SEQUENCE & SERIES

1. DEFINITION :

Sequence :

A succession of terms a_1, a_2, a_3, a_4........ formed according to some rule or law.

Examples are : $1, 4, 9, 16, 25$

$-1, 1, -1, 1, \ldots\ldots$

$\frac{x}{1!}, \frac{x^2}{2!}, \frac{x^3}{3!}, \frac{x^4}{4!}, \ldots\ldots$

A finite sequence has a finite (i.e. limited) number of terms, as in the first example above. An infinite sequence has an unlimited number of terms, i.e. there is no last term, as in the second and third examples.

Series :

The indicated sum of the terms of a sequence. In the case of a finite sequence $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ the corresponding series is $a_1 + a_2 + a_3 + \ldots\ldots + a_n = \sum_{k=1}^{n} a_k$. This series has a finite or limited number of terms and is called a finite series.

2. ARITHMETIC PROGRESSION (A.P.) :

A.P. is a sequence whose terms differ by a fixed number. This fixed number is called the common difference. If a is the first term & d the common difference, then A.P. can be written as

$a,\ a + d,\ a + 2d,\ \ldots\ldots,\ a + (n - 1)\,d,\ \ldots\ldots$

(a) n^{th} term of AP $T_n = a + (n - 1)d$, where $d = t_n - t_{n-1}$

(b) The sum of the first n terms : $S_n = \frac{n}{2}[a + \ell] = \frac{n}{2}[2a + (n-1)d]$

where ℓ is the last term.

Note :

(i) n^{th} term of an A.P. is of the form $An + B$ i.e. a linear expression in 'n', in such a case the coefficient of n is the common difference of the A.P. i.e. A.

(ii) Sum of first 'n' terms of an A.P. is of the form $An^2 + Bn$ i.e. a quadratic expression in 'n', in such case the common difference is twice the coefficient of n^2. i.e. 2A

(iii) Also n^{th} term $T_n = S_n - S_{n-1}$

Illustration 1 :

If $(x + 1)$, $3x$ and $(4x + 2)$ are first three terms of an A.P. then its 5^{th} term is -

(A) 14 (B) 19 (C) 24 (D) 28

Solution :

$(x + 1), 3x, (4x + 2)$ are in AP

$\Rightarrow \quad 3x - (x + 1) = (4x + 2) - 3x \quad \Rightarrow \quad x = 3$

$\therefore \quad a = 4,\ d = 9 - 4 = 5 \quad \Rightarrow \quad T_5 = 4 + 4(5) = 24$ **Ans. (C)**

Illustration 2 :

The sum of first four terms of an A.P. is 56 and the sum of it's last four terms is 112. If its first term is 11 then find the number of terms in the A.P.

Solution :

$a + a + d + a + 2d + a + 3d = 56$

$4a + 6d = 56$

$44 + 6d = 56$ (as $a = 11$)

$6d = 12$ hence $d = 2$

Now sum of last four terms.

$a + (n - 1)d + a + (n - 2)d + a + (n - 3)d + a + (n - 4)d = 112$

$\Rightarrow \quad 4a + (4n - 10)d = 112 \qquad \Rightarrow \quad 44 + (4n - 10)2 = 112$

$\Rightarrow \quad 4n - 10 = 34$

$\Rightarrow \quad n = 11$ **Ans.**

Illustration 3 :

The sum of first n terms of two A.Ps. are in ratio $\frac{7n+1}{4n+27}$. Find the ratio of their 11^{th} terms.

Solution :

Let a_1 and a_2 be the first terms and d_1 and d_2 be the common differences of two A.P.s respectively then

$$\frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{7n+1}{4n+27} \Rightarrow \frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{7n+1}{4n+27}$$

For ratio of 11^{th} terms

$$\frac{n-1}{2} = 10 \Rightarrow n = 21$$

so ratio of 11^{th} terms is $\frac{7(21)+1}{4(21)+27} = \frac{148}{111} = \frac{4}{3}$ **Ans.**

Do yourself - 1 :

(i) Write down the sequence whose n^{th} terms is : (a) $\frac{2^n}{n}$ (b) $\frac{3+(-1)^n}{3^n}$

(ii) For an A.P, show that $t_m + t_{2n+m} = 2t_{m+n}$

(iii) If the sum of p terms of an A.P. is q and the sum of its q terms is p, then find the sum of its (p + q) term.

3. PROPERTIES OF A.P. :

(a) If each term of an A.P. is increased, decreased, multiplied or divided by the same nonzero number, then the resulting sequence is also an A.P.

(b) Three numbers in A.P. : $a - d, a, a + d$

Four numbers in A.P. : $a - 3d, a - d, a + d, a + 3d$

Five numbers in A.P. : $a - 2d, a - d, a, a + d, a + 2d$

Six numbers in A.P. : $a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$ etc.

(c) The common difference can be zero, positive or negative.

(d) k^{th} term from the last = $(n - k + 1)^{th}$ term from the beginning.

(e) The sum of the two terms of an AP equidistant from the beginning & end is constant and equal to the sum of first & last terms. $\Rightarrow T_k + T_{n-k+1} = \text{constant} = a + \ell$.

(f) Any term of an AP (except the first) is equal to half the sum of terms which are equidistant from it.

$a_n = (1/2)(a_{n-k} + a_{n+k}), k < n$

For $k = 1,\ a_n = (1/2)(a_{n-1} + a_{n+1})$; For $k = 2,\ a_n = (1/2)(a_{n-2} + a_{n+2})$ and so on.

(g) If a, b, c are in AP, then $2b = a + c$.

Illustration 4 :

Four numbers are in A.P. If their sum is 20 and the sum of their squares is 120, then the middle terms are -

(A) 2, 4 (B) 4, 6 (C) 6, 8 (D) 8, 10

Solution :

Let the numbers are $a - 3d,\ a - d,\ a + d,\ a + 3d$

given, $a - 3d + a - d + a + d + a + 3d = 20 \Rightarrow 4a = 20 \Rightarrow a = 5$

and $(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 120 \Rightarrow 4a^2 + 20d^2 = 120$

$\Rightarrow 4\ \ 5^2 + 20d^2 = 120 \Rightarrow d^2 = 1 \Rightarrow d = \pm 1$

Hence numbers are 2, 4, 6, 8 **Ans. (B)**

Illustration 5 :

Four different integers form an increasing A.P. One of these numbers is equal to the sum of the squares of the other three numbers. Find the numbers.

Solution :

Let the numbers be $a - d,\ a,\ a + d,\ a + 2d$, where $a, d \in I,\ d > 0$

according to the question; $(a - d)^2 + a^2 + (a + d)^2 = a + 2d$

i.e., $2d^2 - 2d + 3a^2 - a = 0$

$\therefore \quad d = \frac{1}{2}[1 \pm \sqrt{(1 + 2a - 6a^2)}\,]$

Since, d is positive integer,

$\Rightarrow 1 + 2a - 6a^2 > 0 \Rightarrow a^2 - \frac{a}{3} - \frac{1}{6} < 0 \Rightarrow \left(a - \frac{1-\sqrt{7}}{6}\right)\left(a - \frac{1+\sqrt{7}}{6}\right) < 0$

$\therefore \quad \left(\frac{1-\sqrt{7}}{6}\right) < a < \left(\frac{1+\sqrt{7}}{6}\right)$

$\because \quad a \in I$

$\therefore \quad a = 0$

then $d = \frac{1}{2}[1 \pm 1] = 1$ or 0. Since, $d > 0$ $\therefore\ d = 1$

Hence, the numbers are –1, 0, 1, 2

Illustration 6 :

If $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ are in A.P. where $a_i > 0$ for all i, show that :

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots\ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = \frac{(n-1)}{\sqrt{a_1}+\sqrt{a_n}}$$

Solution :

$$\text{L.H.S.} = \frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots\ldots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$$

$$= \frac{1}{\sqrt{a_2}+\sqrt{a_1}} + \frac{1}{\sqrt{a_3}+\sqrt{a_2}} + \ldots\ldots + \frac{1}{\sqrt{a_n}+\sqrt{a_{n-1}}}$$

$$= \frac{\sqrt{a_2}-\sqrt{a_1}}{(a_2 - a_1)} + \frac{\sqrt{a_3}-\sqrt{a_2}}{(a_3 - a_2)} + \ldots\ldots + \frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{a_n - a_{n-1}}$$

Let 'd' is the common difference of this A.P.

then $a_2 - a_1 = a_3 - a_2 = ... = a_n - a_{n-1} = d$

Now L.H.S.

$$= \frac{1}{d}\left\{\sqrt{a_2} - \sqrt{a_1} + \sqrt{a_3} - \sqrt{a_2} + ... + \sqrt{a_{n-1}} - \sqrt{a_{n-2}} + \sqrt{a_n} - \sqrt{a_{n-1}}\right\} = \frac{1}{d}\left\{\sqrt{a_n} - \sqrt{a_1}\right\}$$

$$= \frac{a_n - a_1}{d\left(\sqrt{a_n} + \sqrt{a_1}\right)} = \frac{a_1 + (n-1)d - a_1}{d\left(\sqrt{a_n} + \sqrt{a_1}\right)} = \frac{1}{d}\frac{(n-1)d}{(\sqrt{a_n} + \sqrt{a_1})} = \frac{n-1}{\sqrt{a_n} + \sqrt{a_1}} = \text{R.H.S.}$$

Do yourself - 2 :

(i) Find the sum of first 24 terms of the A.P. a_1, a_2, a_3......, if it is know that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$.

(ii) Find the number of terms common to the two A.P.'s 3, 7, 11, ... 407 and 2, 9, 16, ..., 709

4. GEOMETRIC PROGRESSION (G.P.) :

G.P. is a sequence of non zero numbers each of the succeeding term is equal to the preceeding term multiplied by a constant. Thus in a GP the ratio of successive terms is constant. This constant factor is called the COMMON RATIO of the sequence & is obtained by dividing any term by the immediately previous term. Therefore a, ar, ar^2, ar^3, ar^4 , ... is a GP with 'a' as the first term & 'r' as common ratio.

(a) n^{th} term ; $T_n = a\, r^{n-1}$

(b) Sum of the first n terms; $S_n = \frac{a(r^n - 1)}{r-1}$, if $r \neq 1$

(c) Sum of infinite G.P. , $S_\infty = \frac{a}{1-r}$; $0 < |r| < 1$

5. PROPERTIES OF GP :

(a) If each term of a G.P. be multiplied or divided by the some non-zero quantity, then the resulting sequence is also a G.P.

(b) Three consecutive terms of a GP : a/r, a, ar ;
Four consecutive terms of a GP : a/r^3, a/r, ar, ar^3 & so on.

(c) If a, b, c are in G.P. then $b^2 = ac$.

(d) If in a G.P, the product of two terms which are equidistant from the first and the last term, is constant and is equal to the product of first and last term. $\Rightarrow T_k . T_{n-k+1}$ = constant = $a.\ell$

(e) If each term of a G.P. be raised to the same power, then resulting sequence is also a G.P.

(f) In a G.P., $T_r^2 = T_{r-k} . T_{r+k}$, $k < r$, $r \neq 1$

(g) If the terms of a given G.P. are chosen at regular intervals, then the new sequence is also a G.P.

(h) If a_1, a_2, a_3.....a_n is a G.P. of positive terms, then $\log a_1, \log a_2$,.....$\log a_n$ is an A.P. and vice-versa.

