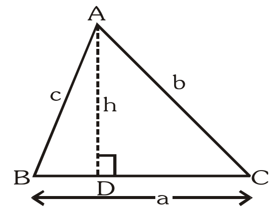
SOLUTIONS OF TRIANGLE

The process of calculating the sides and angles of triangle using given information is called solution of triangle. In a ΔABC, the angles are denoted by capital letters A, B and C and the length of the sides opposite these angle are denoted by small letter a, b and c respectively.

1. SINE FORMULAE :



In any triangle ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \lambda = \frac{abc}{2\Delta} = 2R$$

where R is circumradius and Δ is area of triangle.

Illustration 1 : Angles of a triangle are in 4 : 1 : 1 ratio. The ratio between its greatest side and perimeter is

(A) $\dfrac{3}{2+\sqrt{3}}$ (B) $\dfrac{\sqrt{3}}{2+\sqrt{3}}$ (C) $\dfrac{\sqrt{3}}{2-\sqrt{3}}$ (D) $\dfrac{1}{2+\sqrt{3}}$

Solution : Angles are in ratio 4 : 1 : 1.

$\Rightarrow$ angles are 120 , 30 , 30 .

If sides opposite to these angles are a, b, c respectively, then a will be the greatest side. Now from

sine formula $\dfrac{a}{\sin 120°} = \dfrac{b}{\sin 30°} = \dfrac{c}{\sin 30°}$

$\Rightarrow \quad \dfrac{a}{\sqrt{3}/2} = \dfrac{b}{1/2} = \dfrac{c}{1/2}$

$\Rightarrow \quad \dfrac{a}{\sqrt{3}} = \dfrac{b}{1} = \dfrac{c}{1} = k$ (say)

then a = $\sqrt{3}k$, perimeter = $(2+\sqrt{3})k$

$\therefore$ required ratio = $\dfrac{\sqrt{3}k}{(2+\sqrt{3})k} = \dfrac{\sqrt{3}}{2+\sqrt{3}}$ **Ans. (B)**

Illustration 2 : In triangle ABC, if b = 3, c = 4 and $\angle B = \pi/3$, then number of such triangles is -

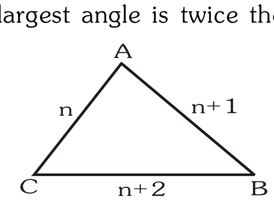
(A) 1 (B) 2 (C) 0 (D) infinite

Solution : Using sine formulae $\dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\Rightarrow \quad \dfrac{\sin \pi/3}{3} = \dfrac{\sin C}{4} \quad \Rightarrow \quad \dfrac{\sqrt{3}}{6} = \dfrac{\sin C}{4} \quad \Rightarrow \quad \sin C = \dfrac{2}{\sqrt{3}} > 1$ which is not possible.

Hence there exist no triangle with given elements. **Ans. (C)**

Illustration 3 : The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one. Determine the sides of the triangle.



Solution : Let the sides be n, n + 1, n + 2 cms.

i.e. AC = n, AB = n + 1, BC = n + 2

Smallest angle is B and largest one is A.

Here, $\angle A = 2\angle B$

Also, $\angle A + \angle B + \angle C = 180$

$\Rightarrow \quad 3\angle B + \angle C = 180 \quad \Rightarrow \angle C = 180 - 3\angle B$

We have, sine law as,

$$\frac{\sin A}{n+2} = \frac{\sin B}{n} = \frac{\sin C}{n+1} \qquad \Rightarrow \qquad \frac{\sin 2B}{n+2} = \frac{\sin B}{n} = \frac{\sin(180-3B)}{n+1}$$

$$\Rightarrow \quad \underset{(i)}{\frac{\sin 2B}{n+2}} = \underset{(ii)}{\frac{\sin B}{n}} = \underset{(iii)}{\frac{\sin 3B}{n+1}}$$

from (i) and (ii);

$$\frac{2\sin B\cos B}{n+2} = \frac{\sin B}{n} \quad \Rightarrow \quad \cos B = \frac{n+2}{2n} \qquad \text{......... (iv)}$$

and from (ii) and (iii);

$$\frac{\sin B}{n} = \frac{3\sin B - 4\sin^3 B}{n+1} \quad \Rightarrow \quad \frac{\sin B}{n} = \frac{\sin B(3-4\sin^2 B)}{n+1}$$

$$\Rightarrow \quad \frac{n+1}{n} = 3 - 4(1-\cos^2 B) \qquad \text{.......... (v)}$$

from (iv) and (v), we get

$$\frac{n+1}{n} = -1 + 4\left(\frac{n+2}{2n}\right)^2 \quad \Rightarrow \quad \frac{n+1}{n} + 1 = \left(\frac{n^2+4n+4}{n^2}\right)$$

$$\Rightarrow \quad \frac{2n+1}{n} = \frac{n^2+4n+4}{n^2} \quad \Rightarrow \quad 2n^2 + n = n^2 + 4n + 4$$

$\Rightarrow \quad n^2 - 3n - 4 = 0 \quad \Rightarrow \quad (n-4)(n+1) = 0$

$n = 4$ or -1

where $n \neq -1$

$\therefore \quad n = 4$. Hence the sides are 4, 5, 6 **Ans.**

Do yourself - 1 :

(i) If in a ΔABC, $\angle A = \frac{\pi}{6}$ and $b : c = 2 : \sqrt{3}$, find $\angle B$.

(ii) Show that, in any ΔABC : $a\sin(B-C) + b\sin(C-A) + c\sin(A-B) = 0$.

(iii) If in a ΔABC, $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$, show that a^2, b^2, c^2 are in A.P.

(iv) If in a ΔABC, $\angle A = 3\angle B$, then prove that $\sin B = \frac{1}{2}\sqrt{\frac{3b-a}{b}}$.

2. COSINE FORMULAE :

(a) $\cos A = \frac{b^2+c^2-a^2}{2bc}$ or $a^2 = b^2 + c^2 - 2bc\cos A$ **(b)** $\cos B = \frac{c^2+a^2-b^2}{2ca}$ **(c)** $\cos C = \frac{a^2+b^2-c^2}{2ab}$

Illustration 4 : In a triangle ABC, if B = 30 and $c = \sqrt{3}\,b$, then A can be equal to -

(A) 45 (B) 60 (C) 90 (D) 120

Solution : We have $\cos B = \frac{c^2+a^2-b^2}{2ca} \Rightarrow \frac{\sqrt{3}}{2} = \frac{3b^2+a^2-b^2}{2\times\sqrt{3}b\times a}$

$\Rightarrow \quad a^2 - 3ab + 2b^2 = 0 \quad \Rightarrow \quad (a-2b)(a-b) = 0$

$\Rightarrow$ Either $a = b \Rightarrow A = 30$

or $a = 2b \Rightarrow a^2 = 4b^2 = b^2 + c^2 \Rightarrow A = 90$. **Ans. (C)**

Illustration 5 : In a triangle ABC, $(a^2 - b^2 - c^2)\tan A + (a^2 - b^2 + c^2)\tan B$ is equal to -

(A) $(a^2 + b^2 - c^2)\tan C$ (B) $(a^2 + b^2 + c^2)\tan C$

(C) $(b^2 + c^2 - a^2)\tan C$ (D) none of these

Solution : Using cosine law :

The given expression is equal to $-2\,bc\cos A\tan A + 2\,ac\cos B\tan B$

$= 2abc\left(-\frac{\sin A}{a}+\frac{\sin B}{b}\right)=0$ **Ans. (D)**

Illustration 6 : If in a triangle ABC, $\frac{2\cos A}{a}+\frac{\cos B}{b}+\frac{2\cos C}{c}=\frac{a}{bc}+\frac{b}{ac}$, find the $\angle A$ =

(A) 90 (B) 60 (C) 30 (D) none of these

Solution : We have $\frac{2\cos A}{a}+\frac{\cos B}{b}+\frac{2\cos C}{c}=\frac{a}{bc}+\frac{b}{ac}$

Multiplying both sides of abc, we get

$\Rightarrow$ $2bc\cos A + ac\cos B + 2ab\cos C = a^2 + b^2$

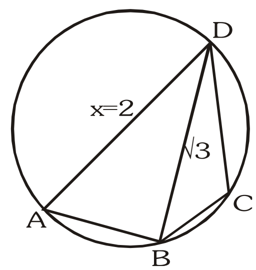
$\Rightarrow$ $(b^2 + c^2 - a^2) + \frac{(a^2+c^2-b^2)}{2}+(a^2+b^2-c^2)=a^2+b^2$

$\Rightarrow$ $c^2 + a^2 - b^2 = 2a^2 - 2b^2$ $\Rightarrow$ $b^2 + c^2 = a^2$

$\therefore$ ΔABC is right angled at A. $\Rightarrow$ $\angle A = 90$ **Ans. (A)**

Illustration 7 : A cyclic quadrilateral ABCD of area $\frac{3\sqrt{3}}{4}$ is inscribed in unit circle. If one of its side AB = 1, and the diagonal $BD=\sqrt{3}$, find lengths of the other sides.

Solution : AB = 1, $BD=\sqrt{3}$, OA = OB = OD = 1



The given circle of radius 1 is also circumcircle of Δ ABD

$\Rightarrow$ R = 1 for ΔABD

$\Rightarrow$ $\frac{a}{\sin A}=2R$ $\Rightarrow$ A = 60

and hence C = 120

Also by cosine rule on ΔABD, $\left(\sqrt{3}\right)^2=1^2+x^2-2x\cos 60°$

$\Rightarrow$ x = 2

Now, area ABCD = $\Delta ABD + \Delta BCD$

$\Rightarrow$ $\frac{3\sqrt{3}}{4}=\frac{1}{2}(1.2.\sin 60°)+\frac{1}{2}\left(c.d.\sin 120°\right)$

$\Rightarrow$ cd = 1, or $c^2d^2 = 1$

Also by cosine rule on triangle BCD we have

$\left(\sqrt{3}\right)^2=c^2+d^2-2cd\cos 120°=c^2+d^2+cd$

$\Rightarrow$ $c^2 + d^2 = 2$ or cd = 1

$\Rightarrow$ c^2 and d^2 are the roots of $t^2 - 2t + 1 = 0$

$\therefore$ $c^2 = d^2 = 1$ $\therefore$ BC = 1 = CD and AD = x = 2.

