STATISTICS

MEASURES OF CENTRAL TENDENCY :

An average value or a central value of a distribution is the value of variable which is representative of the entire distribution, this representative value are called the measures of central tendency.

Generally the following five measures of central tendency.

(a) Mathematical average

(i) Arithmetic mean (ii) Geometric mean (iii) Harmonic mean

(b) Positional average

(i) Median (ii) Mode

1. ARITHMETIC MEAN :

(i) For ungrouped dist. : If $x_1, x_2, \ldots\ldots x_n$ are n values of variate x_i then their A.M. $\bar{x}$ is defined as

$$\bar{x} = \frac{x_1 + x_2 + \ldots.. + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$\Rightarrow$ $\Sigma x_i = n\bar{x}$

(ii) For ungrouped and grouped freq. dist. : If $x_1, x_2, \ldots. x_n$ are values of variate with corresponding frequencies $f_1, f_2, \ldots f_n$ then their A.M. is given by

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \ldots. + f_nx_n}{f_1 + f_2 + \ldots. + f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{N}, \quad \text{where } N = \sum_{i=1}^{n} f_i$$

(iii) By short method : If the value of x_i are large, then the calculation of A.M. by using previous formula is quite tedious and time consuming. In such case we take deviation of variate from an arbitrary point a.

Let $d_i = x_i - a$

$\therefore$ $\bar{x} = a + \dfrac{\Sigma f_i d_i}{N}$, where a is assumed mean

(iv) By step deviation method : Sometime during the application of short method of finding the A.M. If each deviation d_i are divisible by a common number h(let)

Let $u_i = \dfrac{d_i}{h} = \dfrac{x_i - a}{h}$

$\therefore$ $\bar{x} = a + \left(\dfrac{\Sigma f_i u_i}{N}\right)h$

Illustration 1 :

If the mean of the series $x_1, x_2, \ldots\ldots, x_n$ is $\bar{x}$, then the mean of the series $x_i + 2i$, i = 1, 2, ..., n will be-

(1) $\bar{x} + n$ (2) $\bar{x} + n + 1$ (3) $\bar{x} + 2$ (4) $\bar{x} + 2n$

Solution :

As given $\bar{x} = \dfrac{x_1 + x_2 + \ldots.. + x_n}{n}$ ....(1)

If the mean of the series $x_i + 2i$, i = 1, 2, ..., n be $\bar{X}$, then

$$\bar{X} = \frac{(x_1 + 2) + (x_2 + 2.2) + (x_3 + 2.3) + \ldots.. + (x_n + 2.n)}{n}$$

$$= \frac{x_1 + x_2 + \ldots.. + x_n}{n} + \frac{2(1 + 2 + 3 + \ldots.. + n)}{n}$$

$$= \bar{x} + \frac{2n(n+1)}{2n} \quad \text{from (1)}$$

$= \bar{x} + n + 1$ Ans. (2)

Illustration 2 :

Find the A.M. of the following freq. dist.

x_i	5	8	11	14	17
f_i	4	5	6	10	20

Solution :

Here $N = \Sigma f_i = 4 + 5 + 6 + 10 + 20 = 45$

$\Sigma f_i x_i = (5 \quad 4) + (8 \quad 5) + (11 \quad 6) + (14 \quad 10) + (17 \quad 20) = 606$

$\therefore \quad \bar{x} = \dfrac{\Sigma f_i x_i}{N} = \dfrac{606}{45} = 13.47$

Illustration 3 :

Find the mean of the following freq. dist.

x_i	5	15	25	35	45	55
f_i	12	18	27	20	17	6

Solution :

Let assumed mean a = 35, h = 10

here $N = \Sigma f_i = 100, \quad u_i = \dfrac{(x_i - 35)}{10}$

$\therefore \quad \Sigma f_i u_i = (12 \quad -3) + (18 \quad -2) + (27 \quad -1) + (20 \quad 0) + (17 \quad 1) + (6 \quad 2) = -70$

$\therefore \quad \bar{x} = a + \left(\dfrac{\Sigma f_i u_i}{N}\right) h = 35 + \dfrac{(-70)}{100} \quad 10 = 28$

Illustration 4 :

If a variable takes the value 0, 1, 2......n with frequencies proportional to the bionomial coefficients $^nC_0$, $^nC_1$,.....,^nC_n then the mean of the distribution is-

(1) $\dfrac{n(n+1)}{4}$ (2) $\dfrac{n}{2}$ (3) $\dfrac{n(n-1)}{2}$ (4) $\dfrac{n(n+1)}{2}$

Solution :

$N = \sum f_i = k\,[{}^nC_0 + {}^nC_1 + \ldots. + {}^nC_n] = k2^n$

$\sum f_i\, x_i = k\,[1.{}^nC_1 + 2.\,{}^nC_2 + \ldots. + n\,{}^nC_n] = k\sum_{r=1}^{n} r.{}^nC_r = kn\sum_{r=1}^{n} {}^{n-1}C_{r-1} = kn2^{n-1}$

Thus $\bar{x} = \dfrac{1}{2^n}(n \quad 2^{n-1}) = \dfrac{n}{2}$. **Ans. (2)**

(v) Weighted mean : If $w_1, w_2, \ldots\ldots w_n$ are the weights assigned to the values $x_1, x_2, \ldots\ldots x_n$ respectively then their weighted mean is defined as

$$\text{Weighted mean} = \frac{w_1x_1 + w_2x_2 + \ldots.. + w_nx_n}{w_1 + \ldots..+ w_n} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

Illustration 5 :

Find the weighted mean of first n natural numbers when their weights are equal to their squares respectively

Solution :

$$\text{Weighted Mean} = \frac{1.1^2 + 2.2^2 + \ldots.+ n.n^2}{1^2 + 2^2 + \ldots.+ n^2} = \frac{1^3 + 2^3 + \ldots.. + n^3}{1^2 + 2^2 + \ldots.+ n^2} = \frac{[n(n+1)/2]^2}{[n(n+1)(2n+1)/6]} = \frac{3n(n+1)}{2(2n+1)}$$

(vi) Combined mean : If $\bar{x}_1$ and $\bar{x}_2$ be the means of two groups having n_1 and n_2 terms respectively then the mean (combined mean) of their composite group is given by

combined mean = $\dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

If there are more than two groups then, combined mean = $\dfrac{n_1\bar{x}_1 + n_1\bar{x}_2 + n_3\bar{x}_3 + ...}{n_1 + n_2 + n_3 + ...}$

Illustration 6 :

The mean income of a group of persons is Rs. 400 and another group of persons is Rs. 480. If the mean income of all the persons of these two groups is Rs. 430 then find the ratio of the number of persons in the groups.

