# RELATIONS

## INTRODUCTION :

Let A and B be two sets. Then a relation R from A to B is a subset of A   B.

thus, R is a relation from A to B $\Leftrightarrow$ R $\subseteq$ A   B.

**Ex.** If A = {1, 2, 3} and B = {a, b, c}, then R = {(1, b), (2, c), (1, a), (3, a)} being a subset of A   B, is a relation from A to B. Here (1, b), (2, c), (1, a) and (3, a) $\in$ R, so we write 1 Rb, 2Rc, 1Ra and 3Ra. But (2, b) $\notin$ R, so we write $2 \not R\ b$

**Total Number of Realtions :** Let A and B be two non-empty finite sets consisting of m and n elements respectively. Then A   B consists of mn ordered pairs. So, total number of subsets of A   B is $2^{mn}$.

**Domain and Range of a relation :** Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called to domain of R, while the set of all second components or coordinates of the ordered pairs in R is called the range of R.

Thus, Dom (R) = {a : (a, b) $\in$ R}

and, Range (R) = {b : (a, b) $\in$ R}

It is evident from the definition that the domain of a relation from A to B is a subset of A and its range is a subset of B.

e.g. Let A = {1, 3, 5, 7} and B = {2, 4, 6, 8} be two sets and let R be a relation from A to B defined by the phrase "(x, y) $\in$ R $\Leftrightarrow$ x > y". Under this relation R, we have

3R2, 5R2, 5R4, 7R2, 7R4 and 7R6

i.e. R = {(3, 2), (5, 2), (5, 4), (7, 2), (7, 4), (7, 6)}

$\therefore$ Dom (R) = {3, 5, 7} and Range (R) = {2, 4, 6}

**Inverse Relation :** Let A, B be two sets and let R be a relation from a set A to a set B. Then the inverse of R, denoted by $R^{-1}$, is a relation from B to A and is defined by

$$R^{-1} = \{(b, a) : (a, b) \in R\}$$

Clearly, $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$

Also, Dom(R) = Range($R^{-1}$) and Range (R) = Dom ($R^{-1}$)

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***Illustration 1 :***

Let A be the set of first ten natural numbers and let R be a relation on A defined by (x, y) $\in$ R $\Leftrightarrow$ x + 2y = 10, i.e. R = {(x, y) : x $\in$ A, y $\in$ A and x + 2y = 10}. Express R and $R^{-1}$ as sets of ordered pairs. Determine also (i) domain of R and $R^{-1}$ (ii) range of R and $R^{-1}$

***Solution :***

We have $(x, y) \in R \Leftrightarrow x + 2y = 10 \Leftrightarrow y = \dfrac{10-x}{2}$, x, y $\in$ A

where A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Now, $x = 1 \Rightarrow y = \dfrac{10-1}{2} = \dfrac{9}{2} \notin A.$

This shows that 1 is not related to any element in A. Similarly we can observe. that 3, 5, 7, 9 and 10 are not related to any element of A under the defined relation

Further we find that :

For $x = 2,\ y = \dfrac{10-2}{2} = 4 \in A$ $\qquad \therefore\ (2, 4) \in R$

For $x = 4,\ y = \dfrac{10-4}{2} = 3 \in A$ $\qquad \therefore\ (4, 3) \in R$

For $x = 6,\ y = \dfrac{10-6}{2} = 2 \in A$ $\qquad \therefore\ (6, 2) \in R$

For $x = 8,\ y = \dfrac{10-8}{2} = 1 \in A$ $\quad\therefore\ (8, 1) \in R$

Thus, $R = \{(2, 4), (4, 3), (6, 2), (8, 1)\}$

$\Rightarrow\quad R^{-1} = \{(4, 2), (3, 4), (2, 6), (1, 8)\}$

Clearly, $Dom(R) = \{2, 4, 6, 8\} = Range(R^{-1})$

and, $Range(R) = \{4, 3, 2, 1\} = Dom(R^{-1})$

**Do yourself - 1 :**

**(i)** If $A = \{2, 4, 6, 9\}$ and $B = \{4, 6, 18, 27, 54\}$, $a \in A$, $b \in B$, find the set of ordered pairs such that 'a' is a factor of 'b' and a < b.

**(ii)** Find the domain and range of the relation R given by $R = \{(x, y) : y = x + \dfrac{6}{x}$, where $x, y \in N$ and $x < 6\}$

## TYPES OF RELATIONS :

In this section we intend to define various types of relations on a given set A.