Do yourself - 2 :

(i) If a : b : c = 4 : 5 : 6, then show that $\angle C = 2\angle A$.

(ii) In any ΔABC, prove that

(a) $\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=\frac{a^2+b^2+c^2}{2abc}$

(b) $\frac{b^2}{a}\cos A+\frac{c^2}{b}\cos B+\frac{a^2}{c}\cos C=\frac{a^4+b^4+c^4}{2abc}$

3. PROJECTION FORMULAE :

(a) b cos C + c cos B = a **(b)** c cos A + a cos C = b **(c)** a cos B + b cos A = c

Illustration 8 : In a $\Delta ABC, c\cos^2\frac{A}{2}+a\cos^2\frac{C}{2}=\frac{3b}{2}$, then show a, b, c are in A.P.

Solution : Here, $\frac{c}{2}(1+\cos A)+\frac{a}{2}(1+\cos C)=\frac{3b}{2}$

$\Rightarrow$ a + c + (c cos A + a cos C) = 3b

$\Rightarrow$ a + c + b = 3b {using projection formula}

$\Rightarrow$ a + c = 2b

which shows a, b, c are in A.P.

Do yourself - 3 :

(i) In a ΔABC, if $\angle A=\frac{\pi}{4}$, $\angle B=\frac{5\pi}{12}$, show that $a+c\sqrt{2}=2b$.

(ii) In a ΔABC, prove that : (a) $b(a\cos C - c\cos A) = a^2 - c^2$ (b) $2\left(b\cos^2\frac{C}{2}+c\cos^2\frac{B}{2}\right)=a+b+c$

4. NAPIER'S ANALOGY (TANGENT RULE) :

(a) $\tan\left(\frac{B-C}{2}\right)=\frac{b-c}{b+c}\cot\frac{A}{2}$ **(b)** $\tan\left(\frac{C-A}{2}\right)=\frac{c-a}{c+a}\cot\frac{B}{2}$ **(c)** $\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}\cot\frac{C}{2}$

Illustration 9 : In a ΔABC, the tangent of half the difference of two angles is one-third the tangent of half the sum of the angles. Determine the ratio of the sides opposite to the angles.

Solution : Here, $\tan\left(\frac{A-B}{2}\right)=\frac{1}{3}\tan\left(\frac{A+B}{2}\right)$ .... (i)

using Napier's analogy, $\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}.\cot\left(\frac{C}{2}\right)$ .... (ii)

from (i) & (ii) ;

$\frac{1}{3}\tan\left(\frac{A+B}{2}\right)=\frac{a-b}{a+b}.\cot\left(\frac{C}{2}\right) \Rightarrow \frac{1}{3}\cot\left(\frac{C}{2}\right)=\frac{a-b}{a+b}.\cot\left(\frac{C}{2}\right)$

{as A + B + C = π $\therefore$ $\tan\left(\frac{B+C}{2}\right)=\tan\left(\frac{\pi}{2}-\frac{C}{2}\right)=\cot\frac{C}{2}$ }

$\Rightarrow$ $\frac{a-b}{a+b}=\frac{1}{3}$ or 3a – 3b = a + b

$2a = 4b$ or $\frac{a}{b} = \frac{2}{1} \Rightarrow \frac{b}{a} = \frac{1}{2}$

Thus the ratio of the sides opposite to the angles is b : a = 1 : 2. **Ans.**

Do yourself - 4 :

(i) In any ΔABC, prove that $\frac{b-c}{b+c} = \frac{\tan\left(\frac{B-C}{2}\right)}{\tan\left(\frac{B+C}{2}\right)}$

(ii) If ΔABC is right angled at C, prove that : (a) $\tan\frac{A}{2} = \sqrt{\frac{c-b}{c+b}}$ (b) $\sin(A-B) = \frac{a^2-b^2}{a^2+b^2}$

(iii) If in a ΔABC, two sides are a = 3, b = 5 and $\cos(A-B) = \frac{7}{25}$, find $\tan\frac{C}{2}$.

5. HALF ANGLE FORMULAE :

$s = \frac{a+b+c}{2}$ = semi-perimeter of triangle.

(a) (i) $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$ (ii) $\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$ (iii) $\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

(b) (i) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$ (ii) $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$ (iii) $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

(c) (i) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{\Delta}{s(s-a)}$ (ii) $\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}} = \frac{\Delta}{s(s-b)}$ (iii) $\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{\Delta}{s(s-c)}$

(d) Area of Triangle

$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B = \frac{1}{2}ab\sin C = \frac{1}{2}ap_1 = \frac{1}{2}bp_2 = \frac{1}{2}cp_3$, where p_1, p_2, p_3 are altitudes from vertices A,B,C respectively.

Illustration 10 : If in a triangle ABC, CD is the angle bisector of the angle ACB, then CD is equal to -

(A) $\frac{a+b}{2ab}\cos\frac{C}{2}$ (B) $\frac{2ab}{a+b}\sin\frac{C}{2}$ (C) $\frac{2ab}{a+b}\cos\frac{C}{2}$ (D) $\frac{b\sin\angle DAC}{\sin(B+C/2)}$

Solution : $\Delta CAB = \Delta CAD + \Delta CDB$

$\Rightarrow \frac{1}{2}ab\sin C = \frac{1}{2}b.CD.\sin\left(\frac{C}{2}\right) + \frac{1}{2}a.CD\sin\left(\frac{C}{2}\right)$

$\Rightarrow CD(a+b)\sin\left(\frac{C}{2}\right) = ab\left(2\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)\right)$

So $CD = \frac{2ab\cos(C/2)}{(a+b)}$

and in ΔCAD, $\frac{CD}{\sin\angle DAC} = \frac{b}{\sin\angle CDA}$ (by sine rule)

$\Rightarrow CD = \frac{b\sin\angle DAC}{\sin(B+C/2)}$ **Ans. (C,D)**

Illustration 11 : If Δ is the area and 2s the sum of the sides of a triangle, then show $\Delta \le \dfrac{s^2}{3\sqrt{3}}$.

Solution : We have, $2s = a + b + c$, $\Delta^2 = s(s-a)(s-b)(s-c)$

Now, A.M. $\ge$ G.M.

$$\frac{(s-a)+(s-b)+(s-c)}{3} \ge \{(s-a)(s-b)(s-c)\}^{1/3}$$

or $$\frac{3s-2s}{3} \ge \left(\frac{\Delta^2}{s}\right)^{1/3}$$

or $$\frac{s}{3} \ge \left(\frac{\Delta^2}{s}\right)^{1/3}$$

or $$\frac{\Delta^2}{s} \le \frac{s^3}{27} \Rightarrow \Delta \le \frac{s^2}{3\sqrt{3}}$$ **Ans.**

Do yourself - 5 :

(i) Given a = 6, b = 8, c = 10. Find

(a) sinA (b) tanA (c) $\sin\dfrac{A}{2}$ (d) $\cos\dfrac{A}{2}$ (e) $\tan\dfrac{A}{2}$ (f) Δ

(ii) Prove that in any ΔABC, $(abcs)\sin\dfrac{A}{2}.\sin\dfrac{B}{2}.\sin\dfrac{C}{2} = \Delta^2$.

(iii) Show that if $\left(\tan\dfrac{A}{2}+\tan\dfrac{C}{2}\right) = \dfrac{2}{3}\cot\dfrac{B}{2}$, then a, b, c are in A.P.

6. m-n THEOREM :

$(m + n)\cot\theta = m\cot\alpha - n\cot\beta$

$(m + n)\cot\theta = n\cot B - m\cot C$.

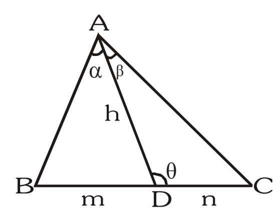


Illustration 12 : The base of a Δ is divided into three equal parts. If t_1, t_2, t_3 be the tangents of the angles subtended by these parts at the opposite vertex, prove that : $\left(\dfrac{1}{t_1}+\dfrac{1}{t_2}\right)\left(\dfrac{1}{t_2}+\dfrac{1}{t_3}\right) = 4\left(1+\dfrac{1}{t_2^2}\right)$

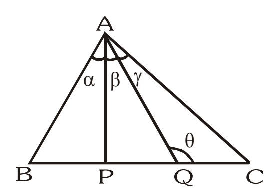
Solution : Let the points P and Q divide the side BC in three equal parts :

Such that BP = PQ = QC = x

Also let,

$\angle BAP = \alpha, \angle PAQ = \beta, \angle QAC = \gamma$

and $\angle AQC = \theta$



From question, $\tan\alpha = t_1$, $\tan\beta = t_2$, $\tan\gamma = t_3$.

