# Yufei Zhao

http://yufeizhao.com yufeiz@mit.edu MIT Department of Mathematics 77 Massachusetts Ave, Room 2-271 Cambridge, MA 02139, USA

Department of Mathematics, Massachusetts Institute of Technology Assistant Professor of Mathematics	Cambridge, MA 2017—
Class of 1956 Career Development Assistant Professor	2018—2021
Previous and Visiting Academic Positions	
Department of Mathematics, Stanford University Visiting Assistant Professor	Stanford, CA Spring 2020
Simons Institute for the Theory of Computing, UC Berkeley Simons-Berkeley Research Fellow	Berkeley, CA Spring 2017
New College, University of Oxford Esmée Fairbairn Junior Research Fellow in Mathematics	Oxford, UK 2015—2017
Education	
Massachusetts Institute of Technology Ph.D. Mathematics. Advisor: Jacob Fox	Cambridge, MA 2011—2015
University of Cambridge M.A.St. Mathematics with Distinction	Cambridge, UK 2010—2011
Massachusetts Institute of Technology S.B. Mathematics, with minor in Economics	Cambridge, MA 2006—2010

#### **Selected Awards and Honors**

S.B. Computer Science and Engineering

NSF CAREER Award, 2021

MIT UROP Outstanding Mentor Award for Faculty, 2020

MIT First Year Advisor Award—Innovative Seminar, 2019

Sloan Research Fellowship, 2019

MIT Future of Science Award, 2018

SIAM Dénes König Prize, 2018

Johnson Prize, MIT Mathematics Department, 2015

Microsoft Research PhD Fellowship, 2013–2015

Morgan Prize Honorable Mention, 2011

Gates Cambridge Scholarship, 2010–2011

MIT Jon A. Bucsela Prize in Mathematics, 2010

Putnam Math Competition: Three-time Putnam Fellow (top five rank) 2006, 2008, 2009; 7th Place 2007

International Mathematical Olympiad: Gold Medal 2005, Silver Medal 2006, Bronze Medal 2004

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#### **Research Interests**

Combinatorics (extremal, probabilistic, additive, graph theory, discrete geometry)

#### **Grants**

NSF CAREER award DMS-2044606	2021—2026
Sloan Research Fellowship	2019—2021
MIT Solomon Buchsbaum Research Fund	2018—
NSF award DMS-1764176	2018—2021
NSF award DMS-1362326	2017—2018

## **Research Internships**

Microsoft Research New England
Mentor: Henry Cohn

Microsoft Research Theory Group
Mentor: Eyal Lubetzky

Cambridge, MA
Summers 2010, 2011, 2013, 2014

Redmond, WA
Summer 2012

#### **Papers**

- 61. Yufei Zhao, Exploring a planet, revisited,

  Amer. Math. Monthly, to appear. arXiv:2110.04376
- 60. Milan Haiman, Carl Schildkraut, Shengtong Zhang, and Yufei Zhao, Graphs with high second eigenvalue multiplicity. arXiv:2109.13131
- Dingding Dong, Nitya Mani, and Yufei Zhao
   Enumerating k-SAT functions,
   ACM-SIAM Symposium on Discrete Algorithms (SODA) 2022 arXiv:2107.09233
- 58. David Conlon, Jacob Fox, Benny Sudakov, and Yufei Zhao, Which graphs can be counted in *C*<sub>4</sub>-free graphs? arXiv:2106.03261
- 57. Jacob Fox and Yufei Zhao, Removal lemmas and approximate homomorphisms. arXiv:2104.11626
- 56. Aaron Berger and Yufei Zhao,  $K_4$ -intersecting families of graphs. arXiv:2103.12671
- 55. Ashwin Sah, Mehtaab Sawhney, Yufei Zhao, The cylindrical width of transitive sets, *Israel J. Math.*, to appear. arXiv:2101.11207
- 54. Ashwin Sah, Mehtaab Sawhney, Yufei Zhao, Paths of given length in tournaments. arXiv:2012.00262
- 53. Jonathan Tidor, Hung-Hsun Hans Yu, and Yufei Zhao, Joints of varieties. arXiv:2008.01610
- 52. Matthew Kwan, Lisa Sauermann, and Yufei Zhao, Extension complexity of low-dimensional polytopes. arXiv:2006.08836
- 51. Zilin Jiang, Jonathan Tidor, Yuan Yao, Shengtong Zhang, and Yufei Zhao, Spherical two-distance sets and eigenvalues of signed graphs. arXiv:2006.06633