**Void Relation** : Let A be a set. Then $\phi \subseteq A \quad A$ and so it is a relation on A. This relation is called the void or empty relation on A.

**Universal Relation** : Let A be a set. Then $A \quad A \subseteq A \quad A$ and so it is a relation on A. This relation is called the universal relation on A.

**Identity Relation** : Let A be a set. Then the relation $I_A = \{(a, a) : a \in A\}$ on A is called the identity relation on A.

In other words, a relation $I_A$ on A is called the identity relation if every element of A is related to itself only.

**e.g.** The relation $I_A = \{(1, 1), (2, 2), (3, 3)\}$ is the identity relation on set $A = \{1, 2, 3\}$. But relations $R_1 = \{(1, 1), (2, 2)\}$ and $R_2 = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$ are not identity relations on A, because $(3, 3) \notin R_1$ and in $R_2$ element 1 is related to elements 1 and 3.

**Reflexive Relation** : A relation R on a set A is said to be reflexive if every element of A is related to itself.

Thus, R on a set A is not reflexive if there exists an element $A \in A$ such that $(a, a) \notin R$.

**e.g.** Let $A = \{1, 2, 3\}$ be a set. Then $R = \{(1, 1), (2, 2), (3, 3), (1, 3), (2, 1)\}$ is a reflexive relation on A. But $R_1 = \{(1, 1), (3, 3), (2, 1), (3, 2)\}$ is not a reflexive relation on A, because $2 \in A$ but $(2, 2) \notin R_1$.

**Note** : Every Identity relation is reflexive but every reflexive ralation is not identity.

**Symmetric Relation** : A relation R on a set A is said to be a symmetric relation iff

$(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$

i.e. $a R b \Rightarrow bRa$ for all $a, b, \in A$.

**e.g.** Let L be the set of all lines in a plane and let R be a relation defined on L by the rule $(x, y) \in R \Leftrightarrow x$ is perpendicular to y. Then R is a symmetric relation on L, because $L_1 \perp L_2 \Rightarrow L_2 \perp L_1$

i.e. $(L_1, L_2) \in R \Rightarrow (L_2, L_1) \in R$.

**e.g.** Let $A = \{1, 2, 3, 4\}$ and Let $R_1$ and $R_2$ be realtion on A given by $R_1 = \{(1, 3), (1, 4), (3, 1), (2, 2), (4, 1)\}$ and $R_2 = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$. Clearly, $R_1$ is a symmetric relation on A. However, $R_2$ is not so, because $(1, 3) \in R_2$ but $(3, 1) \notin R_2$

**Transitive Relation** : Let A be any set. A relation R on A is said to be a transitive relation iff

$(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$

i.e. $a R b$ and $b R c \Rightarrow a R c$ for all $a, b, c \in A$

**e,g.** On the set N of natural numbers, the relation R defined by $x \ R \ y \Rightarrow x$ is less than y is transitive, because for any $x, y, z \in N$

$x < y$ and $y < z \Rightarrow x < z \Rightarrow x \ R \ y$ and $y \ R \ z \Rightarrow x \ R \ z$

**e.g.** Let L be the set of all straight lines in a plane. Then the realtion 'is parallel to' on L is a transitive relation, because from any $\ell_1, \ell_2, \ell_3 \in L$.

$$\ell_1 \parallel \ell_2 \text{ and } \ell_2 \parallel \ell_3 \Rightarrow \ell_1 \parallel \ell_3$$

**Antisymmetric Relation :** Let A be any set. A relation R on set A is said to be an antisymmetric relation iff

$(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$ for all $a, b \in A$

**e.g.** Let R be a relation on the set N of natural numbers defined by

$$x \ R \ y \Leftrightarrow \text{'x divides y' for all } x, y \in N$$

This relation is an antisymmetric relation on N. Since for any two numbers $a, b \in N$

$$a|b \text{ and } b|a \Rightarrow a = b \quad \text{i.e. } a \ R \ b \text{ and } b \ R \ a \Rightarrow a = b$$

**Equivalence Relation :** A relation R on a set A is said to be an equivalence relation on A iff

(i) it is reflexive i.e. $(a, a) \in R$ for all $a \in A$

(ii) it is symmetric i.e. $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$

(iii) it is transitive i.e. $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$.

**e.g.** Let R be a relation on the set of all lines in a plane defined by $(\ell_1, \ell_2) \in R \Leftrightarrow$ line $\ell_1$ is parallel to line $\ell_2$. R is an equivalence relation.

