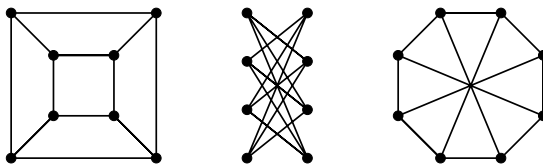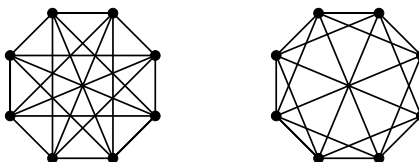## Problem Set 7. Due 10/30

Reminder: You must acknowledge your sources and collaborators (even if it is "none", you must write so). Failure to do so on this problem set will result in an automatic 2-point deduction.
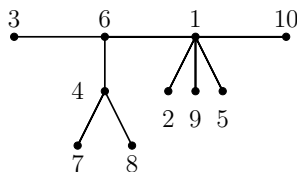
1. Which of the following graphs are isomorphic? Why? (Vertices are drawn with black dots)

2. Are the following graphs isomorphic? Why?

3. (a) What is the Prüfer code of the following tree? What is the map associated with it in Joyal's proof with left end 4 and right end 5? (The correct answer suffices)

   (b) Which labeled tree has Prüfer code (5,1,1,7,7,5)? (The correct answer suffices)

4. Prove that every graph with $n \geq 7$ vertices and at least $5n - 14$ edges contains a subgraph with minimum degree at least 6.

5. Prove that in a connected graph any two paths of maximum length share at least one vertex.

6. Prove that a graph is bipartite if and only if it contains no cycle of odd length.

7. Let T be a tree with $2k$ odd-degree vertices. Prove that T decomposes into k paths (i.e. its edge-set is the disjoint union of k paths).

8. Prove that a connected graph G is a tree if and only if any family of pairwise intersecting (i.e., vertex intersecting) paths $P_1, \ldots, P_k$ in G have a common vertex.