Solution :

Here $\bar{x}_1 = 400$, $\bar{x}_2 = 480$, $\bar{x} = 430$

$\because \quad \bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \Rightarrow 430 = \dfrac{400n_1 + 480n_2}{n_1 + n_2}$

$\Rightarrow \dfrac{n_1}{n_2} = \dfrac{5}{3}$

(vii) Properties of Arithmetic mean :

- Sum of deviations of variate from their A.M. is always zero i.e. $\Sigma(x_i - \bar{x}) = 0$, $\Sigma f_i(x_i - \bar{x}) = 0$
- Sum of square of deviations of variate from their A.M. is minimum i.e. $\Sigma(x_i - \bar{x})^2$ is minimum
- If $\bar{x}$ is the mean of variate x_i then A.M. of $(x_i + \lambda) = \bar{x} + \lambda$
  A.M. of $(\lambda x_i) = \lambda\bar{x}$
  A.M. of $(ax_i + b) = a\bar{x} + b$ (where λ, a, b are constant)
- A.M. is independent of change of assumed mean i.e. it is not effected by any change in assumed mean.

Do yourself - 1 :

(i) If in an examination different weights are assigned to different subjects Physics (2), Chemistry (1), English (1), Mathematics (2) A student scores 60 in Physics, 70 in Chemistry, 70 in English and 80 in Mathematics, then weighted mean is -

(1) 60 (2) 70 (3) 80 (4) 85

(ii) The mean of the following freq. table is 50 and $\Sigma f = 120$

class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
f	17	f_1	32	f_2	19

the missing frequencies are-

(1) 28, 24 (2) 24, 36 (3) 36, 28 (4) None of these

(iii) A student obtained 75%, 80%, 85% marks in three subjects. If the marks of another subject are added then his average marks can not be less than-

(1) 60% (2) 65% (3) 80% (4) 90%

2. GEOMETRIC MEAN :

(i) For ungrouped dist. : If $x_1, x_2, ... x_n$ are n positive values of variate then their geometric mean G is given by

$$G = (x_1 \ x_2 \ ..... \ x_n)^{1/n}$$

$$\Rightarrow G = \text{antilog}\left[\frac{1}{n}\sum_{i=1}^{n}\log x_i\right]$$

(ii) **For freq. dist.** : If x_1, x_2, ... x_n are n positive values with corresponding frequencies f_1, f_2, ... f_n resp. then their G.M.

$$G = (x_i^{f_1} \times x_2^{f_2} \times \ldots\ldots \times x_n^{f_n})^{1/N}$$

$$\Rightarrow \quad G = \text{antilog}\left[\frac{1}{N}\sum_{i=1}^{n} f_i \log x_i\right]$$

Note :- If G_1 and G_2 are geometric means of two series which containing n_1 and n_2 positive values resp. and G is geometric mean of their combined series then

$$G = (G_1^{n_1} \times G_2^{n_2})^{\frac{1}{n_1+n_2}}$$

$$\Rightarrow \quad G = \text{antilog}\left[\frac{n_1 \log G_1 + n_2 \log G_2}{n_1 + n_2}\right]$$

Illustration 7 :

Find the G.M. of 1, 2, 2^2, ..., 2^n

Solution :

$$\text{G.M.} = (1.2.2^2\ldots\ldots2^n)^{\frac{1}{n+1}}$$

$$= \left[2^{\frac{n(n+1)}{2}}\right]^{\frac{1}{n+1}} = 2^{n/2}$$

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3. HARMONIC MEAN :

(i) **For ungrouped dist.** : If x_1, x_2, ... x_n are n non-zero values of variate then their harmonic mean H is defined as

$$H = \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\ldots.+\frac{1}{x_n}} = \frac{n}{\sum_{i=1}^{n}\frac{1}{x_i}}$$

(ii) For freq. dist. : If x_1, x_2, ... x_n are n non-zero values of variate with corresponding frequencies f_1, f_2, ... f_n respectively the their H.M.

$$H = \frac{N}{\frac{f_1}{x_1}+\frac{f_2}{x_2}+\ldots.+\frac{f_n}{x_n}} = \frac{N}{\sum_{i=1}^{n}\frac{f_i}{x_i}}$$

Illustration 8 :

Find the H.M. of $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots.. \frac{1}{17}$

Solution :

$$\text{H.M.} = \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\ldots..+\frac{1}{x_n}} = \frac{16}{2+3+\ldots.+17} = \frac{2}{19}$$

Note :- If A, G, H are A.M. G.M. H.M. of a series respectively then

$$A \geq G \geq H$$

4. MEDIAN :

The median of a series is the value of middle term of the series when the values are written in ascending order. Therefore median, divided an arranged series into two equal parts.

Formulae of median :

(i) For ungrouped distribution : Let n be the number of variate in a series then

$$\text{Median} = \begin{cases} \left(\dfrac{n+1}{2}\right)^{th} \text{ term, (when n is odd)} \\ \text{Mean of} \left(\dfrac{n}{2}\right)^{th} \text{ and } \left(\dfrac{n}{2}+1\right)^{th} \text{ terms, (when n is even)} \end{cases}$$

(ii) For ungrouped freq. dist. : First we prepare the cumulative frequency (c.f.) column and Find value of N then

$$\text{Median} = \begin{cases} \left(\dfrac{N+1}{2}\right)^{th} \text{ term, (when N is odd)} \\ \text{Mean of} \left(\dfrac{N}{2}\right)^{th} \text{ and } \left(\dfrac{N}{2}+1\right)^{th} \text{ terms, (when N is even)} \end{cases}$$

(iii) For grouped freq. dist : Prepare c.f. column and find value of $\frac{N}{2}$ then find the class which contain value of c.f. is equal or just greater to N/2, this is median class

$$\therefore \text{ Median} = \ell + \frac{\left(\frac{N}{2}-F\right)}{f} h$$

where ℓ — lower limit of median class

f — freq. of median class

F — c.f. of the class preceeding median class

h — Class interval of median class

Illustration 9 :

Find the median of following freq. dist.

class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
f	8	30	40	12	10

Solution :

class	f_i	c.f.
0 – 10	8	8
10 – 20	30	38
20 – 30	40	78
30 – 40	12	90
40 – 50	10	100

Here $\frac{N}{2} = \frac{100}{2} = 50$ which lies in the value 78 of c.f. hence corresponding class of this c.f. is 20-30 is the median class, so

$\ell = 20$, $f = 40$, $F = 38$, $h = 10$

$$\therefore \text{ Median} = \ell + \frac{\left(\frac{N}{2}-F\right)}{f} h = 20 + \frac{(50-38)}{40} 10 = 23$$

5. MODE :

In a frequency distribution the mode is the value of that variate which have the maximum frequency

Method for determining mode :

(i) **For ungrouped dist.** : The value of that variate which is repeated maximum number of times

(ii) **For ungrouped freq. dist.** : The value of that variate which have maximum frequency.

(iii) **For grouped freq. dist.** : First we find the class which have maximum frequency, this is model calss

$$\therefore \text{ Mode} = \ell + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \quad h$$

where ℓ — lower limit of model class

f_0 — freq. of the model class

f_1 — freq. of the class preceeding model class

f_2 — freq. of the class succeeding model class

h — class interval of model class

Illustration 10 :

Find the mode of the following frequecy dist

class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
f_i	2	18	30	45	35	20	6	3

Solution :

Here the class 30–40 has maximum freq. so this is the model class

$\ell = 30,\ f_0 = 45,\ f_1 = 30,\ f_2 = 35,\ h = 10$

$$\therefore \text{ Mode} = \ell + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \quad h = 30 + \frac{45 - 30}{2 \times 45 - 30 - 35} \quad 10 = 36$$

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6. RELATION BETWEEN MEAN, MEDIAN AND MODE :

In a moderately asymmetric distribution following relation between mean, median and mode of a distribution. It is known as imprical formula.