**Note :** It is not neccessary that every relation which is symmetric and transitive is also reflexive.

## PARTIAL ORDER RELATION :

A relation R on set A is said to be an partial order relation on A if

(i) R is reflexive i.e. $(a, a) \in R, \ \forall \ a \in A$

(ii) R is antisymmetric i.e. $(a, b) \in R \Rightarrow (b, a) \in R$ only Possible When $a = b \ \ \forall \ a, b \in A$

(iii) R is transitive i.e. $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \ \forall \ a, b, c \in R$

e.g. R be a relation on the set N of natural numbers defined by

$x \ R \ y \Rightarrow$ 'x divides y' $\forall \ x, y \in N$ then R is a partial order Relation.

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*Illustration 2 :*

Three relation $R_1$, $R_2$ and $R_3$ are defined on set $A = \{a, b, c\}$ as follows :

(i) $R_1 \{(a, a), (a, b), (a, c), (b, b), (b, c), (c, a), (c, b), (c, c)\}$ (ii) $R_2 \{(a, b), (b, a), (a, c), (c, a)\}$

(iii) $R_3\{(a, b), (b, c), (c, a)\}$

Find whether each of $R_1$, $R_2$ and $R_3$ is reflexive, symmetric and transitive.

*Solution :*

(i) Reflexive : Clearly, $(a, a), (b, b), (c, c) \in R_1$. So, $R_1$ is reflexive on A.

Symmetric : We observe that $(a, b) \in R_1$ but $(b, a) \notin R_1$. So, $R_1$ is not symmetric on A.

Transitive : We find that $(b, c) \in R_1$ and $(c, a) \in R_1$ but $(b, a) \notin R_1$. So, R is not transitive on A.

(ii) Reflexive : Since $(a, a), (b, b)$ and $(c, c)$ are not in $R_2$. So, it is not a reflexive realtion on A.

Symmetric : We find that the ordered pairs obtained by interchanging the components of ordered pairs in $R_2$ are also in $R_2$. So, $R_2$ is a symmetric relation on A.

Transitive : Clearly $(c, a) \in R_2$ and $(a, b) \in R_2$ but $(c, b) \notin R_2$. So, it is not a transitive relation on $R_2$.

(iii) Reflexive : Since non of $(a, a), (b, b)$ and $(c, c)$ is an element of $R_3$. So, $R_3$ is not reflexive on A.

Symmetric : Clearly, $(b, c) \in R_3$ but $(c, b) \notin R_3$ . so, is not symmetric on A.

Transitive : Clearly, $(b, c) \in R_3$ and $(c, a) \in R_3$ but $(b, a) \notin R_3$. So, $R_3$ is not transitive on A.

*Illustration 3 :*

Prove that the relation R on the set Z of all integers defined by

$$(x, y) \in R \Leftrightarrow x - y \text{ is divisible by } n$$

is an equivalence relation on Z.

*Solution :*

We observe the following properties

Reflexivity : For any $a \in N$, we have

$$a - a = 0 = 0 \quad n \Rightarrow a - a \text{ is divisible by } n \Rightarrow (a, a) \in R$$

Thus, $(a, a) \in R$ for all $a \in Z$

So, R is reflexive on Z

symmetry : Let $(a, b) \in R$. Then,

$(a, b) \in R \Rightarrow (a - b)$ is divisible by n

$\Rightarrow a - b = np$ for some $p \in Z$

$\Rightarrow b - a = n(-p)$

$\Rightarrow b - a$ is divisible by n $\quad [\because p \in Z \Rightarrow -p \in Z]$

$\Rightarrow (b, a) \in R$

Thus, $(a , b) \in R \Rightarrow (b, a) \in R$ for all $a, b, \in Z$

So, R is symmetric on Z.

Transitivity : Let $a, b, c \in Z$ such that $(a, b) \in R$ and $(b, c) \in R$. Then,

$(a , b) \in R \Rightarrow (a - b)$ is divisible by n

$\Rightarrow a - b = np$ for some $p \in Z$

$(b, c) \in R \Rightarrow (b - c)$ is divisible by n

$\Rightarrow b - c = nq$ for some $q \in Z$

$\therefore (a, b) \in R$ and $(b, c) \in R$

$\Rightarrow a - b = np$ and $b - c - nq$

$\Rightarrow (a - b) + (b - c) = np + nq$

$\Rightarrow a - c = n(p + q)$

$\Rightarrow a - c$ is divisible by n $\quad [\because p, q \in Z \Rightarrow p + q = Z]$

$\Rightarrow (a, c) \in R$

thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in Z$. so, R is transitive realtion in Z.

