SETS

SET : A set is a collection of well defined objects which are distinct from each other

Set are generally denoted by capital letters A, B, C, ... etc. and the elements of the set by a, b, c .... etc.

If a is an element of a set A, then we write a $\in$ A and say a belongs to A.

If a does not belong to A then we write a $\notin$ A,

e.g. The collection of first five prime natural numbers is a set containing the elements 2, 3, 5, 7, 11.

SOME IMPORTANT NUMBER SETS :

N = Set of all natural numbers

   = {1, 2, 3, 4, ...}

W = Set of all whole numbers

   = {0, 1, 2, 3, ...}

Z or I set of all integers

   = {.... –3, –2, –1, 0, 1, 2, 3, ...}

Z^+ = Set of all +ve integers

   = {1, 2, 3, ...} = N.

Z^- = Set of all –ve integers

   = (–1, –2, –3, ...}

Z_0 = The set of all non-zero integers.

   = {±1, ±2, ±3, ...}

Q = The set of all rational numbers.

   = $\left\{ \dfrac{p}{q} : p, q \in I, q \neq 0 \right\}$

R = the set of all real numbers.

R–Q = The set of all irrational numbers

e.g. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, ... π, e, log2 etc. are all irrational numbers.

METHODS TO WRITE A SET :

(i) Roster Method : In this method a set is described by listing elements, separated by commas and enclose then by curly brackets

   e.g. The set of vowels of English Alphabet may be described as {a, e, i, o, u}

(ii) Set Builder Form : In this case we write down a property or rule p Which gives us all the element of the set

	A = {x : P(x)}
e.g.	A = {x : x $\in$ N and x = 2n for n $\in$ N}
i.e.	A = {2, 4, 6, ...}
e.g.	B = {x^2 : x $\in$ z}
i.e.	B = {0, 1, 4, 9, ...}

TYPES OF SETS :

Null set or Empty set : A set having no element in it is called an Empty set or a null set or void set it is denoted by ϕ or { }

e.g. A = {x $\in$ N : 5 < x < 6} = ϕ

A set consisting of at least one element is called a non-empty set or a non-void set.

Illustration 1 :

The set A = [x : x ∈ R, x^2 = 16 and 2x = 6] equal-

(1) φ   (2) [14, 3, 4]   (3) [3]   (4) [4]

Solution :

x^2 = 16 ⇒ x = ±4

2x = 6 ⇒ x = 3

There is no value of x which satisfies both the above equations.

Thus, A = φ

Hence (1) is the correct answer

Singleton : A set consisting of a single element is called a singleton set.

e.g. Then set {0}, is a singleton set

Finite Set : A set which has only finite number of elements is called a finite set.

e.g. A = {a, b, c}

Order of a finite set : The number of elements in a finite set is called the order of the set A and is denoted O(A) or n(A). It is also called cardinal number of the set.

e.g. A = {a, b, c, d} ⇒ n(A) = 4

Infinite set : A set which has an infinite number of elements is called an infinite set.

e.g. A = {1, 2, 3, 4, ...} is an infinite set

Equal sets : Two sets A and B are said to be equal if every element of A is a member of B, and every element of B is a member of A.

If sets A and B are equal. We write A = B and A and B are not equal then A ≠ B

e.g. A = {1, 2, 6, 7} and B = {6, 1, 2, 7} ⇒ A = B

Equivalent sets : Two finite sets A and B are equivalent if their number of elements are same ie. n(A) = n(B)

e.g. A = {1, 3, 5, 7}, B = {a, b, c, d}

         n(A) = 4 and n(B) = 4 ⇒ n(A) = n(B)

Note : Equal set always equivalent but equivalent sets may not be equal

Subsets : Let A and B be two sets if every element of A is an element B, then A is called a subset of B if A is a subset of B. we write A ⊆ B

Example :   A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5, 6, 7} ⇒ A ⊆ B

     The symbol "⇒" stands for "implies"