(i) If a_1, a_2, a_3..... and b_1, b_2, b_3..... are two G.P.'s then a_1b_1, a_2b_2, a_3b_3..... & $\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}$.......... is also in G.P.

Illustration 7 :

If a, b, c, d and p are distinct real numbers such that

$\left(a^2+b^2+c^2\right)p^2-2p\left(ab+bc+cd\right)+\left(b^2+c^2+d^2\right)\leq 0$ then a, b, c, d are in

(A) A.P. (B) G.P. (C) H.P. (D) none of these

Solution :

Here, the given condition $\left(a^2+b^2+c^2\right)p^2-2p\left(ab+bc+ca\right)+b^2+c^2+d^2\leq 0$

$\Rightarrow \left(ap-b\right)^2+\left(bp-c\right)^2+\left(cp-d\right)^2\leq 0$

$\because$ a square can not be negative

$\therefore$ $ap-b=0, bp-c=0, cp-d=0 \Rightarrow p=\frac{b}{a}=\frac{c}{b}=\frac{d}{c}\Rightarrow$ a, b, c, d are in G.P. **Ans. (B)**

Illustration 8 :

If a, b, c are in G.P., then the equations $ax^2+2bx+c=0$ and $dx^2+2ex+f=0$ have a common root if $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in -

(A) A.P. (B) G.P. (C) H.P. (D) none of these

Solution :

a, b, c are in G.P $\Rightarrow b^2=ac$

Now the equation $ax^2+2bx+c=0$ can be rewritten as $ax^2+2\sqrt{ac}x+c=0$

$\Rightarrow \left(\sqrt{a}x+\sqrt{c}\right)^2=0\Rightarrow x=-\sqrt{\frac{c}{a}},-\sqrt{\frac{c}{a}}$

If the two given equations have a common root, then this root must be $-\sqrt{\frac{c}{a}}$.

Thus $d\frac{c}{a}-2e\sqrt{\frac{c}{a}}+f=0 \Rightarrow \frac{d}{a}+\frac{f}{c}=\frac{2e}{c}\sqrt{\frac{c}{a}}=\frac{2e}{\sqrt{ac}}=\frac{2e}{b} \Rightarrow \frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in A.P. **Ans. (A)**

Illustration 9 :

A number consists of three digits which are in G.P. the sum of the right hand and left hand digits exceeds twice the middle digit by 1 and the sum of the left hand and middle digits is two third of the sum of the middle and right hand digits. Find the numbers.

Solution :

Let the three digits be a, ar and ar^2 then number is

$100a + 10ar + ar^2$ ....(i)

Given, $a + ar^2 = 2ar + 1$

or $a(r^2 - 2r + 1) = 1$

or $a(r - 1)^2 = 1$ ....(ii)

Also given $a + ar = \frac{2}{3}(ar + ar^2)$

$\Rightarrow$ $3 + 3r = 2r + 2r^2$ $\Rightarrow$ $2r^2 - r - 3 = 0$ $\Rightarrow$ $(r + 1)(2r - 3) = 0$

$\therefore$ $r = -1, 3/2$

for $r = -1$, $a=\frac{1}{(r-1)^2}=\frac{1}{4}\notin I$ $\therefore r\neq -1$

for $r = 3/2$, $a=\frac{1}{\left(\frac{3}{2}-1\right)^2}=4$ {from (ii)}

From (i), number is $400 + 10.4.\frac{3}{2}+4.\frac{9}{4}=469$ **Ans.**

Illustration 10 :

Find the value of $0.32\overline{58}$

Solution :

Let $R = 0.32\overline{58}$ $\Rightarrow$ $R = 0.32585858....$ .... (i)

Here number of figures which are not recurring is 2 and number of figures which are recurring is also 2.

then $100\ R = 32.585858......$ ....(ii)

and $10000\ R = 3258.5858.....$ ....(iii)

Subtracting (ii) from (iii) , we get

$9900\ R = 3226$ $\Rightarrow$ $R = \frac{1613}{4950}$

Aliter Method : $R = .32 + .0058 + .0058 + .000058 + ...$

$$= .32 + \frac{58}{10^4}\left(1 + \frac{1}{10^2} + \frac{1}{10^4} + ...\infty\right)$$

$$= .32 + \frac{58}{10^4}\left(\frac{1}{1 - \frac{1}{100}}\right)$$

$$= \frac{32}{100} + \frac{58}{9900} = \frac{3168 + 58}{9900} = \frac{3226}{9900} = \frac{1613}{4950}$$

Do yourself - 3 :

(i) Find a three digit number whose consecutive digits form a G.P. If we subtract 792 from this number, we get a number consisting of the same digits written in the reverse order. Now, if we increase the second digit of the required number by 2, then the resulting digits will form an A.P.

(ii) If the third term of G.P. is 4, then find the product of first five terms.

(iii) If a, b, c are respectively the p^{th}, q^{th} and r^{th} terms of the given G.P., then show that $(q - r)\log a + (r - p)\log b + (p - q)\log c = 0$, where $a, b, c > 0$.

(iv) Find three numbers in G.P., whose sum is 52 and the sum of whose products in pairs is 624.

(v) The rational number which equals the number $2.\overline{357}$ with recurring decimal is -

(A) $\frac{2357}{999}$ (B) $\frac{2379}{997}$ (C) $\frac{785}{333}$ (D) $\frac{2355}{1001}$

6. HARMONIC PROGRESSION (H.P.) :

A sequence is said to be in H.P. if the reciprocal of its terms are in AP.

If the sequence $a_1, a_2, a_3, ..., a_n$ is an HP then $1/a_1, 1/a_2, ..., 1/a_n$ is an AP . Here we do not have the formula for the sum of the n terms of an HP. The general form of a harmonic progression is $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, ... \frac{1}{a+(n-1)d}$

Note : No term of any H.P. can be zero.

(i) If a, b, c are in HP, then $b = \frac{2ac}{a+c}$ or $\frac{a}{c} = \frac{a-b}{b-c}$

Illustration 11 :

If $\frac{1}{a} + \frac{1}{c} + \frac{1}{a-b} + \frac{1}{c-b} = 0$, prove that a, b, c are in H.P, or b = a + c

Solution :

We have $\frac{1}{a}+\frac{1}{c}+\frac{1}{a-b}+\frac{1}{c-b}=0,$

$\Rightarrow \quad \frac{a+c}{ac}+\frac{c-b+a-b}{(a-b)(c-b)} \quad \Rightarrow \quad \frac{a+c}{ac}+\frac{(a+c)-2b}{ac-b(a+c)+b^2}=0$

Let $\quad a + c = \lambda$

$\therefore \quad \frac{\lambda}{ac}+\frac{\lambda-2b}{ac-b\lambda+b^2}=0$

$\Rightarrow \quad \frac{ac\lambda-b\lambda^2+b^2\lambda+ac\lambda-2abc}{ac(ac-b\lambda+b^2)}=0 \quad \Rightarrow \quad 2ac\lambda - b\lambda^2 + b^2\lambda - 2abc = 0$

$\Rightarrow \quad 2ac(\lambda - b) - b\lambda(\lambda - b) = 0 \quad \Rightarrow \quad (2ac - b\lambda)(\lambda - b) = 0$

$\Rightarrow \quad \lambda = b \text{ or } \lambda = \frac{2ac}{b}$

$\Rightarrow \quad a + c = b \text{ or } a + c = \frac{2ac}{b} \qquad (\because a + c = \lambda)$

$\Rightarrow \quad a + c = b \text{ or } b = \frac{2ac}{a+c}$

$\therefore$ a, b, c are in H.P. or $a + c = b$.

Illustration 12 :

The sum of three numbers are in H.P. is 37 and the sum of their reciprocals is $\frac{1}{4}$. Find the numbers.

Solution :

Three numbers are in H.P. can be taken as

$$\frac{1}{a-d},\frac{1}{a},\frac{1}{a+d}$$

then $\quad \frac{1}{a-d}+\frac{1}{a}+\frac{1}{a+d} = 37 \qquad$ ....(i)

and $\quad a - d + a + a + d = \frac{1}{4} \quad \Rightarrow \quad a = \frac{1}{12}$

from (i), $\quad \frac{12}{1-12d}+12+\frac{12}{1+12d}=37 \quad \Rightarrow \quad \frac{12}{1-12d}+\frac{12}{1+12d}=25$

$\Rightarrow \quad \frac{24}{1-144d^2}=25 \quad \Rightarrow \quad 1-144d^2 = \frac{24}{25}$

$\Rightarrow \quad d^2 = \frac{1}{25\times 144}$

$\therefore \quad d = \pm\frac{1}{60}$

$\therefore$ $a - d, a, a + d$ are $\frac{1}{15},\frac{1}{12},\frac{1}{10}$ or $\frac{1}{10},\frac{1}{12},\frac{1}{15}$

Hence, three numbers in H.P. are 15, 12, 10 or 10, 12, 15 **Ans.**

Illustration 13 :

Suppose a is a fixed real number such that $\frac{a-x}{px}=\frac{a-y}{qy}=\frac{a-z}{rz}$

If p, q, r are in A.P., then prove that x, y, z are in H.P.

Solution :

$\because$ p, q, r are in A.P.

$\therefore$ $q - p = r - q$ .... (i)

$\Rightarrow$ $p - q = q - r = k$ (let)

given $\frac{a-x}{px} = \frac{a-y}{qy} = \frac{a-z}{rz}$ $\Rightarrow$ $\frac{\frac{a}{x}-1}{p} = \frac{\frac{a}{y}-1}{q} = \frac{\frac{a}{z}-1}{r}$

$\Rightarrow$ $\frac{\left(\frac{a}{x}-1\right)-\left(\frac{a}{y}-1\right)}{p-q} = \frac{\left(\frac{a}{y}-1\right)-\left(\frac{a}{z}-1\right)}{q-r}$ (by law of proportion)

$\Rightarrow$ $\frac{\frac{a}{x}-\frac{a}{y}}{k} = \frac{\frac{a}{y}-\frac{a}{z}}{k}$ {from (i)}

$\Rightarrow$ $a\left(\frac{1}{x}-\frac{1}{y}\right) = a\left(\frac{1}{y}-\frac{1}{z}\right)$ $\Rightarrow$ $\frac{1}{x}-\frac{1}{y} = \frac{1}{y}-\frac{1}{z}$

$\therefore$ $\frac{2}{y} = \frac{1}{x}+\frac{1}{z}$

$\therefore$ $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ are in A.P.

Hence x, y, z are in H.P.

Do yourself - 4 :

(i) If the 7^{th} term of a H.P. is 8 and the 8^{th} term is 7. Then find the 28^{th} term.

(ii) In a H.P., if 5^{th} term is 6 and 3^{rd} term is 10. Find the 2^{nd} term.

(iii) If the p^{th}, q^{th} and r^{th} terms of a H.P. are a,b,c respectively, then prove that $\frac{q-r}{a}+\frac{r-p}{b}+\frac{p-q}{c} = 0$.

7. MEANS

(a) ARITHMETIC MEAN :

If three terms are in A.P. then the middle term is called the A.M. between the other two, so if a, b, c are in A.P., b is A.M. of a & c. So A.M. of a and c = $\frac{a+c}{2} = b$.

n-ARITHMETIC MEANS BETWEEN TWO NUMBERS :

If a,b be any two given numbers & a, A_1, A_2, ..., A_n, b are in AP, then A_1, A_2,........A_n are the 'n' A.M's between a & b then. $A_1 = a + d$, $A_2 = a + 2d$,......, $A_n = a + nd$ or $b - d$, where $d = \frac{b-a}{n+1}$

$\Rightarrow A_1 = a + \frac{b-a}{n+1}$, $A_2 = a + \frac{2(b-a)}{n+1}$,.......