Mode = 3 Median – 2 Mean

Note (i) Median always lies between mean and mode

(ii) For a symmetric distribution the mean, median and mode are coincide.

Do yourself - 2 :

(i) Median of the distribution :

${}^{20}C_0,\ {}^{20}C_{19},\ {}^{20}C_2,\ {}^{20}C_{17},\ {}^{20}C_4,\ {}^{20}C_{15},\ {}^{20}C_9$

${}^{20}C_{10},\ {}^{20}C_6,\ {}^{20}C_{13},\ {}^{20}C_{12}$ will be -

(1) ${}^{20}C_6$ (2) ${}^{20}C_{15}$ (3) ${}^{20}C_9$ (4) None

(ii) Let a, b, c and d are real numbers (d > a > b > c). If mean and median of the distribution a, b, c, d are 5 and 6 respectively then the value of $-a + 3d + 3c - b$ is :

(1) 8 (2) 10 (3) 12 (4) None

(iii) Let median of 23 observations is 50 if smallest 13 observations are increased by 2 then median will become :-

(1) 50 (2) 52 (3) Can't say anything (4) None of these

7. MEASURES OF DISPERSION :

The dispersion of a statistical distribution is the measure of deviation of its values about the their average (central) value.

It gives an idea of scatteredness of different values from the average value.

Generally the following measures of dispersion are commonly used.

(i) Range (ii) Mean deviation (iii) Variance and standard deviation

(i) Range : The difference between the greatest and least values of variate of a distribution, are called the range of that distribution.

If the distribution is grouped distribution, then its range is the difference between upper limit of the maximum class and lower limit of the minimum class.

Also, coefficient of range $= \dfrac{\text{difference of extreme values}}{\text{sum of extreme values}}$

Illustration 11 :

Find the range of following numbers 10, 8, 12, 11, 14, 9, 6

Solution :

Here greatest value and least value of the distribution are 14 and 6 resp. therefore

Range = 14 – 6 = 8

(ii) Mean deviation (M.D.) : The mean deviation of a distribution is, the mean of absolute value of deviations of variate from their statistical average (Mean, Median, Mode).

If A is any statistical average of a distribution then mean deviation about A is defined as

$$\text{Mean deviation} = \frac{\sum_{i=1}^{n} |x_i - A|}{n} \qquad \text{(for ungrouped dist.)}$$

$$\text{Mean deviation} = \frac{\sum_{i=1}^{n} f_i\,|x_i - A|}{N} \qquad \text{(for freq. dist.)}$$

Note :- Mean deviation is minimum when it taken about the median

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Illustration 12 :

Find the mean deviation of number 3, 4, 5, 6, 7

Solution :

Here n = 5, $\bar{x} = 5$

$$\therefore \quad \text{Mean deviation} = \frac{\Sigma |x_i - \bar{x}|}{n}$$

$$= \frac{1}{5}[|3-5| + |4-5| + |5-5| + |6-5| + |7-5|]$$

$$= \frac{1}{5}[2+1+0+1+2] = \frac{6}{5} = 1.2$$

Illustration 13 :

Find the mean deviation about mean from the following data

x_i	3	9	17	23	27
f_i	8	10	12	9	5

Solution :

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
3	8	24	12	96
9	10	90	6	60
17	12	204	2	24
23	9	207	8	72
27	5	135	12	60
	N = 44	$\Sigma f_i x_i = 660$		$\Sigma f_i \lvert x_i - \bar{x} \rvert = 312$

Mean $(\bar{x}) = \dfrac{\Sigma f_i x_i}{N} = \dfrac{660}{44} = 15$

Mean deviation $= \dfrac{\Sigma f_i \lvert x_i - \bar{x} \rvert}{N} = \dfrac{312}{44} = 7.09$

Do yourself - 3 :

(i) The mean deviation about median from the following data 340, 150, 210, 240, 300, 310, 320, is-

(1) 52.4 (2) 52.5 (3) 52.8 (4) none of these

(ii) The mean deviation of the series $a, a + d, a + 2d, ..., a + 2nd$ from its mean is-

(1) $\dfrac{n+1}{2n+1}|d|$ (2) $\dfrac{n(n+1)}{2n+1}|d|$ (3) $\dfrac{n(n-1)}{2n+1}|d|$ (4) none of these

(iii) Variance and standard deviation : The variance of a distribution is, the mean of squares of deviation of variate from their mean. It is denoted by σ^2 or var(x).

The positive square root of the variance are called the standard deviation. It is denoted by σ or S.D.

Hence standard deviation $= +\sqrt{\text{variance}}$

Formulae for variance :

(i) for ungrouped dist. :

$$\sigma_x^2 = \frac{\Sigma (x_i - \bar{x})^2}{n}$$

$$\sigma_x^2 = \frac{\Sigma x_i^2}{n} - \bar{x}^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$\sigma_d^2 = \frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2, \quad \text{where } d_i = x_i - a$$

(ii) For freq. dist. :

$$\sigma_x^2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{N}$$

$$\sigma_x^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2 = \frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2$$

$$\sigma_d^2 = \frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2$$

$$\sigma_u^2 = h^2\left[\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2\right] \quad \text{where } u_i = \frac{d_i}{h}$$

(iii) Coefficient of S.D. = $\frac{\sigma}{\bar{x}}$

Coefficient of variation = $\frac{\sigma}{\bar{x}}$ 100 (in percentage)

Note :- $\sigma^2 = \sigma_x^2 = \sigma_d^2 = h^2\sigma_u^2$

Illustration 14 :

Find the variance of first n natural numbers

Solution :

$$\sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 = \frac{\Sigma n^2}{n} - \left(\frac{\Sigma n}{n}\right)^2 = \frac{n(n+1)(2n+1)}{6n} - \left\{\frac{n(n+1)}{2n}\right\}^2 = \frac{n^2-1}{12}$$

Illustration 15 :

If $\sum_{i=1}^{18}(x_i - 8) = 9$ and $\sum_{i=1}^{18}(x_i - 8)^2 = 45$, then find the standard deviation of $x_1, x_2, \ldots\; x_{18}$

Solution :

Let $(x_i - 8) = d_i$

$$\therefore\quad \sigma_x = \sigma_d = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2} = \sqrt{\frac{45}{18} - \left(\frac{9}{18}\right)^2} = \sqrt{\frac{5}{2} - \frac{1}{4}} = \frac{3}{2}$$

Illustration 16 :

Find the coefficient of variation of first n natural numbers

Solution :

For first n natural numbers.