Proper subset : If A is a subset of B and A ≠ B then A is a proper subset of B. and we write A ⊂ B

Note-1 : Every set is a subset of itself i.e. A ⊆ A for all A

Note-2 : Empty set φ is a subset of every set

Note-3 : Clearly N ⊂ W ⊂ Z ⊂ Q ⊂ R ⊂ C

Note-4 : The total number of subsets of a finite set containing n elements is 2^n

Universal set : A set consisting of all possible elements which occur in the discussion is called a Universal set and is denoted by U

Note : All sets are contained in the universal set

e.g. If A = {1, 2, 3}, B = {2, 4, 5, 6}, C = {1, 3, 5, 7} then U = {1, 2, 3, 4, 5, 6, 7} can be taken as the Universal set.

Power set : Let A be any set. The set of all subsets of A is called power set of A and is denoted by P(A)

e.g. Let A = {1, 2} then P(A) = {ϕ, {1}, {2}, {1, 2}}

e.g. Let P(ϕ) = {ϕ}

$\because$ P(P(ϕ)) = {ϕ, {ϕ}}

$\because$ P(P(P(ϕ)) = {ϕ, {ϕ}, {{ϕ}}, {ϕ, {ϕ}}

Note-1 : If A = ϕ then P(A) has one element

Note-2 : Power set of a given set is always non empty

Illustration 2 :

Two finite sets of have m and n elements respectively the total number of elements in power set of first set is 56 more thatn the total number of elements in power set of the second set find the value of m and n respectively.

Solution :

Number of elements in power set of 1st set = 2^m

Number of elements in power set of 2nd set = 2^n

Given $2^m = 2^n + 56$

$\Rightarrow$ $2^m - 2^n = 56$

$\Rightarrow$ $2^n(2^{m-n} - 1) = 2^3(2^3 - 1)$

$\Rightarrow$ n = 3 and m = 6

Do yourself - 1 :

(i)   Write the following set in roaster form :

A = {x | x is a positive integer less than 10 and $2^x - 1$ is an odd number}

(ii)   Write power set of set A = {ϕ, {ϕ}, 1}

Some Operation on Sets :

(i)   **Union of two sets :** A $\cup$ B = {x : x $\in$ A or x $\in$ B}

e.g. A = {1, 2, 3}, B = {2, 3, 4} then A $\cup$ B = {1, 2, 3, 4}

(ii)   **Intersection of two sets :** A $\cap$ B = {x : x $\in$ A and x $\in$ B}

e.g. A = {1, 2, 3, }, B = {2, 3, 4} then A $\cap$ B = {2, 3}

(iii)   **Difference of two sets :** A – B = {x : x $\in$ A and x $\notin$ B}

e.g. A = {1, 2, 3}, B = {2, 3, 4} ;   A – B = {1}

(iv)   **Complement of a set :** A' = {x : x $\notin$ A but x $\in$ U} = U – A

e.g. U = {1, 2, ..., 10}, A = {1, 2, 3, 4, 5} then A' = {6, 7, 8, 9, 10}

(v)   **De-Morgan Laws :** (A $\cup$ B)' = A' $\cap$ B' ; (A $\cap$ B)' = A' $\cup$ B'

(vi)   A – (B $\cup$ C) = (A – B) $\cap$ (A – C) ; A – (B $\cap$ C) = (A – B) $\cup$ (A – C)

(vii)   **Distributive Laws :** A $\cup$ (B $\cap$ C) = (A $\cup$ B) $\cap$ (A $\cup$ C); A $\cap$ (B $\cup$ C) = (A $\cap$ B) $\cup$ (A $\cap$ C)

(viii)   **Commutative Laws :** A $\cup$ B = B $\cup$ A ; A $\cap$ B = B $\cap$ A

(ix)   **Associative Laws :** (A $\cup$ B) $\cup$ C = A $\cup$ (B $\cup$ C) ; (A $\cap$ B) $\cap$ C = A $\cap$ (B $\cap$ C)