Note : Sum of n A.M's inserted between a & b is equal to n times the single A.M. between a & b i.e. $\sum_{r=1}^{n} A_r = nA$ where A is the single A.M. between a & b.

(b) GEOMETRIC MEAN :

If a, b, c are in G.P., then b is the G.M. between a & c, $b^2 = ac$. So G.M. of a and c = $\sqrt{ac} = b$

n-GEOMETRIC MEANS BETWEEN TWO NUMBERS :

If a, b are two given positive numbers & a, G_1, G_2, ..., G_n, b are in G.P. Then G_1, G_2, G_3 ,.......G_n are 'n' G.Ms between a & b.

$G_1 = a(b/a)^{1/n+1}$, $G_2 = a(b/a)^{2/n+1}$, ..., $G_n = a(b/a)^{n/n+1}$

$= ar$, $= ar^2$, ... $= ar^n = b/r$, where $r = (b/a)^{1/n+1}$

Note : The product of n G.Ms between a & b is equal to n^{th} power of the single G.M. between a & b

i.e. $\prod_{r=1}^{n} G_r = (G)^n$ where G is the single G.M. between a & b

(c) HARMONIC MEAN :

If a, b, c are in H.P., then b is H.M. between a & c. So H.M. of a and c $= \dfrac{2ac}{a+c} = b$.

Insertion of 'n' HM's between a and b :

$a, H_1, H_2, H_3, \ldots\ldots, H_n, b \rightarrow$ H.P

$\dfrac{1}{a}, \dfrac{1}{H_1}, \dfrac{1}{H_2}, \dfrac{1}{H_3}, \ldots\ldots\ldots \dfrac{1}{H_n}, \dfrac{1}{b} \rightarrow$ A.P.

$\dfrac{1}{b} = \dfrac{1}{a} + (n+1)D \quad \Rightarrow \quad D = \dfrac{\frac{1}{b} - \frac{1}{a}}{n+1}$

$\dfrac{1}{H_n} = \dfrac{1}{a} + n\left(\dfrac{\frac{1}{b} - \frac{1}{a}}{n+1}\right)$

Important note :

(i) If A, G, H, are respectively A.M., G.M., H.M. between two positive number a & b then

(a) $G^2 = AH$ (A, G, H constitute a GP) (b) $A \geq G \geq H$ (c) $A = G = H \Leftrightarrow a = b$

(ii) Let $a_1, a_2, \ldots\ldots, a_n$ be n positive real numbers, then we define their arithmetic mean (A), geometric mean (G) and harmonic mean (H) as $A = \dfrac{a_1 + a_2 + \ldots.. + a_n}{n}$

$G = (a_1 a_2 \ldots\ldots\ldots a_n)^{1/n}$ and $H = \dfrac{n}{\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots \frac{1}{a_n}\right)}$

It can be shown that $A \geq G \geq H$. Moreover equality holds at either place if and only if $a_1 = a_2 = \ldots\ldots = a_n$

Illustration 14 :

If $2x^3 + ax^2 + bx + 4 = 0$ (a and b are positive real numbers) has 3 real roots, then prove that $a + b \geq 6(2^{1/3} + 4^{1/3})$.

Solution :

Let α, β, γ be the roots of $2x^3 + ax^2 + bx + 4 = 0$. Given that all the coefficients are positive, so all the roots will be negative.

Let $\alpha_1 = -\alpha, \alpha_2 = -\beta, \alpha_3 = -\gamma \quad \Rightarrow \quad \alpha_1 + \alpha_2 + \alpha_3 = \dfrac{a}{2}$

$\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1 = \dfrac{b}{2}$

$\alpha_1\alpha_2\alpha_3 = 2$

Applying AM $\geq$ GM, we have

$\dfrac{\alpha_1 + \alpha_2 + \alpha_3}{3} \geq (\alpha_1\alpha_2\alpha_3)^{1/3} \quad \Rightarrow \quad a \geq 6 \times 2^{1/3}$

Also $\dfrac{\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_1\alpha_3}{3} > (\alpha_1\alpha_2\alpha_3)^{2/3} \quad \Rightarrow \quad b \geq 6 \quad 4^{1/3}$

Therefore $a + b \geq 6(2^{1/3} + 4^{1/3})$.

Illustration 15 :

If $a_i > 0 \ \forall\ i \in N$ such that $\prod_{i=1}^{n} a_i = 1$, then prove that $(1 + a_1)(1 + a_2)(1 + a_3).....(1 + a_n) \ge 2^n$

Solution :

Using A.M. $\ge$ G.M.

$1 + a_1 \ge 2\sqrt{a_1}$

$1 + a_2 \ge 2\sqrt{a_2}$

$\vdots$

$1 + a_n \ge 2\sqrt{a_n} \quad \Rightarrow \quad (1 + a_1)(1 + a_2)........(1 + a_n) \ge 2^n(a_1a_2a_3.....a_n)^{1/2}$

As $a_1a_2a_3.....a_n = 1$

Hence $(1 + a_1)(1 + a_2)..........(1 + a_n) \ge 2^n$.

Illustration 16 :

If a, b, x, y are positive natural numbers such that $\frac{1}{x}+\frac{1}{y}=1$ then prove that $\frac{a^x}{x}+\frac{b^y}{y} \ge ab$.

Solution :

Consider the positive numbers $a^x, a^x,.......y$ times and $b^y, b^y,......x$ times

For all these numbers,

$$AM = \frac{\{a^x + a^x + ...y \text{ time}\} + \{b^y + b^y + ...x \text{ times}\}}{x+y} = \frac{ya^x + xa^y}{(x+y)}$$

$$GM = \left\{\left(a^x.a^x......y \text{ times}\right)\left(b^y.b^y......x \text{ times}\right)\right\}^{\frac{1}{(x+y)}} = \left[\left(a^{xy}\right).\left(b^{xy}\right)\right]^{\frac{1}{(x+y)}} = (ab)^{\frac{xy}{(x+y)}}$$

As $\frac{1}{x}+\frac{1}{y}=1,\ \frac{x+y}{xy}=1$, i.e, $x + y = xy$

So using AM $\ge$ GM $\frac{ya^x + xa^y}{x+y} \ge (ab)^{\frac{xy}{x+y}}$

$\therefore \quad \frac{ya^x + xa^y}{xy} \ge ab$ or $\frac{a^x}{x}+\frac{a^y}{y} \ge ab$.

Do yourself - 5 :

(i) If $\frac{a^n + b^n}{a^{n-1}+b^{n-1}}$ is the G.M. between a & b then find the value of 'n'.

(ii) If b is the harmonic mean between a and c, then prove that $\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{a}+\frac{1}{c}$.

8. ARITHMETICO - GEOMETRIC SERIES :

A series, each term of which is formed by multiplying the corresponding term of an A.P. & G.P. is called the Arithmetico-Geometric Series , e.g. $1 + 3x + 5x^2 + 7x^3 + ...$

Here 1, 3, 5, ... are in A.P. & 1, x, x^2, x^3 .... are in G.P.

(a) SUM OF N TERMS OF AN ARITHMETICO-GEOMETRIC SERIES :

Let $S_n = a + (a+d)r + (a+2d)r^2 + ... + [a+(n-1)d]r^{n-1}$

then $S_n = \frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]\ \ r^n}{1-r}, \ \ r \ne 1$

(b) SUM TO INFINITY :

If $0 < |r| < 1$ & $n \to \infty$, then $\lim_{n\to\infty} r^n = 0,\ S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$

Illustration 17 :

Find the sum of series $4 - 9x + 16x^2 - 25x^3 + 36x^4 - 49x^5 + \ldots\ldots\ldots \infty$.

Solution :

Let $S = 4 - 9x + 16x^2 - 25x^3 + 36x^4 - 49x^5 + \ldots\ldots\ldots \infty$

$-Sx = -4x + 9x^2 - 16x^3 + 25x^4 - 36x^5 + \ldots\ldots\ldots \infty$

On subtraction, we get

$S(1 + x) = 4 - 5x + 7x^2 - 9x^3 + 11x^4 - 13x^5 + \ldots\ldots \infty$

$-S(1 + x)x = -4x + 5x^2 - 7x^3 + 9x^4 - 11x^5 + \ldots\ldots \infty$

On subtraction, we get

$S(1 + x)^2 = 4 - x + 2x^2 - 2x^3 + 2x^4 - 2x^5 + \ldots\ldots \infty$

$= 4 - x + 2x^2 (1 - x + x^2 - \ldots\ldots\ldots\infty) = 4 - x + \dfrac{2x^2}{1+x} = \dfrac{4+3x+x^2}{1+x}$

$S = \dfrac{4+3x+x^2}{(1+x)^3}$ **Ans.**

Illustration 18 :

Find the sum of series upto n terms $\left(\dfrac{2n+1}{2n-1}\right) + 3\left(\dfrac{2n+1}{2n-1}\right)^2 + 5\left(\dfrac{2n+1}{2n-1}\right)^3 + \ldots\ldots\ldots$.

Solution :

For $x \neq 1$, let

$S = x + 3x^2 + 5x^3 + \ldots\ldots + (2n - 3)x^{n-1} + (2n - 1)x^n$ .... (i)

$\Rightarrow$ $xS = x^2 + 3x^3 + \ldots\ldots + (2n - 5)x^{n-1} + (2n - 3)x^n + (2n - 1)x^{n+1}$ .... (ii)

Subtracting (ii) from (i), we get

$(1 - x)S = x + 2x^2 + 2x^3 + \ldots\ldots\ldots + 2x^{n-1} + 2x^n - (2n - 1)x^{n+1} = x + \dfrac{2x^2(1-x^{n-1})}{1-x} - (2n-1)x^{n+1}$

$= \dfrac{x}{1-x}[1 - x + 2x - 2x^n - (2n - 1)x^n + (2n - 1)x^{n+1}]$

$\Rightarrow$ $S = \dfrac{x}{(1-x)^2}[(2n-1)x^{n+1} - (2n + 1)x^n + 1 + x]$

Thus $\left(\dfrac{2n+1}{2n-1}\right) + 3\left(\dfrac{2n+1}{2n-1}\right)^2 + \ldots\ldots + (2n-1)\left(\dfrac{2n+1}{2n-1}\right)^n$

$= \left(\dfrac{2n+1}{2n-1}\right)\left(\dfrac{2n-1}{2}\right)^2\left[(2n-1)\left(\dfrac{2n+1}{2n-1}\right)^{n+1} - (2n+1)\left(\dfrac{2n+1}{2n-1}\right)^n + 1 + \dfrac{2n+1}{2n-1}\right] = \dfrac{4n^2-1}{4}\cdot\dfrac{4n}{2n-1} = n(2n + 1)$ **Ans.**

Do yourself - 6 :

(i) Find sum to n terms of the series $3 + 5 \times \dfrac{1}{4} + 7 \times \dfrac{1}{4^2} + \ldots\ldots$

(ii) If the sum to the infinity of the series $3 + 5r + 7r^2 + \ldots\ldots\ldots$ is $\dfrac{44}{9}$, then find the value of r.

