

Practice Midterm 3

Time: 80 minutes.

5 problems worth 10 points each.

No electronic devices. You may bring **two sheets of notes** on letter-sized paper (total four sides front and back) **in your own handwriting**. Typed, printed, or photocopied notes are **forbidden**.

You must provide justification in your solutions (not just answers). You may quote theorems and facts proved in class, course textbook/notes, or homework, provided that you state the facts that you are using.

1. Determine whether each of the following statement is TRUE or FALSE, and provide a short justification or a counterexample (a correct answer without justification receives zero credit).
 - (a) If G is a connected planar graph, then any planar embedding of G always has the same number of faces.
 - (b) If G is a connected d -regular graph with $d \geq 1$, then its line graph $L(G)$ contains an Eulerian tour.
2. Does there exist a connected graph with a cut vertex whose edge set can be partitioned into perfect matchings?
3. Let G be a bipartite graph with n vertices on both sides and minimum degree at least $n/2$. Prove that G has a perfect matching.
4. Let $k \geq 1$. Let G be a $2k$ -edge-connected graph. Let $s_1, \dots, s_k, t_1, \dots, t_k$ be distinct vertices. Show that there are edge disjoint paths P_1, \dots, P_k such that each P_i starts at s_i and ends at t_i .
5. Let G be the union of k planar graphs. Prove that $\chi(G) \leq 6k$.