Applying

m : n rule in triangle ABC we get,

$(2x + x)\cot\theta = 2x\cot(\alpha + \beta) - x\cot\gamma$ .... (i)

from ΔAPC, we get

$(x + x)\cot\theta = x\cot\beta - x\cot\gamma$ .... (ii) dividing (i) and (ii), we get

$$\frac{3}{2}=\frac{2\cot(\alpha+\beta)-\cot\gamma}{\cot\beta-\cot\gamma}$$

or $\quad 3\cot\beta-\cot\gamma=\dfrac{4\left(\cot\alpha.\cot\beta-1\right)}{\cot\beta+\cot\alpha}$

or $\quad 3\cot^2\beta-\cot\beta\cot\gamma+3\cot\alpha.\cot\beta-\cot\alpha.\cot\gamma=4\cot\alpha.\cot\beta-4$

or $\quad 4+4\cot^2\beta=\cot^2\beta+\cot\alpha\,.\,\cot\beta+\cot\beta\,.\,\cot\gamma+\cot\gamma\,.\,\cot\alpha$

or $\quad 4(1+\cot^2\beta)=(\cot\beta+\cot\alpha)(\cot\beta+\cot\gamma)$

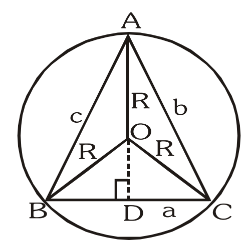
or $\quad 4\left(1+\dfrac{1}{t_2^2}\right)=\left(\dfrac{1}{t_1}+\dfrac{1}{t_2}\right)\left(\dfrac{1}{t_2}+\dfrac{1}{t_3}\right)$

Do yourself - 6 :

(i) The median AD of a ΔABC is perpendicular to AB, prove that $\tan A+2\tan B=0$

7. RADIUS OF THE CIRCUMCIRCLE 'R' :

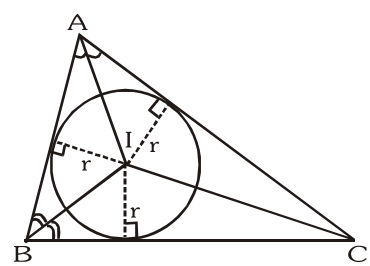
Circumcentre is the point of intersection of perpendicular bisectors of the sides and distance between circumcentre & vertex of triangle is called circumradius 'R'.



$$R=\frac{a}{2\sin A}=\frac{b}{2\sin B}=\frac{c}{2\sin C}=\frac{abc}{4\Delta}.$$

8. RADIUS OF THE INCIRCLE 'r' :

Point of intersection of internal angle bisectors is incentre and perpendicular distance of incentre from any side is called inradius 'r'.



$$r=\frac{\Delta}{s}=(s-a)\tan\frac{A}{2}=(s-b)\tan\frac{B}{2}=(s-c)\tan\frac{C}{2}=4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}.$$

$$=a\frac{\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}}=b\frac{\sin\frac{A}{2}\sin\frac{C}{2}}{\cos\frac{B}{2}}=c\frac{\sin\frac{B}{2}\sin\frac{A}{2}}{\cos\frac{C}{2}}$$

Illustration 13 : In a triangle ABC, if a : b : c = 4 : 5 : 6, then ratio between its circumradius and inradius is-

(A) $\dfrac{16}{7}$ (B) $\dfrac{16}{9}$ (C) $\dfrac{7}{16}$ (D) $\dfrac{11}{7}$

Solution : $\dfrac{R}{r}=\dfrac{abc}{4\Delta}\Big/\dfrac{\Delta}{s}=\dfrac{(abc)s}{4\Delta^2}\quad\Rightarrow\quad\dfrac{R}{r}=\dfrac{abc}{4(s-a)(s-b)(s-c)}$ ....(i)

$\because\quad a:b:c=4:5:6\ \Rightarrow\ \dfrac{a}{4}=\dfrac{b}{5}=\dfrac{c}{6}=k$ (say)

$\Rightarrow\quad a=4k,\ b=5k,\ c=6k$

$\therefore\quad s=\dfrac{a+b+c}{2}=\dfrac{15k}{2},\ s-a=\dfrac{7k}{2},\ s-b=\dfrac{5k}{2},\ s-c=\dfrac{3k}{2}$

using (i) in these values $\dfrac{R}{r}=\dfrac{(4k)(5k)(6k)}{4\left(\frac{7k}{2}\right)\left(\frac{5k}{2}\right)\left(\frac{3k}{2}\right)}=\dfrac{16}{7}$ **Ans. (A)**

Illustration 14 : If A, B, C are the angles of a triangle, prove that : $\cos A + \cos B + \cos C = 1+\frac{r}{R}$.

Solution :

$$\cos A + \cos B + \cos C = 2\cos\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)+\cos C$$

$$= 2\sin\frac{C}{2}.\cos\left(\frac{A-B}{2}\right)+1-2\sin^2\frac{C}{2} = 1+2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)-\sin\left(\frac{C}{2}\right)\right]$$

$$= 1+2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)-\cos\left(\frac{A+B}{2}\right)\right] \qquad \left\{\because \frac{C}{2}=90°-\left(\frac{A+B}{2}\right)\right\}$$

$$= 1+2\sin\frac{C}{2}.2\sin\frac{A}{2}.\sin\frac{B}{2} = 1+4\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}$$

$$= 1+\frac{r}{R} \qquad \{\text{as, } r = 4R\sin A/2 . \sin B/2 . \sin C/2\}$$

$\Rightarrow \quad \cos A + \cos B + \cos C = 1+\frac{r}{R}$. Hence proved.

Do yourself - 7 :

(i) If in ΔABC, a = 3, b = 4 and c = 5, find

(a) Δ (b) R (c) r

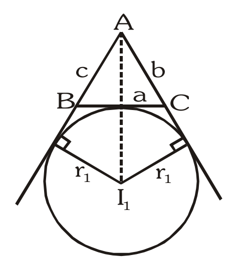
(ii) In a ΔABC, show that :

(a) $\frac{a^2-b^2}{c}=2R\sin(A-B)$ (b) $r\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}=\frac{\Delta}{4R}$ (c) $a+b+c=\frac{abc}{2Rr}$

(iii) Let Δ & Δ' denote the areas of a Δ and that of its incircle. Prove that $\Delta : \Delta' = \left(\cot\frac{A}{2}.\cot\frac{B}{2}.\cot\frac{C}{2}\right) : \pi$

9. RADII OF THE EX-CIRCLES :

Point of intersection of two external angles and one internal angle bisectors is excentre and perpendicular distance of excentre from any side is called exradius. If r_1 is the radius of escribed circle opposite to $\angle A$ of ΔABC and so on, then -



(a) $$r_1=\frac{\Delta}{s-a}=s\tan\frac{A}{2}=4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}=\frac{a\cos\frac{B}{2}\cos\frac{C}{2}}{\cos\frac{A}{2}}$$

(b) $$r_2=\frac{\Delta}{s-b}=s\tan\frac{B}{2}=4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}=\frac{b\cos\frac{A}{2}\cos\frac{C}{2}}{\cos\frac{B}{2}}$$

(c) $$r_3=\frac{\Delta}{s-c}=s\tan\frac{C}{2}=4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}=\frac{c\cos\frac{A}{2}\cos\frac{B}{2}}{\cos\frac{C}{2}}$$

I_1, I_2 and I_3 are taken as ex-centre opposite to vertex A, B, C repsectively.

Illustration 15 : Value of the expression $\frac{b-c}{r_1}+\frac{c-a}{r_2}+\frac{a-b}{r_3}$ is equal to -

(A) 1 (B) 2 (C) 3 (D) 0

Solution : $\frac{(b-c)}{r_1}+\frac{(c-a)}{r_2}+\frac{(a-b)}{r_3}$

$\Rightarrow \quad (b-c)\left(\frac{s-a}{\Delta}\right)+(c-a)\left(\frac{s-b}{\Delta}\right)+(a-b)\cdot\left(\frac{s-c}{\Delta}\right)$

$\Rightarrow \quad \frac{(s-a)(b-c)+(s-b)(c-a)+(s-c)(a-b)}{\Delta}$

$= \frac{s(b-c+c-a+a-b)-[ab-ac+bc-ba+ac-bc]}{\Delta}=\frac{0}{\Delta}=0$

Thus, $\frac{b-c}{r_1}+\frac{c-a}{r_2}+\frac{a-b}{r_3}=0$ **Ans. (D)**

Illustration 16 : If $r_1 = r_2 + r_3 + r$, prove that the triangle is right angled.

Solution : We have, $r_1 - r = r_2 + r_3$

$\Rightarrow \quad \frac{\Delta}{s-a}-\frac{\Delta}{s}=\frac{\Delta}{s-b}+\frac{\Delta}{s-c} \quad \Rightarrow \quad \frac{s-s+a}{s(s-a)}=\frac{s-c+s-b}{(s-b)(s-c)}$

$\Rightarrow \quad \frac{a}{s(s-a)}=\frac{2s-(b+c)}{(s-b)(s-c)}$ {as, $2s = a + b + c$}

$\Rightarrow \quad \frac{a}{s(s-a)}=\frac{a}{(s-b)(s-c)} \quad \Rightarrow \quad s^2 - (b+c)s + bc = s^2 - as$

$\Rightarrow \quad s(-a+b+c) = bc \quad \Rightarrow \quad \frac{(b+c-a)(a+b+c)}{2}=bc$

$\Rightarrow \quad (b+c)^2 - (a)^2 = 2bc \quad \Rightarrow \quad b^2 + c^2 + 2bc - a^2 = 2bc$

$\Rightarrow \quad b^2 + c^2 = a^2$

$\therefore \quad \angle A = 90$. **Ans.**

Do yourself - 8 :

(i) In an equilateral ΔABC, $R = 2$, find

(a) r (b) r_1 (c) a

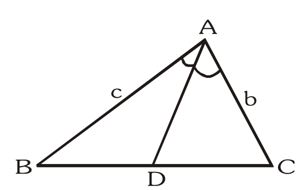
(ii) In a ΔABC, show that

(a) $r_1r_2 + r_2r_3 + r_3r_1 = s^2$ (b) $\frac{1}{4}r^2s^2\left(\frac{1}{r}-\frac{1}{r_1}\right)\left(\frac{1}{r}-\frac{1}{r_2}\right)\left(\frac{1}{r}-\frac{1}{r_3}\right)=\frac{r+r_1+r_2-r_3}{4\cos C}=R$

(c) $\sqrt{rr_1r_2r_3}=\Delta$

10. ANGLE BISECTORS & MEDIANS :

An angle bisector divides the base in the ratio of corresponding sides.