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50. Ashwin Sah, Mehtaab Sawhney, and Yufei Zhao, Cayley graphs without a bounded eigenbasis, *Int. Math. Res. Not. IMRN*, appeared online (2020). arXiv:2005.04502

49. Jacob Fox, Yuval Wigderson, and Yufei Zhao, A short proof of the canonical polynomial van der Waerden theorem, C. R. Math. Acad. Sci. Paris 358 (2020), 957–959. arXiv:2005.04135

48. Jacob Fox, Huy Tuan Pham, and Yufei Zhao,
Tower-type bounds for Roth's theorem with popular differences. arXiv:2004.13690

47. David Conlon, Jacob Fox, Benny Sudakov, and Yufei Zhao, The regularity method for graphs with few 4-cycles, *J. Lond. Math. Soc.*, to appear. arXiv:2004.10180

46. Ashwin Sah, Mehtaab Sawhney, and Yufei Zhao, Patterns without a popular difference, *Discrete Anal.*, 2021:8, 30 pp. arXiv:2004.07722

45. Ashwin Sah, Mehtaab Sawhney, Jonathan Tidor, and Yufei Zhao, A counterexample to the Bollobás-Riordan conjectures on sparse graph limits, *Combin. Probab. Comput.* 30 (2021), 796–799. arXiv:2003.05272

44. Hung-Hsun Hans Yu and Yufei Zhao, Joints tightened. arXiv:1911.08605

43. Jonathan Tidor and Yufei Zhao,
Testing linear-invariant properties,
IEEE Symposium on Foundations of Computer Science (FOCS) 2020. arXiv:1911.06793

42. Jacob Fox, Jonathan Tidor, and Yufei Zhao, Induced arithmetic removal: complexity 1 patterns over finite fields, *Israel J. Math.*, to appear. arXiv:1911.03427

41. Jacob Fox, Huy Tuan Pham, and Yufei Zhao, Common and Sidorenko linear equations, Q. J. Math., appeared online (2021). arXiv:1910.06436

40. Yang Liu and Yufei Zhao,
On the upper tail problem for random hypergraphs,
Random Structures Algorithms 58 (2021), 179–220. arXiv:1910.02916

39. Zilin Jiang, Jonathan Tidor, Yuan Yao, Shengtong Zhang, and Yufei Zhao, Equiangular lines with a fixed angle, *Ann. of Math.* 194 (2021), 729–743. arXiv:1907.12466

38. Yufei Zhao and Yunkun Zhou, Impartial digraphs, Combinatorica 40 (2020), 875–896. arXiv:1906.10482

37. Ashwin Sah, Mehtaab Sawhney, David Stoner, and Yufei Zhao, Exponential improvements for superball packing upper bounds, *Adv. Math.* 365 (2020), 107056. arXiv:1904.11462

36. Jacob Fox, Ashwin Sah, Mehtaab Sawhney, David Stoner, and Yufei Zhao, Triforce and corners, *Math. Proc. Cambridge Philos. Soc.* 169 (2020), 209–223. arXiv:1903.04863

35. Ashwin Sah, Mehtaab Sawhney, David Stoner, and Yufei Zhao, A reverse Sidorenko inequality, *Invent. Math.* 221 (2020), 665–711. arXiv:1809.09462

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34. David Conlon, Jonathan Tidor, and Yufei Zhao, Hypergraph expanders of all uniformities from Cayley graphs, *Proc. Lond. Math. Soc.* 121 (2020), 1311–1336. arXiv:1809.06342