(iii) If the sum to infinity of the series $3 + (3+d).\dfrac{1}{4} + (3+2d).\dfrac{1}{4^2} + \ldots\ldots\ldots$ is $\dfrac{44}{9}$ then find d.

9. SIGMA NOTATIONS (Σ)

THEOREMS :

(a) $\sum_{r=1}^{n}(a_r \pm b_r) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r$ **(b)** $\sum_{r=1}^{n} k\,a_r = k\sum_{r=1}^{n} a_r$ **(c)** $\sum_{r=1}^{n} k = nk$; where k is a constant.

10. RESULTS

(a) $\displaystyle\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$ (sum of the first n natural numbers)

(b) $\displaystyle\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ (sum of the squares of the first n natural numbers)

(c) $\displaystyle\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4} = \left[\sum_{r=1}^{n} r\right]^2$ (sum of the cubes of the first n natural numbers)

(d) $\displaystyle\sum_{r=1}^{n} r^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1)$

(e) $\displaystyle\sum_{r=1}^{n}(2r-1) = n^2$ (sum of first n odd natural numbers)

(f) $\displaystyle\sum_{r=1}^{n} 2r = n(n+1)$ (sum of first n even natural numbers)

Note :

If n^{th} term of a sequence is given by $T_n = an^3 + bn^2 + cn + d$ where a, b, c, d are constants,

then sum of n terms $S_n = \Sigma T_n = a\Sigma n^3 + b\Sigma n^2 + c\Sigma n + \Sigma d$

This can be evaluated using the above results.

Illustration 19 :

Sum up to 16 terms of the series $\dfrac{1^3}{1} + \dfrac{1^3+2^3}{1+3} + \dfrac{1^3+2^3+3^3}{1+3+5} + \ldots$ is

(A) 450 (B) 456 (C) 446 (D) none of these

Solution :

$$t_n = \frac{1^3+2^3+3^3+\ldots+n^3}{1+3+5+\ldots(2n-1)} = \frac{\left\{\dfrac{n(n+1)}{2}\right\}^2}{\dfrac{n}{2}\{2+2(n-1)\}} = \frac{\dfrac{n^2(n+1)^2}{4}}{n^2} = \frac{(n+1)^2}{4} = \frac{n^2}{4}+\frac{n}{2}+\frac{1}{4}$$

$$\therefore \quad S_n = \Sigma t_n = \frac{1}{4}\Sigma n^2 + \frac{1}{2}\Sigma n + \frac{1}{4}\Sigma 1 \qquad = \frac{1}{4}.\frac{n(n+1)(2n+1)}{6} + \frac{1}{2}.\frac{n(n+1)}{2} + \frac{1}{4}.n$$

$$\therefore \quad S_{16} = \frac{16.17.33}{24} + \frac{16.17}{4} + \frac{16}{4} = 446$$

Ans. (C)

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11. METHOD OF DIFFERENCE :

Some times the n^{th} term of a sequence or a series can not be determined by the method, we have discussed earlier. So we compute the difference between the successive terms of given sequence for obtained the n^{th} terms.

If $T_1, T_2, T_3, \ldots\ldots, T_n$ are the terms of a sequence then some times the terms $T_2 - T_1$, $T_3 - T_2, \ldots\ldots$ constitute an AP/GP. n^{th} term of the series is determined & the sum to n terms of the sequence can easily be obtained.

Case 1 :

(a) If difference series are in A.P., then

Let $T_n = an^2 + bn + c$, where a, b, c are constant

(b) If difference of difference series are in A.P.

Let $T_n = an^3 + bn^2 + cn + d$, where a, b, c, d are constant

Case 2 :

(a) If difference are in G.P., then

Let $T_n = ar^n + b$, where r is common ratio & a, b are constant

(b) If difference of difference are in G.P., then

Let $T_n = ar^n + bn + c$, where r is common ratio & a, b, c are constant

Determine constant by putting n = 1, 2, 3 .... n and putting the value of T_1, T_2, T_3 ......

and sum of series $(S_n) = \sum T_n$

Do yourself - 7 :

(i) Find the sum of the series upto n terms $1+\frac{1+2}{2}+\frac{1+2+3}{3}+\frac{1+2+3+4}{4}+$...........

(ii) Find the sum of 'n' terms of the series whose n^{th} term is $t_n = 3n^2 + 2n$.

Miscellaneous Illustration :

Illustration 20 :

If $\sum_{r=1}^{n} T_r = \frac{n}{8}(n+1)(n+2)(n+3)$, then find $\sum_{r=1}^{n}\frac{1}{T_r}$.

Solution : $\because$ $T_n = S_n - S_{n-1}$

$$= \sum_{r=1}^{n} T_r - \sum_{r=1}^{n-1} T_r = \frac{n(n+1)(n+2)(n+3)}{8} - \frac{(n-1)n(n+1)(n+2)}{8} = \frac{n(n+1)(n+2)}{8}[(n+3)-(n-1)]$$

$$T_n = \frac{n(n+1)(n+2)}{8}(4) = \frac{n(n+1)(n+2)}{2}$$

$$\Rightarrow \frac{1}{T_n} = \frac{2}{n(n+1)(n+2)} = \frac{(n+2)-n}{n(n+1)(n+2)} = \frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \quad \text{......... (i)}$$

Let $V_n = \frac{1}{n(n+1)}$

$$\therefore \frac{1}{T_n} = V_n - V_{n+1}$$

Putting n = 1, 2, 3, ... n

$$\Rightarrow \frac{1}{T_1}+\frac{1}{T_2}+\frac{1}{T_3}+.......+\frac{1}{T_n} = (V_1 - V_{n+1}) \Rightarrow \sum_{r=1}^{n}\frac{1}{T_r} = \frac{n^2+3n}{2(n+1)(n+2)}$$

Illustration 21 :

Find the sum of n terms of the series $1 \cdot 3 \cdot 5 + 3 \cdot 5 \cdot 7 + 5 \cdot 7 \cdot 9 + \ldots\ldots.$

Solution :

The n^{th} term is $(2n-1)(2n+1)(2n+3)$

$T_n = (2n-1)(2n+1)(2n+3)$

$$T_n = \frac{1}{8}(2n-1)(2n+1)(2n+3)\{(2n+5)-(2n-3)\}$$

$$= \frac{1}{8}(V_n - V_{n-1}) \qquad [\text{Let } V_n = (2n-1)(2n+1)(2n+3)(2n+5)]$$

$$S_n = \sum T_n = \frac{1}{8}[V_n - V_0]$$

$$\therefore \quad S_n = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{8} + \frac{15}{8} = n(2n^3 + 8n^2 + 7n - 2)$$ **Ans.**

Illustration 22 :

Find the sum of n terms of the series $3 + 7 + 14 + 24 + 37 + \ldots\ldots\ldots$

Solution :

Clearly here the differences between the successive terms are

$7-3,\ 14-7,\ 24-14,\ \ldots\ldots$ i.e. $4, 7, 10, 13, \ldots\ldots\ldots$, which are in A.P.

Let $S = 3 + 7 + 14 + 24 + \ldots\ldots + T_n$

$S = \quad 3 + 7 + 14 + \ldots\ldots + T_{n-1} + T_n$

Subtracting, we get

$0 = 3 + [4 + 7 + 10 + 13 + \ldots\ldots (n-1) \text{ terms}] - T_n$

$\therefore \quad T_n = 3 + S_{n-1}$ of an A.P. whose $a = 4$ and $d = 3$.

$$\therefore \quad T_n = 3 + \left(\frac{n-1}{2}\right)(2.4 + (n-2)3) = \frac{6 + (n-1)(3n+2)}{4} \quad \text{or,} \quad T_n = \frac{1}{2}\left(3n^2 - n + 4\right)$$

Now putting $n = 1, 2, 3, \ldots\ldots, n$ and adding

$$\therefore \quad S_n = \frac{1}{2}\left[3\sum n^2 - \sum n + 4n\right] = \frac{1}{2}\left[3\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 4n\right] = \frac{n}{2}(n^2 + n + 4)$$ **Ans.**

Aliter Method :

Let $T_n = an^2 + bn + c$

Now, $T_1 = 3 = a + b + c$ .....(i)

$T_2 = 7 = 4a + 2b + c$ .....(ii)

$T_3 = 14 = 8a + 3b + c$ .....(iii)

Solving (i), (ii) & (iii) we get

$$a = \frac{3}{2}, b = -\frac{1}{2} \ \& \ c = 2$$

$$\therefore \quad T_n = \frac{1}{2}(3n^2 - n + 4)$$

$$\Rightarrow \quad s_n = \Sigma T_n = \frac{1}{2}\left[3\sum n^2 - \sum n + 4n\right] = \frac{1}{2}\left[3\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 4n\right] = \frac{n}{2}(n^2 + n + 4)$$ **Ans.**

Illustration 23 :

Find the sum of n-terms of the series 1 + 4 + 10 + 22 + ...

Solution :

Let $S = 1 + 4 + 10 + 22 + ... + T_n$ .... (i)

$S = \quad 1 + 4 + 10 + ... + T_{n-1} + T_n$ .... (ii)

(i) − (ii) $\Rightarrow T_n = 1 + (3 + 6 + 12 + ... + T_n - T_{n-1})$

$$T_n = 1 + 3\left(\frac{2^{n-1}-1}{2-1}\right)$$

$$T_n = 3 \,.\, 2^{n-1} - 2$$

So $S_n = \Sigma T_n = 3\Sigma 2^{n-1} - \Sigma 2$

$$= 3\left(\frac{2^n-1}{2-1}\right) - 2n = 3.2^n - 2n - 3$$ **Ans.**

Aliter Method :

Let $T_n = ar^n + b$, where $r = 2$

Now $T_1 = 1 = ar + b$ ....(i)

$T_2 = 4 = ar^2 + b$ ....(ii)

Solving (i) & (ii), we get

$$a = \frac{3}{2}, b = -2$$

$\therefore \quad T_n = 3.2^{n-1} - 2$

$\Rightarrow \quad S_n = \Sigma T_n = 3\Sigma 2^{n-1} - \Sigma 2$

$$= 3\left(\frac{2^n-1}{2-1}\right) - 2n = 3.2^n - 2n - 3$$ **Ans.**

Illustration 24 :

The series of natural numbers is divided into groups (1), (2, 3, 4), (5, 6, 7, 8, 9) .... and so on. Show that the sum of the numbers in n^{th} group is $n^3 + (n-1)^3$

Solution :

The groups are (1), (2, 3, 4), (5, 6, 7, 8, 9) ....

The number of terms in the groups are 1, 3, 5......

$\therefore$ The number of terms in the n^{th} group = $(2n - 1)$

the last term of the n^{th} group is n^2

If we count from last term common difference should be −1

So the sum of numbers in the n^{th} group $= \left(\frac{2n-1}{2}\right)\left\{2n^2 + (2n-2)(-1)\right\}$

$= (2n - 1)(n^2 - n + 1) = 2n^3 - 3n^2 + 3n - 1 = n^3 + (n-1)^3$

Illustration 25 :

Find the natural number 'a' for which $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$, where the function f satisfied $f(x+y) = f(x).\ f(y)$ for all natural number x,y and further $f(1) = 2$.