Mean $(\bar{x}) = \frac{n+1}{2}$, S.D.$(\sigma) = \sqrt{\frac{n^2-1}{12}}$

$$\therefore\quad \text{coefficient of variance} = \frac{\sigma}{\bar{x}}\quad 100 = \sqrt{\frac{n^2-1}{12}}\quad \frac{1}{\left(\frac{n+1}{2}\right)}\quad 100 = \sqrt{\frac{(n-1)}{3(n+1)}}\quad 100$$

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8. MEAN SQUARE DEVIATION :

The mean square deviation of a distrubution is the mean of the square of deviations of variate from assumed mean. It is denoted by S^2

Hence $S^2 = \frac{\Sigma(x_i - a)^2}{n} = \frac{\Sigma d_i^2}{n}$ (for ungrouped dist.)

$S^2 = \frac{\Sigma f_i(x_i - a)^2}{N} = \frac{\Sigma f_i d_i^2}{N}$ (for freq. dist.), where $d_i = (x_i - a)$

Illustration 17 :

The mean square deviation of a set of n observations $x_1, x_2, \ldots, x_n$ about a point c is defined as $\frac{1}{n}\sum_{i=1}^{n}(x_i - c)^2$

The mean square deviation about –2 and 2 are 18 and 10 respectively, then standard deviation of this set of observations is-

(1) 3 (2) 2 (3) 1 (4) None of these

Solution :

$\because \quad \frac{1}{n}\Sigma(x_i+2)^2 = 18$ and $\frac{1}{n}\Sigma(x_i-2)^2 = 10$

$\Rightarrow \quad \Sigma(x_i+2)^2 = 18n$ and $\Sigma(x_i-2)^2 = 10n$

$\Rightarrow \quad \Sigma(x_i+2)^2 + \Sigma(x_i-2)^2 = 28\,n$ and $\Sigma(x_i+2)^2 - \Sigma(x_i-2)^2 = 8\,n$

$\Rightarrow \quad 2\Sigma x_i^2 + 8n = 28\,n$ and $8\Sigma x_i = 8n$

$\Rightarrow \quad \Sigma x_i^2 = 10\,n$ and $\Sigma x_i = n$

$\Rightarrow \quad \frac{\Sigma x_i^2}{n} = 10$ and $\frac{\Sigma x_i}{n} = 1$

$\therefore \quad \sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2} = \sqrt{10-(1)^2} = 3$ **Ans. (1)**

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9. RELATION BETWEEN VARIANCE AND MEAN SQUARE DEVIATION :

$\because \quad \sigma^2 = \frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2$

$\Rightarrow \quad \sigma^2 = s^2 - d^2$, where $d = \overline{x} - a = \frac{\Sigma f_i d_i}{N}$

$\Rightarrow \quad s^2 = \sigma^2 + d^2 \Rightarrow s^2 \geq \sigma^2$

Hence the variance is the minimum value of mean square deviation of a distribution

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Illustration 18 :

Determine the variance of the following frequency dist.

class	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12
f_i	2	7	12	19	9	1

Solution :

Let a = 7, h = 2

class	x_i	f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$	$f_i u_i^2$
0 – 2	1	2	–3	–6	18
2 – 4	3	7	–2	–14	28
4 – 6	5	12	–1	–12	12
6 – 8	7	19	0	0	0
8 – 10	9	9	1	9	9
10 – 12	11	1	2	2	4
		N = 50		$\Sigma f_i u_i = -21$	$\Sigma f_i u_i^2 = 71$

$\therefore \quad \sigma^2 = h^2\left[\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2\right] = 4\left[\frac{71}{50} - \left(\frac{-21}{50}\right)^2\right] = 4[1.42 - 0.1764] = 4.97$

10. MATHEMATICAL PROPERTIES OF VARIANCE :

- Var.$(x_i + \lambda)$ = Var.(x_i)
  Var.$(\lambda x_i) = \lambda^2$.Var$(x_i)$
  Var$(ax_i + b) = a^2$.Var(x_i)
  where λ, a, b, are constant
- If means of two series containing n_1, n_2 terms are $\bar{x}_1$, $\bar{x}_2$ and their variance's are σ_1^2, σ_2^2 respectively and their combined mean is $\bar{x}$ then the variance σ^2 of their combined series is given by following formula

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{(n_1 + n_2)} \quad \text{where } d_1 = \bar{x}_1 - \bar{x},\ d_2 = \bar{x}_2 - \bar{x}$$

i.e.

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1 + n_2)^2}(\bar{x}_1 - \bar{x}_2)^2$$

Do yourself - 4 :

(i) The variance of first 20-natural numbers is-

(1) $\frac{133}{4}$ (2) $\frac{379}{12}$ (3) $\frac{133}{2}$ (4) $\frac{399}{4}$

(ii) The mean and variance of a series containing 5 terms are 8 and 24 respectively. The mean and variance of another series containing 3 terms are also 8 and 24 respectively. The variance of their combined series will be-

(1) 20 (2) 24 (3) 25 (4) 42

(iii) Variance of the data given below is

Size of item	3.5	4.5	5.5	6.5	7.5	8.5	9.5
Frequency	3	7	22	60	85	32	8

(1) 1.29 (2) 2.19 (3) 1.32 (4) none of these

(iv) The mean and variance of 5 observations of an experiment are 4 and 5.2 respectively. If from these observations three are 1, 2 and 6, then the remaining will be-

(1) 2, 9 (2) 5, 6 (3) 4, 7 (4) 3, 8

ANSWERS FOR DO YOURSELF

1.	(i) 2	(ii) 1	(iii) 1	
2.	(i) 2	(ii) 3	(iii) 3	
3.	(i) 3	(ii) 2		
4.	(i) 1	(ii) 2	(iii) 3	(iv) 3

CHECK YOUR GRASP STATISTICS EXERCISE-I

Arithmetic mean, weighted mean, Combined mean

1. Mean of the first n terms of the A.P. a, (a + d), (a + 2d), ... is-

(1) $a+\frac{nd}{2}$ (2) $a+\frac{(n-1)d}{2}$

(3) a + (n − 1) d (4) a + nd

2. The A.M. of first n even natural number is -

(1) n(n + 1) (2) $\frac{n+1}{2}$ (3) $\frac{n}{2}$ (4) n + 1

3. The A.M. of ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ...${}^nC_n$ is -

(1) $\frac{2^n}{n}$ (2) $\frac{2^{n+1}}{n}$ (3) $\frac{2^n}{n+1}$ (4) $\frac{2^{n+1}}{n+1}$

4. If the mean of numbers 27, 31, 89, 107, 156 is 82, then the mean of numbers 130, 126, 68, 50, 1 will be-

(1) 80 (2) 82 (3) 75 (4) 157

5. If the mean of n observations x_1, x_2, ...x_n is $\bar{x}$, then the sum of deviations of observations from mean is :-