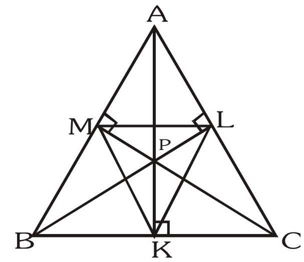
$\frac{BD}{CD}=\frac{c}{b} \quad \Rightarrow \quad BD = \frac{ac}{b+c} \quad \& \quad CD = \frac{ab}{b+c}$

If m_a and β_a are the lengths of a median and an angle bisector from the angle A then,

$m_a = \frac{1}{2}\sqrt{2b^2+2c^2-a^2}$ and $\beta_a = \frac{2bc\cos\frac{A}{2}}{b+c}$

Note that $m_a^2 + m_b^2 + m_c^2 = \frac{3}{4}(a^2 + b^2 + c^2)$

11. ORTHOCENTRE :



(a) Point of intersection of altitudes is orthocentre & the triangle KLM which is formed by joining the feet of the altitudes is called the pedal triangle.

(b) The distances of the orthocentre from the angular points of the ΔABC are 2R cosA, 2R cosB, & 2R cosC.

(c) The distance of P from sides are 2R cosB cosC, 2R cosC cosA and 2R cosA cosB.

Do yourself - 9 :

(i) If x, y, z are the distance of the vertices of ΔABC respectively from the orthocentre, then prove that $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = \frac{abc}{xyz}$

(ii) If p_1, p_2, p_3 are respectively the perpendiculars from the vertices of a triangle to the opposite sides, prove that

(a) $p_1 p_2 p_3 = \frac{a^2 b^2 c^2}{8R^3}$ (b) $\Delta = \sqrt{\frac{1}{2} R p_1 p_2 p_3}$

(iii) In a ΔABC, AD is altitude and H is the orthocentre prove that AH : DH = (tanB + tanC) : tanA

(iv) In a ΔABC, the lengths of the bisectors of the angle A, B and C are x, y, z respectively. Show that

$\frac{1}{x}\cos\frac{A}{2} + \frac{1}{y}\cos\frac{B}{2} + \frac{1}{z}\cos\frac{C}{2} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$. Also show that $\frac{a}{b+c} = \sqrt{1 - \frac{x^2}{bc}}$

12. THE DISTANCES BETWEEN THE SPECIAL POINTS :

(a) The distance between circumcentre and orthocentre is $= R\sqrt{1 - 8\cos A \cos B \cos C}$

(b) The distance between circumcentre and incentre is $= \sqrt{R^2 - 2Rr}$

(c) The distance between incentre and orthocentre is $= \sqrt{2r^2 - 4R^2 \cos A \cos B \cos C}$

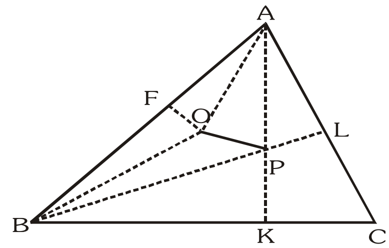
(d) The distances between circumcentre & excentres are

$$OI_1 = R\sqrt{1 + 8\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}} = \sqrt{R^2 + 2Rr_1} \text{ \& so on.}$$

Illustration 17 : Prove that the distance between the circumcentre and the orthocentre of a triangle ABC is $R\sqrt{1 - 8\cos A \cos B \cos C}$.

Solution : Let O and P be the circumcentre and the orthocentre respectively. If OF is the perpendicular to AB, we have $\angle OAF = 90 - \angle AOF = 90 - C$. Also $\angle PAL = 90 - C$.

Hence, $\angle OAP = A - \angle OAF - \angle PAL = A - 2(90 - C) = A + 2C - 180$



$= A + 2C - (A + B + C) = C - B.$

Also OA = R and PA = 2RcosA.

Now in ΔAOP,

$OP^2 = OA^2 + PA^2 - 2OA.\ PA\ \cos OAP$

$= R^2 + 4R^2\cos^2 A - 4R^2\ \cos A\cos(C-B)$

$= R^2 + 4R^2\cos A[\cos A - \cos(C - B)]$

$= R^2 - 4R^2\ \cos A[\cos(B + C) + \cos(C - B)] = R^2 - 8R^2\ \cos A\ \cos B\ \cos C.$

Hence $OP = R\sqrt{1-8\cos A\cos B\cos C}$. **Ans.**

Do yourself - 10 :

(i) Show that in an equilateral triangle, circumcentre, orthocentre and incentre overlap each other.

(ii) If the incentre and circumcentre of a triangle are equidistant from the side BC, show that $\cos B + \cos C = 1$.

13. SOLUTION OF TRIANGLES :

The three sides a,b,c and the three angles A,B,C are called the elements of the triangle ABC. When any three of these six elements (except all the three angles) of a triangle are given, the triangle is known completely; that is the other three elements can be expressed in terms of the given elements and can be evaluated. This process is called the solution of triangles.

* If the three sides a,b,c are given, angle A is obtained from $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

or $\cos A = \frac{b^2+c^2-a^2}{2bc}$ .B and C can be obtained in the similar way.

* If two sides b and c and the included angle A are given, then $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$ gives $\frac{B-C}{2}$. Also

$\frac{B+C}{2} = 90° - \frac{A}{2}$, so that B and C can be evaluated. The third side is given by a = b $\frac{\sin A}{\sin B}$

or $a^2 = b^2 + c^2 - 2bc\cos A$.

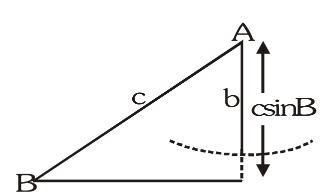
* If two sides b and c and an angle opposite the one of them (say B) are given then

$\sin C = \frac{c}{b}\sin B,\ A = 180° - (B+C)$ and $a = \frac{b\sin A}{\sin B}$ given the remaining elements.

Case I :

b < c sin B.

We draw the side c and angle B. Now it is obvious from the figure that there is no triangle possible.



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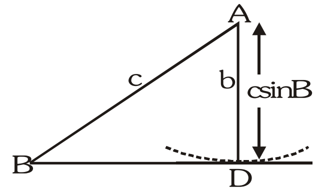
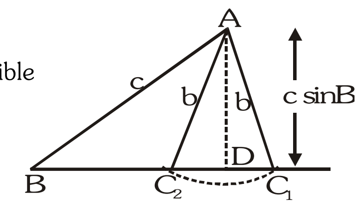
Case II :

$b = c \sin B$ and B is an acute angle, there is only one triangle possible. and it is right-angled at C.



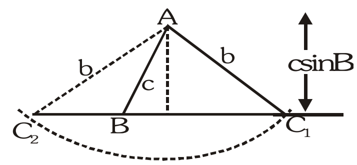
Case III :

$b > c \sin B$, $b < c$ and B is an acute angle, then there are two triangles possible for two values of angle C.



Case IV :

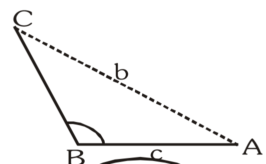
$b > c \sin B$, $c < b$ and B is an acute angle, then there is only one triangle.



Case V :

$b > c \sin B$, $c > b$ and B is an obtuse angle.

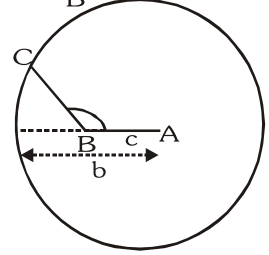
For any choice of point C, b will be greater than c which is a contradication as $c > b$ (given). So there is no triangle possible.



Case VI :

$b > c \sin B$, $c < b$ and B is an obtuse angle.

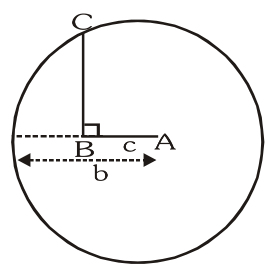
We can see that the circle with A as centre and b as radius will cut the line only in one point. So only one triangle is possible.



Case VII :

$b > c$ and $B = 90$.

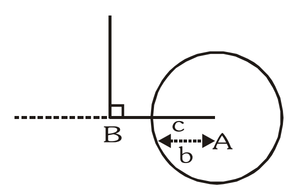
Again the circle with A as centre and b as radius will cut the line only in one point. So only one triangle is possible.



Case VIII :

$b \leq c$ and $B = 90$.

The circle with A as centre and b as radius will not cut the line in any point. So no triangle is possible.



This is, sometimes, called an ambiguous case.

Alternative Method :

By applying cosine rule, we have $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \quad a^2 - (2c \cos B)a + (c^2 - b^2) = 0 \Rightarrow a = c \cos B \pm \sqrt{(c \cos B)^2 - (c^2 - b^2)}$

$\Rightarrow \quad a = c \cos B \pm \sqrt{b^2 - (c \sin B)^2}$

This equation leads to following cases :

Case-I : If $b < c\sin B$, no such triangle is possible.

Case-II: Let $b = c \sin B$. There are further following case :

(a) B is an obtuse angle $\Rightarrow$ cosB is negative. There exists no such triangle.

(b) B is an acute angle $\Rightarrow$ cosB is positive. There exists only one such triangle.

Case-III: Let $b > c \sin B$. There are further following cases :

(a) B is an acute angle $\Rightarrow$ cosB is positive. In this case triangle will exist if and only if $c \cos B > \sqrt{b^2 - (c\sin B)^2}$ or $c > b \Rightarrow$ Two such triangle is possible. If $c < b$, only one such triangle is possible.

