3D-COORDINATE GEOMETRY

POINT

1. INTRODUCTION :

In earlier classes we have learnt about points, lines, circles and conic section in two dimensional geometry. In two dimensions a point represented by an ordered pair (x, y) (where x & y are both real numbers)

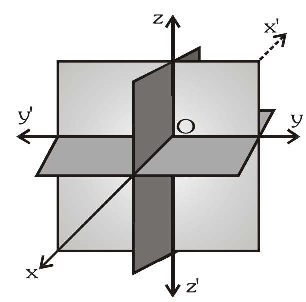
In space, each body has length, breadth and height i.e. each body exist in three dimensional space. Therefore three independent quantities are essential to represent any point in space. Three axes are required to represent these three quantities.

2. RECTANGULAR CO-ORDINATE SYSTEM :

In cartesian system of the three lines are mutually perpendicular, such a system is called rectangular cartesian co-ordinate system.

Co-ordinate axes and co-ordinate planes :

When three mutually perpendicular planes intersect at a point, then mutually perpendicular lines are obtained and these lines also pass through that point. If we assume the point of intersection as origin, then the three planes are known as co-ordinate planes and the three lines are known as co-ordinate axes.

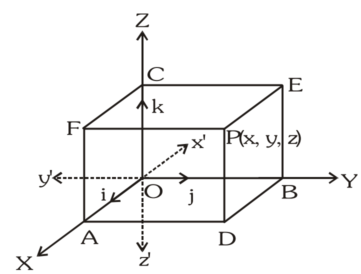


Octants :

Every plane bisects the space. Hence three co-ordinate plane divide the space in eight parts. These parts are known as octants.

3. COORDINATES OF A POINT IN SPACE :

Let O be a fixed point, known as origin and let OX, OY and OZ be three mutually perpendicular lines, taken as x-axis, y-axis and z-axis respectively, in such a way that they form a right handed system.



The planes XOY, YOZ and ZOX are known as xy-plane, yz-plane and zx-plane respectively.

Let P be a point in space and distances of P from yz, zx and xy planes be x, y, z respectively (with proper signs) then we say that coordinates of P are (x, y, z). Also OA = |x|, OB = |y|, OC = |z|

4. DISTANCE FORMULA :

The distance between two points A (x_1, y_1, z_1) and B (x_2, y_2, z_2) is given by

$$AB = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$$

(a) Distance from Origin :

Let O be the origin and P (x, y, z) be any point, then $OP = \sqrt{(x^2 + y^2 + z^2)}$

(b) Distance of a point from coordinate axes :

Let P(x, y, z) be any point in the space. Let PA, PB and PC be the perpendiculars drawn from P to the axes OX, OY and OZ respectively. Then

$$PA = \sqrt{(y^2 + z^2)}\ ; \quad PB = \sqrt{(z^2 + x^2)}\ ; \quad PC = \sqrt{(x^2 + y^2)}$$

Illustration 1 : Prove by using distance formula that the points P (1, 2, 3), Q (–1, –1, –1) and R (3, 5, 7) are collinear.

Solution : We have PQ = $\sqrt{(-1-1)^2+(-1-2)^2+(-1-3)^2} = \sqrt{4+9+16} = \sqrt{29}$

$$QR = \sqrt{(3+1)^2+(5+1)^2+(7+1)^2} = \sqrt{16+36+64} = \sqrt{116} = 2\sqrt{29}$$

and $PR = \sqrt{(3-1)^2+(5-2)^2+(7-3)^2} = \sqrt{4+9+16} = \sqrt{29}$

Since QR = PQ + PR. Therefore the given points are collinear. **Ans.**

Illustration 2 : Find the locus of a point the sum of whose distances from (1, 0, 0) and (–1, 0, 0) is equal to 10.

Solution : Let the points A(1,0,0), B (–1,0,0) and P(x,y,z)

Given : PA + PB =10

$$\sqrt{(x-1)^2+(y-0)^2+(z-0)^2} + \sqrt{(x+1)^2+(y-0)^2+(z-0)^2} = 10$$

$\Rightarrow \quad \sqrt{(x-1)^2+y^2+z^2} = 10 - \sqrt{(x+1)^2+y^2+z^2}$

Squaring both sides, we get ;

$\Rightarrow \quad (x-1)^2+y^2+z^2 = 100+(x+1)^2+y^2+z^2-20\sqrt{(x+1)^2+y^2+z^2}$

$\Rightarrow \quad -4x-100 = -20\sqrt{(x+1)^2+y^2+z^2} \quad \Rightarrow \quad x+25 = 5\sqrt{(x+1)^2+y^2+z^2}$

Again squaring both sides we get $x^2+50x+625 = 25\{(x^2+2x+1)+y^2+z^2\}$

$\Rightarrow \quad 24x^2+25y^2+25z^2-600 = 0$

i.e. required equation of locus **Ans.**

5. SECTION FORMULAE :

Let $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points and let R (x, y, z) divide PQ in the ratio $m_1 : m_2$. Then co-ordinates of R(x, y, z) = $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}, \frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$

If (m_1/m_2) is positive, R divides PQ internally and if (m_1/m_2) is negative, then externally.

Mid-Point : Mid point of PQ is given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

Illustration 3 : Find the ratio in which the plane x – 2y + 3z = 17 divides the line joining the points (–2, 4, 7) and (3, –5, 8).

Solution : Let the required ratio be k : 1

The co-ordinates of the point which divides the join of (–2, 4, 7) and (3, –5, 8) in the ratio k : 1 are $\left(\frac{3k-2}{k+1}, \frac{-5k+4}{k+1}, \frac{8k+7}{k+1}\right)$

Since this point lies on the plane x – 2y + 3z – 17 = 0

$$\therefore \quad \left(\frac{3k-2}{k+1}\right) - 2\left(\frac{-5k+4}{k+1}\right) + 3\left(\frac{8k+7}{k+1}\right) - 17 = 0$$

$\Rightarrow \quad (3k-2)-2(-5k+4)+3(8k+7) = 17k+17$

$\Rightarrow \quad 3k+10k+24k-17k = 17+2+8-21$

$\Rightarrow \quad 37k-17k = 6 \quad \Rightarrow \quad 20k = 6 \; ; \quad k = \frac{6}{20} = \frac{3}{10}$

Hence the required ratio = k : 1 = $\frac{3}{10}$: 1 = 3 : 10 **Ans.**

Do yourself 1:

(i) Find the distance between the points P(3, 4, 5) and Q(–1, 2, –3).

(ii) Show that the points A(0, 7, 10), B(–1, 6, 6) and C(–4, 9, 6) are vertices of an isosceles right angled triangle.

(iii) Find the locus of a point such that the difference of the square of its distance from the points A(3, 4, 5) and B(–1, 3, –7) is equal to $2k^2$.

(iv) Find the co-ordinates of points which trisects the line joining the points A(–3, 2, 4) and B(0, 4, 7)

(v) Find the ratio in which the planes (a) xy (b) yz divide the line joining the points P(–2, 4, 7) and Q(3, –5, 8).

6. CENTROID OF A TRIANGLE :

Let $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ be the vertices of a triangle ABC. Then its centroid G is given by

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

Illustration 4 : If the centroid of a tetrahedron OABC where A, B, C, are given by (a, 2, 3), (1, b, 2) and (2, 1, c) respectively be (1, 2, – 1), then distance of P (a, b, c) from origin is -

(A) $\sqrt{107}$ (B) $\sqrt{14}$ (C) $\sqrt{107}/14$ (D) none of these

Solution : Centroid is $\left(\frac{1}{4}\Sigma x, \frac{1}{4}\Sigma y, \frac{1}{4}\Sigma z\right) = (1, 2, -1)$

$$\Rightarrow \quad \frac{a+1+2+0}{4} = 1, \quad \frac{2+b+1+0}{4} = 2, \quad \frac{3+2+c+0}{4} = -1 \quad \Rightarrow \quad a = 1, b = 5, c = -9$$

$$\therefore \quad OP = \sqrt{a^2 + b^2 + c^2} = \sqrt{107}$$

Ans. (A)

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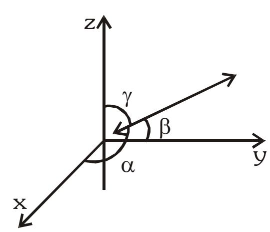
7. DIRECTION COSINES OF LINE :



If α, β, γ be the angles made by a line with x-axis, y-axis & z-axis respectively then cosα, cosβ & cosγ are called direction cosines of a line, denoted by ℓ, m & n respectively.

Note :

(i) If line makes angles α, β, γ with x, y & z axis respectively then π – α, π – β & π – γ is another set of angle that line makes with principle axes. Hence if ℓ, m & n are direction cosines of line then $-\ell$, –m & –n are also direction cosines of the same line.

(ii) Since parallel lines have same direction. So, in case of lines, which do not pass through the origin. We can draw a parallel line passing through the origin and direction cosines of that line can be found.

Important points :

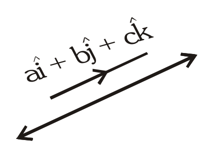
(i) Direction cosines of a line :

Take a vector $\vec{A} = a\tilde{i} + b\tilde{j} + c\tilde{k}$ parallel to a line whose D.C's are to be found out.

$\vec{A}.\tilde{i} = a$

$|\vec{A}| \cos\alpha = a$

$\cos\alpha = \frac{a}{|\vec{A}|}$ similarly, $\cos\beta = \frac{b}{|\vec{A}|}$; $\cos\gamma = \frac{c}{|\vec{A}|}$



$\Rightarrow \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \quad \ell^2 + m^2 + n^2 = 1$

(ii) **Direction cosine of axes :**

Since the positive x-axes makes angle 0 , 90 , 90 with axes of x, y and z respectively,

$\therefore$ D.C.'s of x axes are 1, 0, 0.

D.C.'s of y-axis are 0, 1, 0

D.C.'s of z-axis are 0, 0, 1

8. DIRECTION RATIOS :

Any three numbers a, b, c proportional to direction cosines ℓ, m, n are called direction ratios of the line.

i.e. $\frac{\ell}{a} = \frac{m}{b} = \frac{n}{c}$

There can be infinitely many sets of direction ratios for a given line.

Direction ratios and Direction cosines of the line joining two points :

Let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be two points, then d.r.'s of AB are $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$ and the d.c.'s of AB are $\frac{1}{r}(x_2 - x_1)$, $\frac{1}{r}(y_2 - y_1)$, $\frac{1}{r}(z_2 - z_1)$ where $r = \sqrt{[\Sigma(x_2 - x_1)^2]}$

9. RELATION BETWEEN D.C'S & D.R'S :

$$\frac{\ell}{a} = \frac{m}{b} = \frac{n}{c}$$

$$\therefore \quad \frac{\ell^2}{a^2} = \frac{m^2}{b^2} = \frac{n^2}{c^2} = \frac{\ell^2 + m^2 + n^2}{a^2 + b^2 + c^2}$$

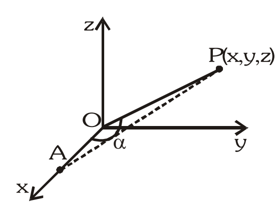
$$\therefore \quad \ell = \frac{\pm a}{\sqrt{a^2 + b^2 + c^2}} \; ; \quad m = \frac{\pm b}{\sqrt{a^2 + b^2 + c^2}} \; ; \quad n = \frac{\pm c}{\sqrt{a^2 + b^2 + c^2}}$$

Important point :

Direction cosines of a line are unique but Dr's of a line in no way unique but can be infinite.

7. PROJECTIONS :

(a) **Projection of line segment OP on co-ordinate axes :**



Let line segment make angle α with x-axis

Thus, the projections of line segment OP on axes are the absolute values of the co-ordinates of P. i.e.

Projection of OP on x-axis = $|x|$

Projection of OP on y-axis = $|y|$

Projection of OP on z-axis = $|z|$

Now, in ΔOAP, angle A is a right angle and OA = x

$OP = \sqrt{x^2 + y^2 + z^2}$

$$\therefore \quad \cos\alpha = \frac{x}{\sqrt{x^2 + y^2 + z^2}} = \frac{x}{|OP|}$$

if $|OP| = r$, then $x = |OP|\cos\alpha = \ell r$

Similarly $y = |OP|\cos\beta = mr$, $z = nr$, where ℓ, m, n are DC's of line

(b) **Projection of a line segment AB on coordinate axes :**

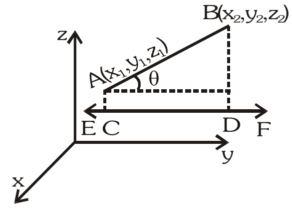
Projection of the point $A(x_1, y_1, z_1)$ on x-axis is $E(x_1, 0, 0)$. Projection of point $B(x_2, y_2, z_2)$ on x-axis is $F(x_2, 0, 0)$.

Hence projection of AB on x-axis is $EF = |x_2 - x_1|$.

Similarly, projection of AB on y and z-axis are $|y_2 - y_1|$, $|z_2 - z_1|$ respectively.

Note : Projection is only a length therefore it is always taken as positive.

(c) Projection of line segment AB on a line having direction cosines ℓ, m, n :



Let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$.

Now projection of AB on EF = CD = AB $\cos\theta$

$$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}\ \frac{|(x_2-x_1)\ell+(y_2-y_1)m+(z_2-z_1)n|}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$

$$= |(x_2-x_1)\ell+(y_2-y_1)m+(z_2-z_1)n|$$

Illustration 5 : A line OP makes with the x-axis an angle of measure 120 and with y-axis an angle of measure 60 . Find the angle made by the line with the z-axis.

Solution : $\alpha = 120$ and $\beta = 60$

$\therefore \quad \cos\alpha = \cos 120 = -\frac{1}{2}$ and $\cos\beta = \cos 60 = \frac{1}{2}$ but $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore \quad \left(\frac{-1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\gamma = 1$

$\cos^2\gamma = 1 - \frac{1}{4} - \frac{1}{4} = \frac{1}{2} \quad \Rightarrow \quad \cos\gamma = \pm\frac{1}{\sqrt{2}}$

$\therefore \quad \gamma = 45$ or 135 **Ans.**

Illustration 6 : Find the projection of the line segment joining the points (–1, 0, 3) and (2, 5, 1) on the line whose direction ratios are 6, 2, 3.