(x)   A $\cap$ ϕ = ϕ ; A $\cap$ U = A

A $\cup$ ϕ = A ; A $\cup$ U = U

(xi)   A $\cap$ B $\subseteq$ A ; A $\cap$ B $\subseteq$ B

(xii)   A $\subseteq$ A $\cup$ B ; B $\subseteq$ A $\cup$ B

(xiii)   A $\subseteq$ B $\Rightarrow$ A $\cap$ B = A

(xiv)   A $\subseteq$ B $\Rightarrow$ A $\cup$ B = B

Illustration 3 :

Let A = {x : x ∈ R, |x| < 1} ; B = [x : x ∈ R, |x – 1| ≥ 1] and A ∪ B = R – D, then the set D is-

(1) [x : 1 < x ≤ 2]   (2) [x : 1 ≤ x < 2]   (3) [x : 1 ≤ x ≤ 2]   (4) none of these

Solution :

A = [x : x ∈ R, –1 < x < 1]

B = [x : x ∈ R : x – 1 ≤ –1 or x – 1 ≥ 1]

   = [x : x ∈ R : x ≤ 0 or x ≥ 2]

∴ A ∪ B = R – D

where D = [x : x ∈ R, 1 ≤ x < 2]

Thus (2) is the correct answer.

Disjoint Sets :

IF A ∩ B = φ, then A, B are disjoint.

e.g. if A = {1, 2, 3}, B = {7, 8, 9} then A ∩ B = φ

Note : A ∩ A' = φ   ∴ A, A' are disjoint.

Symmetric Difference of Sets :

A Δ B = (A – B) ∪ (B – A)

● (A')' = A

● A ⊆ B ⇔ B' ⊆ A'

If A and B are any two sets, then

(i)   A – B = A ∩ B'

(ii) B – A = B ∩ A'

(iii) A – B = A ⇔ A ∩ B = φ

(iv) (A – B) ∪ B = A ∪ B

(v) (A – B) ∩ B = φ

(vi) (A – B) ∪ (B – A) = (A ∪ B) – (A ∩ B)

Venn Diagrame :

$A \cup B$ $A \cap B$ $A - B$ $B - A$

Clearly (A – B) ∪ (B – A) ∪ (A ∪ B) = A ∪ B

A'   (AΔB)=(A–B) ∪ (B–A)   Disjoint Sets
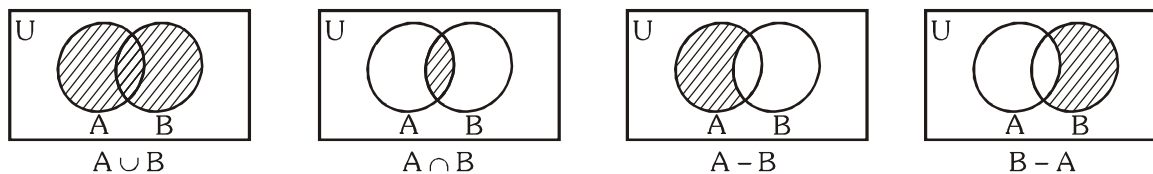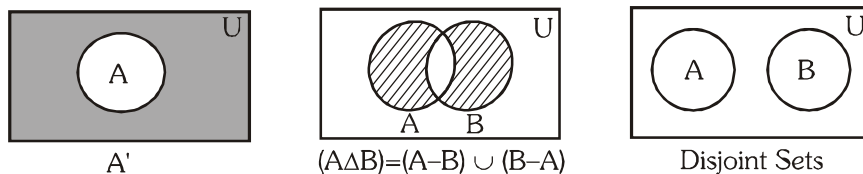
Note : A ∩ A' = φ, A ∪ A' = U

SOME IMPORTANT RESULTS ON NUMBER OF ELEMENTS IN SETS :

If A, B and C are finite sets, and U be the finite universal set, then

(i)   n(A ∪ B) = n(A) + n(B) – n(A ∩ B)

(ii) n(A ∪ B) = n(A) + n(B) ⇔ A, B are disjoint non-void sets.