(1) 0 (2) $n\bar{x}$

(3) $\frac{\bar{x}}{n}$ (4) None of these

6. The mean of 9 terms is 15. if one new term is added and mean become 16, then the value of new term is :-

(1) 23 (2) 25 (3) 27 (4) 30

7. If the mean of first n natural numbers is equal to $\frac{n+7}{3}$, then n is equal to-

(1) 10 (2) 11

(3) 12 (4) none of these

8. The mean of first three terms is 14 and mean of next two terms is 18. The mean of all the five terms is-

(1) 15.5 (2) 15.0 (3) 15.2 (4) 15.6

9. If the mean of five observations x, x + 2 , x + 4, x+ 6 and x + 8 is 11, then the mean of last three obsevations is-

(1) 11 (2) 13 (3) 15 (4) 17

10. The mean of a set of numbers is $\bar{x}$. If each number is decreased by λ, the mean of the new set is-

(1) $\bar{x}$ (2) $\bar{x}+\lambda$ (3) $\lambda-\bar{x}$ (4) $\bar{x}-\lambda$

11. The mean of 50 observations is 36. If its two observations 30 and 42 are deleted, then the mean of the remaining observations is-

(1) 48 (2) 36

(3) 38 (4) none of these

12. In a frequency dist. , if d_i is deviation of variates from a number ℓ and mean = $\ell + \frac{\Sigma f_i d_i}{\Sigma f_i}$, then ℓ is :-

(1) Lower limit (2) Assumed mean

(3) Number of observation (4) Class interval

13. The A.M. of n observation is $\bar{x}$. If the sum of n − 4 observations is K, then the mean of remaining observations is-

(1) $\frac{\bar{x}-K}{4}$ (2) $\frac{n\bar{x}-K}{n-4}$

(3) $\frac{n\bar{x}-K}{4}$ (4) $\frac{n\bar{x}-(n-4)K}{4}$

14. The mean of values $1,\frac{1}{2},\frac{1}{3},......\frac{1}{n}$ which have frequencies 1, 2, 3, ... n resp., is :-

(1) $\frac{2n+1}{3}$ (2) $\frac{2}{n}$ (3) $\frac{n+1}{2}$ (4) $\frac{2}{n+1}$

15. The sum of squares of deviation of variates from their A.M. is always :-

(1) Zero (2) Minimum

(3) Maximum (4) Nothing can be said

16. If the mean of following feq. dist. is 2.6, then the value of f is :-

x_i	1	2	3	4	5
f_i	5	4	f	2	3

(1) 1 (2) 3

(3) 8 (4) None of these

17. The weighted mean (W.M.) is computed by the formula ?

(1) W.M. = $\frac{\Sigma x_i}{\Sigma w_i}$ (2) $W.M.=\frac{\Sigma w_i}{\Sigma x_i}$

(3) W.M. = $\frac{\Sigma w_i x_i}{\Sigma x_i}$ (4) W.M.= $\frac{\Sigma w_i x_i}{\Sigma w_i}$

18. The weighted mean of first n natural numbers when their weights are equal to corresponding natural number, is :-

(1) $\frac{n+1}{2}$ (2) $\frac{2n+1}{3}$

(3) $\frac{(n+1)(2n+1)}{6}$ (4) None of these

19. The average income of a group of persons is $\bar{x}$ and that of another group is $\bar{y}$. If the number of persons of both group are in the ratio 4 : 3, then average income of combined group is :-

(1) $\frac{\bar{x}+\bar{y}}{7}$ (2) $\frac{3\bar{x}+4\bar{y}}{7}$

(3) $\frac{4\bar{x}+3\bar{y}}{7}$ (4) None of these

20. In a group of students, the mean weight of boys is 65 kg. and mean weight of girls is 55 kg. If the mean weight of all students of group is 61 kg, then the ratio of the number of boys and girls in the group is :-

(1) 2 : 3 (2) 3 : 1 (3) 3 : 2 (4) 4 : 3

Geometric mean, Harmonic mean

21. The G.M. of n positive terms $x_1, x_2, ... x_n$ is :-

(1) $(x_1 \quad x_2 \quad ..... \quad x_n)^n$ (2) $\frac{1}{n}(x_1 \quad x_2 \quad ... \quad x_n)$

(3) $(x_1 \quad x_2 \quad ... \quad x_n)^{1/n}$ (4) None of these

22. The G.M. of numbers 4, 5, 10, 20, 25 is :-

(1) 12.8 (2) 10

(3) 7.8 (4) None of these

23. The geometric mean of the first n terms of the series a, ar, ar^2, ..., is-

(1) $ar^{n/2}$ (2) ar^n (3) $ar^{(n-1)/2}$ (4) ar^{n-1}

24. If G_1 and G_2 are geometric mean of two series of sizes n_1 and n_2 resp. and G is geometric mean of their combined series, then log G is equal to :-

(1) $\log G_1 + \log G_2$ (2) $n_1 \log G_1 + n_2 \log G_2$

(3) $\frac{\log G_1 + \log G_2}{n_1 + n_2}$ (4) $\frac{n_1 \log G_1 + n_2 \log G_2}{n_1 + n_2}$

25. The Harmonic mean of 3, 7, 8, 10, 14 is-

(1) $\frac{3+7+8+10+14}{5}$

(2) $\frac{5}{3+7+8+10+14}$

(3) $\frac{\frac{1}{3}+\frac{1}{7}+\frac{1}{8}+\frac{1}{10}+\frac{1}{14}}{5}$

(4) $\frac{5}{\frac{1}{3}+\frac{1}{7}+\frac{1}{8}+\frac{1}{10}+\frac{1}{14}}$

26. The H.M. of the numbers 2, 3, 4 is :-

(1) 3 (2) $2(3)^{1/3}$ (3) $\frac{36}{13}$ (4) $\frac{13}{36}$

27. The H.M. of following freq. dist. is :-

x_i	3	6	9	12
f_i	1	2	3	4

(1) 9 (2) 3

(3) 7.5 (4) None of these

28. A boy goes to school from his home at a speed of x km/hr. and comes back at a speed of y km/hr. then the average speed of the boy is :-

(1) $\frac{x+y}{2}$ km / hr (2) $\sqrt{xy}$ km / hr

(3) $\frac{2xy}{x+y}$ km / hr (4) $\frac{x+y}{2xy}$ km / hr

Median, Mode

29. The median of an arranged series of n even observations, will be :-

(1) $\left(\frac{n+1}{2}\right)$ th term

(2) $\left(\frac{n}{2}\right)$ th term

(3) $\left(\frac{n}{2}+1\right)$ th term

(4) Mean of $\left(\frac{n}{2}\right)$ th and $\left(\frac{n}{2}+1\right)$ th terms

30. The median of the numbers 6, 14, 12, 8, 10, 9, 11, is :-

(1) 8 (2) 10 (3) 10.5 (4) 11

31. Median of the following freq. dist.

x_i	3	6	10	12	7	15
f_i	3	4	2	8	13	10

(1) 7 (2) 10

(3) 8.5 (4) None of these

32. Median is independent of change of :-

(1) only Origin

(2) only Scale

(3) Origin and scale both

(4) Neither origin nor scale

33. A series which have numbers three 4's, four 5's, five 6's, eight 7's, seven 8's and six 9's then the mode of numbers is :-