Solution :

It is given that

$f(x+y) = f(x)\, f(y)$ and $f(1) = 2$

$f(1+1) = f(1)\, f(1) \Rightarrow f(2) = 2^2$, $f(1+2) = f(1)\, f(2) \Rightarrow f(3) = 2^3$, $f(2+2) = f(2)\, f(2) \Rightarrow f(4) = 2^4$

Similarly $f(k) = 2^k$ and $f(a) = 2^a$

Hence , $\sum_{k=1}^{n} f(a+k) = \sum_{k=1}^{n} f(a)f(k) = f(a)\sum_{k=1}^{n} f(k) = 2^a \sum_{k=1}^{n} 2^k = 2^a\{2^1 + 2^2 + \ldots\ldots\ldots + 2^n\}$

$$= 2^a \left\{\frac{2(2^n - 1)}{2-1}\right\} = 2^{a+1}(2^n - 1)$$

But $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$

$2^{a+1}(2^n - 1) = 16\,(2^n - 1)$

$\therefore\ 2^{a+1} = 2^4$

$\therefore\ a+1 = 4 \Rightarrow a = 3$ **Ans.**

ANSWERS FOR DO YOURSELF

1 : (i) (a) $\frac{2}{1},\frac{4}{2},\frac{8}{3},\frac{16}{4},\ldots\ldots$, (b) $\frac{2}{3},\frac{4}{9},\frac{2}{27},\frac{4}{81},\ldots\ldots$; (iii) $-(p+q)$

2 : (i) 900 (ii) 14

3 : (i) 931 (ii) 4^5 (iv) 4, 12, 36 (v) C

4 : (i) 2 (ii) 15

5 : (i) $\frac{1}{2}$

6 : (i) $4 + \frac{8}{9}\left(1 - \frac{1}{4^{n-1}}\right) - \left(\frac{2n+1}{3\times 4^{n-1}}\right)$ (ii) $\frac{1}{4}$ (iii) 2

7 : (i) $\frac{n(n+3)}{4}$ (ii) $\frac{n(n+1)(2n+3)}{2}$

EXERCISE - 01 CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. The maximum value of the sum of the A.P. 50, 48, 46, 44, ... is -

(A) 325 (B) 648 (C) 650 (D) 652

2. Let T_r be the r^{th} term of an A.P. for r = 1, 2, 3, ... If for some positive integers m, n we have $T_m = \frac{1}{n}$ & $T_n = \frac{1}{m}$, then T_{mn} equals -

(A) $\frac{1}{mn}$ (B) $\frac{1}{m}+\frac{1}{n}$ (C) 1 (D) 0

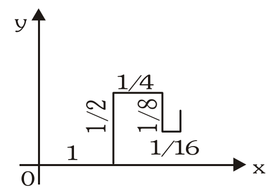
3. The interior angles of a convex polygon are in AP . The smallest angle is 120 & the common difference is 5 . Find the number of sides of the polygon -

(A) 9 (B) 16 (C) 12 (D) none of these

4. The first term of an infinitely decreasing G.P. is unity and its sum is S. The sum of the squares of the terms of the progression is -

(A) $\frac{S}{2S-1}$ (B) $\frac{S^2}{2S-1}$ (C) $\frac{S}{2-S}$ (D) S^2

5. A particle begins at the origin and moves successively in the following manner as shown, 1 unit to the right, 1/2 unit up, 1/4 unit to the right, 1/8 unit down, 1/16 unit to the right etc. The length of each move is half the length of the previous move and movement continues in the 'zigzag' manner indefinitely. The co-ordinates of the point to which the 'zigzag' converges is -



(A) (4/3, 2/3) (B) (4/3, 2/5) (C) (3/2, 2/3) (D) (2, 2/5)

6. Let a_n be the n^{th} term of a G.P. of positive numbers. Let $\sum_{n=1}^{100} a_{2n} = \alpha$ & $\sum_{n=1}^{100} a_{2n-1} = \beta$ such that $\alpha \neq \beta$. Then the common ratio of the G.P. is -

(A) $\frac{\alpha}{\beta}$ (B) $\frac{\beta}{\alpha}$ (C) $\sqrt{\frac{\alpha}{\beta}}$ (D) $\sqrt{\frac{\beta}{\alpha}}$

7. If p, q, r in harmonic progression and p & r be different having same sign then the roots of the equation $px^2 + qx + r = 0$ are -

(A) real and equal (B) real and distinct (C) irrational (D) imaginary

8. If x > 1, y > 1, z >1 are in G.P., then $\frac{1}{1+\ell n x}$, $\frac{1}{1+\ell n y}$, $\frac{1}{1+\ell n z}$ are in -

(A) A.P. (B) H.P. (C) G.P. (D) none of above

9. If ln (a + c) , ln (c − a), ln (a − 2b + c) are in A.P., then :

(A) a, b, c are in A.P. (B) a^2, b^2, c^2 are in A.P

(C) a, b, c are in G.P. (D) a, b, c are in H.P.

10. If the $(m + 1)^{th}$, $(n + 1)^{th}$ & $(r + 1)^{th}$ terms of an AP are in GP & m, n, r are in HP, then the ratio of the common difference to the first term of the AP is -

(A) $\frac{1}{n}$ (B) $\frac{2}{n}$ (C) $-\frac{2}{n}$ (D) none of these

11. The sum of roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of squares of their reciprocals. Then bc^2, ca^2 and ab^2 are in -

(A) AP (B) GP (C) HP (D) none of these

12. The quadratic equation whose roots are the A.M. and H.M. between the roots of the equation, $2x^2 - 3x + 5 = 0$ is -

(A) $4x^2 - 25x + 10 = 0$ (B) $12x^2 - 49x + 30 = 0$
(C) $14x^2 - 12x + 35 = 0$ (D) $2x^2 + 3x + 5 = 0$

13. If the sum of the first n natural numbers is 1/5 times the sum of the their squares, then the value of n is -

(A) 5 (B) 6 (C) 7 (D) 8

14. Suppose p is the first of n(n > 1) AM's between two positive numbers a and b, then value of p is -

(A) $\frac{na+b}{n+1}$ (B) $\frac{na-b}{n+1}$ (C) $\frac{nb+a}{n+1}$ (D) $\frac{nb-a}{n+1}$

15. If $\frac{1}{a}+\frac{1}{a-2b}+\frac{1}{c}+\frac{1}{c-2b}=0$ and a, b, c are not in A.P., then -

(A) a, b, c are in G.P. (B) a, $\frac{b}{2}$, c are in A.P. (C) a, $\frac{b}{2}$, c are in H.P. (D) a, 2b, c are in H.P.

16. The sum to n terms of the series $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\ldots\ldots\ldots$ is -

(A) $\frac{3n}{n+1}$ (B) $\frac{6n}{n+1}$ (C) $\frac{9n}{n+1}$ (D) $\frac{12n}{n+1}$

17. If $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\ldots\ldots+$ to $\infty=\frac{\pi^4}{90}$, then $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\ldots\ldots+$ to ∞ is equals to -

(A) $\frac{\pi^4}{96}$ (B) $\frac{\pi^4}{45}$ (C) $\frac{89\pi^4}{90}$ (D) none of these

18. If $\sum_{s=1}^{n}\left\{\sum_{r=1}^{s} r\right\}=an^3+bn^2+cn$, then find the value of a + b + c.

(A) 1 (B) 0 (C) 2 (D) 3

19. If a, b, c are positive numbers in G.P. and $\log\left(\frac{5c}{a}\right)$, $\log\left(\frac{3b}{5c}\right)$ and $\log\left(\frac{a}{3b}\right)$ are in A.P., then a, b, c forms the sides of a triangle which is -

(A) equilateral (B) right angled (C) isosceles (D) none of these

<u>SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)</u>

20. If sum of n terms of a sequence is given by $S_n = 3n^2 - 5n + 7$ & t_r represents its r^{th} term, then -

(A) $t_7 = 34$ (B) $t_2 = 7$ (C) $t_{10} = 34$ (D) $t_8 = 40$

21. If 10 harmonic means H_1, H_2, H_3 .... H_{10} are inserted between 7 and $-\frac{1}{3}$, then -

(A) $H_1 = -7$ (B) $H_2 = \frac{3}{7}$ (C) $H_1 = -\frac{1}{7}$ (D) $H_{10} = -\frac{7}{19}$

22. If t_n be the n^{th} term of the series 1 + 3 + 7 + 15 + ..., then -

(A) $t_5 + 1 = 32$ (B) $t_7 = 2^7 + 1$ (C) $t_{10} = 2^{10} - 1$ (D) $t_{100} = 2^{50} + 1$

23. Indicate the correct alternative(s), for $0 < \phi < \frac{\pi}{2}$, if $x = \sum_{n=0}^{\infty} \cos^{2n}\phi$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and $z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi$, then -

(A) $xyz = xz + y$ (B) $xyz = xy + z$ (C) $xyz = x + y + z$ (D) $xyz = yz + x$

BRAIN TEASERS **ANSWER KEY** EXERCISE-2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	C	C	A	B	B	A	D	B	D	C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A	B	C	A	D	B	A	A	D	A,D
Que.	21	22	23							
Ans.	A,D	A,C	B,C							

EXERCISE - 02

BRAIN TEASERS

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. Consider an A.P. with first term 'a' and the common difference d. Let S_k denote the sum of the first K terms. Let $\frac{S_{kx}}{S_x}$ is independent of x, then -

(A) a = d/2 (B) a = d (C) a = 2d (D) none of these

2. Let α, β, γ be the roots of the equation $x^3 + 3ax^2 + 3bx + c = 0$. If α, β, γ are in H.P. then β is equal to -

(A) $-c/b$ (B) c/b (C) $-a$ (D) a

3. $\sum_{r=1}^{\infty}(2r-1)\left(\frac{9}{11}\right)^r$ is equal to -

(A) 45 (B) 55

(C) sum of first nine natural numbers (D) sum of first ten natural numbers

4. For the A.P. given by a_1, a_2, ..., a_n, ..., with non-zero common difference, the equations satisfied are-

(A) $a_1 + 2a_2 + a_3 = 0$ (B) $a_1 - 2a_2 + a_3 = 0$

(C) $a_1 + 3a_2 - 3a_3 - a_4 = 0$ (D) $a_1 - 4a_2 + 6a_3 - 4a_4 + a_5 = 0$

5. If a, a_1, a_2,.....,a_{10} , b are in A.P. and a, g_1, g_2,.....g_{10} , b are in G.P. and h is the H.M. between a and b, then $\frac{a_1+a_2+.....+a_{10}}{g_1g_{10}}+\frac{a_2+a_3+.....+a_9}{g_2g_9}+.......+\frac{a_5+a_6}{g_5g_6}$ is -

(A) $\frac{10}{h}$ (B) $\frac{15}{h}$ (C) $\frac{30}{h}$ (D) $\frac{5}{h}$

6. The sum of the first n terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ is $\frac{n(n+1)^2}{2}$, when n is even. When n is odd, the sum is -

(A) $\frac{n^2(n+1)}{2}$ (B) $\frac{n(n+1)(2n+1)}{6}$ (C) $\frac{n(n+1)^2}{2}$ (D) $\frac{n^2(n+1)^2}{2}$

7. If (1 + 3 + 5 +...+ a) + (1 + 3 + 5 +...+ b) = (1 + 3 + 5 + ... + c), where each set of parentheses contains the sum of consecutive odd integers as shown such that - (i) a + b + c = 21, (ii) a > 6

If G = Max{a, b, c} and L = Min{a, b, c}, then -

(A) G – L = 4 (B) b – a = 2 (C) G – L = 7 (D) a – b = 2

8. If a, b and c are distinct positive real numbers and $a^2 + b^2 + c^2 = 1$, then ab + bc + ca is -

(A) equal to 1 (B) less than 1 (C) greater than 1 (D) any real number

9. Let p, q, r $\in R^+$ and $27\,pqr \geq (p + q + r)^3$ and $3p + 4q + 5r = 12$ then $p^3 + q^4 + r^5$ is equal to -

(A) 2 (B) 6 (C) 3 (D) none of these

10. The sum of the first 100 terms common to the series 17, 21, 25, ... and 16, 21, 26, ...is -

(A) 101100 (B) 111000 (C) 110010 (D) 100101

11. If a, b, c are positive such that $ab^2c^3 = 64$ then least value of $\left(\frac{1}{a}+\frac{2}{b}+\frac{3}{c}\right)$ is -

(A) 6 (B) 2 (C) 3 (D) 32

12. If $a_1, a_2, \ldots\ldots a_n \in R^+$ and $a_1.a_2\ldots\ldots a_n = 1$ then the least value of $(1+a_1+a_1^2)(1+a_2+a_2^2)\ldots..(1+a_n+a_n^2)$ is -

(A) 3^n (B) $n3^n$ (C) 3^{3n} (D) data inadequate

13. Let $a_1, a_2, a_3, \ldots\ldots$ and $b_1, b_2, b_3, \ldots\ldots$ be arithmetic progression such that $a_1 = 25$, $b_1 = 75$ and $a_{100} + b_{100} = 100$, then -

(A) The common difference in progression 'a_i' is equal but opposite in sign to the common difference in progression 'b_j'.