(b) B is an obtuse angle $\Rightarrow$ cosB is negative. In this case triangle will exist if and only if $\sqrt{b^2 - (c\sin B)^2} > |c \cos B| \Rightarrow b > c$. So in this case only one such triangle is possible. If $b < c$ there exists no such triangle.

This is called an ambiguous case.

* If one side a and angles B and C are given, then $A = 180 - (B + C)$, and $b = \dfrac{a\sin B}{\sin A}, c = \dfrac{a\sin C}{\sin A}$.

* If the three angles A,B,C are given, we can only find the ratios of the sides a,b,c by using sine rule (since there are infinite similar triangles possible).

Illustration 18 : In the ambiguous case of the solution of triangles, prove that the circumcircles of the two triangles are of same size.

Solution : Let us say b,c and angle B are given in the ambiguous case. Both the triangles will have b and its opposite angle as B. so $\dfrac{b}{\sin B} = 2R$ will be given for both the triangles. So their circumradii and therefore their sizes will be same.

Illustration 19 : If a,b and A are given in a triangle and c_1, c_2 are the possible values of the third side, prove that $c_1^2 + c_2^2 - 2c_1c_2 \cos 2A = 4a^2\cos^2A$.

Solution :
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$\Rightarrow c^2 - 2bc \cos A + b^2 - a^2 = 0$.

$c_1 + c_2 = 2b\cos A$ and $c_1c_2 = b^2 - a^2$.

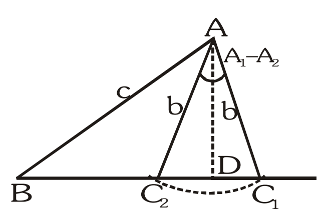
$\Rightarrow c_1^2 + c_2^2 - 2c_1c_2\cos 2A = (c_1 + c_2)^2 - 2c_1c_2(1 + \cos 2A)$

$= 4b^2 \cos^2A - 2(b^2 - a^2)2 \cos^2A = 4a^2\cos^2A$.

Illustration 20 : If b,c,B are given and $b < c$, prove that $\cos\left(\dfrac{A_1 - A_2}{2}\right) = \dfrac{c\sin B}{b}$.

Solution : $\angle C_2AC_1$ is bisected by AD.

$\Rightarrow$ In ΔAC_2D, $\cos\left(\dfrac{A_1 - A_2}{2}\right) = \dfrac{AD}{AC_2} = \dfrac{c\sin B}{b}$



Hence proved.

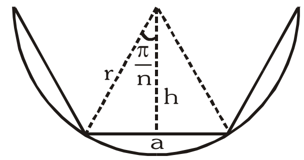
Do yourself - 11 :

(i) If b,c,B are given and b<c, prove that $\sin\left(\frac{A_1 - A_2}{2}\right) = \frac{a_1 - a_2}{2b}$

(ii) In a ΔABC, b,c,B (c > b) are gives. If the third side has two values a_1 and a_2 such that $a_1 = 3a_2$, show that $\sin B = \sqrt{\frac{4b^2 - c^2}{3c^2}}$.

14. REGULAR POLYGON :

A regular polygon has all its sides equal. It may be inscribed or circumscribed.

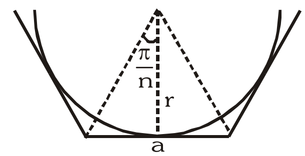


(a) Inscribed in circle of radius r :

(i) $a = 2h\tan\frac{\pi}{n} = 2r\sin\frac{\pi}{n}$

(ii) Perimeter (P) and area (A) of a regular polygon of n sides inscribed in a circle of radius r are given by $P = 2nr\sin\frac{\pi}{n}$ and $A = \frac{1}{2}nr^2\sin\frac{2\pi}{n}$

(b) Circumscribed about a circle of radius r :



(i) $a = 2r\tan\frac{\pi}{n}$

(ii) Perimeter (P) and area (A) of a regular polygon of n sides circumscribed about a given circle of radius r is given by $P = 2nr\tan\frac{\pi}{n}$ and $A = nr^2\tan\frac{\pi}{n}$

Do yourself - 12 :

(i) If the perimeter of a circle and a regular polygon of n sides are equal, then prove that $\frac{\text{area of the circle}}{\text{area of polygon}} = \frac{\tan\frac{\pi}{n}}{\frac{\pi}{n}}$.

(ii) The ratio of the area of n-sided regular polygon, circumscribed about a circle, to the area of the regular polygon of equal number of sides inscribed in the circle is 4 : 3. Find the value of n.

15. IMPORTANT POINTS :

(a) (i) If a cos B = b cos A, then the triangle is isosceles.

(ii) If a cos A = b cos B, then the triangle is isosceles or right angled.

(b) In right angle triangle

(i) $a^2 + b^2 + c^2 = 8R^2$ (ii) $\cos^2 A + \cos^2 B + \cos^2 C = 1$

(c) In equilateral triangle

(i) $R = 2r$ (ii) $r_1 = r_2 = r_3 = \frac{3R}{2}$

(iii) $r : R : r_1 = 1 : 2 : 3$ (iv) area $= \frac{\sqrt{3}a^2}{4}$ (v) $R = \frac{a}{\sqrt{3}}$

(d) (i) The circumcentre lies (1) inside an acute angled triangle (2) outside an obtuse angled triangle & (3) mid point of the hypotenuse of right angled triangle.

(ii) The orthocentre of right angled triangle is the vertex at the right angle.

(iii) The orthocentre, centroid & circumcentre are collinear & centroid divides the line segment joining orthocentre & circumcentre internally in the ratio 2 : 1 except in case of equilateral triangle. In equilateral triangle, all these centres coincide

(e) Area of a cyclic quadrilateral $= \sqrt{s(s-a)(s-b)(s-c)(s-d)}$

where a, b, c, d are lengths of the sides of quadrilateral and $s = \frac{a+b+c+d}{2}$.

Illustration 21 : For a ΔABC, it is given that $\cos A + \cos B + \cos C = 3/2$. Prove that the triangle is equilateral.

Solution : If a, b, c are the sides of the ΔABC, then $\cos A + \cos B + \cos C = 3/2$

$$\Rightarrow \quad \frac{b^2+c^2-a^2}{2bc}+\frac{a^2+c^2-b^2}{2ac}+\frac{a^2+b^2-c^2}{2ab}=\frac{3}{2}$$

$$\Rightarrow \quad ab^2 + ac^2 - a^3 + bc^2 + ba^2 - b^3 + ca^2 + cb^2 - c^3 = 3abc$$

$$\Rightarrow \quad ab^2 + ac^2 + bc^2 + ba^2 + ca^2 + cb^2 - 6abc = a^3 + b^3 + c^3 - 3abc$$

$$\Rightarrow \quad a(b-c)^2 + b(c-a)^2 + c(a-b)^2 = \frac{(a+b+c)}{2}\left\{(a-b)^2+(b-c)^2+(c-a)^2\right\}$$

$$\Rightarrow \quad (a+b-c)(a-b)^2 + (b+c-a)(b-c)^2 + (c+a-b)(c-a)^2 = 0 \quad \ldots\ldots\ldots \text{(i)}$$

as we know $a + b > c$, $b + c > a$, $c + a > b$

$\therefore$ each term on the left side of equation (i) has positive coefficient multiplied by perfect square, each must be separately zero.

$\Rightarrow \quad a = b = c.$

Hence Δ is equilateral. **Ans.**

Illustration 22 : In a triangle ABC, if $\cos A + 2\cos B + \cos C = 2$. Prove that the sides of the triangle are in A.P.

Solution : $\cos A + 2\cos B + \cos C = 2$ or $\cos A + \cos C = 2(1 - \cos B)$

$$\Rightarrow \quad 2\cos\left(\frac{A+C}{2}\right).\cos\left(\frac{A-C}{2}\right) = 4\sin^2 B/2$$

$$\Rightarrow \quad \cos\left(\frac{A-C}{2}\right) = 2\sin\frac{B}{2} \qquad \left\{\text{as } \cos\left(\frac{A+C}{2}\right) = \cos\left(\frac{\pi}{2}-\frac{B}{2}\right) = \sin\frac{B}{2}\right\}$$

$$\Rightarrow \quad \cos\left(\frac{A-C}{2}\right) = 2\cos\left(\frac{A+C}{2}\right)$$

$$\Rightarrow \quad \cos\frac{A}{2}.\cos\frac{C}{2}+\sin\frac{A}{2}.\sin\frac{C}{2} = 2\cos\frac{A}{2}.\cos\frac{C}{2}-2\sin\frac{A}{2}.\sin\frac{C}{2}$$

$$\Rightarrow \quad \cot\frac{A}{2}.\cot\frac{C}{2} = 3 \quad \Rightarrow \quad \sqrt{\frac{s(s-a)}{(s-b)(s-c)}}.\sqrt{\frac{s(s-c)}{(s-a)(s-b)}} = 3$$

$$\Rightarrow \quad \frac{s}{(s-b)} = 3 \Rightarrow s = 3s-3b \Rightarrow \quad 2s = 3b$$

$\Rightarrow \quad a + c = 2b, \quad \therefore \quad$ a, b, c are in A.P. **Ans.**

ANSWERS FOR DO YOURSELF

1 : (i) 90

4 : (iii) $\frac{1}{3}$

5 : (i) (a) $\frac{3}{5}$ (b) $\frac{3}{4}$ (c) $\frac{1}{\sqrt{10}}$ (d) $\frac{3}{\sqrt{10}}$ (e) $\frac{1}{3}$ (f) 24

7 : (i) (a) 6 (b) $\frac{5}{2}$ (c) 1

8 : (i) (a) 1 (b) 3 (c) $2\sqrt{3}$

12 : (ii) 6

EXERCISE - 01 CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. In a triangle $\angle A = 55°$ and $\angle B = 15°$, then $\frac{c^2 - a^2}{ab}$ is equal to -