Solution : The direction cosines ℓ, m, n of the line are given by $\frac{\ell}{6} = \frac{m}{2} = \frac{n}{3} = \frac{\sqrt{\ell^2+m^2+n^2}}{\sqrt{6^2+2^2+3^2}} = \frac{1}{\sqrt{49}} = \frac{1}{7}$

$\therefore \quad \ell = \frac{6}{7},\ m = \frac{2}{7},\ n = \frac{3}{7}$

The required projection is given by

$$= |\ell(x_2-x_1)+m(y_2-y_1)+n(z_2-z_1)| = \left|\frac{6}{7}[2-(-1)]+\frac{2}{7}(5-0)+\frac{3}{7}(1-3)\right|$$

$$= \left|\frac{6}{7}\times 3+\frac{2}{7}\times 5+\frac{3}{7}\times -2\right| = \left|\frac{18}{7}+\frac{10}{7}-\frac{6}{7}\right| = \left|\frac{18+10-6}{7}\right| = \frac{22}{7}$$ **Ans.**

Do yourself - 2 :

(i) Find the projections of the line segment joining the origin O to the point P(3, 2, –5) on the axes.

(ii) Find the projections of the line joining the points P(3, 2, 5) and Q(0, –2, 8) on the axes.

(iii) Find the direction ratios & direction cosines of the line joining the points O(0, 0, 0) and P(2, 3, 4).

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11. ANGLE BETWEEN TWO LINES :

Let θ be the angle between the lines with d.c.'s ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 then $\cos\theta = \ell_1\ell_2 + m_1m_2 + n_1n_2$. If a_1, b_1, c_1 and a_2, b_2, c_2 be D.R.'s of two lines then angle θ between them is given by

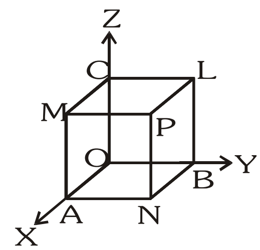
$$\cos\theta = \frac{(a_1a_2+b_1b_2+c_1c_2)}{\sqrt{(a_1^2+b_1^2+c_1^2)}\sqrt{(a_2^2+b_2^2+c_2^2)}}$$

Illustration 7 : If a line makes angles α, β, γ, δ with four diagonals of a cube, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$ equals -

(A) 3 (B) 4 (C) 4/3 (D) 3/4

Solution : Let OA, OB, OC be coterminous edges of a cube and OA = OB = OC = a, then co-ordinates of its vertices are O(0, 0, 0), A(a, 0, 0), B(0, a, 0), C(0, 0, a), L(0, a, a), M(a, 0, a), N(a, a, 0) and P (a, a, a)

Direction ratio of diagonal AL, BM, CN and OP are



$$\left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right),\left(\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right),\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}}\right),\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$$

Let ℓ, m, n be the direction cosines of the given line, then

$$\cos\alpha = \ell\left(-\frac{1}{\sqrt{3}}\right)+m\left(\frac{1}{\sqrt{3}}\right)+n\left(\frac{1}{\sqrt{3}}\right)=\frac{-\ell+m+n}{\sqrt{3}}$$

Similarly $\cos\beta = \dfrac{\ell-m+n}{\sqrt{3}}$, $\cos\gamma=\dfrac{\ell+m-n}{\sqrt{3}}$ and $\cos\delta=\dfrac{\ell+m+n}{\sqrt{3}}$

$\therefore$ $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \dfrac{4}{3}$ **Ans. (C)**

Illustration 8 : (a) Find the acute angle between two lines whose direction ratios are 2, 3, 6 and 1, 2, 2 respectively.

(b) Find the measure of the angle between the lines whose direction ratios are 1, –2, 7 and 3, –2, –1.

Solution : (a) $a_1 = 2$, $b_1 = 3$, $c_1 = 6$; $a_2 = 1$, $b_2 = 2$, $c_2 = 2$.

If θ be the angle between two lines whose d.r's are given, then

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} = \frac{2\times1+3\times2+6\times2}{\sqrt{2^2+3^2+6^2}\sqrt{1^2+2^2+2^2}} = \frac{2+6+12}{7\times3}=\frac{20}{21}$$

$\therefore$ $\theta=\cos^{-1}\left(\dfrac{20}{21}\right)$

(b) $\sqrt{1^2+(-2)^2+7^2}=\sqrt{54}$

$\sqrt{3^2+(-2)^2+(-1)^2}=\sqrt{14}$

$\therefore$ The actual direction cosines of the lines are

$$\frac{1}{\sqrt{54}},\frac{-2}{\sqrt{54}},\frac{7}{\sqrt{54}} \quad\text{and}\quad \frac{3}{\sqrt{14}},\frac{-2}{\sqrt{14}},\frac{-1}{\sqrt{14}}$$

If θ is the angle between the lines, then

$$\cos\theta=\left(\frac{1}{\sqrt{54}}\right)\left(\frac{3}{\sqrt{14}}\right)+\left(\frac{-2}{\sqrt{54}}\right)\left(\frac{-2}{\sqrt{14}}\right)+\left(\frac{7}{\sqrt{54}}\right)\left(\frac{-1}{\sqrt{14}}\right)$$

$= \dfrac{3+4-7}{\sqrt{54}.\sqrt{14}} = 0 \Rightarrow \theta=90^\circ$ **Ans.**

12. PERPENDICULAR AND PARALLEL LINES :

Let the two lines have their d.c.'s given by ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 respectively then they are perpendicular if $\theta = 90$ i.e. $\cos\theta = 0$, i.e. $\ell_1\ell_2 + m_1m_2 + n_1n_2 = 0$.

Also the two lines are parallel if $\theta = 0$ i.e. $\sin\theta = 0$, i.e. $\frac{\ell_1}{\ell_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

Note: If instead of d.c.'s, d.r.'s a_1, b_1, c_1 and a_2, b_2, c_2 are given, then the lines are perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ and parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Illustration 9 : If the lines whose direction cosines are given by $a\ell + bm + cn = 0$ and $fmn + gn\ell + h\ell m = 0$ are perpendicular, then $\frac{f}{a} + \frac{g}{b} + \frac{h}{c}$ equals -

(A) 0 (B) -1 (C) 1 (D) none of these

Solution : Eliminating n between the given relations, we find that $(fm + g\ell)\left(\frac{-a\ell - bm}{c}\right) + h\ell m = 0$

or $$ag\left(\frac{\ell}{m}\right)^2 + (af + bg - ch)\left(\frac{\ell}{m}\right) + bf = 0 \qquad \text{........(i)}$$

Let $\frac{\ell_1}{m_1}$ and $\frac{\ell_2}{m_2}$, are roots of (i), then $\frac{\ell_1}{m_1}\cdot\frac{\ell_2}{m_2} = \frac{bf}{ag}$

$$\Rightarrow \frac{\ell_1\ell_2}{f/a} = \frac{m_1m_2}{g/b} \qquad \text{........(ii)}$$

Similarly $$\frac{m_1m_2}{g/b} = \frac{n_1n_2}{h/c} \qquad \text{........(iii)}$$

From (ii) and (iii), we get $\frac{\ell_1\ell_2}{f/a} = \frac{m_1m_2}{g/b} = \frac{n_1n_2}{h/c} = \lambda$

$\Rightarrow$ $\ell_1\ell_2 = \lambda.f/a$; $m_1m_2 = \lambda.g/b$; $n_1n_2 = \lambda.h/c$

$\Rightarrow$ $\ell_1\ell_2 + m_1m_2 + n_1n_2 = \lambda\left(\frac{f}{a} + \frac{g}{b} + \frac{h}{c}\right)$

$\Rightarrow$ $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ $\{\because \ell_1\ell_2 + m_1m_2 + n_1n_2 = 0\}$ **Ans. (A)**

Do yourself - 3 :

(i) Find the angle between the lines whose direction ratios are 1, -2, 1 and 4, 3, 2.

(ii) If a line makes α, β and γ angle with axes, then prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

(iii) Find the direction cosines of the line which is perpendicular to the lines with direction cosines proportional to $(1, -2, -2)$ & $(0, 2, 1)$.

PLANE

13. DEFINITION :

A geometrical locus is a plane, such that if P and Q are any two points on the locus, then every point on the line PQ is also a point on the locus.

14. EQUATIONS OF A PLANE :

The equation of every plane is of the first degree i.e. of the form ax + by + cz + d = 0, in which a, b, c are constants, not all zero simultaneously.

(a) Equation of plane passing through a fixed point :

Vector form : If $\vec{a}$ be the position vector of a point on the plane and $\vec{n}$ be a vector normal to the plane then it's vectorial equation is given by $(\vec{r}-\vec{a}).\vec{n}=0 \Rightarrow \vec{r}.\vec{n}=d$, where $d=\vec{a}.\vec{n}=$ constant.

Cartesian form : If $\vec{a}(x_1, y_1, z_1)$ and $\vec{n}=a\hat{i}+b\hat{j}+c\hat{k}$, then cartesian equation of plane will be

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

(b) Plane Parallel to the Coordinate Planes :

(i) Equation of yz plane is x = 0.

(ii) Equation of zx plane is y = 0.

(iii) Equation of xy plane is z = 0.

(iv) Equation of the plane parallel to xy plane at a distance c is z = c or z = –c.

(v) Equation of the plane parallel to yz plane at a distance c is x = c or x = –c

(vi) Equation of the plane parallel to zx plane at a distance c is y = c or y = –c.

(c) Equations of Planes Parallel to the Axes :

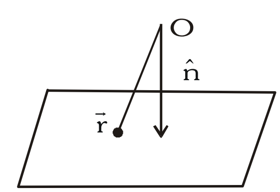
If a = 0, the plane is parallel to x-axis i.e. equation of the plane parallel to x-axis is **by + cz + d = 0**.

Similarly, equations of planes parallel to y-axis and parallel to z-axis are **ax + cz + d = 0** and **ax + by + d = 0**, respectively.

(d) Equation of a Plane in Intercept Form :

Equation of the plane which cuts off intercepts a, b, c from the axes x, y, z respectively is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

(e) Equation of a Plane in Normal Form :



Vector form : If $\hat{n}$ is a unit vector normal to the plane from the origin and d be the perpendicular distance of plane from origin then its vector equation is $\vec{r}.\hat{n}=d$.

Cartesian form : If the length of the perpendicular distance of the plane from the origin is p and direction cosines of this perpendicular are (ℓ, m, n), then the equation of the plane is $\ell x + my + nz = p$.

(f) Equation of a Plane through three points :

Vector form : If A, B, C are three points having P.V.'s $\vec{a}, \vec{b}, \vec{c}$ respectively, then vector equation of the plane is $[\vec{r}\ \vec{a}\ \vec{b}]+[\vec{r}\ \vec{b}\ \vec{c}]+[\vec{r}\ \vec{c}\ \vec{a}]=[\vec{a}\ \vec{b}\ \vec{c}]$.

Cartesian form :The equation of the plane through three non-collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$

Illustration 10 : Find the equation of the plane through the points A(2, 2, –1), B(3, 4, 2) and C(7, 0, 6).

Solution : The general equation of a plane passing through (2, 2, –1) is

$a(x-2) + b(y-2) + c(z+1) = 0$ ....(i)

It will pass through B (3, 4, 2) and C (7, 0, 6) if

$a(3-2) + b(4-2) + c(2+1) = 0$ or $a + 2b + 3c = 0$ ....(ii)

and $a(7-2) + b(0-2) + c(6+1) = 0$ or $5a - 2b + 7c = 0$ ....(iii)

Solving (ii) and (iii) by cross-multiplication, we have

$$\frac{a}{14+6} = \frac{b}{15-7} = \frac{c}{-2-10} \quad \text{or} \quad \frac{a}{5} = \frac{b}{2} = \frac{c}{-3} = \lambda \quad \text{(say)}$$

$\Rightarrow$ $a = 5\lambda$, $b = 2\lambda$ and $c = -3\lambda$

Substituting the values of a, b and c in (i), we get

$5\lambda(x-2) + 2\lambda(y-2) - 3\lambda(z+1) = 0$

or $5(x-2) + 2(y-2) - 3(z+1) = 0$

$\Rightarrow$ $5x + 2y - 3z = 17$, which is the required equation of the plane **Ans.**

Illustration 11 : A plane meets the co-ordinates axis in A,B,C such that the centroid of the Δ ABC is the point (p,q,r) show that the equation of the plane is $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$

Solution : Let the required equation of plane be :

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \text{......(i)}$$

Then, the co-ordinates of A, B and C are A(a, 0, 0), B(0, b, 0), C(0, 0, c) respectively

So the centroid of the triangle ABC is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

But the co-ordinate of the centroid are (p,q,r)

$$\frac{a}{3} = p, \quad \frac{b}{3} = q, \quad \frac{c}{3} = r$$

Putting the values of a, b and c in (i), we get the required plane as $\frac{x}{3p} + \frac{y}{3q} + \frac{z}{3r} = 1$

$\Rightarrow$ $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$ **Ans.**

Do yourself - 4 :

(i) Equation of a plane is $3x + 4y + 5z = 7$.

(a) Find the direction cosines of its normal

(b) Find the points where it intersects the axes.

(c) Find its intercept form.

(d) Find its equation in normal form (in cartesian as well as in vector form)

(ii) Find the equation of the plane passing through the points (2, 3, 1), (3, 0, 2) and (–1, 2, 3).

15. ANGLE BETWEEN TWO PLANES :

Vector form : If $\vec{r}.\vec{n}_1 = d_1$ and $\vec{r}.\vec{n}_2 = d_2$ be two planes, then angle between these planes is the angle between their normals

$$\cos\theta = \frac{\vec{n}_1 . \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$

$\therefore$ Planes are perpendicular if $\vec{n}_1 . \vec{n}_2 = 0$ and they are parallel if $\vec{n}_1 = \lambda \vec{n}_2$.

Cartesian form : Consider two planes ax + by + cz + d = 0 and a' x + b' y + c' z + d' = 0. Angle between these planes is the angle between their normals.

$$\cos\theta = \frac{aa' + bb' + cc'}{\sqrt{a^2 + b^2 + c^2}\sqrt{a'^2 + b'^2 + c'^2}}$$

$\therefore$ Planes are perpendicular if aa' + bb' + cc' = 0 and they are parallel if $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$.

Planes parallel to a given Plane :

Equation of a plane parallel to the plane ax + by + cz + d = 0 is ax + by + cz + d' = 0. d' is to be found by other given condition.

Illustration 12 : Find the angle between the planes x + y + 2z = 9 and 2x – y + z = 15

Solution : We know that the angle between the planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by $\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

Therefore, angle between x + y + 2z = 9 and 2x – y + z = 15 is given by

$$\cos\theta = \frac{(1)(2) + (1)(-1) + (2)(1)}{\sqrt{1^2 + 1^2 + 2^2}\sqrt{2^2 + (-1)^2 + 1^2}} = \frac{1}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{3}$$ **Ans.**

Illustration 13 : Find the equation of the plane through the point (1, 4, –2) and parallel to the plane –2x + y – 3z = 7.