- 33. Asaf Ferber, Vishesh Jain, and Yufei Zhao, On the number of Hadamard matrices via anti-concentration, *Combin. Probab. Comput.*, to appear. arXiv:1808.07222
- 32. Ashwin Sah, Mehtaab Sawhney, David Stoner, and Yufei Zhao, The number of independent sets in an irregular graph, *J. Combin. Theory Ser. B* 138 (2019), 172–195. arXiv:1805.04021.
- 31. Jacob Fox, László Miklós Lovász, and Yufei Zhao, A fast new algorithm for weak graph regularity, *Combin. Probab. Comput.* 28 (2019), 777–790. arXiv:1801.05037
- 30. Noga Alon, Jacob Fox, and Yufei Zhao, Efficient arithmetic regularity and removal lemmas for induced bipartite patterns, *Discrete Anal.* 2019:3, 14 pp. arXiv:1801.04675
- 29. Yufei Zhao, Group representations that resist worst-case sampling. arXiv:1705.04675
- 28. Yufei Zhao, Extremal regular graphs: independent sets and graph homomorphisms, *Amer. Math. Monthly* 124 (2017), 827–843. arXiv:1610.09210
- 27. Bhaswar B. Bhattacharya, Shirshendu Ganguly, Xuancheng Shao, and Yufei Zhao, Upper tails for arithmetic progressions in a random set, *Int. Math. Res. Not. IMRN* 2020, 167–213. arXiv:1605.02994
- Jacob Fox, László Miklós Lovász, and Yufei Zhao,
   On regularity lemmas and their algorithmic applications,
   Combin. Probab. Comput. 26 (2017), 481–505. arXiv:1604.00733
- 25. David Conlon and Yufei Zhao, Quasirandom Cayley graphs, *Discrete Anal.* 2017:6, 14 pp. arXiv:1603.03025
- 24. Bhaswar B. Bhattacharya, Shirshendu Ganguly, Eyal Lubetzky, and Yufei Zhao, Upper tails and independence polynomials in random graphs, *Adv. Math.* 319 (2017), 313–347. arXiv:1507.04074
- László Miklós Lovász and Yufei Zhao,
   On derivatives of graphon parameters,
   J. Combin. Theory Ser. A 145 (2017), 364–368. arXiv:1505.07448
- 22. Yufei Zhao, On the lower tail variational problem for random graphs, *Combin. Probab. Comput.* 26 (2017), 301–320. arXiv:1502.00867
- 21. Christian Borgs, Jennifer T. Chayes, Henry Cohn, and Yufei Zhao, An *L*<sup>p</sup> theory of sparse graph convergence II: LD convergence, quotients, and right convergence, *Ann. Probab.* 46 (2018), 337–396. arXiv:1408.0744
- 20. David Conlon, Jacob Fox, and Yufei Zhao, The Green-Tao theorem: an exposition, EMS Surv. Math. Sci. 1 (2014), 249–282. arXiv:1403.2957
- 19. Eyal Lubetzky and Yufei Zhao, On the variational problem for upper tails in sparse random graphs, Random Structures Algorithms 50 (2017), 420–436. arXiv:1402.6011
- 18. Christian Borgs, Jennifer T. Chayes, Henry Cohn, and Yufei Zhao, An  $L^p$  theory of sparse graph convergence I: limits, sparse random graph models, and power law

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distributions,

Trans. Amer. Math. Soc. 372 (2019), 3019-3062. arXiv:1401.2906

17. Yufei Zhao, An arithmetic transference proof of a relative Szemerédi theorem, *Math. Proc. Cambridge Philos. Soc.* 156 (2014), 255–261. arXiv:1307.4959

16. Jacob Fox and Yufei Zhao,

A short proof of the multidimensional Szemerédi theorem in the primes, *Amer. J. Math.* 137 (2015), 1139–1145. arXiv:1307.4679

15. David Conlon, Jacob Fox, and Yufei Zhao,

A relative Szemerédi theorem,

Geom. Funct. Anal. 25 (2015), 733-762. arXiv:1305.5440

14. Yufei Zhao, Hypergraph limits: a regularity approach, *Random Structures Algorithms* 47 (2015), 205–226. arXiv:1302.1634