(A) 4 (B) 3 (C) 2 (D) 1

2. In a triangle ABC a : b : c = $\sqrt{3}$: 1 : 1, then the triangle is -

(A) right angled triangle (B) obtuse angled triangle
(C) acute angled triangle, which is not isosceles (D) Equilateral triangle

3. The sides of a triangle ABC are x, y, $\sqrt{x^2 + y^2 + xy}$ respectively. The size of the greatest angle in radians is -

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) none of these

4. In a $\Delta ABC \left(\frac{a^2}{\sin A} + \frac{b^2}{\sin B} + \frac{c^2}{\sin C}\right) . \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$ simplifies to -

(A) 2Δ (B) Δ (C) $\frac{\Delta}{2}$ (D) $\frac{\Delta}{4}$

(where Δ is the area of triangle)

5. If p_1, p_2, p_3 are the altitudes of a triangle from its vertices A, B, C and Δ, the area of the triangle ABC, then $\frac{1}{p_1} + \frac{1}{p_2} - \frac{1}{p_3}$ is equal to -

(A) $\frac{s}{\Delta}$ (B) $\frac{s-c}{\Delta}$ (C) $\frac{s-b}{\Delta}$ (D) $\frac{s-a}{\Delta}$

6. If in a triangle ABC angle B = 90 then $\tan^2 A/2$ is -

(A) $\frac{b-c}{a}$ (B) $\frac{b-c}{b+c}$ (C) $\frac{b+c}{b-c}$ (D) $\frac{b+c}{a}$

7. In a triangle ABC, if $\frac{a^3 + b^3 + c^3}{\sin^3 A + \sin^3 B + \sin^3 C} = 343$, the diameter of the circle circumscribing the triangle is -

(A) 7 units (B) 14 units (C) 21 units (D) none of these

8. In a ΔABC if b + c = 3a then $\cot\frac{B}{2} . \cot\frac{C}{2}$ has the value equal to -

(A) 4 (B) 3 (C) 2 (D) 1

9. If $\frac{a}{\sin A}$ = K, then the area of ΔABC in terms of K and sines of the angles is -

(A) $\frac{K^2}{4} \sin A \sin B \sin C$ (B) $\frac{K^2}{2} \sin A \sin B \sin C$
(C) $2K^2 \sin A \sin B \sin(A + B)$ (D) none

10. In a ΔABC, $\angle C = 60°$ & $\angle A = 75°$. If D is a point on AC such that the area of the ΔBAD is $\sqrt{3}$ times the area of the ΔBCD, then the $\angle ABD$ =

(A) 60 (B) 30 (C) 90 (D) none of these

11. In a ΔABC, a semicircle is inscribed, whose diameter lies on the side c. Then the radius of the semicircle is (Where Δ is the area of the triangle ABC)

(A) $\frac{2\Delta}{a+b}$ (B) $\frac{2\Delta}{a+b-c}$ (C) $\frac{2\Delta}{s}$ (D) $\frac{c}{2}$

12. In a triangle ABC, right angled at B, the inradius is -

(A) $\frac{AB+BC-AC}{2}$ (B) $\frac{AB+AC-BC}{2}$ (C) $\frac{AB+BC+AC}{2}$ (D) none

13. In triangle ABC where A, B, C are acute, the distances of the orthocentre from the sides are in the proportion

(A) cos A : cos B : cos C (B) sin A : sin B : sin C

(C) sec A : sec B : sec C (D) tan A : tan B : tan C

14. In a ΔABC, the value of $\frac{a\cos A+b\cos B+c\cos C}{a+b+c}$ is equal to -

(A) $\frac{r}{R}$ (B) $\frac{R}{2r}$ (C) $\frac{R}{r}$ (D) $\frac{2r}{R}$

15. If the orthocentre and circumcentre of a triangle ABC be at equal distances from the side BC and lie on the same side of BC then $\tan B\tan C$ has the value equal to -

(A) 3 (B) $\frac{1}{3}$ (C) -3 (D) $-\frac{1}{3}$

16. In an equilateral triangle, inradius r, circumradius R & ex-radius r_1 are in -

(A) A.P. (B) G.P. (C) H.P. (D) none of these

17. With usual notation in a ΔABC $\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\left(\frac{1}{r_2}+\frac{1}{r_3}\right)\left(\frac{1}{r_3}+\frac{1}{r_1}\right)=\frac{K\ R^3}{a^2\ b^2\ c^2}$ then K has value equal to -

(A) 1 (B) 16 (C) 64 (D) 128

18. In a triangle ABC, $\frac{r_1+r_2}{1+\cos C}$ is equal to -

(A) $2ab/c\Delta$ (B) $(a+b)/c\Delta$ (C) $abc/2\Delta$ (D) abc/Δ^2

19. With usual notations in a triangle ABC, if $r_1 = 2r_2 = 2r_3$ then -

(A) $4a = 3b$ (B) $3a = 2b$ (C) $4b = 3a$ (D) $2a = 3b$

20. If r_1, r_2, and r_3 be the radii of excircles of the triangle ABC, then $\frac{\sum r_1}{\sqrt{\sum r_1r_2}}$ is equal to -

(A) $\sum\cot\frac{A}{2}$ (B) $\sum\cot\frac{A}{2}\cot\frac{B}{2}$ (C) $\sum\tan\frac{A}{2}$ (D) $\prod\tan\frac{A}{2}$

21. If in a triangle PQR, sin P, sin Q, sin R are in A.P., then - [JEE 98]

(A) the altitudes are in A.P. (B) the altitudes are in H.P.

(C) the medians are in G.P. (D) the medians are in A.P.

22. In ΔABC, if $r : r_1 : R = 2 : 12 : 5$, where all symbols have their usual meaning, then -

(A) ΔABC is an acute angled triangle (B) ΔABC is an obtuse angled triangle

(C) ΔABC is right angled which is not isosceles (D) ΔABC is isosceles which is not right angled

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

23. In a ΔABC, $A = \frac{\pi}{3}$ and $b : c = 2 : 3$. If $\tan\alpha = \frac{\sqrt{3}}{5}, 0 < \alpha < \frac{\pi}{2}$, then -

(A) $B = 60° + \alpha$ (B) $C = 60° + \alpha$ (C) $B = 60° - \alpha$ (D) $C = 60° - \alpha$

24. In a triangle ABC, points D and E are taken on sides BC such that DB = DE = EC. If $\angle ADE = \angle AED = \theta$, then -

(A) $\tan\theta = 3\tan B$ (B) $\tan\theta = 3\tan C$ (C) $\tan A = \frac{6\tan\theta}{\tan^2\theta - 9}$ (D) $9\cot^2\frac{A}{2} = \tan^2\theta$

25. If a, b, A are given in a triangle and c_1 and c_2 are two possible values of third side such that $c_1^2 + c_1c_2 + c_2^2 = a^2$, then A is equal to -

(A) 30 (B) 60 (C) 90 (D) 120

26. In a ΔABC, AD is the bisector of the angle A meeting BC at D. If I is the incentre of the triangle, then AI : DI is equal to -

(A) (sinB + sinC) : sinA (B) (cosB + cosC) : cosA

(C) $\cos\left(\frac{B-C}{2}\right) : \cos\left(\frac{B+C}{2}\right)$ (D) $\sin\left(\frac{B-C}{2}\right) : \sin\left(\frac{B+C}{2}\right)$

CHECK YOUR GRASP — ANSWER KEY — EXERCISE-1

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	B	A	B	B	B	A	C	B	B
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A	A	C	A	A	A	C	C	C	C
Que.	21	22	23	24	25	26				
Ans.	B	C	B,C	A,B,C,D	B	A,C				

EXERCISE - 02

BRAIN TEASERS

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. If A, B, C are angles of a triangle which of the following will not imply it is equilateral -

(A) $\tan A + \tan B + \tan C = 3\sqrt{3}$ (B) $\cot A + \cot B + \cot C = \sqrt{3}$

(C) $a + b + c = 2R$ (D) $a^2 + b^2 + c^2 = 9R^2$

2. In a ΔABC, $\frac{s}{R}$ is equal to -

(A) $\sin A + \sin B + \sin C$ (B) $4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ (C) $4\sin A\ \sin B\sin C$ (D) $\frac{\Delta s}{abc}$

3. If $\cos A + \cos B + 2\cos C = 2$ then the sides of the ΔABC are in-

(A) A.P. (B) G.P. (C) H.P. (D) none

4. The line $\frac{x}{6}+\frac{y}{8}=1$ cuts the co-ordinate axis at A & B. If O is origin, then $\prod\sin\frac{A}{2}$ for the triangle OAB is -

(A) 5/6 (B) 1/10 (C) 5/4 (D) none of above

5. In a triangle ABC, CD is the bisector of the angle C. If $\cos\frac{C}{2}$ has the value $\frac{1}{3}$ and $\ell(CD) = 6$, then $\left(\frac{1}{a}+\frac{1}{b}\right)$ has the value equal to -

(A) $\frac{1}{9}$ (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) none

6. In the triangle ABC, CH and CM are the lengths of the altitude and median to the base AB. If a = 10, b = 26, c = 32 then length HM is -

(A) 5 (B) 7 (C) 9 (D) none

7. D, E, F are the foot of the perpendiculars from vertices A, B, C to sides BC, CA, AB respectively, and H is the orthocentre of acute angled triangle ABC; where a, b, c are the sides of triangle ABC, then

(A) H is the incentre of triangle DEF

(B) A, B, C are excentres of triangle DEF

(C) Perimeter of ΔDEF is $a\cos A + b\cos B + c\cos C$

(D) Circumradius of triangle DEF is $\frac{R}{2}$, where R is circumradius of ΔABC.