Solution : Let the equation of a plane parallel to the plane –2x + y – 3z = 7 be –2x + y – 3z + k = 0

This passes through (1, 4, –2), therefore (–2) (1) + 4 – 3 (–2) + k = 0

$\Rightarrow$ –2 + 4 + 6 + k = 0 $\Rightarrow$ k = –8

Putting k = –8 in (i), we obtain –2x + y – 3z – 8 = 0 or –2x + y – 3z = 8 **Ans.**

This is the equation of the required plane.

Do yourself - 5 :

(i) Prove that the planes 3x – 2y + z + 17 = 0 and 4x + 3y – 6z – 25 = 0 are perpendicular.

(ii) Find the angle between the planes 3x + 4y + z + 7 = 0 and –x + y – 2z = 5

16. A PLANE THROUGH THE LINE OF INTERSECTION OF TWO GIVEN PLANES :

Consider two planes u ≡ ax + by + cz + d = 0 and v ≡ a' x + b' y + c' z + d' = 0.

The equation u + λv = 0, λ a real parameter, represents the plane passing through the line of intersection of given planes and if planes are parallel, this represents a plane parallel to them.

Illustration 14 : Find the equation of plane containing the line of intersection of the plane x + y + z – 6 = 0 and 2x + 3y + 4z + 5 = 0 and passing through (1,1,1).

Solution : The equation of the plane through the line of intersection of the given planes is,

$$(x + y + z - 6) + \lambda\,(2x + 3y + 4z + 5) = 0 \qquad \ldots\ldots(i)$$

If it is passes through (1,1,1)

$$\Rightarrow \quad (1 + 1 + 1 - 6) + \lambda\,(2 + 3 + 4 + 5) = 0 \quad \Rightarrow \quad \lambda = \frac{3}{14}$$

Putting $\lambda = 3/14$ in (i); we get $(x + y + z - 6) + \frac{3}{14}\,(2x + 3y + 4z + 5) = 0$

$\Rightarrow \quad 20x + 23y + 26z - 69 = 0$ **Ans.**

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17. PERPENDICULAR DISTANCE OF A POINT FROM THE PLANE :

Vector form : If $\vec{r}.\vec{n} = d$ be the plane, then perpendicular distance p, of the point $A(\vec{a})$

$$p = \frac{|\vec{a}.\vec{n} - d|}{|\vec{n}|}$$

Distance between two parallel planes $\vec{r}.\vec{n} = d_1$ & $\vec{r}.\vec{n} = d_2$ is $\left|\frac{d_1 - d_2}{|\vec{n}|}\right|$.

Cartesian form : Perpendicular distance p, of the point $A(x_1, y_1, z_1)$ from the plane ax + by + cz + d = 0 is given by $p = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{(a^2 + b^2 + c^2)}}$

Distance between two parallel planes $ax + by + cz + d_1 = 0$ & $ax + by + cz + d_2 = 0$ is $\left|\frac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}}\right|$

Illustration 15 : Find the perpendicular distance of the point (2, 1, 0) from the plane 2x + y +2z + 5 = 0

Solution : We know that the perpendicular distance of the point (x_1, y_1, z_1) from the plane ax + by + cz + d = 0 is $\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$

so required distance $= \frac{|2 \times 2 + 1 \times 1 + 2 \times 0 + 5|}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{10}{3}$ **Ans.**

Illustration 16 : Find the distance between the parallel planes 2x – y + 2z + 3 = 0 and 4x – 2y + 4z + 5 = 0.

Solution : Let $P(x_1, y_1, z_1)$ be any point on 2x – y + 2z + 3 = 0, then $2x_1 - y_1 + 2z_1 + 3 = 0$

The length of the perpendicular from $P(x_1, y_1, z_1)$ to 4x – 2y + 4z + 5 = 0 is

$$\left|\frac{4x_1 - 2y_1 + 4z_1 + 5}{\sqrt{4^2 + (-2)^2 + 4^2}}\right| = \left|\frac{2(2x_1 - y_1 + 2z_1) + 5}{\sqrt{36}}\right| = \frac{|2(-3) + 5|}{6} = \frac{1}{6} \qquad \text{[using (i)]}$$

Therefore, the distance between the two given parallel planes is $\frac{1}{6}$ **Ans.**

Do yourself - 6 :

(i) Find the perpendicular distance of the point P(1, 2, 3) from the plane 2x + y + z + 1 = 0.

(ii) Find the equation of the plane passing through the line of intersection of the planes x + y + z = 5 and 2x + 3y + z + 5 = 0 and passing through the point (0, 0, 0).

18. BISECTORS OF ANGLES BETWEEN TWO PLANES :

Let the equations of the two planes be $ax + by + cz + d = 0$ and $a_1 x + b_1 y + c_1 z + d_1 = 0$.

Then equations of bisectors of angles between them are given by

$$\frac{ax+by+cz+d}{\sqrt{(a^2+b^2+c^2)}} = \pm\frac{a_1x+b_1y+c_1z+d_1}{\sqrt{(a_1^2+b_1^2+c_1^2)}}$$

(a) **Equation of bisector of the angle containing origin** : First make both constant terms positive. Then positive sign give the bisector of the angle which contains the origin.

(b) **Bisector of acute/obtuse angle** : First making both constant terms positive,

$aa_1 + bb_1 + cc_1 > 0 \quad \Rightarrow \quad$ origin lies in obtuse angle

$aa_1 + bb_1 + cc_1 < 0 \quad \Rightarrow \quad$ origin lies in acute angle

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Illustration 17 : Find the equation of the bisector planes of the angles between the planes $2x - y + 2z + 3 = 0$ and $3x - 2y + 6z + 8 = 0$ and specify the plane which bisects the acute angle and the plane which bisects the obtuse angle.

Solution : The two given planes are $2x - y + 2z + 3 = 0$ and $3x - 2y + 6z + 8 = 0$

where $d_1, d_2 > 0$

and $a_1a_2 + b_1b_2 + c_1c_2 = 6 + 2 + 12 > 0$

$$\therefore \quad \frac{a_1x+b_1y+c_1z+d_1}{\sqrt{a_1^2+b_1^2+c_1^2}} = -\frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}} \quad \text{(acute angle bisector)}$$

$$\text{and} \quad \frac{a_1x+b_1y+c_1z+d}{\sqrt{a_1^2+b_1^2+c_1^2}} = \frac{a_2x+b_2y+c_2z+d_2}{\sqrt{a_2^2+b_2^2+c_2^2}} \quad \text{(obtuse angle bisector)}$$

$$\text{i.e.,} \quad \frac{2x-y+2z+3}{\sqrt{4+1+4}} = \pm\frac{3x-2y+6z+8}{\sqrt{9+4+36}}$$

$\Rightarrow \quad (14x - 7y + 14z + 21) = \pm (9x - 6y + 18z + 24)$

Taking positive sign on the right hand side,

we get $\quad 5x - y - 4z - 3 = 0 \quad$ (obtuse angle bisector)

and taking negative sign on the right hand side,

we get $\quad 23x - 13y + 32z + 45 = 0 \quad$ (acute angle bisector) **Ans.**

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19. POSITION OF TWO POINTS W.R.T. A PLANE :

Two points $P(x_1, y_1, z_1)$ & $Q(x_2, y_2, z_2)$ are on the same or opposite sides of a plane $ax + by + cz + d = 0$ according to $ax_1 + by_1 + cz_1 + d$ & $ax_2 + by_2 + cz_2 + d$ are of same or opposite signs. The plane divides the line joining the points P & Q externally or internally according to P and Q lying on same or opposite sides of the plane.

Do yourself - 7 :

(i) Find the position of the point $P(2, -2, 1)$, $Q(3, 0, 1)$ and $R(-12, 1, 8)$ w.r.t. the plane $2x - 3y + 4z - 7 = 0$.

(ii) Two given planes are $-2x + y - 2z + 5 = 0$ and $6x - 2y + 3z - 7 = 0$. Find

(a) equation of plane bisecting the angle between the planes.

(b) equation of a plane parallel to the plane bisecting the angle between both the two planes and passing through the point (3, 2, 0).

(c) specify which plane is acute angle bisector and which one is obtuse angle bisector.

STRAIGHT LINE

20. DEFINITION :

A straight line in space is characterised by the intersection of two planes which are not parallel and, therefore, the equation of a straight line is present as a solution of the system constituted by the equations of the two planes : $a_1 x + b_1 y + c_1 z + d_1 = 0$; $a_2 x + b_2 y + c_2 z + d_2 = 0$

This form is also known as **unsymmetrical form**.

Some particular straight lines :

	Straight lines	Equation
(i)	Through the origin	$y = mx,\ z = nx$
(ii)	x-axis	$y = 0,\ z = 0$ or $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$
(iii)	y-axis	$x = 0,\ z = 0$ or $\frac{x}{0} = \frac{y}{1} = \frac{z}{0}$
(iv)	z-axis	$x = 0,\ y = 0$ or $\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$
(v)	parallel to x-axis	$y = p,\ z = q$
(vi)	parallel to y-axis	$x = h,\ z = q$
(vii)	parallel to z-axis	$x = h,\ y = p$

21. EQUATION OF A STRAIGHT LINE IN SYMMETRICAL FORM :

(a) **One point form** : Let $A(x_1, y_1, z_1)$ be a given point on the straight line and ℓ, m, n be the d.c's of the line, then its equation is

$$\frac{x - x_1}{\ell} = \frac{y - y_1}{m} = \frac{z - z_1}{n} = r \qquad \text{(say)}$$

It should be noted that $P(x_1 + \ell r,\ y_1 + mr,\ z_1 + nr)$ is a general point on this line at a distance r from the point $A(x_1, y_1, z_1)$ i.e. AP = r. One should note that for AP = r; ℓ, m, n must be d.c.'s not d.r.'s. If a, b, c are direction ratios of the line, then equation of the line is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = r \qquad \text{but here } AP \neq r$$

(b) Equation of the line through two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

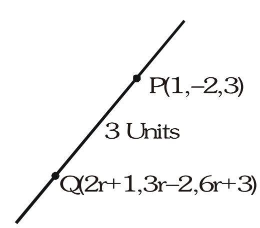
Illustration 18 : Find the co-ordinates of those points on the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{6}$ which is at a distance of 3 units from point (1,–2, 3).

Solution : Here, $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{6}$ ....(i)

is the given straight line

Let, P = (1,–2,3) on the straight line

Here direction ratios of line (i) are (2,3,6)



$\therefore$ Direction cosines of line (i) are : $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$

$\Rightarrow$ Equations of line(i) any may be written as

$$\frac{x-1}{2/7}=\frac{y+2}{3/7}=\frac{z-3}{6/7} \qquad \text{......(ii)}$$

Co-ordinates of any point on the line (ii) may be taken as $\left(\frac{2}{7}r+1, \frac{3}{7}r-2, \frac{6}{7}r+3\right)$

Let, $Q\left(\frac{2}{7}r+1, \frac{3}{7}r-2, \frac{6}{7}r+3\right)$

Given $|\vec{r}| = 3$, $\therefore$ $r = \pm 3$

Putting the value of r, we have $Q\left(\frac{13}{7}, -\frac{5}{7}, \frac{39}{7}\right)$ or $Q = \left(\frac{1}{7}, -\frac{23}{7}, \frac{3}{7}\right)$ **Ans.**

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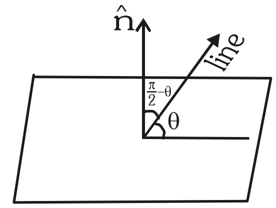
22. ANGLE BETWEEN A LINE AND A PLANE :

Let equations of the line and plane be $\frac{x-x_1}{\ell}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ and $ax + by + cz + d = 0$ respectively and θ be the angle which line makes with the plane. Then $(\pi/2 - \theta)$ is the angle between the line and the normal to the plane.



So, $\sin\theta = \frac{a\ell+bm+cn}{\sqrt{(a^2+b^2+c^2)}\sqrt{(\ell^2+m^2+n^2)}}$

Line is parallel to plane if $\theta = 0$ i.e. if $a\ell + bm + cn = 0$.

Line is perpendicular to the plane if line is parallel to the normal of the plane i.e. if $\frac{a}{\ell}=\frac{b}{m}=\frac{c}{n}$.

Illustration 19 : Find the angle between the line $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{-2}$ and the plane $3x + 4y + z + 5 = 0$.

Solution : The given line is $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{-2}$ .... (i)

and the given plane is $3x + 4y + z + 5 = 0$ .... (ii)

If the line (i) makes angle θ with the plane (ii), then the line (i) will make angle $(90 - \theta)$ with the normal to the plane (i). Now direction-ratios of line (i) are $< 3, -1, -2 >$ and direction-ratios of normal to plane (ii) are $< 3, 4, 1 >$

$$\therefore \quad \cos(90°-\theta)=\frac{(3)(3)+(-1)(4)+(-2)(1)}{\sqrt{9+1+4}\sqrt{9+16+1}} \quad \Rightarrow \quad \sin\theta=\frac{9-4-2}{\sqrt{14}\sqrt{26}}=\frac{3}{\sqrt{14}\sqrt{26}}$$

Hence $\theta = \sin^{-1}\left(\frac{3}{\sqrt{14}\sqrt{26}}\right)$ **Ans.**

Do yourself - 8 :

(i) Find the equation of the line passing through the point (4, 2, 3) and having direction ratios 1, –1, 2

(ii) Find the symmetrical form of the line $x - y + 2z = 5$, $3x + y + z = 6$.

(iii) Find the angle between the plane $3x + 4y + 5 = 0$ and the line $\frac{x-1}{2}=\frac{y-2}{0}=\frac{z-1}{1}$.

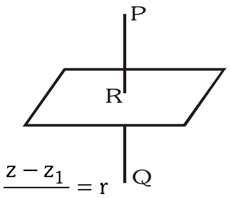
(iv) Prove that the line $\frac{x-3}{2}=\frac{y-4}{3}=\frac{z-5}{4}$ is parallel to the plane $4x + 4y - 5z + 2 = 0$.

23. CONDITION IN ORDER THAT THE LINE MAY LIE ON THE GIVEN PLANE :

The line $\frac{x-x_1}{\ell}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ will lie on the plane Ax + By + Cz + D = 0 if

(a) $A\ell + Bm + Cn = 0$ and (b) $Ax_1 + By_1 + Cz_1 + D = 0$

24. IMAGE OF A POINT IN THE PLANE :



In order to find the image of a point $P(x_1, y_1, z_1)$ in a plane ax + by + cz + d = 0, assume it as a mirror. Let $Q(x_2, y_2, z_2)$ be the image of the point $P(x_1, y_1, z_1)$ in the plane, then

(a) Line PQ is perpendicular to the plane. Hence equation of PQ is $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=r$

(b) Hence, Q satisfies the equation of line then $\frac{x_2-x_1}{a}=\frac{y_2-y_1}{b}=\frac{z_2-z_1}{c}=r$. The plane passes through the middle point of line PQ and the middle point satisfies the equation of the plane i.e. $a\left(\frac{x_2+x_1}{2}\right)+b\left(\frac{y_2+y_1}{2}\right)+c\left(\frac{z_2+z_1}{2}\right)+d=0$. The co-ordinates of Q can be obtained by solving these equations.