*Illustration 4 :*

Show that the relation is congruent to' on the set of all triangles in a plane is an equivalence relation.

*Solution :*

Let S be the set of all triangles in a plane and let R be the relation on S defined by $(\Delta_1, \Delta_2) \in R \Leftrightarrow$ triangle $\Delta_1$ is congruent to triangle $\Delta_2$. We observe the following properties.

**Reflexivity** : For each triangle $\Delta \in S$, we have

$\Delta \cong \Delta \Rightarrow (\Delta, \Delta) \in R$ for all $\Delta \in S \Rightarrow R$ is reflexive on S

**Symmetry** : Let $\Delta_1, \Delta_2 \in S$ such that $(\Delta_1, \Delta_2) \in R$. Then, $(\Delta_1, \Delta_2) \in R \Rightarrow \Delta_1 \cong \Delta_2 \Rightarrow \Delta_2 \cong \Delta_1 \Rightarrow (\Delta_2, \Delta_1) \in R$

So, R is symmetric on S

**Transitivity :** Let $\Delta_1, \Delta_2, \Delta_3 \in S$ such that $(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R$. Then,

$(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R \Rightarrow \Delta_1 \cong \Delta_2$ and $\Delta_2 \cong \Delta_3 \Rightarrow \Delta_1 \cong \Delta_3 \Rightarrow (\Delta_1, \Delta_3) \in R$

So, R is transitive on S.

Hence, R being reflexive, symmetric and transitive, is an equivalence relation on S.

**Do yourself - 2 :**

(i) Show that the relation R defined on the set N of natural number by $xRy \Leftrightarrow 2x^2 - 3xy + y^2 = 0$, i.e. by $R = \{(x, y); x, y \in N \text{ and } 2x^2 - 3xy + y^2 = 0\}$ is not symmetric but it is reflexive.

## ANSWERS FOR DO YOURSELF

1. (i) $\{(2, 4), (2, 6), (2, 18), (2, 54), (6, 18), (6, 54), (9, 18), (9, 27), (9, 54)\}$

(ii) Domain of R = {1, 2, 3}, Range of R = {7, 5}

## CHECK YOUR GRASP — RELATIONS — EXERCISE-I

**1.** If R is a relation from a finite set A having m elements to a finite set B having n elements, then the number of relations from A to B is-
(1) $2^{mn}$ (2) $2^{mn}-1$ (3) $2mn$ (4) $m^n$

**2.** In the set A = {1, 2, 3, 4, 5}, a relation R is defined by R = {(x, y) | x, y ∈ A and x < y}. Then R is-
(1) Reflexive (2) Symmetric
(3) Transitive (4) None of these

**3.** For real numbers x and y, we write $x\ R\ y \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then the relation R is-
(1) Reflexive (2) Symmetric
(3) Transitive (4) none of these

**4.** Let X = {1, 2, 3, 4} and Y = {1, 3, 5, 7, 9}. Which of the following is relations from X to Y-
(1) $R_1$ = {(x, y) | y = 2 + x, x ∈ X, y ∈ Y}
(2) $R_2$ = {(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)}
(3) $R_3$ = {(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)}
(4) $R_4$ = {(1, 3), (2, 5), (2, 4), (7, 9)}

**5.** Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha\ R\ \beta \Leftrightarrow \alpha \perp \beta$, $\alpha, \beta \in L$. Then R is-
(1) Reflexive (2) Symmetric
(3) Transitive (4) none of these

**6.** Let R be a relation defined in the set of real numbers by $a\ R\ b \Leftrightarrow 1 + ab > 0$. Then R is-
(1) Equivalence relation (2) Transitive
(3) Symmetric (4) Anti-symmetric

**7.** Which one of the following relations on R is equivalence relation-
(1) $x\ R_1 y \Leftrightarrow |x| = |y|$ (2) $x\ R_2 y \Leftrightarrow x \geq y$
(3) $x\ R_3 y \Leftrightarrow x \mid y$ (4) $x\ R_4 y \Leftrightarrow x < y$

**8.** Two points P and Q in a plane are related if OP = OQ, where O is a fixed point. This relation is-
(1) Reflexive but symmetric
(2) Symmetric but not transitive
(3) An equivalence relation
(4) none of these