8. If x, y and z are the distances of incentre from the vertices of the triangle ABC respectively then $\frac{abc}{xyz}$ is equal to -

(A) $\prod\tan\frac{A}{2}$ (B) $\sum\cot\frac{A}{2}$ (C) $\sum\tan\frac{A}{2}$ (D) $\prod\cot\frac{A}{2}$

9. The medians of a ΔABC are 9 cm, 12 cm and 15 cm respectively. Then the area of the triangle is -

(A) 96 sq cm (B) 84 sq cm (C) 72 sq cm (D) 60 sq cm

10. In an isosceles ΔABC, if the altitudes intersect on the inscribed circle then the cosine of the vertical angle 'A' is :

(A) $\frac{1}{9}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) none

11. In triangle ABC, cosA + 2cosB + cosC = 2, then -

(A) $\tan\frac{A}{2}\tan\frac{C}{2}=3$ (B) $\cot\frac{A}{2}\cot\frac{C}{2}=3$ (C) $\cot\frac{A}{2}+\cot\frac{C}{2}=2\cot\frac{B}{2}$ (D) $\tan\frac{A}{2}\tan\frac{C}{2}=0$

12. If in a triangle ABC p, q, r are the altitudes from the vertices A, B, C to the opposite sides, then which of the following does not hold good ?

(A) $(\Sigma p)\left(\Sigma\frac{1}{p}\right) = (\Sigma a)\left(\Sigma\frac{1}{a}\right)$ (B) $(\Sigma p)\ (\Sigma a) = \left(\Sigma\frac{1}{p}\right)\left(\Sigma\frac{1}{a}\right)$

(C) $(\Sigma p)\ (\Sigma pq)\ (\Pi a) = (\Sigma a)\ (\Sigma ab)\ (\Pi p)$ (D) $\left(\Sigma\frac{1}{p}\right)\Pi\left(\frac{1}{p}+\frac{1}{q}-\frac{1}{r}\right)\Pi a^2 = 16R^2$

13. AD, BE and CF are the perpendiculars from the angular points of a ΔABC upon the opposite sides. The perimeters of the ΔDEF and ΔABC are in the ratio -

(A) $\frac{2r}{R}$ (B) $\frac{r}{2R}$ (C) $\frac{r}{R}$ (D) $\frac{r}{3R}$

Where r is the inradius and R is circum-radius of the ΔABC

14. If 'O' is the circum centre of the ΔABC and R_1, R_2 and R_3 are the radii of the circumcircles of triangles OBC, OCA and OAB respectively then $\frac{a}{R_1}+\frac{b}{R_2}+\frac{c}{R_3}$ has the value equal to -

(A) $\frac{abc}{2R^3}$ (B) $\frac{R^3}{abc}$ (C) $\frac{4\Delta}{R^2}$ (D) $\frac{abc}{R^3}$

15. In a triangle ABC, $(r_1 - r)\ (r_2 - r)\ (r_3 - r)$ is equal to -

(A) $4Rr^2$ (B) $\frac{4abc.\Delta}{(a+b+c)^2}$

(C) $16R^3(\cos A + \cos B + \cos C - 1)$ (D) $r^3\operatorname{cosec}\frac{A}{2}\operatorname{cosec}\frac{B}{2}\operatorname{cosec}\frac{C}{2}$

16. Two rays emanate from the point A and form an angle of 43 with one another. Lines L_1, L_2 and L_3 (no two of which are parallel) each form an isosceles triangle with the original rays. The largest angle of the triangle formed by lines L_1, L_2 and L_3 is -

(A) 127 (B) 129 (C) 133 (D) 137

BRAIN TEASERS — ANSWER KEY — EXERCISE-2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	A,B	A	B	A	C	A,B,C,D	B,D	C	A
Que.	11	12	13	14	15	16				
Ans.	B,C	B	C	C,D	A,B,D	B				

EXERCISE - 03 MISCELLANEOUS TYPE QUESTIONS

TRUE / FALSE

1. If external angle bisector of any angle of triangle ABC is parallel to the opposite base then triangle is isosceles.
2. Sides of the pedal triangle of any acute or obtuse angle triangle are given by Rsin2A, Rsin2B and Rsin2C.
3. In the triangle ABC, the altitudes p_1, p_2, p_3 are in AP, then a, b, c are in HP.
4. In a triangle ABC, if $a^4 - 2(b^2 + c^2)a^2 + b^4 + b^2c^2 + c^4 = 0$, then $\angle A$ is 60 or 120

FILL IN THE BLANKS

1. In a ΔABC, tan A : tan B : tan C = 1 : 2 : 3. Hence sinA : sinB : sinC = ____________.
2. In triangle ABC, if a = 2, b = 3 and tan A = $\sqrt{\frac{3}{5}}$ then the two possible values of the side c are $K_1\sqrt{10}$ and $K_2\sqrt{10}$ then K_1 and K_2 are equal to ___________ and _____________.
3. If f, g and h are the lengths of the perpendiculars from the circumcentre on the sides a, b and c of a triangle ABC respectively then $\frac{a}{f} + \frac{b}{g} + \frac{c}{h} = K\frac{abc}{fgh}$ where K has the value equal to ______________.

MATCH THE COLUMN

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE** statement in **Column-II**.

1. If p_1, p_2, p_3 are altitudes of a triangle ABC from the vertices A, B, C respectively and Δ is the area of the triangle and s is semi perimeter of the triangle, then match the columns

	Column-I		Column-I
(A)	If $\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}=\frac{1}{2}$ then the least value of $p_1p_2p_3$ is	(p)	$\frac{1}{R}$
(B)	The value of $\frac{\cos A}{p_1}+\frac{\cos B}{p_2}+\frac{\cos C}{p_3}$ is	(q)	216
(C)	The minimum value of $\frac{b^2p_1}{c}+\frac{c^2p_2}{a}+\frac{a^2p_3}{b}$ is	(r)	6Δ
(D)	The value of $p_1^{-2} + p_2^{-2} + p_3^{-2}$ is	(s)	$\frac{\Sigma a^2}{4\Delta^2}$

ASSERTION & REASON

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.
(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.
(C) Statement-I is true, Statement-II is false.
(D) Statement-I is false, Statement-II is true.

1. **Statement-I** : If two sides of a triangle are 4 and 5, then its area lies in (0, 10]

Because :

Statement-II : Area of a triangle $=\frac{1}{2}$ ab sinC and sinC $\in$ (0, 1]

(A) A (B) B (C) C (D) D

2. **Statement-I** : Perimeter of a regular pentagon inscribed in a circle with centre O and radius a cm equals $10 a \sin 36°$ cm

Because :

Statement-II : Perimeter of a regular polygon inscribed in a circle with centre O and radius a cm equals

$(3n - 5)\ \sin\left(\frac{360°}{2n}\right)$ cm, then it is n sided, where $n \geq 3$

(A) A (B) B (C) C (D) D

3. **Statement-I** : The statement that circumradius and inradius of a triangle are 12 and 8 respectively can not be correct.

Because :

Statement-II : Circumradius $\geq$ 2 (inradius)

(A) A (B) B (C) C (D) D

4. **Statement-I** : In any triangle ABC, the minimum value of $\frac{r_1 + r_2 + r_3}{r}$ is 9

Because :

Statement-II : For any three numbers AM $\geq$ GM

(A) A (B) B (C) C (D) D

5. **Statement-I** : Area of triangle having sides greater than 9 can be smaller than area of triangle having sides less than 3.

Because :

Statement-II : Sine of an angle of triangle can take any value in (0, 1]

(A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1

Let A_n be the area that is outside a n-sided regular polygon and inside it's circumscribing circle. Also B_n is the area inside the polygon and outside the circle inscribed in the polygon. Let R be the radius of the circle circumscribing n-sided polygon.

On the basis of above information, answer the following questions :

1. If n = 6 then A_n is equal to-

(A) $R^2\left(\frac{\pi - \sqrt{3}}{2}\right)$ (B) $R^2\left(\frac{2\pi - 6\sqrt{3}}{2}\right)$ (C) $R^2\left(\pi - \sqrt{3}\right)$ (D) $R^2\left(\frac{2\pi - 3\sqrt{3}}{2}\right)$

2. If n = 4 then B_n is equal to -

(A) $R^2\frac{(4-\pi)}{2}$ (B) $R^2\frac{(4-\pi\sqrt{2})}{2}$ (C) $R^2\frac{(4\sqrt{2}-\pi)}{2}$ (D) none of these

3. $\frac{A_n}{B_n}$ is equal to $\left(\theta=\frac{\pi}{n}\right)$ -

(A) $\frac{2\theta-\sin 2\theta}{\sin 2\theta-\theta\cos^2\theta}$ (B) $\frac{2\theta-\sin\theta}{\sin 2\theta-\theta\cos^2\theta}$ (C) $\frac{\theta-\cos\theta\sin\theta}{\cos\theta(\sin\theta-\theta\cos\theta)}$ (D) none of these

MISCELLANEOUS TYPE QUESTION **ANSWER KEY** EXERCISE-3

- ***True / False***

  1. T 2. F 3. T 4. T

- ***Fill in the Blanks***

  1. $\sqrt{5}:2\sqrt{2}:3$ 2. 1 and 1/2 3. $\frac{1}{4}$

- ***Match the Column***

  1. (A) $\rightarrow$ (q), (B) $\rightarrow$ (p), (C) $\rightarrow$ (r), (D) $\rightarrow$ (s)

- ***Assertion & Reason***

  1. A 2. C 3. A 4. C 5. A

- ***Comprehension Based Questions***

  Comprehension # 1 : 1. D 2. A 3. C

EXERCISE - 04 [A]

CONCEPTUAL SUBJECTIVE EXERCISE

1. Prove that : $4R \sin A \sin B \sin C = a\cos A + b\cos B + c\cos C$.

2. Prove that : $a\cos B\cos C + b\cos C\cos A + c\cos A\cos B = \dfrac{\Delta}{R}$

3. If p_1, p_2, p_3 are the altitudes of a triangle from the vertices A, B, C & Δ denotes the area of the triangle, prove that $\dfrac{1}{p_1}+\dfrac{1}{p_2}-\dfrac{1}{p_3}=\dfrac{2ab}{(a+b+c)\Delta}\cos^2\dfrac{C}{2}$.