(iii) n(A – B) = n(A) – n(A ∩ B) i.e. n(A – B) + n(A ∩ B) = n(A)

(iv) n(A Δ B) = No. of elements which belong to exactly one of A or B

$$= n((A - B) \cup (B - A))$$

$$= n(A - B) + n(B - A) \quad\quad [\because (A - B) \text{ and } (B - A) \text{ are disjoint}]$$

$$= n(A) - n(A \cap B) + n(B) - n(A \cap B)$$

$$= n(A) + n(B) - 2n(A \cap B)$$

$$= n(A) + n(B) - 2n(A \cap B)$$

(v) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(vi) Number of elements in exactly two of the sets A, B, C

$$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$$

(vii) number of elements in exactly one of the sets A, B, C

$$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$$

(viii) $n(A' \cup B') = n((A \cap B)') = n(U) - n(A \cap B)$

(ix) $n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B)$

Illustration 4 :

In a group of 1000 people, there are 750 who can speak Hindi and 400 who can speak Bengali. How many can speak Hindi only ?How many can spak Bengali ? How many can spak both Hindi and Bengali?

Solution :

Let A and B be the sets of persons who can speak Hindi and Bengali respectively.

then n(A ∩ B) = 1000, n(A) = 750, n(B) = 400.

Number of persons whos can speak both Hindi and Bengali

$$= n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$= 750 + 400 - 1000$$

Number of persons who can speak Hindi only = 150

$$= n(A - B) = n(A) - n(A \cap B) = 750 - 150 = 600$$

Number of persons Whos can speak Bengali only

$$= n(B - A) = n(B) - n(A \cap B) = 400 - 150 = 250$$

Do yourself - 2 :

(i) If aN = {ax : x ∈ N}, then the set 6N ∩ 8N is equal to-

(1) 8N   (2) 48N   (3) 12N   (4) 24N

(ii) If P, Q and R subsets of a set A, then R (P' ∪ Q')' =

(1) (R P) ∩ (R Q)   (2) (R Q) ∩ (R P)   (3) (R P) ∪ (R Q)   (4) none of these

(iii) If A = {x, y}, then the power set of A is-

(1) $\{x^y, y^x\}$ (2) {φ, x, y}   (3) {φ, {x} {2y}}   (4) {φ, {x}, {y}, {x, y}}

ANSWERS FOR DO YOURSELF

1. (i) {1, 2, 3, 4, 5, 6, 7, 8, 9}   (ii) {φ, {φ}, {{φ}}, {1}, {φ{φ}}, {{φ}, 1}, {φ, 1}, {φ, {φ}, 1}}
2. (i) 4   (ii) 1   (iii) 4

CHECK YOUR GRASP   SETS   EXERCISE-I

1. If A and B are two sets, then A $\cap$ (A $\cup$ B)' is equal to-
   (1) A
   (2) B
   (3) ϕ
   (4) none of these

2. If A is any set, then-
   (1) A $\cup$ A' = ϕ
   (2) A $\cup$ A' = U
   (3) A $\cap$ A' = U
   (4) none of these

3. If A, B be any two sets, then (A $\cup$ B)' is equal to-
   (1) A' $\cup$ B'
   (2) A' $\cap$ B'
   (3) A $\cap$ B
   (4) A $\cup$ B

4. If A and B be any two sets, then (A $\cap$ B)' is equal to-
   (1) A' $\cap$ B'   (2) A' $\cup$ B'   (3) A $\cap$ B   (4) A $\cup$ B

5. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A={1, 2, 5}, B = {6, 7} then A $\cap$ B' is-
   (1) B'
   (2) A
   (3) A'
   (4) B.

