18.S997 (FALL 2017) PROBLEM SET 1

1. (a) Let s and r be positive integers. Show that there is some integer $n = n(s, r)$ so that if every edge of the complete graph K_n on n vertices is colored with one of r colors, then there is a monochromatic copy of K_s.
   (b) Let $s \geq 3$ be a positive integer. Show that if the edges of the complete graph on $\binom{2s-2}{s-1}$ vertices are colored with 2 colors, then there is a monochromatic copy of K_s.
2. Show that a graph with n vertices and m edges has at least
$$\frac{4m}{3n}\left(m - \frac{n^2}{4}\right)$$
many triangles.
3. Let S be a set of n points in the plane, with the property that no two points are at distance greater than 1. Show that S has at most $\lfloor n^2/3 \rfloor$ pairs of points at distance greater than $1/\sqrt{2}$. Also, show that the bound $\lfloor n^2/3 \rfloor$ is tight (i.e., cannot be improved).

. . . to be continued . . . check back later (last updated: September 12, 2017)