13. Henry Cohn and Yufei Zhao,

Sphere packing bounds via spherical codes,

Duke Math. J. 163 (2014), 1965–2002. arXiv:1212.5966

12. Henry Cohn and Yufei Zhao,

Universally optimal error-correcting codes,

IEEE Trans. Inform. Theory 60 (2014), 7442-7450. arXiv:1212.1913

11. Eyal Lubetzky and Yufei Zhao,

On replica symmetry of large deviations in random graphs,

Random Structures Algorithms 47 (2015) 109–146. arXiv:1210.7013

10. Jacob Fox, Po-Shen Loh, and Yufei Zhao,

The critical window for the classical Ramsey-Turán problem,

Combinatorica 35 (2015) 435-476. arXiv:1208.3276

9. David Conlon, Jacob Fox, and Yufei Zhao,

Extremal results in sparse pseudorandom graphs,

Adv. Math. 256 (2014), 206–290. arXiv:1204.6645

8. Yufei Zhao, The bipartite swapping trick on graph homomorphisms, *SIAM J. Discrete Math.* 25 (2011), 660–680. arXiv:1104.3704

7. Yufei Zhao, Sets characterized by the number of missing sums and differences,

J. Number Theory 11 (2011), 2107–2134. arXiv:0911.2292

6. David Galvin and Yufei Zhao,

The number of independent sets in graphs with small maximum degree, *Graphs Combin.* 27 (2011), 177–186. arXiv:1007.4803

5. Yufei Zhao, Counting MSTD sets in finite abelian groups,

J. Number Theory 130 (2010), 2308-2322. arXiv:0911.2288

4. Yufei Zhao, Constructing numerical semigroups of a given genus, *Semigroup Forum* 80 (2010), 242–254. arXiv:0910.2075

3. Yufei Zhao, Constructing MSTD sets using bidirectonal ballot sequences,

J. Number Theory 130 (2010), 1212–1220. arXiv:0908.4442

2. Yufei Zhao, The number of independent sets in a regular graph, *Combin. Probab. Comput.* 19 (2010), 315–320. arXiv:0909.3354

1. Yufei Zhao, The coefficients of a truncated Fibonacci power series, *Fibonacci Quart.* 46/47 (2009), 53–55.

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# **Invited Talks**

2022	Conference on Random Structures & Algorithms (RS&A): Plenary Speaker	Gniezno, Poland
	Workshop on Critical and Collective Effects in Graphs and Networks (CCEGN-V	Cape Cod, MA
2021	Ohio State University Mathematics Colloquium	Columbus, OH
	Ohio State University Combinatorics & Probability Seminar	Columbus, OH
	National University of Singapore (NUS) Young Mathematician Lecture Series	Online
	Canadian Discrete and Algorithmic Math Conference (CanaDAM): Plenary Lect	ure Online
	Workshop on Extremal and Algorithmic Aspects of Partition Functions	Online
	Copenhagen–Jerusalem Combinatorics Seminar	Online
	Simons Collaboration: Algorithms & Geometry Monthly Meeting	Online
	Caltech/UCLA Joint Analysis Seminar	Online
	Joint Math Meetings MAA Invited Paper Session "Coding Theory and Geometry	" Online
2020	Warwick Centre for Discrete Mathematics and its Applications seminar	Online
	Virtual Harmonic Analysis Seminar	Online
	University of Wisconsin Number Theory / Representation Theory Seminar	Online
	Princeton Discrete Mathematics Seminar	Online
	Big Seminar by Laboratory of Combinatorial and Geometric Structures	Online
	SCMS Combinatorics Seminar	Online
	Cumberland Conference Plenary speaker (Canceled due to COVID-19)	Williamsburg, VA
	Simons Collaboration: Algorithms & Geometry Annual Conference Plenary Lee to COVID-19)	cture (Canceled due New York, NY
	Webinar in Additive Combinatorics	Online
	Stanford Online Combinatorics Seminar	Online
	Stanford Math Department Colloquium	Stanford, CA
	Oberwolfach workshop: Combinatorics Ob	erwolfach, Germany
2019	Shanghai Center for Mathematical Sciences (Fudan) Discrete Math. Seminar	Shanghai, China
	Conference on Graph Theory and its Applications: A Tribute to Prof. Fan Chung	g Sanya, China
	Atlanta Lectures Series in Combinatorics and Graph Theory at Emory	Atlanta, GA
	Princeton Discrete Mathematics Seminar	Princeton, NJ
	Banff workshop: Probabilistic and Extremal Combinatorics	Banff, AB
	ETH Zurich Theory of Combinatorial Algorithms Mittagsseminar	Zürich, Switzerland
	Oberwolfach workshop: Combinatorics, Probability and Computing Ob	erwolfach, Germany
	Rutgers Discrete Math Seminar	Piscataway, NJ
	Yale Combinatorics Seminar	New Haven, CT
	Stanford Combinatorics Seminar	Stanford, CA
2018	Clay Math Institute workshop: Recent Advances in Extremal Combinatorics	Oxford, UK
	$ICM\ satellite\ workshop\ -\!\!\!\!\!-\ Combinatorics:\ Extremal,\ Probabilistic\ and\ Additive$	São Paulo, Brazil