25. FOOT, LENGTH AND EQUATION OF PERPENDICULAR FROM A POINT TO A LINE :

Let equation of the line be $\frac{x-x_1}{\ell}=\frac{y-y_1}{m}=\frac{z-z_1}{n}=r$ (say) .... (i)

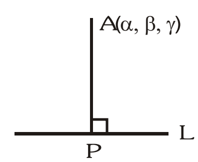
and A (α, β, γ) be the point. Any point on the line (i) is $P(\ell r + x_1, mr + y_1, nr + z_1)$ .... (ii)

If it is the foot of the perpendicular, from A on the line, then AP is $\perp$ to the line, so

$\ell(\ell r + x_1 - \alpha) + m(mr + y_1 - \beta) + n(nr + z_1 - \gamma) = 0$

i.e. $r = (\alpha - x_1)\ell + (\beta - y_1)m + (\gamma - z_1)n$

since $\ell^2 + m^2 + n^2 = 1$



Putting this value of r in (ii), we get the foot of perpendicular from point A to the line.

Length : Since foot of perpendicular P is known, length of perpendicular,

$$AP=\sqrt{[(\ell r+x_1-\alpha)^2+(mr+y_1-\beta)^2+(nr+z_1-\gamma)^2]}$$

Equation of perpendicular is given by

$$\frac{x-\alpha}{\ell r+x_1-\alpha}=\frac{y-\beta}{mr+y_1-\beta}=\frac{z-\gamma}{nr+z_1-\gamma}$$

Illustration 20 : Find the co-ordinates of the foot of the perpendicular from (1, 1, 1) on the line joining (5, 4, 4) and (1, 4, 6).

Solution : Let A (1, 1, 1), B (5, 4, 4) and C (1, 4, 6) be the given points. Let M be the foot of the perpendicular from A on BC.

If M divides BC in the ratio $\lambda : 1$, then

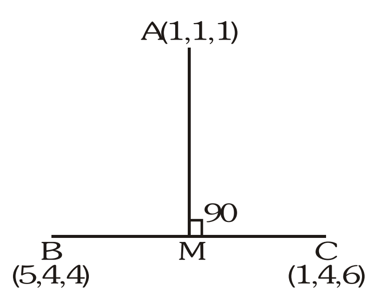
co-ordinates of M are $\left(\frac{\lambda+5}{\lambda+1},\frac{4\lambda+4}{\lambda+1},\frac{6\lambda+4}{\lambda+1}\right)$

Direction ratios of BC are 1 – 5, 4 – 4, 6 – 4

i.e. –4, 0, 2

D.R.'s of AM are $\frac{\lambda+5}{\lambda+1}-1,\frac{4\lambda+4}{\lambda+1}-1,\frac{6\lambda+4}{\lambda+1}-1$

$\Rightarrow \frac{4}{\lambda+1},\frac{3\lambda+3}{\lambda+1},\frac{5\lambda+3}{\lambda+1} \Rightarrow 4, 3\lambda + 3, 5\lambda + 3$



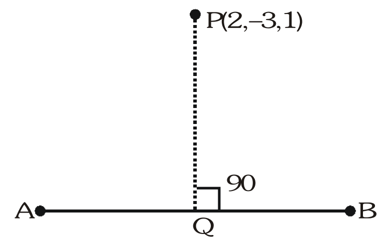
Since AM $\perp$ BC

$\therefore$ $2(4) + 0(3\lambda + 3) - 1(5\lambda + 3) = 0 \Rightarrow 8 - 5\lambda - 3 = 0 \Rightarrow \lambda = 1$

Hence the co-ordinates of M are (3, 4, 5) **Ans.**

Illustration 21 : Find the length of perpendicular from P(2, – 3,1) to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z+2}{-1}$

Solution : Given line is $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z+2}{-1}$ ....(i)



and P(2, – 3,1)

Co-ordinates of any point on (i) may be taken as

(2r–1,3r+3,–r–2)

Let Q = (2r –1, 3r + 3,–r – 2)

Direction ratio's of PQ are : (2r –3, 3r +6, –r –3)

Direction ratio's of AB are : (2,3, –1)

Since, PQ ⊥ AB

$$2(2r-3)+3(3r+6)-1(-r-3)=0$$

$$\Rightarrow \quad r=-\frac{15}{14}$$

$$\therefore \quad Q=\left(-\frac{22}{7},-\frac{3}{14},-\frac{13}{14}\right)$$

$$PQ^2=\left(2+\frac{22}{7}\right)^2+\left(-3+\frac{3}{14}\right)^2+\left(1+\frac{13}{14}\right)^2=\frac{531}{14}$$

$$PQ=\sqrt{\frac{531}{14}} \text{ units}$$

Ans.

Do yourself - 9 :

(i) Find the image of point P(1, 3, 2) in the plane 2x – y + z + 3 = 0 as well as the foot of the perpendicular drawn from the point (1, 3, 2).

(ii) Find the distance of the point (1, –2, 3) from the plane x – y + z = 5 measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$

(iii) Prove that $\frac{x+1}{-2}=\frac{y+2}{3}=\frac{z+5}{4}$ lies in the plane x + 2y – z = 0.

26. EQUATION OF PLANE CONTAINING TWO INTERSECTING LINES :

Let the two lines be

$$\frac{x-\alpha_1}{\ell_1}=\frac{y-\beta_1}{m_1}=\frac{z-\gamma_1}{n_1} \quad \text{.......... (i)}$$

and $$\frac{x-\alpha_2}{\ell_2}=\frac{y-\beta_2}{m_2}=\frac{z-\gamma_2}{n_2} \quad \text{.......... (ii)}$$

These lines will coplanar if $\begin{vmatrix}\alpha_2-\alpha_1 & \beta_2-\beta_1 & \gamma_2-\gamma_1\\ \ell_1 & m_1 & n_1\\ \ell_2 & m_2 & n_2\end{vmatrix}=0$ (It is condition for intersection of two lines)

the plane containing the two lines is $\begin{vmatrix}x-\alpha_1 & y-\beta_1 & z-\gamma_1\\ \ell_1 & m_1 & n_1\\ \ell_2 & m_2 & n_2\end{vmatrix}=0$

Illustration 22 : Find the equation of the plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+1}{2}$ and parallel to the line $\frac{x-4}{2}=\frac{y-1}{-3}=\frac{z+3}{5}$.

Solution : Any plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+1}{2}$ is

$a(x-1)+b(y+6)+c(z+1)=0$ .... (i)

where, $3a+4b+2c=0$ .... (ii)

Also, it is parallel to the second line and hence, its normal is perpendicular to this line

$\therefore$ $2a-3b+5c=0$ .... (iii)

Solving (ii) & (iii) by cross multiplication, we get $\frac{a}{26}=\frac{b}{-11}=\frac{c}{-17}=k$

$\Rightarrow$ $a=26k,\ b=-11k$ & $c=-17k$

Putting these values in (i), we get $26k(x-1)-11k(y+6)-17k(z+1)=0$

$\Rightarrow$ $26x-11y-17z=109$, which is the required equation of the plane.

27. LINE OF GREATEST SLOPE :

Consider two planes G-plane and H-plane. H-plane is treated as a horizontal plane or reference plane. G-plane is a given plane. Let AB be the line of intersection of G-plane & H-plane. Line of greatest slope is a line which is contained by G-plane & perpendicular to line of intersection of G-plane & H-plane. Obviously, infinitely many such lines of greatest slopes are contained by G-plane. Generally an additional information is given in problem so that a unique line of greatest slope can be found out.

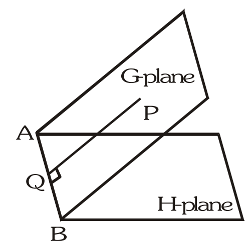


Illustration 23 : Assuming the plane $4x-3y+7z=0$ to be horizontal, find the equation of the line of greatest slope through the point (2,1,1) in the plane $2x+y-5z=0$.

Solution : The required line passing through the point P (2,1,1) in the plane $2x+y-5z=0$ and is having greatest slope, so it must be perpendicular to the line of intersection of the planes

$2x+y-5z=0$ ......(i)

and $4x-3y+7z=0$ ......(ii)

Let the d.r'.s of the line of intersection of (i) and (ii) be a, b, c

$\Rightarrow$ $2a+b-5c=0$ and $4a-3b+7c=0$

{as dr'.s of straight line (a, b,c) is perpendicular to d.r'.s of normal to both the planes}

$\Rightarrow$ $\frac{a}{4}=\frac{b}{17}=\frac{c}{5}$

Now let the direction ratio of required line be proportional to ℓ, m, n then its equation be

$$\frac{x-2}{\ell}=\frac{y-1}{m}=\frac{z-1}{n}$$

where $2\ell+m-5n=0$ and $4\ell+17m+5n=0$

so, $\frac{\ell}{3}=\frac{m}{-1}=\frac{n}{1}$

Thus the required line is $\frac{x-2}{3}=\frac{y-1}{-1}=\frac{z-1}{1}$ **Ans.**

28. AREA OF TRIANGLE :

To find the area of a triangle in terms of its projections on the co-ordinates planes.

Let Δ_x, Δ_y, Δ_z be the projections of the plane area of the triangle on the planes yOz, zOx, xOy respectively. Let ℓ, m, n be the direction cosines of the normal to the plane of the triangle.

Then the angle between the plane of the triangle and yOz plane is the angle between the normal to the plane of the triangle and the x-axis.

$\therefore \quad \Delta_x = \Delta\ell$

Similarly $\quad \Delta_y = \Delta m$; $\Delta_z = \Delta n \quad \Rightarrow \quad \Delta = \sqrt{\Delta_x^2 + \Delta_y^2 + \Delta_z^2}$

If $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ be the three vertices of the triangle then

$$\Delta_x = \frac{1}{2}\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix}, \quad \Delta_y = \frac{1}{2}\begin{vmatrix} x_1 & z_1 & 1 \\ x_2 & z_2 & 1 \\ x_3 & z_3 & 1 \end{vmatrix}, \quad \Delta_z = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Do yourself - 10 :

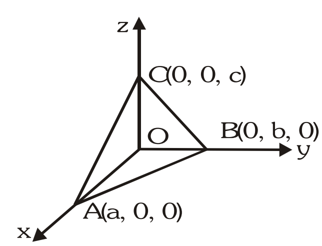
(i) Prove that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ are coplanar. Find their point of intersection.

(ii) Find the area of the triangle whose vertices are the points (1, 2, 3), (–2, 1, –4), (3, 4, –2).

Miscellaneous Illustrations :

Illustration 24 : If a variable plane cuts the coordinate axes in A, B and C and is at a constant distance p from the origin, find the locus of the centroid of the tetrahedron OABC.

Solution : Let A ≡ (a, 0, 0), B ≡ (0, b, 0) and C ≡ (0, 0, c)



$\therefore \quad$ Equation of plane ABC is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Now p = length of perpendicular from O to plane (i)

$$= \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} \quad \text{or} \quad p^2 = \frac{1}{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}$$

Let $G(\alpha, \beta, \gamma)$ be the centroid of the tetrahedron OABC, then

$$\alpha = \frac{a}{4}, \ \beta = \frac{b}{4}, \ \gamma = \frac{c}{4} \qquad \left[\because \alpha = \frac{a+0+0+0}{4} = \frac{a}{4}\right]$$

or, $\quad a = 4\alpha$, $b = 4\beta$, $c = 4\gamma$

Putting these values of a, b, c in equation (ii), we get

$$p^2 = \frac{16}{\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}\right)} \quad \text{or} \quad \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = \frac{16}{p^2}$$

$\therefore \quad$ locus of (α, β, γ) is $x^{-2} + y^{-2} + z^{-2} = 16\,p^{-2}$ **Ans.**

Illustration 25 : Through a point $P(h, k, \ell)$ a plane is drawn at right angles to OP to meet the coordinate axes in A, B and C. If OP = p, show that the area of ΔABC is $\frac{p^5}{2|hk\ell|}$.