**9.** The relation R defined in A = {1, 2, 3} by a R b if $|a^2 - b^2| \leq 5$. Which of the following is false-
(1)R ={(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (2, 3), (3, 2)
(2) $R^{-1} = R$
(3) Domain of R = {1, 2, 3}
(4) Range of R = {5}

**10.** Let a relation R is the set N of natural numbers be defined as (x, y) ∈ R if and only if $x^2 - 4xy + 3y^2 = 0$ for all x, y ∈ N. The relation R is-
(1) Reflexive
(2) Symmetric
(3) Transitive
(4) An equivalence relation

**11.** Let A = {2, 3, 4, 5} and let R = {(2, 2), (3, 3), (4, 4), (5, 5), (2, 3), (3, 2), (3, 5), (5, 3)} be a relation in A. Then R is-
(1) Reflexive and transitive
(2) Reflexive and symmetric
(3) Reflexive and antisymmetric
(4) none of these

**12.** If A = {2, 3} and B = {1, 2}, then A  B is equal to-
(1) {(2, 1), (2, 2), (3, 1), (3, 2)}
(2) {(1, 2), (1, 3), (2, 2), (2, 3)}
(3) {(2, 1), (3, 2)}
(4) {(1, 2), (2, 3)}

**13.** Let R be a relation over the set N  N and it is defined by (a, b) R (c, d) ⇒ a + d = b + c. Then R is-
(1) Reflexive only
(2) Symmetric only
(3) Transitive only
(4) An equivalence relation

**14.** Let N denote the set of all natural numbers and R be the relation on N  N defined by (a, b) R (c, d) if ad (b + c) = bc(a + d), then R is-
(1) Symmetric only
(2) Reflexive only
(3) Transitive only
(4) An equivalence relation

**15.** If A = {1, 2, 3}, B = {1, 4, 6, 9} and R is a relation from A to B defined by 'x is greater than y'. Then range of R is-
(1) {1, 4, 6, 9} (2) {4, 6, 9}
(3) {1} (4) none of these

**16.** Let L be the set of all straight lines in the Euclidean plane. Two lines $\ell_1$ and $\ell_2$ are said to be related by the relation R if $\ell_1$ is parallel to $\ell_2$. Then the relation R is-
(1) Reflexive (2) Symmetric
(3) Transitive (4) Equivalence

**17.** A and B are two sets having 3 and 4 elements respectively and having 2 elements in common. The number of relations which can be defined from A to B is-

(1) $2^5$ (2) $2^{10} - 1$

(3) $2^{12} - 1$ (4) none of these

**18.** For n, m $\in$ N, n|m means that n is a factor of m, the relation | is-

(1) reflexive and symmetric

(2) transitive and symmetric

(3) reflexive, transitive and symmetric

(4) reflexive, transitive and not symmetric

**19.** Let R = $\{(x, y) : x, y \in A, x + y = 5\}$ where A = $\{1, 2, 3, 4, 5\}$ then

(1) R is not reflexive, symmetric and not transitive

(2) R is an equivalence relation

(3) R is reflexive, symmetric but not transitive

(4) R is not reflexive, not symmetric but transitive

**20.** Let R be a relation on a set A such that $R = R^{-1}$ then R is-

(1) reflexive

(2) symmetric

(3) transitive

(4) none of these

**21.** Let x, y $\in$ I and suppose that a relation R on I is defined by x R y if and only if $x \le y$ then

(1) R is partial order ralation

(2) R is an equivalence relation

(3) R is reflexive and symmetric

(4) R is symmetric and transitive

**22.** Let R be a relation from a set A to a set B, then-

(1) $R = A \cup B$ (2) $R = A \cap B$

(3) $R \subseteq A \quad B$ (4) $R \subseteq B \quad A$

**23.** Given the relation R = = $\{(1, 2), (2, 3)\}$ on the set A = $\{1, 2, 3\}$, the minimum number of ordered pairs which when added to R make it an equivalence relation is-

(1) 5 (2) 6 (3) 7 (4) 8

**24.** Let P = $\{(x, y) \mid x^2 + y^2 = 1, x, y \in R\}$ Then P is-

(1) reflexive (2) symmetric

(3) transitive (4) anti-symmetric

**25.** Let X be a family of sets and R be a relation on X defined by 'A is disjoint from B'. Then R is-

(1) reflexive (2) symmetric

(3) anti-symmetric (4) transitive

**26.** In order that a relation R defined in a non-empty set A is an equivalence relation, it is sufficient that R

(1) is reflexive

(2) is symmetric

(3) is transitive

(4) possesses all the above three properties

**27.** If R be a relation '<' from A = $\{1, 2, 3, 4\}$ to B = $\{1, 3, 5\}$ i.e. $(a, b) \in R$ iff $a < b$, then $ROR^{-1}$ is-