(B) $a_n + b_n = 100$ for any n.

(C) $(a_1 + b_1), (a_2 + b_2), (a_3 + b_3), \ldots\ldots$ are in A.P.

(D) $\sum_{r=1}^{100}(a_r + b_r) = 10^4$

14. If the AM of two positive numbers be three times their geometric mean then the ratio of the numbers is -

(A) $3 \pm 2\sqrt{2}$ (B) $\sqrt{2} \pm 1$ (C) $17+12\sqrt{2}$ (D) $\left(3-2\sqrt{2}\right)^{-2}$

15. If first and $(2n - 1)^{th}$ terms of an A.P., G.P. and H.P. are equal and their n^{th} terms are a, b, c respectively, then -

(A) $a + c = 2b$ (B) $a \geq b \geq c$ (C) $a + c = b$ (D) $b^2 = ac$

16. Let a, x, b be in A.P. ; a, y, b be in G.P. and a, z, b be in H.P. If $x = y + 2$ and $a = 5z$ then -

(A) $y^2 = xz$ (B) $x > y > z$ (C) $a = 9, b = 1$ (D) $a = \frac{9}{4}, b = \frac{1}{4}$

17. The p^{th} term T_p of H.P. is $q(q + p)$ and q^{th} term T_q is $p(p + q)$ when $p > 1, q > 1$, then -

(A) $T_{p+q} = pq$ (B) $T_{pq} = p + q$ (C) $T_{p+q} > T_{pq}$ (D) $T_{pq} > T_{p+q}$

18. a, b, c are three distinct real numbers, which are in G.P. and $a + b + c = xb$, then -

(A) $x < -1$ (B) $-1 < x < 2$ (C) $2 < x < 3$ (D) $x > 3$

19. Let $a_1, a_2, \ldots\ldots, a_{10}$ be in A.P. & $h_1, h_2, \ldots\ldots h_{10}$ be in H.P. . If $a_1 = h_1 = 2$ & $a_{10} = h_{10} = 3$ then a_4h_7 is -

(A) 2 (B) 3 (C) 5 (D) 6

BRAIN TEASERS **ANSWER KEY** EXERCISE-2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	A	A,C	B,D	C	A	A,D	B	C	A
Que.	11	12	13	14	15	16	17	18	19	
Ans.	C	A	A,B,C,D	C,D	B,D	A,B,C	A,B,C	A,D	D	

EXERCISE - 03

MISCELLANEOUS TYPE QUESTIONS

FILL IN THE BLANKS

1. The sum of n terms of two A.P.'s are in the ratio of (n + 7) : (3n + 11). The ratio of their 9th term is ________.
2. The sum of the first nineteen terms of an A.P. a_1, a_2, a_3 .... if it is known that $a_4 + a_8 + a_{12} + a_{16} = 224$, is ______________.
3. If $x \in R$ and the numbers $(5^{1+x} + 5^{1-x})$, a/2, $(25^x + 25^{-x})$ form an A.P. then 'a' must lie in the interval _________.
4. If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots$ upto $\infty = \frac{\pi^2}{6}$, then $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots =$ ____________.
5. When 9^{th} term of an A.P. is divided by its 2^{nd} term the quotient is 5 & when 13^{th} term is divided by the 6^{th} term, the quotient is 2 and remainder is 5. The first term and the common difference of the A.P. are _____________ & ________ respectively.
6. The sum to infinity of the series $\frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots\ldots$ is equal to _______________.
7. If sin (x – y), sin x and sin (x + y) are in H.P., then sin x. sec $\frac{y}{2}$ = ______________.

MATCH THE COLUMN

Following questions contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE** statement in **Column-II**.

1.

	Column-I		Column-II
(A)	If a_i's are in A.P. and $a_1 + a_3 + a_4 + a_5 + a_7 = 20$, a_4 is equal to	(p)	21
(B)	Sum of an infinite G.P. is 6 and it's first term is 3. then harmonic mean of first and third terms of G.P. is	(q)	4
(C)	If roots of the equation $x^3 - ax^2 + bx + 27 = 0$, are in G.P. with common ratio 2, then a + b is equal to	(r)	24
(D)	If the roots of $x^4 - 8x^3 + ax^2 + bx + 16 = 0$ are positive real numbers then a is	(s)	6/5

2.

	Column-I		Column-II
(A)	n^{th} term of the series 4, 11, 22, 37, 56, 79,.......	(p)	$2n^2 + n$
(B)	$\|1^2 - 2^2 + 3^2 - 4^2 \ldots\ldots\ldots 2n \text{ terms}\|$ is equal to	(q)	$2n^2 + n + 1$
(C)	sum to n terms of the series 3, 7, 11, 15,....... is	(r)	$-(n^2 + n)$
(D)	coefficient of x^n in 2x(x – 1)(x – 2) .... (x – n) is	(s)	$\frac{1}{2}(n^2 + n)$

ASSERTION & REASON

These questions contains, Statement-I (assertion) and Statement-II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement-I** : If a, b, c are three distinct positive number in H.P., then $\left(\frac{a+b}{2a-b}\right)+\left(\frac{c+b}{2c-b}\right) > 4$

Because

Statement-II : Sum of any number and it's reciprocal is always greater than or equal to 2.

(A) A (B) B (C) C (D) D

2. **Statement-I** : If $x^2y^3 = 6(x, y > 0)$, then the least value of $3x + 4y$ is 10

Because

Statement-II : If $m_1, m_2 \in N$, $a_1, a_2 > 0$ then $\frac{m_1a_1+m_2a_2}{m_1+m_2} \ge (a_1^{m_1}a_2^{m_2})^{\frac{1}{m_1+m_2}}$ and equality holds when $a_1 = a_2$.

(A) A (B) B (C) C (D) D

3. **Statement-I** : For $n \in N$, $2^n > 1 + n\left(\sqrt{(2^{n-1})}\right)$

Because

Statement-II : G.M. > H.M. and (AM) (HM) = $(GM)^2$

(A) A (B) B (C) C (D) D

4. **Statement-I** : If a, b, c are three positive numbers in G.P., then $\left(\frac{a+b+c}{3}\right).\left(\frac{3abc}{ab+bc+ca}\right) = \left(\sqrt[3]{abc}\right)^2$

Because

Statement-II : (A.M.) (H.M.) = $(G.M.)^2$ is true for any set of positive numbers.

(A) A (B) B (C) C (D) D

5. **Statement-I** : n^{th} term (T_n) of the sequence (1, 6, 18, 40, 75, 126,....) is $an^3 + bn^2 + cn + d$, and $6a + 2b - d$ is = 4.

Because

Statement-II If the second successive differences (Differences of the differences) of a series are in A.P., then T_n is a cubic polynomial in n.

(A) A (B) B (C) C (D) D

6. **Statement-I** : The format of n^{th} term (T_n) of the sequence ($\ell n2$, $\ell n4$, $\ell n32$, $\ell n1024$.......) is $an^2 + bn + c$.

Because

Statement-II : If the second successive differences between the consecutive terms of the given sequence are in G.P., then $T_n = a + bn + cr^{n-1}$, where a, b, c are constants and r is common ratio of G.P.

(A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1

There are 4n + 1 terms in a sequence of which first 2n + 1 are in Arithmetic Progression and last 2n + 1 are in Geometric Progression the common difference of Arithmetic Progression is 2 and common ratio of Geometric Progression is 1/2. The middle term of the Arithmetic Progression is equal to middle term of Geometric Progression. Let middle term of the sequence is T_m and T_m is the sum of infinite Geometric Progression whose sum of first two terms is $\left(\frac{5}{4}\right)^2 n$ and ratio of these terms is $\frac{9}{16}$.

On the basis of above information, answer the following questions :

1. Number of terms in the given sequence is equal to -

(A) 9 (B) 17 (C) 13 (D) none

2. Middle term of the given sequence, i.e. T_m is equal to -

(A) 16/7 (B) 32/7 (C) 48/7 (D) 16/9

3. First term of given sequence is equal to -

(A) −8/7, −20/7 (B) −36/7 (C) 36/7 (D) 48/7

4. Middle term of given A. P. is equal to -
(A) 6/7 (B) 10/7 (C) 78/7 (D) 11

5. Sum of the terms of given A. P. is equal to -
(A) 6/7 (B) 7 (C) 3 (D) 6

Comprehension # 2 :

If $a_i > 0$, $i = 1, 2, 3, \ldots. n$ and $m_1, m_2, m_3, \ldots., m_n$ be positive rational numbers, then

$$\left(\frac{m_1a_1+m_2a_2+\ldots..+m_na_n}{m_1+m_2+\ldots.+m_n}\right) \geq \left(a_1^{m_1}a_2^{m_2}\ldots..a_n^{m_n}\right)^{1/(m_1+m_2+\ldots..m_n)} \geq \frac{(m_1+m_2+\ldots.+m_n)}{\frac{m_1}{a_1}+\frac{m_2}{a_2}+\ldots+\frac{m_n}{a_n}}$$

is called weighted mean theorem

where $A^* = \dfrac{m_1a_1+m_2a_2+\ldots.+m_na_n}{m_1+m_2+\ldots.+m_n}$ = Weighted arithmetic mean

$G^* = \left(a_1^{m_1}a_2^{m_2}\ldots.a_n^{m_n}\right)^{1/(m_1+m_2+\ldots.+m_n)}$ = Weighted geometric mean

and $H^* = \dfrac{m_1+m_2+\ldots..+m_n}{\frac{m_1}{a_1}+\frac{m_2}{a_2}+\ldots.\frac{m_n}{a_n}}$ = Weighted harmonic mean

i.e., $A^* \geq G^* \geq H^*$

Now, let $a + b + c = 5(a, b, c > 0)$ and $x^2y^3 = 243(x > 0, y > 0)$

On the basis of above information, answer the following questions :