4. Prove that : $\dfrac{abc}{s}\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}=\Delta$

5. For any triangle ABC, if B = 3C , show that $\cos C = \sqrt{\dfrac{b+c}{4c}}$ & $\sin\dfrac{A}{2}=\dfrac{b-c}{2c}$.

6. ABC is a triangle. D is the mid point of BC. If AD is perpendicular to AC, then prove that $\cos A \,.\, \cos C = \dfrac{2(c^2-a^2)}{3ac}$.

7. Let $1 < m < 3$. In a triangle ABC , if $2b = (m+1)a$ & $\cos A = \dfrac{1}{2}\sqrt{\dfrac{(m-1)(m+3)}{m}}$ prove that there are two values to the third side, one of which is m times the other.

8. Prove that : $\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}=\dfrac{1}{r}$

9. Prove that : $r_1 + r_2 + r_3 - r = 4R$

10. Prove that : $\dfrac{b-c}{r_1}+\dfrac{c-a}{r_2}+\dfrac{a-b}{r_3}=0$

11. Consider a ΔDEF, the pedal triangle of the ΔABC such that A–F–B and B–D–C are collinear . If H is the incentre of ΔDEF and R_1, R_2 , R_3 are the circumradii of the quadarilaterals AFHE; BDHF and CEHD respectively, then prove that $\sum R_1 = R + r$ where R is the circumradius and r is the inradius of ΔABC.

12. DEF is the triangle formed by joining the points of contact of the incircle with the sides of the triangle ABC, prove that

    (a) its sides are $2r\cos\dfrac{A}{2}$, $2r\cos\dfrac{B}{2}$ and $2r\cos\dfrac{C}{2}$ where r is the radius of incircle of ΔABC.

    (b) its angles are $\dfrac{\pi}{2}-\dfrac{A}{2}$, $\dfrac{\pi}{2}-\dfrac{B}{2}$ and $\dfrac{\pi}{2}-\dfrac{C}{2}$

    (c) its area is $\dfrac{r^2 s}{2R}$ where 's' is the semiperimeter and R is the circumradius of the ΔABC.

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

1. If sides a, b, c, of the triangle ABC are in A.P., then prove that

$\sin^2\frac{A}{2}\operatorname{cosec}2A$; $\sin^2\frac{B}{2}\operatorname{cosec}2B$; $\sin^2\frac{C}{2}\operatorname{cosec}2C$ are in H.P.

2. Sides a, b, c of the triangle ABC are in H.P., then prove that
cosecA (cosecA + cot A) ; cosec B (cosecB + cotB) & cosecC (cosecC + cot C) are in A.P.

3. In a Δ ABC, GA,GB,GC makes angles α, β, γ with each other where G is the centroid to the Δ ABC then show that, cotA + cotB + cotC + cot α + cot β +cot γ =0.

4. In a triangle ABC, the median to the side BC is of length $\frac{1}{\sqrt{11-6\sqrt{3}}}$ & it divides the angle A into angles of 30 & 45 . Find the length of the side BC.

5. Prove that : $\frac{r_1}{bc}+\frac{r_2}{ca}+\frac{r_3}{ab}=\frac{1}{r}-\frac{1}{2R}$

6. Prove that : $\frac{\tan\frac{A}{2}}{(a-b)(a-c)}+\frac{\tan\frac{B}{2}}{(b-a)(b-c)}+\frac{\tan\frac{C}{2}}{(c-a)(c-b)}=\frac{1}{\Delta}$

7. Prove that in a triangle $\frac{bc}{r_1}+\frac{ca}{r_2}+\frac{ab}{r_3}=2R\left[\left(\frac{a}{b}+\frac{b}{a}\right)+\left(\frac{b}{c}+\frac{c}{b}\right)+\left(\frac{c}{a}+\frac{a}{c}\right)-3\right]$.

8. In a triangle the angles A, B, C are in A.P. Show that $2\cos\frac{A-C}{2}=\frac{a+c}{\sqrt{a^2-ac+c^2}}$.

9. In a scalene triangle ABC the altitudes AD & CF are dropped form the vertices A & C to the sides BC & AB. The area of ΔABC is known to be equal to 18, the area of triangle BDF is equal to 2 and length of segment DF is equal to $2\sqrt{2}$. Find the radius of the circle circumscribing ΔABC.

10. With reference to a given circle, A_1 and B_1 are the areas of the inscribed and circumscribed regular polygons of n sides, A_2 and B_2 are corresponding quantities for regular polygons of 2n sides : Prove that
(a) A_2 is a geometric mean between A_1 and B_1
(b) B_2 is a harmonic mean between A_2 and B_1

11. Let a, b, c be the sides of a trianlge & Δ its area. Prove that $a^2+b^2+c^2 \geq 4\sqrt{3}\Delta$, and find when does the equality hold ?

12. If in a triangle of base 'a', the ratio of the other two sides is r (< 1) . Show that the altitude of the triangle is less than or equal to $\frac{ar}{1-r^2}$.

13. If the bisector of angle C of triangle ABC meets AB in D & the circumcircle in E prove that , $\frac{CE}{DE}=\frac{(a+b)^2}{c^2}$.

BRAIN STORMING SUBJECTIVE EXERCISE	ANSWER KEY	EXERCISE-4(B)
4. a = 2	**9.** 9/2 units	**11.** b = c & A = 60

EXERCISE - 05

JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

1. If Δ is the area of a triangle with side lengths a, b, c, then show that: $\Delta \le \frac{1}{4}\sqrt{(a+b+c)abc}$

Also show that equality occurs in the above inequality if and only if a = b = c. [JEE 2001]

2. Which of the following pieces of data does NOT uniquely determine an acute-angled triangle ABC(R being the radius of the circumcircle)?

(A) a, sinA, sinB (B) a, b, c (C) a, sinB, R (D) a, sinA, R

[JEE 2002 (Scr), 3]

3. If I_n is the area of n sided regular polygon inscribed in a circle of unit radius and O_n be the area of the polygon circumscribing the given circle, prove that $I_n = \frac{O_n}{2}\left(1+\sqrt{1-\left(\frac{2I_n}{n}\right)^2}\right)$

[JEE 2003, Mains, 4 out of 60]

4. The ratio of the sides of a triangle ABC is $1 : \sqrt{3} : 2$. The ratio A : B : C is [JEE 2004 (Screening)]

(A) 3 : 5 : 2 (B) $1 : \sqrt{3} : 2$ (C) 3 : 2 : 1 (D) 1 : 2 : 3

5. (a) In ΔABC, a, b, c are the lengths of its sides and A, B, C are the angles of triangle ABC. The correct relation is [JEE 2005 (Screening)]

(A) $(b-c)\sin\left(\frac{B-C}{2}\right) = a\cos\left(\frac{A}{2}\right)$ (B) $(b-c)\cos\left(\frac{A}{2}\right) = a\sin\left(\frac{B-C}{2}\right)$

(C) $(b+c)\sin\left(\frac{B+C}{2}\right) = a\cos\left(\frac{A}{2}\right)$ (D) $(b-c)\cos\left(\frac{A}{2}\right) = 2a\sin\left(\frac{B+C}{2}\right)$

(b) Circles with radii 3, 4 and 5 touch each other externally if P is the point of intersection of tangents to these circles at their points of contact. Find the distance of P from the points of contact. [JEE 2005 (Mains), 2]

6. (a) Given an isosceles triangle, whose one angle is 120 and radius of its incircle is $\sqrt{3}$. Then the area of triangle in sq. units is [JEE 2006, 3]

(A) $7 + 12\sqrt{3}$ (B) $12 - 7\sqrt{3}$ (C) $12 + 7\sqrt{3}$ (D) 4π

(b) Internal bisector of $\angle$A of a triangle ABC meets side BC at D. A line drawn through D perpendicular to AD intersects the side AC at E and the side AB at F. If a, b, c represent sides of ΔABC then

(A) AE is HM of b and c (B) $AD = \frac{2bc}{b+c}\cos\frac{A}{2}$

(C) $EF = \frac{4bc}{b+c}\sin\frac{A}{2}$ (D) the triangle AEF is isosceles [JEE 2006, 5]

7. Let ABC and ABC′ be two non-congruent triangles with sides AB = 4, AC = AC′ = $2\sqrt{2}$ and angle B = 30 . The absolute value of the difference between the areas of these triangles is

[JEE 2009, 5]

8. (a) If the angle A,B and C of a triangle are in an arithmetic progression and if a,b and c denote the length of the sides opposite to A,B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$, is -

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) 1 (D) $\sqrt{3}$

(b) Consider a triangle ABC and let a,b and c denote the length of the sides opposite to vertices A,B and C respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then r^2 is equal to

(c) Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a,b and c denote the lengths of the sides opposite to A,B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is/are [JEE 2010, 3+3+3]

(A) $-(2+\sqrt{3})$ (B) $1+\sqrt{3}$ (C) $2+\sqrt{3}$ (D) $4\sqrt{3}$

9. Let PQR be a triangle of area Δ with a = 2, $b = \frac{7}{2}$ and $c = \frac{5}{2}$, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals

[JEE 2012, 3M, –1M]

(A) $\frac{3}{4\Delta}$ (B) $\frac{45}{4\Delta}$ (C) $\left(\frac{3}{4\Delta}\right)^2$ (D) $\left(\frac{45}{4\Delta}\right)^2$

PREVIOUS YEARS QUESTIONS				ANSWER KEY				EXERCISE-5	
2.	D	4.	D	5.	(a) B; (b) $\sqrt{5}$	6.	(a) C, (b) A, B, C, D	7.	4
8.	(a) D, (b) 3, (c) B			9.	C				