(1) 9 (2) 8 (3) 7 (4) 6

34. Mode of the following freqency distribution

x :	4	5	6	7	8
f :	6	7	10	8	3

(1) 5 (2) 6 (3) 8 (4) 10

35. The mode of the following freq. dist is :-

Class	1–10	11–20	21–30	31–40	41–50
f_i	5	7	8	6	4

(1) 24 (2) 23.83

(3) 27.16 (4) None of these

Symmetric and asymmetric distribution, Range

36. For a normal dist :-
(1) mean = median
(2) median = mode
(3) mean = mode
(4) mean = median = mode

37. The relationship between mean, median and mode for a moderately skewed distribution is-
(1) mode = median – 2 mean
(2) mode = 2 median – mean
(3) mode = 2 median – 3 mean
(4) mode = 3 median – 2 mean

38. The range of observations 2, 3, 5, 9, 8, 7, 6, 5, 7, 4, 3 is :-
(1) 6 (2) 7 (3) 5.5 (4) 11

Mean Deviation

39. The mean deviation of a frequency dist. is equal to :-
(1) $\frac{\Sigma d_i}{\Sigma f_i}$ (2) $\frac{\Sigma |d_i|}{\Sigma f_i}$
(3) $\frac{\Sigma f_i d_i}{\Sigma f_i}$ (4) $\frac{\Sigma f_i |d_i|}{\Sigma f_i}$

40. Mean deviation from the mean for the observation –1, 0, 4 is-
(1) $\sqrt{\frac{14}{3}}$ (2) $\frac{2}{3}$
(3) 2 (4) none of these

41. Mean deviation of the observations 70, 42, 63, 34, 44, 54, 55, 46, 38, 48 from median is :-
(1) 7.8 (2) 8.6
(3) 7.6 (4) 8.8

42. Mean deviation of 5 observations from their mean 3 is 1.2, then coefficient of mean deviation is :-
(1) 0.24 (2) 0.4
(3) 2.5 (4) None of these

43. The mean deviation from median is
(1) greater than the mean deviation from any other central value
(2) less than the mean deviation from any other central value
(3) equal to the mean deviation from any other central value
(4) maximum if all values are positive

Variance and Standard Deviation

44. The variate x and u are related by $u = \frac{x-a}{h}$ then correct relation between σ_x and σ_u is :-
(1) $\sigma_x = h\sigma_u$ (2) $\sigma_x = h + \sigma_u$
(3) $\sigma_u = h\sigma_x$ (4) $\sigma_u = h + \sigma_x$

45. The S.D. of the first n natural numbers is-
(1) $\sqrt{\frac{n^2-1}{2}}$ (2) $\sqrt{\frac{n^2-1}{3}}$
(3) $\sqrt{\frac{n^2-1}{4}}$ (4) $\sqrt{\frac{n^2-1}{12}}$

46. The variance of observations 112, 116, 120, 125, 132 is :-
(1) 58.8 (2) 48.8
(3) 61.8 (4) None of these

47. If $\sum_{i=1}^{10}(x_i - 15) = 12$ and $\sum_{i=1}^{10}(x_i - 15)^2 = 18$ then the S.D. of observations x_1, x_2 .... x_{10} is :-
(1) $\frac{2}{5}$ (2) $\frac{3}{5}$
(3) $\frac{4}{5}$ (4) None of these

48. The S.D. of 7 scored 1, 2, 3, 4, 5, 6, 7 is-
(1) 4 (2) 2
(3) $\sqrt{7}$ (4) none of these

49. The variance of series a, a + d, a + 2d, ..., a + 2nd is :-
(1) $\frac{n(n+1)}{2}d^2$ (2) $\frac{n(n+1)}{3}d^2$
(3) $\frac{n(n+1)}{6}d^2$ (4) $\frac{n(n+1)}{12}d^2$

50. Variance is independent of change of-
(1) only origin
(2) only scale
(3) origin and scale both
(4) none of these

51. If the coefficient of variation and standard deviation of a distribution are 50% and 20 respectively, then its mean is-

(1) 40 (2) 30
(3) 20 (4) None of these

52. If each observation of a dist. whose S.D. is σ, is increased by λ, then the variance of the new observations is -

(1) σ (2) $\sigma + \lambda$
(3) σ^2 (4) $\sigma^2 + \lambda$

53. The variance of 2, 4, 6, 8, 10 is-

(1) 8 (2) $\sqrt{8}$
(3) 6 (4) none of these

54. If each observation of a dist., whose variance is σ^2, is multiplied by λ, then the S.D. of the new new observations is-

(1) σ (2) $\lambda\sigma$
(3) $|\lambda|\sigma$ (4) $\lambda^2 \sigma$

55. The standard deviation of variate x_i is σ. Then standard deviation of the variate $\frac{ax_i + b}{c}$, where a , b, c are constants is-

(1) $\left(\frac{a}{c}\right)\sigma$ (2) $\left|\frac{a}{c}\right|\sigma$
(3) $\left(\frac{a^2}{c^2}\right)\sigma$ (4) None of these

CHECK YOUR GRASP **ANSWER-KEY** **EXERCISE-I**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	4	3	3	1	2	2	4	2	4	2	2	3	4	2	1	4	2	3	3
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	3	2	3	4	4	3	3	3	4	2	3	4	3	2	2	4	4	2	4	3
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55					
Ans.	2	2	2	1	4	2	2	2	2	1	1	3	1	3	2					

BRAIN TEASERS — STATISTICS — EXERCISE-II

1. The A.M. of the series 1, 2, 4, 8, 16, ..., 2^n is-
(1) $\frac{2^n-1}{n}$ (2) $\frac{2^{n+1}-1}{n+1}$
(3) $\frac{2^n-1}{n+1}$ (4) $\frac{2^{n+1}-1}{n}$

2. If the mean of n observations $1^2, 2^2, 3^2, \ldots\ldots n^2$ is $\frac{46n}{11}$, then n is equal to-
(1) 11 (2) 12
(3) 23 (4) 22

3. The weighted mean of first n natural numbers whose weights are equal, is :-
(1) $\frac{n+1}{2}$ (2) $\frac{2n+1}{2}$
(3) $\frac{2n+1}{3}$ (4) $\frac{(2n+1)(n+1)}{6}$

4. The average age of a group of men and women is 30years. If average age of men is 32 and that of women is 27, then the percentage of women in the group is-
(1) 60 (2) 50
(3) 40 (4) 30

5. The geometric mean of the observations 2, 4, 8, 16, 32, 64 is-
(1) $2^{5/2}$ (2) $2^{7/2}$
(3) 33 (4) None of these

6. The H.M. of the reciprocal of first n natural numbers is :-
(1) $\frac{n+1}{2}$ (2) $\frac{n}{\left(1+\frac{1}{2}+\frac{1}{3}+\ldots.+\frac{1}{n}\right)}$
(3) $\frac{2}{n+1}$ (4) None of these

7. Product of n positive numbers is unit. The sum of these numbers can not be less than-
(1) 1 (2) n
(3) n^2 (4) none of these

8. The A.M. of first n terms of the series 1.3.5, 3.5.7, 5.7.9,....., is-
(1) $3n^3 + 6n^2 + 7n - 1$ (2) $n^3 + 8n^2 + 7n - 1$
(3) $2n^3 + 8n^2 - 7n - 2$ (4) $2n^3 + 8n^2 + 7n - 2$