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	Simons Institute workshop: Pseudorandomness Reunion	Berkeley, CA
	MIT Workshop on Local Algorithms (WOLA 2018)	Cambridge, MA
	MIT workshop on Sublinear Algorithms: bootcamp tutorial	Cambridge, MA
	SIAM Conference on Discrete Mathematics: minisymposium	Denver, CO
	SIAM Conference on Discrete Mathematics: Dénes König Prize Lecture	Denver, CO
	Georgia Tech workshop: Algorithms and Randomness	Atlanta, GA
	Northeastern U. Network Science Institute Talk	Boston, MA
	AMS Sectional Meeting at Northeastern University	Boston, MA
	Rutgers Discrete Math Seminar	Piscataway, NJ
	Tsinghua YMSC minicourse	Beijing, China
	CMU ACO Seminar	Pittsburgh, PA
	Harvard CMSA workshop: Probabilistic and Extremal Combinatorics	Cambridge, MA
	UCLA Combinatorics Seminar	Los Angeles, CA
2017	Harvard CMSA workshop: Additive Combinatorics	Cambridge, MA
	Birmingham workshop: Interactions with Combinatorics	Birmingham, UK
	BGSMath workshop: Random Discrete Structures and Beyond	Barcelona, Spain
	SFSU: ACG Seminar	San Francisco, CA
	Stanford Math Department Colloquium	Stanford, CA
	Simons Institute workshop: Structure and Randomness	Berkeley, CA
	MIT Combinatorics Seminar	Cambridge, MA
	UC Berkeley Combinatorics Seminar	Berkeley, CA
	Simons Institute workshop: Pseudorandomness Boot Camp	Berkeley, CA
	Stanford Combinatorics Seminar	Stanford, CA
	Oberwolfach workshop: Combinatorics	Oberwolfach, Germany
2016	Turing Institute workshop: Large-scale structures in random graphs	London, UK
	Birmingham Combinatorics Seminar	Birmingham, UK
	IHÉS Seminar	Bures-sur-Yvette, France
	Warwick DIMAP Seminar	Coventry, UK
	LSE/Queen Mary Colloquia in Combinatorics	London, UK
	Oberwolfach workshop: Combinatorics, Probability and Computing	Oberwolfach, Germany
	Simons Symposium: Analysis of Boolean Functions	Krün, Germany
	British Mathematical Colloquium: Combinatorics Workshop	Bristol, UK
	Oxford Mathematical Institute North meets South Colloquium	Oxford, UK
	AMS-MAA Joint Mtgs: AMS Spec. Session on Pseudorandomness and Its	Applications Seattle, WA
2015	London School of Economics Discrete Mathematics and Game Theory Se	minar London, UK
	Queen Mary Combinatorics Seminar	London, UK
	Warwick Combinatorics Seminar	Coventry, UK