6. If A and B are two sets, then A $\cup$ B = A $\cap$ B iff-
   (1) A $\subseteq$ B
   (2) B $\subseteq$ A
   (3) A = B
   (4) none of these

7. Let A and B be two sets in the universal set. Then A – B equals-
   (1) A $\cap$ B'
   (2) A' $\cap$ B
   (3) A $\cap$ B
   (4) none of these

8. Two sets A, B are disjoint iff-
   (1) A $\cup$ B = ϕ
   (2) A $\cap$ B $\neq$ ϕ
   (3) A $\cap$ B = ϕ
   (4) A – B = A

9. Which of the following is a null set ?
   (1) {0}
   (2) {x : x > 0 or x < 0}
   (3) {x : x^2 = 4 or x = 3}
   (4) {x : x^2 + 1 = 0, x $\in$ R}

10. If A $\subseteq$ B, then A $\cap$ B is equal to-
    (1) A
    (2) B
    (3) A'
    (4) B'

11. If A and B are any two sets, then A $\cup$ (A $\cap$ B) is equal to-
    (1) A
    (2) B
    (3) A'
    (4) B'

12. If A and B are not disjoint, then n(A $\cup$ B) is equal to-
    (1) n(A) + n(B)
    (2) n(A) + n(B) $-$ n(A $\cap$ B)
    (3) n(A) + n(B) + n(A $\cap$ B)
    (4) n(A).n(B)

13. If A = {2, 4, 5}, B = {7, 8, 9} then n(A B) is equal to-
    (1) 6   (2) 9   (3) 3   (4) 0

14. Let A and B be two sets such that n(A) = 70, n(B) = 60 and n(A $\cup$ B) = 110. Then n(A $\cap$ B) is equal to-
    (1) 240   (2) 20   (3) 100   (4) 120

15. Which set is the subset of all given sets ?
    (1) {1, 2, 3, 4, ...}
    (2) {1}
    (3) {0}
    (4) { }

16. If Q = $\left\{ x : x = \dfrac{1}{y}, \text{ where } y \in N \right\}$, then-
    (1) 0 $\in$ Q   (2) 1 $\in$ Q   (3) 2 $\in$ Q   (4) $\dfrac{2}{3}$ $\in$ Q

17. A = {x : x $\neq$ x} represents-
    (1) {0}   (2) { }   (3) {1}   (4) {x}

18. Which of the following statements is true ?
    (1) 3 $\subseteq$ {1, 3, 5}
    (2) 3 $\in$ {1, 3, 5}
    (3) {3} $\in$ {1, 3, 5}
    (4) {3, 5} $\in$ {1, 3, 5}

19. Which of the following is a null set ?
    (1) A = {x : x > 1 and x < 1]
    (2) B = {x : x + 3 = 3}
    (3) C = {ϕ}
    (4) D = {x : x $\geq$ 1 and x $\leq$ 1}

20. P(A) = P(B) $\Rightarrow$
    (1) A $\subseteq$ B
    (2) B $\subseteq$ A
    (3) A = B
    (4) none of these

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	3	2	2	2	2	3	1	3	4	1	1	2	2	2	4
Que.	16	17	18	19	20										
Ans.	2	2	2	1	3										

1. The set $S : \{1, 2, 3, ..., 12\}$ is to be partitioned into three sets A, B, C of equal size. Thus $A \cup B \cup C = S$, $A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is- **[AIEEE - 2007]**

   (1) $12!/3!(4!)^3$ (2) $12!/3!(3!)^4$

   (3) $12!/(4!)^3$ (4) $12!/(3!)^4$

2. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then :- **[AIEEE- 2009]**

   (1) B = C   (2) $A \cap B = \phi$

   (3) A = B   (4) A = C

3. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is : **[AIEEE - 2012]**

   (1) 5^3 (2) 5^2 (3) 3^5 (4) 2^5

ANSWER KEY

Que.	1	2	3												
Ans.	3	1	3												