1. The greatest value of ab^3c is -
(A) 3 (B) 9 (C) 27 (D) 81

2. Which statement is correct -
(A) $\dfrac{1}{5} \geq \dfrac{1}{\frac{1}{a}+\frac{3}{b}+\frac{1}{c}}$ (B) $\dfrac{1}{25} \geq \dfrac{1}{\frac{1}{a}+\frac{9}{b}+\frac{1}{c}}$ (C) $\dfrac{1}{5} \geq \dfrac{1}{\frac{1}{a}+\frac{9}{b}+\frac{1}{c}}$ (D) $\dfrac{1}{25} \geq \dfrac{1}{\frac{1}{a}+\frac{6}{b}+\frac{1}{c}}$

3. The least value of $x^2 + 3y + 1$ is -
(A) 15 (B) greater than 15 (C) 3 (D) less than 15

4. Which statement is correct -
(A) $\dfrac{2x+3y}{5} \geq 3 \geq \dfrac{5}{\frac{3}{x}+\frac{2}{y}}$ (B) $\dfrac{2x+3y}{5} \geq 3 \geq \dfrac{5xy}{3x+2y}$
(C) $\dfrac{2x+3y}{5} \geq 3 \geq \dfrac{5xy}{3x+4y}$ (D) $\dfrac{2x+3y}{5} \geq 3 \geq \dfrac{5xy}{2x+3y}$

MISCELLANEOUS TYPE QUESTION | **ANSWER KEY** | EXERCISE-3

- ***Fill in the Blanks***

1. 12 : 31 2. 1064 3. [12, ∞] 4. $\pi^2/8$ 5. a = 3 d = 4 6. 2 7. $\pm\sqrt{2}$

- ***Match the Column***

1. (A) → (q), (B) → (s), (C) → (p), (D) → (r) 2. (A) → (q), (B) → (p), (C) → (p), (D) → (r)

- ***Assertion & Reason***

1. C 2. A 3. C 4. C 5. A 6. B

- ***Comprehension Based Questions***

Comprehension # 1 : 1. C 2. C 3. B 4. A 5. D
Comprehension # 2 : 1. C 2. C 3. B 4. B

EXERCISE - 04 [A]

CONCEPTUAL SUBJECTIVE EXERCISE

1. Given that $a^x = b^y = c^z = d^u$ & a , b , c , d are in GP, show that x , y , z , u are in HP .

2. There are n AM's between 1 & 31 such that 7th mean : (n − 1)th mean = 5 : 9, then find the value of n.

3. Find the sum of the series , 7 + 77 + 777 + ... to n terms.

4. If the p^{th}, q^{th} & r^{th} terms of an AP are in GP. Show that the common ratio of the GP is $\frac{q-r}{p-q}$.

5. Express the recurring decimal $0.1\overline{576}$ as a rational number using concept of infinite geometric series .

6. If one AM 'a' & two GM's p & q be inserted between any two given numbers then show that $p^3 + q^3 = 2apq$.

7. Find three numbers a , b , c between 2 & 18 which satisfy following conditions :

(i) their sum is 25

(ii) the numbers 2, a, b are consecutive terms of an AP &

(iii) the numbers b , c , 18 are consecutive terms of a GP.

8. Find the sum of the first n terms of the series : $1 + 2\left(1+\frac{1}{n}\right) + 3\left(1+\frac{1}{n}\right)^2 + 4\left(1+\frac{1}{n}\right)^3 + ...$

9. Let $a_1, a_2, a_3 \ldots\ldots a_n$ be an AP . Prove that :

$$\frac{1}{a_1 a_n} + \frac{1}{a_2 a_{n-1}} + \frac{1}{a_3 a_{n-2}} + \ldots\ldots\ldots + \frac{1}{a_n a_1} = \frac{2}{a_1 + a_n}\left[\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots\ldots\ldots + \frac{1}{a_n}\right]$$

10. The harmonic mean of two numbers is 4 . The arithmetic mean A & the geometric mean G satisfy the relation 2 A + G = 27 . Find the two numbers .

11. Prove that : $(ab + xy)(ax + by) \geq 4abxy$ where a, b, x, y $\in R^+$

12. If a, b, c $\in R^+$ & a + b + c = 1; then show that $(1 - a)(1 - b)(1 - c) \geq 8abc$

13. If a, b, c are sides of a scalene triangle then show that $(a + b + c)^3 > 27\,(a + b - c)(b + c - a)(c + a - b)$

14. For positive number a, b, c show that $\frac{bc}{a} + \frac{ac}{b} + \frac{ab}{c} \geq a + b + c$

15. The odd positive numbers are written in the form of a triangle

1

3 5

7 9 11

13 15 17 19

....

.... find the sum of terms in n^{th} row.

CONCEPTUAL SUBJECTIVE EXERCISE	ANSWER KEY	EXERCISE-4(A)

2. 14 **3.** $S = (7/81)(10^{n+1} - 9n - 10)$ **5.** 35/222 **7.** a = 5, b = 8, c = 12

8. n^2 **10.** 6, 3 **15.** n^3

EXERCISE - 04 [B]

BRAIN STORMING SUBJECTIVE EXERCISE

1. In a A.P. & an H.P. have the same first term, the same last term & the same number of terms; prove that the product of the r^{th} term from the beginning in one series & the r^{th} term from the end in the other is independent of r.

2. Sum the following series to n terms and to infinity :

(a) $\frac{1}{1.4.7}+\frac{1}{4.7.10}+\frac{1}{7.10.13}+\ldots\ldots\ldots$ (b) $\sum_{r=1}^{n} r(r+1)(r+2)(r+3)$ (c) $\sum_{r=1}^{n}\frac{1}{4r^2-1}$

3. Find the value of the sum $\sum_{r=1}^{n}\sum_{s=1}^{n}\delta_{rs}2^r3^s$ where δ_{rs} is zero if $r \neq s$ & δ_{rs} is one if $r = s$.

4. Find the sum $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j}1$.

5. If there be 'm' A.P's beginning with unity whose common difference is 1 , 2 , 3 .... m. Show that the sum of their n^{th} terms is (m/2) (mn − m + n + 1).

6. If $a_1, a_2, a_3 \ldots a_n$ are in H.P., then prove that $a_1a_2 + a_2a_3 + \ldots + a_{n-1}a_n = (n-1)a_1a_n$.

7. If a, b, c are in H.P., b, c, d are in G.P. & c, d, e are in A.P., then Show that e = ab /(2a − b) .

8. The value of x + y + z is 15, if a , x , y , z , b are in A.P. while the value of ; (1/x)+(1/y)+(1/z) is 5/3 if a , x , y , z , b are in H.P. Find a & b.

9. Prove that the sum of the infinite series $\frac{1.3}{2}+\frac{3.5}{2^2}+\frac{5.7}{2^3}+\frac{7.9}{2^4}+\ldots\ldots\ldots\infty = 23$.

10. If a , b, c be in G.P. & $\log_c a$, $\log_b c$, $\log_a b$ be in A.P., then show that the common difference of the A.P. must be 3/2.

11. Find the sum to n terms : (a) $\frac{1}{x+1}+\frac{2x}{(x+1)(x+2)}+\frac{3x^2}{(x+1)(x+2)(x+3)}+\ldots\ldots$

(b) $\frac{a_1}{1+a_1}+\frac{a_2}{(1+a_1)(1+a_2)}+\frac{a_3}{(1+a_1)(1+a_2)(1+a_3)}+\ldots\ldots$

12. In a G.P., the ratio of the sum of the first eleven terms to the sum of the last eleven terms is 1/8 and the ratio of the sum of all the terms without the first nine to the sum of all the terms without the last nine is 2 . Find the number of terms in the G.P.

13. Prove that the number $\underbrace{444\ldots\ldots\ldots.4}_{n\text{ digits}}\underbrace{888\ldots\ldots\ldots.8}_{(n-1)\text{ digits}}9$ is a perfect square of the number $\underbrace{666\ldots\ldots\ldots.6}_{(n-1)\text{ digits}}7$.

14. Find the nth term and the sum to 'n' terms of the series :
(a) 1 + 5 + 13 + 29 + 61 + ... (b) 6 + 13 + 22 + 33 + ...

15. If a, b, c are three positive real number then prove that : $\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}\geq\frac{3}{2}$

16. If a, b, c are the sides of a triangle and $s=\frac{a+b+c}{2}$, then prove that 8(s − a)(s − b)(s − c) ≤ abc.

BRAIN STORMING SUBJECTIVE EXERCISE **ANSWER KEY** EXERCISE-4(B)

2. (a) $\frac{1}{24}-\frac{1}{6(3n+1)(3n+4)}$, $\frac{1}{24}$ (b) $\frac{n(n+1)(n+2)(n+3)(n+4)}{5}$ (c) $\frac{n}{2n+1}$, $\frac{1}{2}$

3. $\frac{6}{5}(6^n-1)$ 4. [n(n+1)(n+2)]/6 8. a = 1, b = 9 or b = 1, a = 9

11. (a) $1-\frac{x^n}{(x+1)(x+2)\ldots\ldots\ldots(x+n)}$ (b) $1-\frac{1}{(1+a_1)(1+a_2)\ldots\ldots\ldots(1+a_n)}$

12. n = 38 14. (a) $2^{n+1}-3$; $2^{n+2}-4-3n$ (b) n^2+4n+1 ; $\frac{1}{6}n(n+1)(2n+13)+n$

EXERCISE - 05 [A]

JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

1. If 1, $\log_3 \sqrt{3^{1-x}+2}$, $\log_3(4.3^x - 1)$ are in A.P. then x equals. [AIEEE 2002]
(1) $\log_3 4$ (2) $1 - \log_3 4$ (3) $1 - \log_4 3$ (4) $\log_4 3$

2. Sum of infinite number of terms in G.P. is 20 and sum of their square is 100. The common ratio of G.P. is- [AIEEE 2002]
(1) 5 (2) 3/5 (3) 8/5 (4) 1/5

3. Fifth term of a G.P. is 2, then the product of its 9 terms is- [AIEEE 2002]
(1) 256 (2) 512 (3) 1024 (4) None of these

4. The sum of the series $1^3 - 2^3 + 3^3 - \ldots + 9^3 =$ [AIEEE 2002]
(1) 300 (2) 125 (3) 425 (4) 0

5. Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers m,n, $m \neq n$, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then a – d equals [AIEEE 2004]
(1) 0 (2) 1 (3) $\frac{1}{mn}$ (4) $\frac{1}{m}+\frac{1}{n}$

6. If AM and GM of two roots of a quadratic equation are 9 and 4 respectively, then this quadratic equation is- [AIEEE 2004]
(1) $x^2 - 18x + 16 = 0$ (2) $x^2 + 18x - 16 = 0$ (3) $x^2 + 18x + 16 = 0$ (4) $x^2 - 18x - 16 = 0$

7. If a_1 , a_2, a_3 , ... a_n , ... are in G.P. then the value of the determinant $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$, is- [AIEEE 04, 05]
(1) 0 (2) 1 (3) 2 (4) –2

8. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} C^n$ where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$ then x, y, z are in- [AIEEE 2005]
(1) HP (2) Arithmetic - Geometric Progression (3) AP (4) GP

9. Let a_1, a_2, a_3,....... be terms of an A.P. If $\frac{a_1 + a_2 + \ldots + a_p}{a_1 + a_2 + \ldots + a_q} = \frac{p^2}{q^2}$, $p \neq q$ then $\frac{a_6}{a_{21}}$ equals-[AIEEE-2006]
(1) $\frac{2}{7}$ (2) $\frac{11}{41}$ (3) $\frac{41}{11}$ (4) $\frac{7}{2}$

