Chapter 1 Lecture Notes
MATH 1104
Winter 2023

1 System of Linear Equations

Some definitions to start with

Definition 1. A linear equation in variables $x_1, x_2, \cdots x_n$ is an equation that can be written in the form $a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = b$, where $a_1, a_2, \cdots a_n \in \mathbb{R}$

Definition 2. A system of linear equations (or linear system) is a collection of one or more linear equations involving the same variables , $x_1, x_2, \cdots, x_n$

$$a_{1_1} x_1 + a_{1_2} x_2 + \cdots + a_{1_n} x_n = b_1$$

$$a_{m_1} x_1 + a_{m_2} x_2 + \cdots + a_{m_n} x_n = b_m$$

Definition 3. A solution of the system is a list $(S_1, S_2, \cdots S_n)$ of numbers that makes each equation true when $S_1, S_2, \cdots S_n$ are substituted for $x_1, x_2, \cdots x_n$

Example 1.

$$x_1 - 2x_2 = -1$$
$$-x_1 + 3x_2 = 3$$

A solution here would be $(3, 2)$, where $x_1 = 3$ and $x_2 = 2$. Note that this satisfies BOTH equations.

To draw the graph set all the variables but one to 0. Repeat for the remaining variables. Only works for linear equations

A system of linear equations has either

1. No solutions (Inconsistent) (Parallel)

2. Unique solution (Consistent)

3. Infinite solutions (Consistent) (Same equation)

$$x_1 - 2x_2 + x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$-4x_1 + 5x_2 + 9x_3 = -9$$

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ -4 & 5 & 9 \end{bmatrix}$$ This is a coefficient matrix

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 8 \\ 9 \end{bmatrix}$$

Combine the A and b

Fundamental matrix questions:

1. Does a solution of a linear system exist?

2. How many solutions does it have if it is consistent?