Solution : $OP = \sqrt{h^2+k^2+\ell^2} = p$

Direction cosines of OP are $\frac{h}{\sqrt{h^2+k^2+\ell^2}}, \frac{k}{\sqrt{h^2+k^2+\ell^2}}, \frac{\ell}{\sqrt{h^2+k^2+\ell^2}}$

Since OP is normal to the plane, therefore, equation of the plane will be,

$$\frac{h}{\sqrt{h^2+k^2+\ell^2}}x + \frac{k}{\sqrt{h^2+k^2+\ell^2}}y + \frac{\ell}{\sqrt{h^2+k^2+\ell^2}}z = \sqrt{h^2+k^2+\ell^2}$$

or, $hx + ky + \ell z = h^2 + k^2 + \ell^2 = p^2$

$\therefore$ $A \equiv \left(\frac{p^2}{h}, 0, 0\right)$, $B \equiv \left(0, \frac{p^2}{k}, 0\right)$, $C \equiv \left(0, 0, \frac{p^2}{\ell}\right)$

Now area of ΔABC, $\Delta^2 = A_{xy}^2 + A_{yz}^2 + A_{zx}^2$

Now A_{xy} = area of projection of ΔABC on xy-plane = area of ΔAOB

$$= \text{Mod of } \frac{1}{2}\begin{vmatrix} \frac{p^2}{h} & 0 & 1 \\ 0 & \frac{p^2}{k} & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2}\frac{p^4}{|hk|}$$

Similarly, $A_{yz} = \frac{1}{2}\frac{p^4}{|k\ell|}$ and $A_{zx} = \frac{1}{2}\frac{p^4}{|\ell h|}$

$\therefore$ $\Delta^2 = \frac{1}{4}\frac{p^8}{h^2k^2} + \frac{1}{4}\frac{p^8}{k^2\ell^2} + \frac{1}{4}\frac{p^8}{h^2\ell^2} = \frac{p^{10}}{4h^2k^2\ell^2}$

or $\Delta = \frac{p^5}{2|hk\ell|}$ **Ans.**

Illustration 26 : Find the locus of a point, the sum of squares of whose distances from the planes : $x - z = 0$, $x - 2y + z = 0$ and $x + y + z = 0$ is 36

Solution : Given planes are $x - z = 0$, $x - 2y + z = 0$ and, $x + y + z = 0$

Let the point whose locus is required be $P(\alpha,\beta,\gamma)$. According to question

$$\frac{|\alpha-\gamma|^2}{2} + \frac{|\alpha-2\beta+\gamma|^2}{6} + \frac{|\alpha+\beta+\gamma|^2}{3} = 36$$

or $3(\alpha^2+\gamma^2-2\alpha\gamma)+\alpha^2+4\beta^2+\gamma^2-4\alpha\beta-4\beta\gamma+2\alpha\gamma+2(\alpha^2+\beta^2+\gamma^2+2\alpha\beta+2\beta\gamma+2\alpha\gamma) = 36 \quad 6$

or $6\alpha^2 + 6\beta^2 + 6\gamma^2 = 36 \quad 6$

or $\alpha^2 + \beta^2 + \gamma^2 = 36$

Hence, the required equation of locus is $x^2 + y^2 + z^2 = 36$ **Ans.**

Illustration 27 : Direction ratios of normal to the plane which passes through the point (1, 0, 0) and (0, 1, 0) which makes angle $\pi/4$ with $x + y = 3$ are -

(A) 1, 1, 2 (B) $\sqrt{2}$, 1, 1 (C) 1, $\sqrt{2}$, 1 (D) 1, 1, $\sqrt{2}$

Solution : The plane by intercept form is $\frac{x}{1}+\frac{y}{1}+\frac{z}{c}=1$

d.r.'s of normal are $1, 1, \frac{1}{c}$ and of given plane are 1, 1, 0.

$$\therefore \quad \cos\frac{\pi}{4}=\frac{1.1+1.1+0\cdot\frac{1}{c}}{\sqrt{1+1+\frac{1}{c^2}}\ \sqrt{1+1+0}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}}=\frac{2}{\sqrt{2+\frac{1}{c^2}}\ \sqrt{2}} \quad \Rightarrow \quad 2+\frac{1}{c^2}=4 \quad \Rightarrow \quad c=\frac{1}{\sqrt{2}}$$

$\therefore$ d.r.'s are 1, 1, $\sqrt{2}$ **Ans. (D)**

ANSWERS FOR DO YOURSELF

1: (i) $2\sqrt{21}$ (iii) $8x + 2y + 24z \pm 2k^2 + 9 = 0$ (iv) $\left(-2, \frac{8}{3}, 5\right)$ & $\left(-1, \frac{10}{3}, 6\right)$

(v) (a) 7 : 8, externally (b) 2 : 3 internally

2 : (i) 3, 2, 5 (ii) 3, 4, 3 (iii) 2, 3, 4 & $\frac{2}{\sqrt{29}}, \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}}$

3 : (i) $\theta=\frac{\pi}{2}$ (iii) $\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

4 : (i) (a) $\frac{3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}, \frac{1}{\sqrt{2}}$ (b) $\left(\frac{7}{3}, 0, 0\right)$, $\left(0, \frac{7}{4}, 0\right)$ & $\left(0, 0, \frac{7}{5}\right)$ (c) $\frac{x}{7/3}+\frac{y}{7/4}+\frac{z}{7/5}=1$

(d) $\frac{3x}{5\sqrt{2}}+\frac{4y}{5\sqrt{2}}+\frac{z}{\sqrt{2}}=\frac{7}{5\sqrt{2}}$ & $\vec{r}.\left(\frac{3}{5\sqrt{2}}\tilde{i}+\frac{4}{5\sqrt{2}}\tilde{j}+\frac{1}{\sqrt{2}}\tilde{k}\right)=\frac{7}{5\sqrt{2}}$

(ii) $x + y + 2z = 7$

5: (ii) $\theta=\cos^{-1}\left(\frac{1}{\sqrt{156}}\right)$

6 : (i) $\frac{8}{\sqrt{6}}$ (ii) $3x + 4y + 2z = 0$

7 : (i) P, Q same side & R opposite side

(ii) (a) $4x + y - 5z + 14 = 0$ & $32x - 13y + 23z - 56 = 0$

(b) $4x + y - 5z - 14 = 0$ & $32x - 13y + 23z - 70 = 0$

(c) $4x + y - 5z + 14 = 0$ (acute angle bisector) & $32x - 13y + 23z - 56 = 0$ (obtuse angle bisector)

8 : (i) $\frac{x-4}{1}=\frac{y-2}{-1}=\frac{z-3}{2}$ (ii) $\frac{x-11/4}{-3}=\frac{y+9/4}{5}=\frac{z-0}{4}$ (iii) $\theta=\sin^{-1}\left(\frac{6}{5\sqrt{5}}\right)$

9 : (i) $\left(\frac{-5}{3}, \frac{13}{3}, \frac{2}{3}\right)$ & $\left(\frac{-1}{3}, \frac{11}{3}, \frac{4}{3}\right)$ (ii) 1

10: (i) $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$ (ii) $\frac{\sqrt{1218}}{2}$

EXERCISE - 01 CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. The plane XOZ divides the join of (1, –1, 5) and (2, 3, 4) in the ratio $\lambda : 1$, then λ is -

(A) –3 (B) –1/3 (C) 3 (D) 1/3

2. Let ABCD be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the area of triangles ABC, ACD and ADB be 3, 4 and 5 sq. units respectively. Then the area of the triangle BCD, is -

(A) $5\sqrt{2}$ (B) 5 (C) $\frac{5}{\sqrt{2}}$ (D) $\frac{5}{2}$

3. Which one of the following statement is INCORRECT ?

(A) If $\vec{n}\cdot\vec{a} = 0$, $\vec{n}\cdot\vec{b} = 0$ and $\vec{n}\cdot\vec{c} = 0$ for some non zero vector $\vec{n}$, then $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

(B) there exist a vector having direction angles $\alpha = 30$ and $\beta = 45$

(C) locus of point in space for which x = 3 and y = 4 is a line parallel to the z-axis whose distance from the z-axis is 5

(D) In a regular tetrahedron OABC where 'O' is the origin, the vector $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$ is perpendicular to the plane ABC.

4. Consider the following 5 statements

(I) There exists a plane containing the points (1, 2, 3) and (2, 3, 4) and perpendicular to the vector $\vec{V}_1 = \tilde{i} + \tilde{j} - \tilde{k}$

(II) There exist no plane containing the point (1, 0, 0); (0, 1, 0); (0, 0, 1) and (1, 1, 1)

(III) If a plane with normal vector $\vec{N}$ is perpendicular to a vector $\vec{V}$ then $\vec{N}\ \ \vec{V} = 0$

(IV) If two planes are perpendicular then every line in one plane is perpendicular to every line on the other plane

(v) Let P_1 and P_2 are two perpendicular planes. If a third plane P_3 is perpendicular to P_1 then it must be either parallel or perpendicular or at an angle of 45 to P_2.

Choose the correct alternative.

(A) exactly one is false (B) exactly 2 are false (C) exactly 3 are false (D) exactly four are false

5. Let L_1 be the line $\vec{r}_1 = 2\tilde{i} + \tilde{j} - \tilde{k} + \lambda(\tilde{i} + 2\tilde{k})$ and let L_2 be the line $\vec{r}_2 = 3\tilde{i} + \tilde{j} + \mu(\tilde{i} + \tilde{j} - \tilde{k})$.

Let Π be the plane which contains the line L_1 and is parallel to L_2. The distance of the plane Π from the origin is -

(A) $\sqrt{2/7}$ (B) 1/7 (C) $\sqrt{6}$ (D) none of these

6. The intercept made by the plane $\vec{r}\cdot\vec{n} = q$ on the x-axis is -

(A) $\frac{q}{\tilde{i}.\vec{n}}$ (B) $\frac{\tilde{i}.\vec{n}}{q}$ (C) $(\tilde{i}.\vec{n})\,q$ (D) $\frac{q}{|\vec{n}|}$

7. If from the point P(f, g, h) perpendiculars PL, PM be drawn to yz and zx planes then the equation to the plane OLM is -

(A) $\frac{x}{f} + \frac{y}{g} + \frac{z}{h} = 0$ (B) $\frac{x}{f} + \frac{y}{g} - \frac{z}{h} = 0$

(C) $\frac{x}{f} - \frac{y}{g} + \frac{z}{h} = 0$ (D) $-\frac{x}{f} + \frac{y}{g} + \frac{z}{h} = 0$

8. The line which contains all points (x, y, z) which are of the form (x, y, z) = (2, –2, 5) + λ(1, –3, 2) intersects the plane 2x – 3y + 4z = 163 at P and intersects the YZ plane at Q. If the distance PQ is $a\sqrt{b}$, where a, b ∈ N and a > 3 then (a + b) equals -

(A) 23 (B) 95 (C) 27 (D) none of these

9. A plane passes through the point P(4, 0, 0) and Q(0, 0, 4) and is parallel to the y-axis. The distance of the plane from the origin is -

(A) 2 (B) 4 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

10. The distance between the parallel planes given by the equations, $\vec{r}\cdot(2\hat{i} - 2\hat{j} + \hat{k}) + 3 = 0$ and $\vec{r}\cdot(4\hat{i} - 4\hat{j} + 2\hat{k}) + 5 = 0$ is -

(A) 1/2 (B) 1/3 (C) 1/4 (D) 1/6

11. If the plane 2x – 3y + 6z – 11 = 0 makes an angle $\sin^{-1}(k)$ with x-axis, then k is equal to -

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{2}{7}$ (C) $\frac{\sqrt{2}}{3}$ (D) 1

12. A variable plane forms a tetrahedron of constant volume $64K^3$ with the coordinate planes and the origin, then locus of the centroid of the tetrahedron is -

(A) $x^3 + y^3 + z^3 = 6K^2$ (B) $xyz = 6k^3$ (C) $x^2 + y^2 + z^2 = 4K^2$ (D) $x^{-2} + y^{-2} + z^{-2} = 4k^{-2}$

13. The expression in the vector form for the point $\vec{r}_1$ of intersection of the plane $\vec{r}\ \vec{n} = d$ and the perpendicular line $\vec{r} = \vec{r}_0 + t\vec{n}$ where t is a parameter given by -

(A) $\vec{r}_1 = \vec{r}_0 + \left(\frac{d - \vec{r}_0\ \vec{n}}{\vec{n}^2}\right)\vec{n}$ (B) $\vec{r}_1 = \vec{r}_0 - \left(\frac{\vec{r}_0\ \vec{n}}{\vec{n}^2}\right)\vec{n}$

(C) $\vec{r}_1 = \vec{r}_0 - \left(\frac{\vec{r}_0\ \vec{n} - d}{|\vec{n}|}\right)\vec{n}$ (D) $\vec{r}_1 = \vec{r}_0 + \left(\frac{\vec{r}_0\ \vec{n}}{|\vec{n}|}\right)\vec{n}$

14. The equation of the plane containing the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point (0, 7, –7) is -

(A) x + y + z = 1 (B) x + y + z = 2 (C) x + y + z = 0 (D) none of these

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

15. Consider the plane $\vec{r}.\vec{n}_1 = d_1$ and $\vec{r}.\vec{n}_2 = d_2$, then which of the follwoing are true -

(A) they are perpendicular if $\vec{n}_1.\vec{n}_2 = 0$

(B) angle between them is $\cos^{-1}\left(\frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right)$

(C) normal form of the equation of plane are $\vec{r}.\vec{n}_1 = \frac{d_1}{|\vec{n}_1|}$ & $\vec{r}.\vec{n}_2 = \frac{d_2}{|\vec{n}_2|}$

(D) none of these

16. The equation of the plane which contains the lines $\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and $\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \mu(\hat{i} + \hat{j} + 3\hat{k})$ must be -

(A) $\vec{r}.(7\hat{i} - 4\hat{j} - \hat{k}) = 0$ (B) 7(x – 1) – 4(y – 2) – (z + 1) = 0

(C) $\vec{r}.(\hat{i} + 2\hat{j} - \hat{k}) = 0$ (D) $\vec{r}.(\hat{i} + \hat{j} + 3\hat{k}) = 0$

17. The plane containing the lines $\vec{r} = \vec{a} + t\vec{a}'$ and $\vec{r} = \vec{a}' + s\vec{a}$ -

(A) must be parallel to $\vec{a} \times \vec{a}'$

(B) must be the perpendicular to $\vec{a} \times \vec{a}'$

(C) must be $[\vec{r}, \vec{a}, \vec{a}'] = 0$

(D) $(\vec{r} - \vec{a}).(\vec{a} \times \vec{a}') = 0$

18. The points A(5, –1, 1), B(7, –4, 7), C(1, –6, 10) and D(–1, –3, 4) are the vertices of a -

(A) parallelogram (B) rectangle (C) rhombus (D) square

19. If P_1, P_2, P_3 denotes the perpendicular distances of the plane $2x - 3y + 4z + 2 = 0$ from the parallel planes $2x - 3y + 4z + 6 = 0$, $4x - 6y + 8z + 3 = 0$ and $2x - 3y + 4z - 6 = 0$ respectively, then -

(A) $P_1 + 8P_2 - P_3 = 0$

(B) $P_3 = 16P_2$

(C) $8P_2 = P_1$

(D) $P_1 + 2P_2 + 3P_3 = \sqrt{29}$

CHECK YOUR GRASP — **ANSWER KEY** — EXERCISE-1

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	A	B	D	A	A	B	A	D	D
Que.	11	12	13	14	15	16	17	18	19	
Ans.	B	B	A	C	A,B	A,B	B,C,D	A,C	A,B,C,D	

EXERCISE - 02

BRAIN TEASERS

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. If the line $\vec{r} = 2\hat{i} - \hat{j} + 3\hat{k} + \lambda(\hat{i} + \hat{j} + \sqrt{2}\,\hat{k})$ makes angles α, β, γ with xy, yz and zx planes respectively then which one of the following are not possible ?

(A) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$ and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(B) $\tan^2\alpha + \tan^2\beta + \tan^2\gamma = 7$ and $\cot^2\alpha + \cot^2\beta + \cot^2\gamma = 5/3$

(C) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 1$ and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 2$

(D) $\sec^2\alpha + \sec^2\beta + \sec^2\gamma = 10$ and $\mathrm{cosec}^2\alpha + \mathrm{cosec}^2\beta + \mathrm{cosec}^2\gamma = 14/3$

2. A plane meets the coordinate axes in A, B, C such that the centroid of the triangle ABC is the point $(1, r, r^2)$. The plane passes through the point $(4, -8, 15)$ if r is equal to -

(A) -3 (B) 3 (C) 5 (D) -5

3. Indicate the correct order statements -

(A) The lines $\frac{x-4}{-3} = \frac{y+6}{-1} = \frac{z+6}{-1}$ and $\frac{x-1}{-1} = \frac{y-2}{-2} = \frac{z-3}{2}$ are orthogonal

(B) The planes $3x - 2y - 4z = 3$ and the plane $x - y - z = 3$ are orthogonal.

