $[n]$ so that $A \cap B \neq \emptyset$. In how many different ways can we do this?
5. A class is attended by n sophomores, n juniors, and n seniors. In how many ways can these students form n groups of three people each if each group is to contain a sophomore, a junior, and a senior?
6. Let P be a convex polygon in which no three diagonals intersect in one point. How many intersection points do the diagonals of P have?
7. A student will study 26 hours in preparation for an exam. She will due this in the course of six consecutive days. On each of these days, she will study either four hours, or five hours, or six hours. In how many different ways is this possible?
8. How many $n \times n$ square matrices are there whose entries are 0 or 1 and in which each row and column has an even sum?