10. If a_1, a_2 ,....., a_n are in H.P., then the expression $a_1a_2 + a_2a_3 + \ldots + a_{n-1}a_n$ is equal to-[AIEEE-2006]
(1) na_1a_n (2) $(n - 1)a_1a_n$ (3) $n(a_1 - a_n)$ (4) $(n - 1)(a_1 - a_n)$

11. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals- [AIEEE-2007]
(1) $\frac{1}{2}\sqrt{5}$ (2) $\sqrt{5}$ (3) $\frac{1}{2}(\sqrt{5}-1)$ (4) $\frac{1}{2}(1 - \sqrt{5})$

12. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is [AIEEE 2008]

(1) –4 (2) –12 (3) 12 (4) 4

13. The sum to infinity of the series $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+....$ is :- [AIEEE-2009]

(1) 4 (2) 6 (3) 2 (4) 3

14. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference –2, then the time taken by him to count all notes is :- [AIEEE-2010]

(1) 24 minutes (2) 34 minutes (3) 125 minutes (4) 135 minutes

15. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after :- [AIEEE-2011]

(1) 20 months (2) 21 months (3) 18 months (4) 19 months

16. Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is: [AIEEE-2011]

(1) $\frac{\alpha-\beta}{200}$ (2) $\alpha - \beta$ (3) $\frac{\alpha-\beta}{100}$ (4) $\beta - \alpha$

17. **Statement–1 :** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement–2 : $\sum_{k=1}^{n}\left(k^3-(k-1)^3\right)=n^3$, for any natural number n. [AIEEE-2012]

(1) Statement–1 is true, Statement–2 is false.
(2) Statement–1 is false, Statement–2 is true.
(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.
(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

18. If 100 times the 100^{th} term of an A.P. with non-zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this A.P. is : [AIEEE-2012]

(1) zero (2) –150
(3) 150 times its 50^{th} term (4) 150

19. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is : [JEE-MAIN 2013]

(1) $\frac{7}{81}(179-10^{-20})$ (2) $\frac{7}{9}(99-10^{-20})$ (3) $\frac{7}{81}(179+10^{-20})$ (4) $\frac{7}{9}(99-10^{-20})$

PREVIOUS YEARS QUESTIONS **ANSWER KEY** EXERCISE-5 [A]

1. 2	2. 2	3. 2	4. 3	5. 1	6. 1	7. 1
8. 1	9. 2	10. 2	11. 3	12. 2	13. 4	14. 2
15. 2	16. 3	17. 3	18. 1	19. 3		

EXERCISE - 05 [B] JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

1. (a) Consider an infinite geometric series with first term 'a' and common ratio r. If the sum is 4 and the second term is 3/4, then - [JEE 2000, Screening, 1+1M out of 35]

(A) $a=\frac{7}{4}$, $r=\frac{3}{7}$ (B) $a = 2$, $r=\frac{3}{8}$ (C) $a=\frac{3}{2}$, $r=\frac{1}{2}$ (D) $a = 3$, $r=\frac{1}{4}$

(b) If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then $M = (a + b)(c + d)$ satisfies the relation -

(A) $0 \le M \le 1$ (B) $1 \le M \le 2$ (C) $2 \le M \le 3$ (D) $3 \le M \le 4$

(c) The fourth power of the common difference of an arithmetic progression with integer entries is added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer. [JEE 2000, Mains, 4M out of 100]

2. (a) Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integer values of p and q respectively, are - [JEE 2001 Screening 1+1+1M out of 35]

(A) –2, –32 (B) –2, 3 (C) –6, 3 (D) –6, –32

(b) If the sum of the first 2n terms of the A.P. 2, 5, 8 .... is equal to the sum of the first n terms of the A.P. 57, 59, 61, ... then n equals -

(A) 10 (B) 12 (C) 11 (D) 13

(c) Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are

(A) not in A.P./G.P./H.P. (B) in A.P.
(C) in G.P. (D) in H.P.

(d) Let a_1, a_2.......... be positive real numbers in G.P.. For each n, let A_n, G_n, H_n, be respectively, the arithmetic mean, geometric mean and harmonic mean of a_1, a_2, a_3,........... a_n. Find an expression for the G.M. of $G_1, G_2, \ldots\ldots G_n$ in terms of $A_1, A_2, \ldots\ldots A_n$, $H_1, H_2, \ldots\ldots H_n$ [JEE 2001 (Mains) ; 5M]

3. (a) Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. If $a < b < c$ and $a+b+c=\frac{3}{2}$, then the value of a is -

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{2\sqrt{3}}$ (C) $\frac{1}{2}-\frac{1}{\sqrt{3}}$ (D) $\frac{1}{2}-\frac{1}{\sqrt{2}}$

[JEE 2002 (Screening), 3M]

(b) Let a, b be positive real numbers. If a, A_1, A_2, b are in A.P. ; a, G_1, G_2, b are in G.P. and a, H_1, H_2, b are in H.P., show that $\frac{G_1G_2}{H_1H_2}=\frac{A_1+A_2}{H_1+H_2}=\frac{(2a+b)(a+2b)}{9ab}$. [JEE 2002, Mains, 5M out of 60]

4. If a, b, c are in A.P., a^2, b^2, c^2 are in H.P., then prove that either $a = b = c$ or a, b, $-\frac{c}{2}$ form a G.P.

[JEE 2003, Mains, 4M out of 60]

5. If a, b, c are positive real numbers, then prove that $[(1 + a)(1 + b)(1 + c)]^7 > 7^7 a^4 b^4 c^4$. [JEE 2004, 4M]

6. The first term of an infinite geometric progression is x and its sum is 5. Then - [JEE 2004]

(A) $0 \le x \le 10$ (B) $0 < x < 10$ (C) $-10 < x < 0$ (D) $x > 10$

7. If total number of runs scored in n matches is $\left(\frac{n+1}{4}\right)(2^{n+1} - n - 2)$ where $n > 1$, and the runs scored in the k^{th} match are given by $k.\ 2^{n+1-k}$, where $1 \le k \le n$. Find n. **[JEE-05, Mains-2M out of 60]**

8. In quadratic equation $ax^2 + bx + c = 0$, if α, β are roots of equation, $\Delta = b^2 - 4ac$ and $\alpha + \beta$, $\alpha^2 + \beta^2$, $\alpha^3 + \beta^3$ are in G.P. then **[JEE 2005 (screening)]**

(A) $\Delta \neq 0$ (B) $b\Delta = 0$ (C) $c\Delta = 0$ (D) $\Delta = 0$

9. If $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \ldots (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $b_n = 1 - a_n$ then find the minimum natural number n_0 such that $b_n > a_n \forall n \ge n_0$ **[JEE 2006, 6M out of 184]**

Comprehension Based Question

Comprehension # 1

Let V_r denote the sum of first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r - 1)$.

Let $T_r = V_{r+1} - V_r - 2$ and $Q_r = T_{r+1} - T_r$ for $r = 1, 2, \ldots\ldots$

10. The sum $V_1 + V_2 + \ldots + V_n$ is : **[JEE 2007, 4M]**

(A) $\frac{1}{12}n(n + 1)(3n^2 - n + 1)$ (B) $\frac{1}{12}n(n + 1)(3n^2 + n + 2)$

(C) $\frac{1}{2}n(2n^2 - n + 1)$ (D) $\frac{1}{3}(2n^3 - 2n + 3)$

11. T_r is always : **[JEE 2007, 4M]**

(A) an odd number (B) an even number

(C) a prime number (D) a composite number

12. Which one of the following is a correct statement ? **[JEE 2007, 4M]**

(A) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 5

(B) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 6

(C) $Q_1, Q_2, Q_3, \ldots$ are in A.P. with common difference 11

(D) $Q_1 = Q_2 = Q_3 = \ldots$

Comprehension # 2

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \ge 2$, let A_{n-1} and H_{n-1} has arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively :

13. Which one of the following statements is correct ? **[JEE 2007, 4M]**

(A) $G_1 > G_2 > G_3 > \ldots$

(B) $G_1 < G_2 < G_3 < \ldots$

(C) $G_1 = G_2 = G_3 = \ldots$

(D) $G_1 < G_2 < G_3 < \ldots$ and $G_4 > G_5 > G_6 > \ldots$

14. Which one of the following statements is correct ? **[JEE 2007, 4M]**

(A) $A_1 > A_2 > A_3 > \ldots$ (B) $A_1 < A_2 < A_3 < \ldots$

(C) $A_1 > A_3 > A_5 > \ldots$ and $A_2 < A_4 < A_6 < \ldots$ (D) $A_1 < A_3 < A_5 < \ldots$ and $A_2 > A_4 > A_6 > \ldots$

15. Which one of the following statements is correct ? **[JEE 2007, 4M]**

(A) $H_1 > H_2 > H_3 > \ldots$ (B) $H_1 < H_2 < H_3 < \ldots$

(C) $H_1 > H_3 > H_5 > \ldots$ and $H_2 < H_4 < H_6 > \ldots$ (D) $H_1 < H_3 < H_5 < \ldots$ and $H_2 > H_4 > H_6 > \ldots$

16. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

Statement -I : The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

and

Statement -II : The numbers b_1, b_2, b_3, b_4 are in H.P. **[JEE 2008, 3M, –1M]**

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

17. If the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is **[JEE 2009, 3M, –1M]**

(A) $\frac{n(4n^2-1)c^2}{6}$ (B) $\frac{n(4n^2+1)c^2}{3}$ (C) $\frac{n(4n^2-1)c^2}{3}$ (D) $\frac{n(4n^2+1)c^2}{6}$

18. Let S_k, k = 1,2,.......,100, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!}+\sum_{k=1}^{100}\left|(k^2-3k+1)S_k\right|$ is **[JEE 10, 3M]**

19. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3,4,\ldots,11$.

If $\frac{a_1^2+a_2^2+\ldots+a_{11}^2}{11}=90$, then the value of $\frac{a_1+a_2+\ldots+a_{11}}{11}$ is equal to **[JEE 10, 3M]**

20. The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} with $a > 0$ is **[JEE 2011,4]**

21. Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i, 1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is **[JEE 2011, 4]**

22. Let $a_1, a_2, a_3, \ldots$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is **[JEE 2012, 3 (–1)]**

(A) 22 (B) 23 (C) 24 (D) 25

23. Let $S_n = \sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2$. Then S_n can take value(s) **[JEE-Advanced 2013, 4, (–1)]**

(A) 1056 (B) 1088 (C) 1120 (D) 1332

24. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller to the numbers on the removed cards is k, then k – 20 = **[JEE-Advanced 2013, 4, (–1)]**

PREVIOUS YEARS QUESTIONS **ANSWER KEY** EXERCISE-5 [B]

1. (a) D, (b) A **2.** (a) A, (b) C, (c) D, (d) $\left[(A_1, A_2, \ldots A_n)(H_1, H_2, \ldots H_n)\right]^{\frac{1}{2n}}$

3. (a) D	**6.** B	**7.** n = 7	**8.** C	**9.** 6	**10.** B	**11.** D	**12.** B
13. C	**14.** A	**15.** B	**16.** C	**17.** C	**18.** 3	**19.** 0	
20. 8	**21.** 9 or 3	**22.** D	**23.** A,D	**24.** 5			