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	Oxford Combinatorial Theory Seminar		Oxford, UK
	Northeastern U. workshop: Random Graphs, Simplicial Complexes, and the	neir Appl'ns	Boston, MA
	U. of Chicago Combinatorics and Theoretical Computer Science Seminar		Chicago, IL
	Rutgers Discrete Math Seminar	Pi	scataway, NJ
	ICERM workshop: Crystals, Quasicrystals and Random Networks	P	rovidence, RI
2014	Atlanta Lectures Series in Combinatorics and Graph Theory at Emory		Atlanta, GA
	GSU Colloquium		Atlanta, GA
	CRM workshop: New Topics in Additive Combinatorics	I	Montreal, QC
	IMA workshop: Additive and Analytic Combinatorics	Min	neapolis, MN
	Clay Math Institute workshop: Extremal and Probabilistic Combinatorics		Oxford, UK
	Georgia Tech Combinatorics Seminar		Atlanta, GA
	IAS Computer Science/Discrete Mathematics Seminar	]	Princeton, NJ
	Oxford Combinatorial Theory Seminar		Oxford, UK
	London School of Economics Discrete Mathematics and Game Theory Sen	ninar	London, UK
	Eurandom: Minicourse on Graph Limits (6-hour minicourse co-taught with Christian Borgs)	Eindhoven,	Netherlands
	Oberwolfach workshop: Combinatorics	Oberwolfa	ch, Germany
2013	Simons Institute workshop: Neo-Classical Methods in Discrete Analysis		Berkeley, CA
	Rutgers Discrete Math Seminar	Pi	scataway, NJ
	MIT Combinatorics Seminar	Ca	mbridge, MA
	Yale Combinatorics and Probability Seminar	Ne	w Haven, CT
	Microsoft Research Theory Reading Group	Ca	mbridge, MA
	Oberwolfach workshop: Combinatorics and Probability	Oberwolfa	ch, Germany
2012	MIT Combinatorics Seminar	Ca	mbridge, MA
	SIAM Conference on Discrete Mathematics		Halifax, NS
2009	MIT Combinatorics Seminar	Ca	mbridge, MA

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# **Teaching**

#### **Graph Theory and Additive Combinatorics** (graduate, MIT)

Term	Enrollment (credit + listener)	Instructor evaluation (max 7)
Fall 2021	39 + 7	
Fall 2019	30 + 14	6.9
Fall 2017	31 + 17	7.0

#### **Probabilistic Methods in Combinatorics** (graduate, MIT)

Fall 2020	25 + 16	Not rated due to COVID
Spring 2019	47 + 25	6.9

## Combinatorial Analysis (undergraduate, MIT)

Fall 2018 22 + 7	6.8
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#### Additional teaching:

Mathematical Problem Solving (Putnam Seminar), MIT undergrad first-year advising seminar, every Fall since 2017

Polynomial Method in Combinatorics, graduate-level, Oxford, 2016

Undergraduate tutorials in geometry, Oxford, 2016

# Advising and mentorship

Current PhD students:

Aaron Berger

Ashwin Sah

Mehtaab Sawhney

Jonathan Tidor

Former PhD students:

Benjamin Gunby (Harvard PhD '21) → Rutgers postdoc

Undergraduate research supervised:

Yang Liu (2018) → Stanford PhD student

Ryan Alweiss (2018) → Princeton PhD student

Yunkun Zhou (2018–2019) → Stanford PhD student

Mehtaab Sawhney (2018–2020), *Morgan Prize winner* → MIT PhD student

Ashwin Sah (2018–2020), Morgan Prize winner  $\rightarrow$  MIT PhD student

David Stoner (2018–2019), Morgan Prize honorable mention  $\rightarrow$  Stanford PhD student

Yuan Yao (2019–2020) → MIT PhD student

Shengtong Zhang (2019–)

Hung-Hsun Yu (2019–2021) → Cambridge Part III student

Mihir Singhal (2019)

Zachary Chroman (2019) → Cambridge Part III student

Carl Schildkraut (2020–)

Milan Haiman (2020-)

Anqi Li (2021-)

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Dain Kim (2021-)

Postdoctoral researchers mentored:

Zilin Jiang (2018–2020)  $\rightarrow$  Assistant Professor at Arizona State University László Miklós Lovász (2018–2020)  $\rightarrow$  working in industry

#### **Service**

Undergraduate first-year advising, Fall 2017—current
Co-organizer of MIT Combinatorics Seminar, Fall 2017—current
Organizer of the MIT team for the Putnam Competition, Fall 2017—current
AMS—Simons Travel Grants Committee Member, 2021—2024

# Other Experiences and Activities

Organizer and Chief Coordinator of Cyberspace Mathematical Competition (CMC) 2020

Quantitative Research Intern, D. E. Shaw & Co., New York

MIT Lusztig PRIMES Mentor

Research Experience for Undergraduates at Duluth participant (mentor: Joe Gallian)

Deputy Leader for Canadian IMO Team

Instructor at Canadian IMO Training Camps

Mentor at AwesomeMath Summer Program, Dallas

Trainer at US Math Olympiad Summer Program, Lincoln, Nebraska

Teacher at Spirit of Math Schools, Toronto

CV updated: November 16, 2021