(C) The function $f(x) = \ell n(e^{-2} + e^x)$ is monotonic increasing $\forall\ x \in R$.

(D) If g is the inverse of the function, $f(x) = \ell n(e^{-2} + e^x)$ then $g(x) = \ell n(e^x - e^{-2})$

4. The coordinates of a point on the line $\frac{x-1}{2} = \frac{y+1}{-3} = z$ at a distance $4\sqrt{14}$ from the point $(1, -1, 0)$ are-

(A) $(9, -13, 4)$ (B) $(8\sqrt{14}+1,\ -12\sqrt{14}-1,\ 4\sqrt{14})$

(C) $(-7, 11, -4)$ (D) $(-8\sqrt{14}+1,\ 12\sqrt{14}-1,\ -4\sqrt{14})$

5. Let $6x + 4y - 5z = 4$, $x - 5y + 2z = 12$ and $\frac{x-9}{2} = \frac{y+4}{-1} = \frac{z-5}{1}$ be two lines then-

(A) the angle between them must be $\frac{\pi}{3}$ (B) the angle between them must be $\cos^{-1}\frac{5}{6}$

(C) the plane containing them must be $x + y - z = 0$ (D) they are non-coplanar

6. The lines $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{\lambda}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z+1}{-1}$ are -

(A) coplanar for all λ (B) coplanar for $\lambda = 19/3$

(C) if coplanar then intersect at $\left(-\frac{1}{5}, -\frac{2}{5}, -\frac{4}{5}\right)$ (D) intersect at $\left(\frac{1}{2}, -\frac{1}{2}, -1\right)$

7. If two pairs of opposite edges of a tetrahedron are perpendicular then -

(A) the third is also perpendicular (B) the third pair is inclined at 60

(C) the third pair is inclined at 45 (D) (B), (C) are false

8. The equation of a plane bisecting the angle between the plane $2x - y + 2z + 3 = 0$ and $3x - 2y + 6z + 8 = 0$ is -

(A) $5x - y - 4z - 45 = 0$ (B) $5x - y - 4z - 3 = 0$

(C) $23x - 13y + 32z + 45 = 0$ (D) $23x - 13y + 32z + 5 = 0$

9. A non-zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}$, $\hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}$, $\hat{i}-\hat{k}$,. The possible angle between $\vec{a}$ and $\hat{i}-2\hat{j}+2\hat{k}$ is -

(A) $\pi/3$ (B) $\pi/4$ (C) $\pi/6$ (D) $3\pi/4$

10. If ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2 are DCs of the two lines inclined to each other at an angle θ, then the DCs of the bisector of the angle between these lines are-

(A) $\frac{\ell_1+\ell_2}{2\sin\theta/2}, \frac{m_1+m_2}{2\sin\theta/2}, \frac{n_1+n_2}{2\sin\theta/2}$

(B) $\frac{\ell_1+\ell_2}{2\cos\theta/2}, \frac{m_1+m_2}{2\cos\theta/2}, \frac{n_1+n_2}{2\cos\theta/2}$

(C) $\frac{\ell_1-\ell_2}{2\sin\theta/2}, \frac{m_1-m_2}{2\sin\theta/2}, \frac{n_1-n_2}{2\sin\theta/2}$

(D) $\frac{\ell_1-\ell_2}{2\cos\theta/2}, \frac{m_1-m_2}{2\cos\theta/2}, \frac{n_1-n_2}{2\cos\theta/2}$

11. Points that lie on the lines bisecting the angle between the lines $\frac{x-2}{2}=\frac{y-3}{3}=\frac{z-6}{6}$ and $\frac{x-2}{3}=\frac{y-3}{6}=\frac{z-6}{2}$ are -

(A) (7, 12, 14) (B) (0, –3, 14) (C) (1, 0, 10) (D) (–3, –6, –2)

BRAIN TEASERS — ANSWER KEY — EXERCISE -2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A,B,D	B,C	C,D	A,C	A,C	B,C	A,D	B,C	B,D	B,C
Que.	11									
Ans.	A,B,C,D									

EXERCISE - 03

MISCELLANEOUS TYPE QUESTIONS

TRUE / FALSE

1. If the plane xbc + yac + zab = abc cuts x, y & z-axis in A, B & C respectively then area of ΔABC is $\sqrt{a^2b^2+b^2c^2+c^2a^2}$.

2. The angle between the line $\vec{r} = \vec{a} + \lambda\vec{b}$ and plane $\vec{r}\ \ \vec{n} = d$ is $\frac{\pi}{2} - \cos^{-1}\left(\frac{\vec{b}\ \ \vec{n}}{|\vec{b}||\vec{n}|}\right)$

3. The perpendicular distance of the plane $\vec{r}\ \ \hat{n} = d$, from the origin is d where $d > 0$

4. If A(1, 2, –1), B(2, 6, 2) and C(λ, –2, –4) are collinear then value of λ is 0.

5. The projection of line segment on the axes of reference are 3, 4 and 12 respectively. The length of such a line segment is $\sqrt{13}$

MATCH THE COLUMN

Following questions contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE** statement in **Column-II**.

1. Match the following pair of planes with their lines of intersections :

	Column-I		Column-II
(A)	x + y = 0 = y + z	(p)	$\frac{x-2}{0} = \frac{y-2007}{-1} = \frac{z+2004}{1}$
(B)	x = 2, y = 3	(q)	$\frac{x-2}{0} = \frac{y}{-1} = \frac{z-1}{1}$
(C)	x = 2, y + z = 3	(r)	x = –y = z
(D)	x = 2, x + y + z = 3	(s)	$\frac{x-2}{0} = \frac{y-3}{0} = \frac{z}{1}$

2. Consider three planes

$P_1 \equiv 2x + y + z = 1$

$P_2 \equiv x - y + z = 2$

$P_3 \equiv \alpha x - y + 3z = 5$

The three planes intersects each other at point P on XOY plane and at point Q on YOZ plane. O is the origin.

	Column-I		Column-II
(A)	The value of α is	(p)	1
(B)	The length of projection of PQ on x-axis is		
(C)	If the co-ordinates of point R situated at a minimum distance from point 'O' on the line PQ are (a, b, c), then value of 7a + 14b + 14c is	(q)	2
		(r)	4
(D)	If the area of ΔPOQ is $\sqrt{\frac{a}{b}}$, then value of a – b is	(s)	3

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE OR MORE** statement(s) in **Column-II**.

3. Consider the following four pairs of lines in column–I and match them with one or more entries in column–II

	Column-I		Column-II
(A)	L_1 : $x=1+t,\ y=t,\ z=2-5t$ L_2 : $\vec{r} = (2, 1,-3) + \lambda\ (2,2,-10)$	(p)	non coplanar lines
(B)	$L_1 : \dfrac{x-1}{2} = \dfrac{y-3}{2} = \dfrac{z-2}{-1}$ $L_2 : \dfrac{x-2}{1} = \dfrac{y-6}{-1} = \dfrac{z+2}{3}$	(q)	lines lie in a unique plane
(C)	L_1 : $x = -6t,\ y=1+9t,\ z=-3t$ L_2 : $x=1+2s,\ y=4-3s,\ z=s$	(r)	infinite planes containing both the lines
(D)	$L_1 : \dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3}$ $L_2 : \dfrac{x-3}{-4} = \dfrac{y-2}{-3} = \dfrac{z-1}{2}$	(s)	lines are not intersecting

ASSERTION & REASON

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement - I** : If a plane contains point $A(\vec{a})$ and is parallel to vectors $\vec{b}$ and $\vec{c}$, then its vector equation is $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$, where λ & μ are parameters and $\vec{b} \nparallel \vec{c}$.

Because

Statement - II : If three vectors are co-planar, then any one can be expressed as the linear combination of other two.

(A) A (B) B (C) C (D) D

2. **Statement - I** : If $ax + by + cz = \sqrt{a^2+b^2+c^2}$ be a plane and (x_1, y_1, z_1) and (x_2, y_2, z_2) be two points on this plane then $a(x_1 - x_2) + b(y_1 - y_2) + c(z_1 - z_2) = 0$.

Because

Statement - II : If two vectors $p_1\hat{i} + p_2\hat{j} + p_3\hat{k}$ and $q_1\hat{i} + q_2\hat{j} + p_3\hat{k}$ are orthogonal then $p_1q_1 + p_2q_2 + p_3q_3 = 0$.

(A) A (B) B (C) C (D) D

3. **Statement - I** : If the lines $\dfrac{x-x_1}{a_1} = \dfrac{y-y_1}{b_1} = \dfrac{z-z_1}{c_1}$ and $\dfrac{x-x_2}{a_2} = \dfrac{y-y_2}{b_2} = \dfrac{z-z_2}{c_2}$ are coplanar then

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} x_2 & y_2 & z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

Because

Statement - II : If the two lines are coplanar then shortest distance between them is zero.

(A) A (B) B (C) C (D) D

4. **Statement - I** : $ABCDA_1B_1C_1D_1$ is a cube of edge 1 unit. P and Q are the mid points of the edges B_1A_1, and B_1C_1 respectively. Then the distance of the vertex D from the plane PBQ is $\frac{8}{3}$.

Because

Statement - II : Perpendicular distance of point (x_1, y_1, z_1) from the plane $ax + by + cz + d = 0$ is given by $\left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$.

(A) A (B) B (C) C (D) D

5. **Statement - I** : If $2a + 3b + 6c = 14$, where a, b & c $\in$ R, then the minimum value of $a^2 + b^2 + c^2$ is 4.

Because

Statement - II : The perpendicular distance of the plane $px + qy + rz = 1$ from origin is $\frac{1}{\sqrt{p^2 + q^2 + r^2}}$.

(A) A (B) B (C) C (D) D

6. Consider following two planes

$$P_1 \equiv [\vec{r} - \vec{p}\ \ \vec{a}\ \ \vec{b}] = 0$$

$$P_2 \equiv [\vec{r} - \vec{p}\ \ \vec{c}\ \ \vec{d}] = 0$$

such that $|(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})| \neq 0$ & let $\vec{x}$ be any vector in space.

Statement-I : $\vec{x}.\{(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})\} = 0 \Rightarrow \vec{x}.\{t_1\vec{a} + t_2\vec{b}\} = 0, \forall\ t_1, t_2 \in R$

Because

Statement-II : $\vec{x}.\{t_1\vec{a} + t_2\vec{b}\} = 0\ \forall\ t_1, t_2 \in R \Rightarrow \vec{x}.\{(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})\} = 0$.

(A) A (B) B (C) C (D) D

7. Consider planes $P_1 : (\vec{r} - \hat{i}).\{(\hat{i} - \hat{j} - \hat{k}) \times (\hat{i} - 2\hat{k})\} = 0$ and $P_2 : (\vec{r} - (2\hat{i} - \hat{j} - \hat{k})).\{(\hat{i} - 2\hat{k}) \times (2\hat{i} - \hat{j} - 3\hat{k})\} = 0$

and line L : $\vec{r} = 5\hat{i} + \lambda(\hat{i} - \hat{j} - \hat{k})$

Statement-I : P_1 & P_2 are parallel planes.

Because

Statement-II : L is parallel to both P_1 & P_2.

(A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1 :

Given four points A(2, 1, 0); B(1, 0, 1); C(3, 0, 1) and D(0, 0, 2). The point D lies on a line L orthogonal to the plane determined by the point A, B, and C

On the basis of above information, answer the following questions :

1. Equation of the plane ABC is -

(A) $x + y + z - 3 = 0$ (B) $y + z - 1 = 0$ (C) $x + z - 1 = 0$ (D) $2y + z - 1 = 0$

2. Equation of the line L is -

(A) $\vec{r} = 2\hat{k} + \lambda(\hat{i} + \hat{k})$ (B) $\vec{r} = 2\hat{k} + \lambda(2\hat{j} + \hat{k})$

(C) $\vec{r} = 2\hat{k} + \lambda(\hat{j} + \hat{k})$ (D) none of these

3. Perpendicular distance of D from the plane ABC, is -

(A) $\sqrt{2}$ (B) $\frac{1}{2}$ (C) 2 (D) $\frac{1}{\sqrt{2}}$

Comprehension # 2 :

If a line passes through P (x_1, y_1, z_1) and having Dr's a, b, c, then the equation of line is $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ and equation of plane perpendicular to it and passing through P is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$.
Further equation of plane through the intersection of the two planes
$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is
$(a_1x + b_1y + c_1z + d_1) + k(a_2x + b_2y + c_2z + d_2) = 0$

On the basis of above information, answer the following questions :

1. The distance of the point (1, –2, 3) from the plane x – y + z = 5 measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z-3}{-4}$ is -

(A) $\frac{1}{5}\sqrt{21}$ (B) $\frac{1}{5}\sqrt{29}$ (C) $\frac{1}{5}\sqrt{13}$ (D) $\frac{2}{\sqrt{5}}$

2. The equation of the plane through (0, 2, 4) and containing the line $\frac{x+3}{3} = \frac{y-1}{4} = \frac{z-2}{-2}$ is -

(A) $x - 2y + 4z - 12 = 0$ (B) $5x + y + 9z - 38 = 0$
(C) $10x - 12y - 9z + 60 = 0$ (D) $7x + 5y - 3z + 2 = 0$

3. The plane x – y – z = 2 is rotated through 90 about its line of intersection with the plane x + 2y + z = 2. Then equation of this plane in new position is -

(A) $5x + 4y + z - 10 = 0$ (B) $4x + 5y + 3z = 0$ (C) $2x + y + 2z = 9$ (D) $3x + 4y - 5z = 9$

Comprehension # 3 :

Consider a triangular pyramid ABCD the position vectors of whose angular point are A(3, 0, 1); B(–1, 4, 1); C(5, 2, 3) and D(0, –5, 4). Let G be the point of intersection of the medians of the triangle BCD.