9. The observations 29, 32, 48, 50, x, x + 2, 72, 78, 84, 95 are arranged in ascending order and their median is 63 then the value of x is :-
(1) 61 (2) 62 (3) 62.5 (4) 63

10. If the mode of a distribution is 18 and the mean is 24, then median is-
(1) 18 (2) 24 (3) 22 (4) 21

11. If the mean and S.D. of n observations $x_1, x_2, \ldots. x_n$ are $\overline{x}$ and σ resp, then the sum of squares of observations is :-
(1) $n(\sigma^2 + \overline{x}^2)$ (2) $n(\sigma^2 - \overline{x}^2)$
(3) $n(\overline{x}^2 - \sigma^2)$ (4) None of these

12. The variance of observations 8, 12, 13, 15, 22, is :-
(1) 21 (2) 21.2
(3) 21.4 (4) None of these

13. If the mean of a set of observations $x_1, x_2, \ldots\ldots, x_{10}$ is 20, then the mean of x_1+4, x_2+8, $x_3+12, \ldots, x_{10}+40$ is-
(1) 34 (2) 42 (3) 38 (4) 40

14. The mean of values 0, 1, 2, ..., n when their weights are 1, ${}^nC_1$, ${}^nC_2$, ..., ${}^nC_n$, resp., is
(1) $\frac{2^n}{n+1}$ (2) $\frac{n+1}{2}$ (3) $\frac{2^{n+1}}{n(n+1)}$ (4) $\frac{n}{2}$

15. The G.M. of first n natural numbers is :-
(1) $\frac{n+1}{2}$ (2) $(n!)^n$
(3) $(n!)^{1/n}$ (4) None of these

16. If a variable takes the discrete values $\alpha + 4$, $\alpha - \frac{7}{2}$, $\alpha - \frac{5}{2}$, $\alpha - 3$, $\alpha - 2$, $\alpha + \frac{1}{2}$, $\alpha - \frac{1}{2}$, $\alpha + 5 (\alpha > 0)$, then the median of these values-
(1) $\alpha - \frac{5}{4}$ (2) $\alpha - \frac{1}{2}$
(3) $\alpha - 2$ (4) $\alpha + \frac{5}{4}$

17. The S.D. of first n odd natural numbers is :-
(1) $\sqrt{\frac{n^2-1}{2}}$ (2) $\sqrt{\frac{n^2-1}{3}}$
(3) $\sqrt{\frac{n^2-1}{6}}$ (4) $\sqrt{\frac{n^2-1}{12}}$

18. If the sum and sum of squares of 10 observations are 12 and 18 resp., then, The S.D. of observations is :-

(1) $\frac{1}{5}$ (2) $\frac{2}{5}$ (3) $\frac{3}{5}$ (4) $\frac{4}{5}$

19. The mean of n values of a distribution is $\bar{x}$. If its first value is increased by 1 , second by 2, ... then the mean of new values will be-

(1) $\bar{x}+n$ (2) $\bar{x}+n/2$

(3) $\bar{x}+\left(\frac{n+1}{2}\right)$ (4) None of these

20. The mean of the series $x_1, x_2, ..., x_n$ is $\bar{X}$. If x_2 is replaced by λ, then the new mean is-

(1) $\frac{\bar{X}-x_2+\lambda}{n}$ (2) $\frac{n\bar{X}+x_2-\lambda}{n}$

(3) $\frac{(n-1)\bar{X}+\lambda}{n}$ (4) $\frac{n\bar{X}-x_2+\lambda}{n}$

21. Let G_1 and G_2 be the geometric means of two series $x_1, x_2, ..., x_n$ and $y_1, y_2 ..... y_n$ respectively. If G is the geometric mean of series x_i/y_i, i = 1, 2,..., n, then G is equal to-

(1) $G_1 - G_2$ (2) $\log G_1/\log G_2$

(3) $\log (G_1/G_2)$ (4) G_1/G_2

22. The mean deviation of the numbers 1, 2, 3, 4, 5 is-

(1) 0 (2) 1.2

(3) 2 (4) 1.4

23. If mean = (3 median – mode) x, then the value of x is-

(1) 1 (2) 2 (3) 1/2 (4) 3/2

24. A man spends equal ammount on purchasing three kinds of pens at the rate 5 Rs/pen, 10 Rs/pen, 20 Rs/pen, then average cost of one pen is :-

(1) 10 Rs (2) $\frac{35}{3}$ Rs

(3) $\frac{60}{7}$ Rs (4) None of these

25. The median of 21 observation is 40. if each observations greater than the median are increased by 6, then the median of the observations will be-

(1) 40 (2) 46

(3) 46 + 40/21 (4) 46 – 40/21

26. The coefficient of range of the following distribution 10, 14, 11, 9, 8, 12, 6

(1) 0.4 (2) 2.5

(3) 8 (4) 0.9

27. The S.D. of the following freq. dist. :-

Class	0 – 10	10 – 20	20 – 30	30 – 40
f_i	1	3	4	2

(1) 7.8 (2) 9

(3) 8.1 (4) 0.9

28. The mean of a dist. is 4. if its coefficient of variation is 58%. Then the S.D. of the dist. is :-

(1) 2.23 (2) 3.23

(3) 2.32 (4) None of these

29. The mean of a set of observations is $\bar{x}$. If each observation is divided by α, ($\alpha \neq 0$) and then is increased by 10, then the mean of the new set is

(1) $\frac{\bar{x}}{\alpha}$ (2) $\frac{\bar{x}+10}{\alpha}$

(3) $\frac{\bar{x}+10\alpha}{\alpha}$ (4) $\frac{\alpha\bar{x}+10}{\alpha}$

30. The average age of a teacher and three students is 20 years. If all students are of equal age and the difference between the age of the teacher and that of a student is 20 years, then the age of the teacher is-

(1) 25 years (2) 30 years

(3) 35 years (4) 45 years

31. If a , b, c are any three positive numbers, then the least value of (a + b+ c) $\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ is-

(1) 3 (2) 6

(3) 9 (4) None of these

32. Median of ${}^{2n}C_0, {}^{2n}C_1, {}^{2n}C_2, ..., {}^{2n}C_n$ (when n is even) is-

(1) ${}^{2n}C_{\frac{n-1}{2}}$ (2) ${}^{2n}C_{\frac{n}{2}}$

(3) ${}^{2n}C_{\frac{n+1}{2}}$ (4) None of these

33. The mean deviation from mean of observations 5, 10, 15, 20, ...85 is :-

(1) 43.71 (2) 21.17

(3) 38.7 (4) None of these

34. If standard deviation of variate x_i is 10, then variance of the variate ($50 + 5x_i$) will be-

(1) 50 (2) 250

(3) 500 (4) 2500

35. The S.D. of the numbers 31, 32, 33, ... 47 is-

(1) $2\sqrt{6}$ (2) $4\sqrt{3}$

(3) $\sqrt{\frac{47^2-1}{12}}$ (4) None of these

36. The sum of the squares of deviation of 10 observations from their mean 50 is 250, then coefficient of variation is-

(1) 10% (2) 40%

(3) 50% (4) None of these

37. The median and standard deviation (S.D.) of a distribution will be, If each term is increased by 2 -

(1) median and S.D. will increased by 2

(2) median will increased by 2 but S.D. will remain same

(3) median will remain same but S.D. will increased by 2

(4) median and S.D. will remain same

38. If $\bar{X}_1$ and $\bar{X}_2$ are the means of two series such that $\bar{X}_1 < \bar{X}_2$ and $\bar{X}$ is the mean of the combined series, then-