(1) $\{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\}$

(2) $\{(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)\}$

(3) $\{(3, 3), (3, 5), (5, 3), (5, 5)\}$

(4) $\{(3, 3), (3, 4), (4, 5)\}$

**28.** If R is an equivalence relation in a set A, then $R^{-1}$ is-

(1) reflexive but not symmetric

(2) symmetric but not transitive

(3) an equivalence relation

(4) none of these

**29.** Let R and S be two equivalence relations in a set A. Then-

(1) $R \cup S$ is an equivalence relation in A

(2) $R \cap S$ is an equivalence relation in A

(3) R – S is an equivalence relation in A

(4) none of these

**30.** Let A = $\{p, q, r\}$. Which of the following is an equivalence relation in A ?

(1) $R_1 = \{(p, q), (q, r), (p, r), (p, p)\}$

(2) $R_2 = \{(r, q) (r, p), (r, r), (q, q)\}$

(3) $R_3 = \{(p, p), (q, q), (r, r), (p, q)\}$

(4) none of these

## ANSWER KEY

| Que. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | 1 | 3 | 1 | 1 | 2 | 3 | 1 | 3 | 4 | 1 | 2 | 1 | 4 | 4 | 3 |
| Que. | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| Ans. | 4 | 4 | 4 | 1 | 2 | 1 | 3 | 3 | 2 | 2 | 4 | 3 | 3 | 2 | 4 |

## PREVIOUS YEAR QUESTIONS — RELATIONS — EXERCISE-II

**1.** Let R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} be a releation on the set A = {1, 2, 3, 4}. The relation R is- **[AIEEE - 2004]**

(1) transitive (2) not symmetric

(3) reflexive (4) a function

**2.** Let R = {(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)} be relation on the set A = {3, 6, 9, 12). The relation is- **[AIEEE - 2005]**

(1) rflexive and transitive only

(2) reflexive only

(3) an equilvalence relation

(4) reflexive and symmetric only

**3.** Let W denote the words in the English dictionary. Define the relation R by : R = {(x, y) ∈ W W| the words x and y have at least one letter in common}. Then R is- **[AIEEE - 2006]**

(1) reflexive, symmetric and not transitive

(2) reflexive, symmetric and transitive

(3) reflexive, not symmetric and transtive

(4) not reflexive, symmetric and transitive

**4.** Consider the following relations :-

R = {(x, y) | x, y are real numbers and x = wy for some rational number w} ;

S = $\{(\frac{m}{n}, \frac{p}{q})$ | m, n, p and q are integers such that n, q $\neq$ 0 and qm = pn}.

Then : **[AIEEE - 2010]**

(1) R is an equivalence relation but S is not an equivalence relation

(2) Neither R nor S is an equivalence relation

(3) S is an equivalence relation but R is not an equivalence relation

(4) R and S both are equivalence relations

**5.** Let R be the set of real numbers.

**Statement-1:**
A = {(x, y) ∈ R R : y − x is an integer} is an equivalence relation on R. **[AIEEE - 2011]**

**Statement-2:**
B = {(x, y)∈ R R : x = αy for some rational number α} is an equivalence relation on R.

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

**6.** Consider the following relation R on the set of real square matirces of order 3.

R={(A, B)|A=$P^{-1}$ BP for some invertible matrix P}.

**Statement - 1:**
R is an equivalence relation.

**Statement - 2:**
For any two invertible 3 3 martices M and N, $(MN)^{-1} = N^{-1}M^{-1}$ **[AIEEE - 2011]**

(1) Statement-1 is false, statement-2 is true.

(2) Statement-1 is true, statement-2 is true; Statement-2 is correct explanation for statement-1.

(3) Statement-1 is true, statement-2 is true; Statement-2 is not a correct explanation for statement-1.

(4) Statement-1 is true, statement-2 is false.

## ANSWER KEY

| Que. | 1 | 2 | 3 | 4 | 5 | 6 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Ans. | 2 | 1 | 1 | 3 | 1 | 1 | | | | | | | | | |