On the basis of above information, answer the following questions :

1. The length of the vector $\overrightarrow{AG}$ is-

(A) $\sqrt{17}$ (B) $\frac{\sqrt{51}}{3}$ (C) $\frac{\sqrt{51}}{9}$ (D) $\frac{\sqrt{59}}{4}$

2. Area of the triangle ABC in sq. units is-

(A) 24 (B) $8\sqrt{6}$ (C) $4\sqrt{6}$ (D) none of these

3. The length of the perpendicular from the vertex D on the opposite face is -

(A) $\frac{14}{\sqrt{6}}$ (B) $\frac{2}{\sqrt{6}}$ (C) $\frac{3}{\sqrt{6}}$ (D) none of these

4. Equation of the plane ABC is -

(A) $x + y + 2z = 5$ (B) $x - y - 2z = 1$ (C) $2x + y - 2z = 4$ (D) $x + y - 2z = 1$

MISCELLANEOUS TYPE QUESTION **ANSWER KEY** EXERCISE-03

- ***True / False***
  1. F 2. T 3. T 4. T 5. F
- ***Match the Column***
  1. (A) → (r); (B) → (s); (C) → (p); (D) → (q) 2. (A) → (r); (B) → (p); (C) → (q); (D) → (s)
  3. (A) → (r); (B) → (q); (C) → (q,s); (D) → (p,s)
- ***Assertion & Reason***
  1. C 2. A 3. C 4. D 5. A 6. D 7. B
- ***Comprehension Based Questions***
  Comprehension # 1 : 1. B 2. C 3. D Comprehension # 2 : 1. B 2. C 3. A
  Comprehension # 3 : 1. B 2. C 3. A 4. D

EXERCISE - 04 [A]

CONCEPTUAL SUBJECTIVE EXERCISE

1. Find the angle between the two straight lines whose direction cosines ℓ, m, n are given by $2\ell + 2m - n = 0$ and $mn + n\ell + \ell m = 0$.
2. A variable plane is at a constant distance p from the origin and meets the coordinate axes in points A, B and C respectively. Through these points, planes are drawn parallel to the coordinates planes. Find the locus of their point of intersection.
3. P is any point on the plane $\ell x + my + nz = p$. A point Q taken on the line OP (where O is the origin) such that $OP.OQ = p^2$. Show that the locus of Q is $p(\ell x + my + nz) = x^2 + y^2 + z^2$.
4. The plane $\ell x + my = 0$ is rotated about its line of intersection with the plane $z = 0$ through an angle θ. Prove that the equation to the plane in new position is $\ell x + my \pm z\sqrt{\ell^2 + m^2}\tan\theta = 0$
5. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at an angle of $\frac{\pi}{3}$
6. Find the equation of the line which is reflection of the line $\frac{x-1}{9} = \frac{y-2}{-1} = \frac{z+3}{-3}$ in the plane $3x - 3y + 10z = 26$
7. Find the point where the line of intersection of the planes $x - 2y + z = 1$ and $x + 2y - 2z = 5$, intersects the plane $2x + 2y + z + 6 = 0$
8. Find the foot and hence the length of the perpendicular from the point (5, 7, 3) to the line $\frac{x-15}{3} = \frac{y-29}{8} = \frac{5-z}{5}$. Also find the equation of the plane in which the perpendicular and the given straight line lie.
9. Find the equation of the plane containing the straight line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z}{5}$ are perpendicular to the plane $x - y + z + 2 = 0$
10. Find the equation of the plane containing the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z}{2}$ and parallel to the line $\frac{x-3}{2} = \frac{y}{5} = \frac{z-2}{4}$. Find also the S.D. between two lines.

CONCEPTUAL SUBJECTIVE EXERCISE	ANSWER KEY	EXERCISE-04(A)
1. $\theta = 90$	2. $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$	5. $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ or $\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$
6. $\frac{x-4}{9} = \frac{y+1}{-1} = \frac{z-7}{-3}$	7. (1, – 2, –4)	8. (9, 13, 15) ; 14; $9x - 4y - z = 14$
9. $2x + 3y + z + 4 = 0$	10. $x - 2y + 2z - 1 = 0$; 2 units	

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

1. Through a point P(f, g, h), a plane is drawn at right angles to OP where 'O' is the origin, to meet the coordinate axes in A, B, C. Prove that the area of the triangle ABC is $\frac{r^5}{2fgh}$ where OP = r.

2. Find the equations to the line which can be drawn from the point (2, –1, 3) perpendicular to the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{4} = \frac{y}{5} = \frac{z+3}{3}$ at right angles.

3. The position vectors of the four angular points of a tetrahedron OABC are (0, 0, 0); (0, 0, 2); (0, 4, 0) and (6, 0, 0) respectively. A point P inside the tetrahedron is at the same distance 'r' from the four plane faces of the tetrahedron. Find the values of 'r'.

4. The line $\frac{x+6}{5} = \frac{y+10}{3} = \frac{z+14}{8}$ is the hypotenuse of an isosceles right angled triangle whose opposite vertex is (7, 2, 4). Find the equation of the remaining sides.

5. If two straight lines having direction cosines ℓ, m, n satisfy $a\ell + bm + cn = 0$ and $fmn + gn\ell + h\ell m = 0$ are perpendicular, then show that $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$.

6. Find the equations to the line of greatest slope through the point (7, 2, –1) in the plane x – 2y + 3z = 0 assuming that the axes are so placed that the plane 2x + 3y – 4z = 0 is horizontal.

7. Let ABCD be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the area of triangles ABC, ACD and ADB be denoted by x, y and z sq. units respectively. Find the area of the triangle BCD.

BRAIN STORMING SUBJECTIVE EXERCISE	ANSWER KEY	EXERCISE-04(B)
2. $\frac{x-2}{11} = \frac{y+1}{-10} = \frac{z-3}{2}$	3. $\frac{2}{3}$	4. $\frac{x-7}{3} = \frac{y-2}{6} = \frac{z-4}{2}$; $\frac{x-7}{2} = \frac{y-2}{-3} = \frac{z-4}{6}$
6. $\frac{x-7}{22} = \frac{y-2}{5} = \frac{z+1}{-4}$	7. $\sqrt{(x^2+y^2+z^2)}$	

EXERCISE - 05 [A]

JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

1. If the line $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-5}{1}=\frac{z-6}{-5}$ are perpendicular to each other then k = [AIEEE-2002]

(1) $\frac{5}{7}$ (2) $\frac{7}{5}$ (3) $\frac{-7}{10}$ (4) $\frac{-10}{7}$

2. The angle between the lines, whose direction ratios are 1, 1, 2 and $\sqrt{3}-1$, $-\sqrt{3}-1$, 4, is- [AIEEE-2002]

(1) 45 (2) 30 (3) 60 (4) 90

3. The acute angle between the planes $2x - y + z = 6$ and $x + y + 2z = 3$ is- [AIEEE-2002]

(1) 30 (2) 45 (3) 60 (4) 75

4. The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is- [AIEEE-2003]

(1) 39 (2) 26 (3) $11\frac{4}{13}$ (4) 13

5. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if- [AIEEE-2003]

(1) k = 3 or –3 (2) k = 0 or –1 (3) k = 1 or –1 (4) k = 0 or –3

6. The radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$ is- [AIEEE-2003]

(1) 4 (2) 1 (3) 2 (4) 3

7. A tetrahedron has vertices at O(0, 0, 0), A(1, 2, 1), B(2, 1, 3) and C(–1, 1, 2). Then the angle between the faces OAB and ABC will be- [AIEEE-2003]

(1) 90 (2) $\cos^{-1}\left(\frac{19}{35}\right)$ (3) $\cos^{-1}\left(\frac{17}{31}\right)$ (4) 30

8. The two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ will be perpendicular, if and only if- [AIEEE-2003]

(1) $aa' + cc' + 1 = 0$ (2) $aa' + bb' + cc' + 1=0$

(3) $aa' + bb' + cc' = 0$ (4) $(a + a')(b + b') + (c + c') = 0$

9. A line makes the same angle θ, with each of the x and z axis. If the angle β, which it makes with y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then $\cos^2\theta$ equals- [AIEEE-2004]

(1) 2/3 (2) 1/5 (3) 3/5 (4) 2/5

10. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is- [AIEEE-2004]

(1) 3/2 (2) 5/2 (3) 7/2 (4) 9/2

11. A line with direction cosines proportional to 2, 1, 2 meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. The coordinates of each of the points on intersection are given by- [AIEEE-2004]

(1) (3a, 3a, 3a), (a, a, a) (2) (3a, 2a, 3a), (a, a, a)

(3) (3a, 2a, 3a), (a, a, 2a) (4) (2a, 3a, 3a), (2a, a, a)

12. If the straight lines $x = 1 + s$, $y = -3 - \lambda s$, $z=1 + \lambda s$ and $x = \frac{t}{2}$, $y = 1 + t$, $z = 2 - t$, with parameters s and t respectively are coplanar then λ equals- [AIEEE-2004]

(1) –2 (2) –1 (3) $-\frac{1}{2}$ (4) 0

13. The intersection of the spheres $x^2 + y^2 + z^2 + 7x - 2y - z = 13$ and $x^2 + y^2 + z^2 - 3x + 3y + 4z = 8$ is the same as the intersection of one of the sphere and the plane- **[AIEEE-2004]**

(1) $x - y - z = 1$ (2) $x - 2y - z = 1$ (3) $x - y - 2z = 1$ (4) $2x - y - z = 1$

14. if the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{\lambda}\,z + 4 = 0$ is such that $\sin\theta = \frac{1}{3}$ the value of λ is- **[AIEEE-2005]**

(1) $\frac{5}{3}$ (2) $\frac{-3}{5}$ (3) $\frac{3}{4}$ (4) $\frac{-4}{3}$

15. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is- **[AIEEE-2005]**

(1) 0 (2) 90 (3) 45 (4) 30

16. If the plane $2ax - 3ay + 4az + 6 = 0$ passes through the midpoint of the line joining the centres of the spheres $x^2 + y^2 + z^2 + 6x - 8y - 2z = 13$ and $x^2 + y^2 + z^2 - 10x + 4y - 2z = 8$ then a equals- **[AIEEE-2005]**

(1) -1 (2) 1 (3) -2 (4) 2

17. The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $\vec{r}\cdot(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is- **[AIEEE-2005]**

(1) $\frac{10}{9}$ (2) $\frac{10}{3\sqrt{3}}$ (3) $\frac{3}{10}$ (4) $\frac{10}{3}$

18. The plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius- **[AIEEE-2005]**

(1) 3 (2) 1 (3) 2 (4) $\sqrt{2}$

19. The two lines $x = ay + b$, $z = cy + d$; and $x = a'y + b$; $z = c'y + d'$ are perpendicular to each other if- **[AIEEE-2006]**

(1) $aa' + cc' = 1$ (2) $\frac{a}{a'} + \frac{c}{c'} = -1$ (3) $\frac{a}{a'} + \frac{c}{c'} = 1$ (4) $aa' + cc' = -1$

20. The image of the point $(-1, 3, 4)$ in the plane $x - 2y = 0$ is - **[AIEEE-2006]**

(1) (15, 11, 4) (2) $\left(-\frac{17}{3}, -\frac{19}{3}, 1\right)$ (3) (8, 4, 4) (4) None of these

21. Let L be the line of intersection of the planes $2x + 3y + z = 1$ and $x + 3y + 2z = 2$. If L makes an angle α with the positive x-axis, then $\cos\alpha$ equals - **[AIEEE-2007]**

(1) $1/\sqrt{3}$ (2) $1/2$ (3) 1 (4) $1/\sqrt{2}$

22. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is- **[AIEEE-2007]**

(1) $\pi/6$ (2) $\pi/3$ (3) $\pi/4$ (4) $\pi/2$

23. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are- **[AIEEE-2007]**

(1) $(4, 9, -3)$ (2) $(4, -3, 3)$ (3) $(4, 3, 5)$ (4) $(4, 3, -3)$

24. The line passing through the points (5, 1, a) and (3, b, 1) crosses the yz-plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$. Then- **[AIEEE-2008]**

(1) a = 2, b = 8 (2) a = 4, b = 6 (3) a = 6, b = 4 (4) a = 8, b = 2

25. If the straight lines $\frac{x-1}{k} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{k} = \frac{z-1}{2}$ intersect at a point, then the integer k is equal to- **[AIEEE-2008]**

(1) -5 (2) 5 (3) 2 (4) -2

26. Let the line $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ lie in the plane $x + 3y - \alpha z + \beta = 0$. Then (α, β) equals : **[AIEEE-2009]**

(1) (5, – 15) (2) (–5, 5) (3) (6, –17) (4) (–6, 7)

27. The projections of a vector on the three coordinate axis are 6, –3, 2 respectively. The direction cosines of the vector are :- **[AIEEE-2009]**

(1) $\frac{6}{7},\frac{-3}{7},\frac{2}{7}$ (2) $\frac{-6}{7},\frac{-3}{7},\frac{2}{7}$ (3) 6, –3, 2 (4) $\frac{6}{5},\frac{-3}{5},\frac{2}{5}$

28. A line AB in three-dimensional space makes angle 45º and 120º with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals :- **[AIEEE-2010]**

(1) 30 (2) 45 (3) 60 (4) 75

29. **Statement–1** : The point A(3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane x – y + z = 5. **[AIEEE-2010]**

Statement–2 : The plane x – y + z = 5 bisects the line segment joining A(3, 1, 6) and B(1, 3, 4).

(1) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.

(2) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for Statement–1.

(3) Statement–1 is true, Statement–2 is false.

(4) Statement–1 is false, Statement–2 is true.

30. If the angle between the line $x=\frac{y-1}{2}=\frac{z-3}{\lambda}$ and the plane x + 2y + 3z = 4 is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals - **[AIEEE-2011]**

(1) $\frac{2}{5}$ (2) $\frac{5}{3}$ (3) $\frac{2}{3}$ (4) $\frac{3}{2}$

31. **Statement-1:** The point A(1, 0, 7) is the mirror image of the point B(1, 6, 3) in the line : $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$

Statement-2: The line : $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ bisects the line segment joining A (1, 0, 7) and B(1, 6, 3). **[AIEEE-2011]**

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

32. The length of the perpendicular drawn from the point (3, –1, 11) to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is : **[AIEEE-2011]**

(1) $\sqrt{66}$ (2) $\sqrt{29}$ (3) $\sqrt{33}$ (4) $\sqrt{53}$

33. The distance of the point (1, –5, 9) from the plane x – y + z = 5 measured along a straight line x = y = z is: **[AIEEE-2011]**

(1) $3\sqrt{5}$ (2) $10\sqrt{3}$ (3) $5\sqrt{3}$ (4) $3\sqrt{10}$

34. An equation of a plane parallel to the plane x – 2y + 2z – 5 = 0 and at a unit distance from the origin is : **[AIEEE-2012]**

(1) x – 2y + 2z + 5 = 0 (2) x – 2y + 2z – 3 = 0

(3) x – 2y + 2z + 1 = 0 (4) x – 2y + 2z – 1 = 0

35. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k is equal to :- [AIEEE-2012]

(1) 0 (2) -1 (3) $\frac{2}{9}$ (4) $\frac{9}{2}$

36. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is :- [JEE-MAIN 2013]

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$ (3) $\frac{7}{2}$ (4) $\frac{9}{2}$

37. If the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, then k can have : [JEE-MAIN 2013]

(1) any value (2) exactly one value

(3) exactly two values (4) exactly three values.