(1) $\bar{X} < \bar{X}_1$ (2) $\bar{X} > \bar{X}_2$

(3) $\bar{X}_1 < \bar{X} < \bar{X}_2$ (4) $\bar{X} = \frac{\bar{X}_1+\bar{X}_2}{2}$

39. The median of 19 observations of a group is 30. If two observations with values 8 and 32 are further included, then the median of the new group of 21 observation will be

(1) 28 (2) 30

(3) 32 (4) 34

40. The coefficient of mean deviation from median of observations 40, 62, 54, 90, 68, 76 is :-

(1) 2.16 (2) 0.2

(3) 5 (4) None of these

41. A group of 10 observations has mean 5 and S.D. $2\sqrt{6}$. another group of 20 observations has mean 5 and S.D. $3\sqrt{2}$, then the S.D. of combined group of 30 observations is :-

(1) $\sqrt{5}$ (2) $2\sqrt{5}$

(3) $3\sqrt{5}$ (4) None of these

42. For the values x_1, x_2 ...... x_{101} of a distribution $x_1 < x_2 < x_3 < ..... < x_{100} < x_{101}$. The mean deviation of this distribution with respect to a number k will be minimum when k is equal to-

(1) x_1 (2) x_{51}

(3) x_{50} (4) $\frac{x_1+x_2+.....+x_{101}}{101}$

43. In any discrete series (when all the value are not same) the relationship between M.D. about mean and S.D. is-

(1) M.D. = S.D. (2) M.D. > S.D.

(3) M.D. < S.D. (4) M.D. ≤ S.D.

BRAIN TEASERS — ANSWER-KEY — EXERCISE-II

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	1	1	3	2	3	2	4	2	3	1	2	2	4	3	1	2	3	3	4
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	4	2	3	3	1	1	2	3	3	3	3	2	2	4	1	1	2	3	2	2
Que.	41	42	43																	
Ans.	2	2	3																	

PREVIOUS YEAR QUESTIONS **STATISTICS** **EXERCISE-III**

1. The mean of Mathematics marks of 100 students of a class is 72. If the number of boys is 70 and the mean of their marks is 75. Then the mean of the marks of girls in the class will be- [AIEEE-2002]

(1) 60 (2) 62 (3) 65 (4) 68

2. In an experiment with 15 observations of x, the following results were available $\sum x^2 = 2830$, $\sum x = 170$. One observation that was 20 was found to be wrong and it was replaced by it's correct value 30. Then the corrected variance is- [AIEEE-2003]

(1) 8.33 (2) 78 (3) 188.66 (4) 177.33

3. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively. Then P(X = 1) is- [AIEEE-2003]

(1) $\frac{1}{4}$ (2) $\frac{1}{32}$ (3) $\frac{1}{16}$ (4) $\frac{1}{8}$

4. The median of a set of 9 distinct observations is 20.5. If each of the largest four observations of the set is increased by 2, then the median of the new set- [AIEEE-2003]

(1) remains the same as that of the original set

(2) is increased by 2

(3) is decreased by 2

(4) is two times the original median

5. Consider the following statements- [AIEEE-2004]

(a) Mode can be computed from histogram

(b) median is not independent of change of scale

(c) variance is independent of change of origin and scale

which of these are correct-

(1) only (a) and (b) (2) only (b)

(3) only (a) (4) (a), (b) and (c)

6. In a series of 2n observations, half of them equal a and remaining half equal –a. If the standard deviation of the observations is 2, then |a| equals- [AIEEE-2004]

(1) 2 (2) $\sqrt{2}$ (3) $\frac{1}{n}$ (4) $\frac{\sqrt{2}}{n}$

7. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 sucesses is- [AIEEE-2004]

(1) $\frac{128}{256}$ (2) $\frac{219}{256}$ (3) $\frac{37}{256}$ (4) $\frac{28}{256}$

8. If in a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately- [AIEEE-2005]

(1) 24.0 (2) 25.5

(3) 20.5 (4) 22.0

9. Let $x_1, x_2, \ldots\ldots, x_n$ be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$. Then a possible value of n among the following is- [AIEEE-2005]

(1) 12 (2) 9 (3) 18 (4) 15

10. Suppose a polulation A has 100 observations 101, 102, ... 200 and other polulation B has 100 observations 151, 152, ... 250. If V_A and V_B represent the variance of two population respectively then $\frac{V_A}{V_B}$ is- [AIEEE-2006]

(1) 9/4 (2) 4/9 (3) 2/3 (4) 1

11. The average marks of boys in a class 52 and that of girls is 42. The average marks of boys and girls combined is 50 then the parcentage of boys in the class is- [AIEEE-2007]

(1) 20 (2) 80 (3) 60 (4) 40

12. The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80 then which one of the following gives possible values of a and b ? [AIEEE-2008]

(1) a = 0, b = 7 (2) a = 5, b = 2

(3) a = 1, b = 6 (4) a = 3, b = 4

13. **Statement–1** :

The variance of first n even natural numbers is $\frac{n^2-1}{4}$.

Statement–2 :

The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$. [AIEEE-2009]

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for statement–1.

14. If the mean deviation of the numbers 1, 1 + d, 1 + 2d, ..., 1 + 100d from their mean is 255, then that d is equal to :- [AIEEE-2009]

(1) 10.1 (2) 20.2 (3) 10.0 (4) 20.0

15. For two data sets each of size is 5, the variances are given to be 4 and 5 and the corresponding measn are given to be 2 and 4 respectively, then the variance of the combined data set is :- [AIEEE-2010]

(1) $\frac{5}{2}$ (2) $\frac{11}{2}$ (3) 6 (4) $\frac{13}{2}$

16. If the mean deviation about the median of the numbers a, 2a, ..., 50a is 50, then |a| equals:- [AIEEE-2011]

(1) 4 (2) 5 (3) 2 (4) 3

17. A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 gm and a standard deviation of 2 gm. Later, it was found that the measuring scale was misaligned and always under reported every fish weight by 2 gm. The correct mean and standard deviation (in gm) of fishes are respectively : [AIEEE-2011]

(1) 28, 4 (2) 32, 2 (3) 32, 4 (4) 28, 2

18. Let $x_1, x_2, \ldots, x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be their variance.

Statement–1 : Variance of $2x_1, 2x_2, \ldots, 2x_n$ is $4\sigma^2$.

Statement–2 : Arithmetic mean of $2x_1, 2x_2, \ldots, 2x_n$ is $4\bar{x}$. [AIEEE-2012]

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

PREVIOUS YEARS QUESTIONS **ANSWER-KEY** **EXERCISE-III**

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans	3	2	2	1	1	1	4	1	3	4	2	4	2	1	2
Que.	16	17	18												
Ans	1	2	1												