PREVIOUS YEARS QUESTIONS — **ANSWER KEY** — EXERCISE-5 [A]

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans	4	3	3	4	4	4	2	1	3	3	2	1	4	1	2
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans	3	2	2	4	4	1	4	1	3	1	4	1	3	2	
Que.	31	32	33	34	35	36	37								
Ans	4	4	2	2	4	3	3								

EXERCISE - 05 [B] JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

1. (a) Find the equation of the plane passing through the points (2,1,0),(5,0,1) and (4,1,1)
(b) If P is the point (2, 1, 6) then find the point Q such that PQ is perpendicular to the plane in (a) and the mid point of PQ lies on it. [JEE 03, 4M out of 60]

2. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ are intersecting each other then 'k' is -

(A) $\frac{2}{9}$ (B) $\frac{9}{2}$ (C) 1 (D) $\frac{3}{2}$

[JEE 04 (screening)]

3. T is a parallelopiped in which A, B, C and D are vertices of one face. And the face just above it has corresponding vertices A' , B', C', D'. T is now compressed to S with face ABCD remaining same and A', B', C', D' shifted to A'', B'', C'' and D'' in S. The volume of S is reduced to 90% of T. Prove that locus of A'' is a plane. [JEE 04 (Mains) 2M]

4. A plane is parallel to two lines whose direction ratios (1, 0, –1) & (–1, 1, 0) and it contains the point (1, 1, 1). If it cuts the coordinate axes at A, B, C. then find the volume of tetrahedron OABC, where O is the origin. [JEE 04 (Mains) 2M]

5. P_1 and P_2 are planes passing through origin. L_1 and L_2 are two line on P_1 and P_2 respectively such that their intersection is origin. Show that there exists points A, B, C, whose permutation A', B', C' can be chosen such that (i) A is on L_1, B on P_1 but not on L_1 and C not on P_1 (ii) A' is on L_2, B' on P_2 but not on L_2 and C' not on P_2. [JEE 04 (Mains) 4M]

6. A variable plane at a distance of 1 unit from the origin cut the coordinate axis at A, B & C. If centroid of triangle ABC is D(x, y, z) satisfy the relation $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k$, then value of k is - [JEE 05 (screening) 3M]

(A) 3 (B) 1 (C) 1/3 (D) 9

7. Find the equation of the plane containing the line 2x – y + z – 3 = 0, 3x + y + z = 5 and at a distance of $\frac{1}{\sqrt{6}}$ from the point (2, 1, –1) [JEE 05 (Mains) 2M]

8. A plane passes through (1, –2, 1) and is perpendicular to two planes 2x – 2y + z = 0 and x – y + 2z = 4. The distance of the plane from the point (1, 2, 3) is - [JEE 06, 3M]

(A) 0 (B) 1 (C) $\sqrt{2}$ (D) $2\sqrt{2}$

9. Match the following [JEE 06, 6M]

	Column-I		Column-II
(A)	Two rays in the first quadrant x + y = |a| and ax – y = 1 intersects each other in the interval $a \in (a_0, \infty)$, the value of a_0 is	(p)	2
(B)	Point (α, β, γ) lies on the plane x + y + z = 2. Let $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$, $\hat{k}\times(\hat{k}\times\vec{a})=0$, then γ =	(q)	4/3
(C)	$\left\|\int_0^1(1-y^2)dy\right\|+\left\|\int_1^0(y^2-1)dy\right\|$	(r)	$\left\|\int_0^1\sqrt{1-x}\,dx\right\|+\left\|\int_{-1}^0\sqrt{1+x}\,dx\right\|$
(D)	If sinAsinBsinC + cosAcosB = 1, then the value of sinC =	(s)	1

10. Match the following **[JEE 06, 6M]**

	Column-I		Column-II
(A)	$\sum_{i=1}^{\infty} \tan^{-1}\left(\frac{1}{2i^2}\right) = t$, then tant =	(p)	0
(B)	Sides a, b, c of a triangle ABC are in A.P. and $\cos\theta_1 = \frac{a}{b+c}$, $\cos\theta_2 = \frac{b}{a+c}$, $\cos\theta_3 = \frac{c}{a+b}$, then $\tan^2\frac{\theta_1}{2} + \tan^2\frac{\theta_3}{2} =$	(q)	1
		(r)	$\frac{\sqrt{5}}{3}$
(C)	A line is perpendicular to x + 2y + 2z = 0 and passes through (0, 1, 0).The perpendicular distance of this line from the origin is	(s)	2/3

11. Consider the following linear equations
$ax + by + cz = 0$; $bx + cy + az = 0$; $cx + ay + bz = 0$ **[JEE 2007, 6M]**

	Column-I		Column-II
(A)	$a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(p)	the equations represent planes meeting only at a single point
(B)	$a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(q)	the equations represent the line x = y = z
(C)	$a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(r)	the equations represent identical planes.
(D)	$a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(s)	the equations represent the whole of the three dimensional space

12. Consider the planes 3x – 6y – 2z = 15 and 2x + y – 2z = 5. **[JEE 2007, 3M]**
Statement-1 : The parametric equations of the line of intersection of the given planes are x = 3 + 14t, y = 1 + 2t, z = 15t.
because

Statement-2 : The vector $14\tilde{i}+2\tilde{j}+15\tilde{k}$ is parallel to the line of intersection of given planes.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

13. Consider three planes **[JEE 2008 (4M, –1M)]**

$$P_1 : x - y + z = 1$$
$$P_2 : x + y - z = -1$$
$$P_3 : x - 3y + 3z = 2$$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, and P_1 and P_2, respectively.
Statement-1 : At least two of the lines L_1, L_2 and L_3 are non-parallel.
because
Statement-2 : The three planes do not have a common point.
(A) Statement-1 is True, Statement-2 is True ; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT a correct explanation for Statement-1.
(C) Statement-1 is True, Statement-2 is False.
(D) Statement-1 is False, Statement-2 is True.

Paragraph for Question 14 to 16

Consider the lines $L_1 : \frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}$, $L_2 : \frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{3}$

14. The unit vector perpendicular to both L_1 and L_2 is :- [JEE 2008 (4M, –1M)]

(A) $\frac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$ (B) $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (C) $\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$ (D) $\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$

15. The shortest distance between L_1 and L_2 is :- [JEE 2008 (4M, –1M)]

(A) 0 (B) $\frac{17}{\sqrt{3}}$ (C) $\frac{41}{5\sqrt{3}}$ (D) $\frac{17}{5\sqrt{3}}$

16. The distance of the point (1, 1, 1) from the plane passing through the point (–1, –2, –1) and whose normal is perpendicular to both the lines L_1 and L_2 is :- [JEE 2008 (4M, –1M)]

(A) $\frac{2}{\sqrt{75}}$ (B) $\frac{7}{\sqrt{75}}$ (C) $\frac{13}{\sqrt{75}}$ (D) $\frac{23}{\sqrt{75}}$

17. A line with positive direction cosines passes through the point P (2, –1, 2) and makes equal angles with the coordinate axes. The line meets the plane 2x + y + z = 9 at point Q. The length of the line segment PQ equals [JEE 2009, 3M, –1M]

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

18. Let P(3, 2, 6) be a point in space and Q be a point on the line $\vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\mu(-3\hat{i}+\hat{j}+5\hat{k})$. Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane x – 4y + 3z = 1 is :- [JEE 2009, 3M, –1M]

(A) $\frac{1}{4}$ (B) $-\frac{1}{4}$ (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$

19. Match the statements/ expressions given in **Column I** with the values given in **Column II** [JEE 2009, 8M]

	Column-I		Column-II
(A)	The number of solutions of the equation $xe^{\sin x}-\cos x=0$ in the interval $\left(0,\frac{\pi}{2}\right)$	(P)	1
		(Q)	2
(B)	Value(s) of k for which the planes kx + 4y + z = 0, 4x + ky + 2z = 0 and 2x + 2y + z = 0 intersect in a straight line	(R)	3
(C)	Value(s) of k for which $\lvert x-1\rvert+\lvert x-2\rvert+\lvert x+1\rvert+\lvert x+2\rvert=4k$ has integer solution(s)	(S)	4
(D)	If y' = y +1 and y(0) =1, then value(s) of y (ln 2)	(T)	5

20. Equation of the plane containing the straight line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3}=\frac{y}{4}=\frac{z}{2}$ and $\frac{x}{4}=\frac{y}{2}=\frac{z}{3}$ is [JEE 10, 3M, –1M]

(A) x + 2y – 2z = 0 (B) 3x + 2y – 2z = 0
(C) x – 2y + z = 0 (D) 5x + 2y – 4z = 0

21. If the distance of the point P(1,–2,1) from the plane x + 2y – 2z = α, where $\alpha > 0$, is 5, then the foot of the perpendicular from P to the plane is- **[JEE 10, 5M, –2M]**

(A) $\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$ (B) $\left(\frac{4}{3},-\frac{4}{3},\frac{1}{3}\right)$ (C) $\left(\frac{1}{3},\frac{2}{3},\frac{10}{3}\right)$ (D) $\left(\frac{2}{3},-\frac{1}{3},\frac{5}{2}\right)$

22. If the distance between the plane Ax – 2y + z = d and the plane containing the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ is $\sqrt{6}$, then |d| is **[JEE 10, 3M]**

23. Match the statements in **Column-I** with the values in **Column-II**. **[JEE 10, 8M]**

	Column-I		Column-II
(A)	A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If length PQ = d, then d^2 is	(p)	–4
(B)	The values of x satisfying $\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}\left(\frac{3}{5}\right)$ are	(q)	0
(C)	Non-zero vectors $\vec{a},\vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b}=0$, $(\vec{b}-\vec{a}).(\vec{b}+\vec{c})=0$ and $2\left\|(\vec{b}+\vec{c})\right\|=\left\|(\vec{b}-\vec{a})\right\|$. If $\vec{a}=\mu\vec{b}+4\vec{c}$, then the possible values of μ are	(r)	4
(D)	Let f be the function on $[-\pi,\pi]$ given by f(0)= 9 and f(x) = $\sin\left(\frac{9x}{2}\right)\Big/\sin\left(\frac{x}{2}\right)$ for x $\neq$ 0. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi}f(x)dx$ is	(s)	5
		(t)	6

24. **(a)** The point P is the intersection of the straight line joining the points Q(2,3,5) and R(1,–1,4) with the plane 5x – 4y – z = 1. If S is the foot of the perpendicular drawn from the point T(2,1,4) to QR, then the length of the line segment PS is -

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

(b) The equation of a plane passing through the line of intersection of the planes x + 2y + 3z = 2 and x – y + z = 3 and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, –1) is

(A) 5x – 11y + z = 17 (B) $\sqrt{2}x+y=3\sqrt{2}-1$

(C) x + y + z = $\sqrt{3}$ (D) $x-\sqrt{2}y=1-\sqrt{2}$

(c) If the straight lines $\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is(are)

(A) y + 2z = –1 (B) y + z = –1 (C) y – z = –1 (D) y – 2z = –1

[JEE 2012, 3+3+4]

25. Perpendiculars are drawn from points on the line $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z}{3}$ to the plane x + y + z = 3. The feet of perpendiculars lie on the line **[JEE-Advanced 2013, 2]**

(A) $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$ (B) $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$ (C) $\frac{x}{4}=\frac{y-1}{3}=\frac{z-2}{-7}$ (D) $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

26. A line ℓ passing through the origin is perpendicular to the lines

$$\ell_1:(3+t)\tilde{i}+(-1+2t)\tilde{j}+(4+2t)\tilde{k},-\infty<t<\infty$$

$$\ell_2:(3+2s)\tilde{i}+(3+2s)\tilde{j}+(2+s)\tilde{k},-\infty<s<\infty$$

Then , the coordinate(s) of the point(s) on ℓ_2 at a distance of $\sqrt{17}$ from the point of intersection of ℓ and ℓ_1 is(are) - **[JEE-Advanced 2013, 4, (–1)]**

(A) $\left(\frac{7}{3},\frac{7}{3},\frac{5}{3}\right)$ (B) (–1,–1,0) (C) (1,1,1) (D) $\left(\frac{7}{9},\frac{7}{9},\frac{8}{9}\right)$

27. Two lines $L_1: x=5, \frac{y}{3-\alpha}=\frac{z}{-2}$ and $L_2: x=\alpha, \frac{y}{-1}=\frac{z}{2-\alpha}$ are coplanar. Then α can take value(s)

[JEE-Advanced 2013, 3, (–1)]

(A) 1 (B) 2 (C) 3 (D) 4

28. Consider the lines $L_1:\frac{x-1}{2}=\frac{y}{-1}=\frac{z+3}{1}, L_2:\frac{x-4}{1}=\frac{y+3}{1}=\frac{z+3}{2}$ and the planes $P_1: 7x+y+2z=3$, $P_2: 3x+5y-6z=4$. Let ax + by + cz = d be the equation of the plane passing through the point of intersection of lines L_1 and L_2 and perpendicular to planes P_1 and P_2.

Match List-I with List-II and select the correct answer using the code given below the lists.

	List-I		List-II
P.	a =	1.	13
Q.	b =	2.	–3
R.	c =	3.	1
S.	d =	4.	–2

Codes :

	P	Q	R	S
(A)	3	2	4	1
(B)	1	3	4	2
(C)	3	2	1	4
(D)	2	4	1	3

[JEE-Advanced 2013, 3, (–1)]

PREVIOUS YEARS QUESTIONS **ANSWER KEY** EXERCISE-05

1. (a) x + y – 2z = 3 ; (b) (6, 5, –2) **2.** B **4.** $\frac{9}{2}$ cubic unit **6.** D
7. 2x –y + z – 3 = 0, 62x + 29y + 19z – 105 = 0 **8.** D **9.** (A)→(s) ; (B)→(p) ; (C)→(q,r) ; (D)→(s)
10. (A)→(q) ; (B)→(s) ; (C)→(r) **11.** (A)→(r) ; (B)→(q) ; (C)→(p) ; (D)→(s) **12.** D **13.** D
14. B **15.** D **16.** C **17.** C **18.** A **19.** (A) → (P) ; (B) → (Q, S) ; (C) → (Q, R, S, T) ; (D) → (R)
20. C **21.** A **22.** 6 **23.** (A) → (t), (B) → (p,r), (C) → (q,s), (D) → (r) **24.** (a) A; (b) A; (c) B,C
25. D **26.** B,D **27.** A,D **28.** A