TRIGONOMETRIC RATIOS & IDENTITIES

1. INTRODUCTION TO TRIGONOMETRY :

The word 'trigonometry' is derived from the Greek words 'trigon' and 'metron' and it means 'measuring the sides of a triangle'. The subject was originally developed to solve geometric problems involving triangles. It was studied by sea captains for navigation, surveyor to map out the new lands, by engineers and others. Currently, trigonometry is used in many areas such as the science of seismology, designing electric circuits, describing the state of an atom, predicting the heights of tides in the ocean, analysing a musical tone and in many other areas.

(a) **Measurement of angles** : There are three systems of measurement of angles.

(i) **Sexagesimal or English System** : Here 1 right angle = 90 (degrees)

1 = 60' (minutes)

1' = 60" (seconds)

(ii) **Centesimal or French System** : Here 1 right angle = 100^g (grades)

1^g = 100' (minutes)

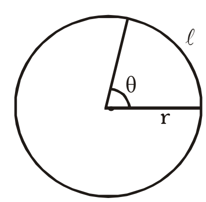
1' = 100" (seconds)

(iii) **Circular system** : Here an angle is measured in radians. One radian corresponds to the angle subtended by an arc of length 'r' at the centre of the circle of radius r. It is a constant quantity and does not depend upon the radius of the circle.

(b) Relation between the three systems : $\frac{D}{90} = \frac{G}{100} = \frac{R}{\pi/2}$

(c) If θ is the angle subtended at the centre of a circle of radius 'r',

by an arc of length 'ℓ' then $\frac{\ell}{r} = \theta$.



Note that here ℓ, r are in the same units and θ is always in radians.

Illustration 1 : If the arcs of same length in two circles subtend angles of 60 and 75 at their centres. Find the ratio of their radii.

Solution : Let r_1 and r_2 be the radii of the given circles and let their arcs of same length s subtend angles of 60 and 75 at their centres.

Now, 60 $= \left(60 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$ and $75° = \left(75 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{12}\right)^c$

$\therefore \quad \frac{\pi}{3} = \frac{s}{r_1}$ and $\frac{5\pi}{12} = \frac{s}{r_2}$

$\Rightarrow \quad \frac{\pi}{3} r_1 = s$ and $\frac{5\pi}{12} r_2 = s \quad \Rightarrow \quad \frac{\pi}{3} r_1 = \frac{5\pi}{12} r_2 \quad \Rightarrow 4r_1 = 5r_2 \quad \Rightarrow \quad r_1 : r_2 = 5 : 4$ **Ans.**

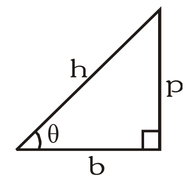
Do yourself - 1 :

(i) Express in the three systems of angular measurement, the magnitude of the angle of a regular decagon.

(ii) The radius of a circle is 30 cm. Find the length of an arc of this circle if the length of the chord of the arc is 30 cm.

2. T-RATIOS (or Trigonometric functions) :

In a right angle triangle $\sin\theta = \frac{p}{h}$; $\cos\theta = \frac{b}{h}$; $\tan\theta = \frac{p}{b}$; $\operatorname{cosec}\theta = \frac{h}{p}$; $\sec\theta = \frac{h}{b}$ and $\cot\theta = \frac{b}{p}$



'p' is perpendicular ; 'b' is base and 'h' is hypotenuse.

Note : The quantity by which the cosine falls short of unity i.e. $1 - \cos\theta$, is called the versed sine θ of θ and also by which the sine falls short of unity i.e. $1 - \sin\theta$ is called the coversed sine of θ.

3. BASIC TRIGONOMETRIC IDENTITIES :

(1) $\sin\theta . \operatorname{cosec}\theta = 1$

(2) $\cos\theta . \sec\theta = 1$

(3) $\tan\theta . \cot\theta = 1$

(4) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ & $\cot\theta = \frac{\cos\theta}{\sin\theta}$

(5) $\sin^2\theta + \cos^2\theta = 1$ or $\sin^2\theta = 1 - \cos^2\theta$ or $\cos^2\theta = 1 - \sin^2\theta$

(6) $\sec^2\theta - \tan^2\theta = 1$ or $\sec^2\theta = 1 + \tan^2\theta$ or $\tan^2\theta = \sec^2\theta - 1$

(7) $\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$

(8) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$ or $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$ or $\cot^2\theta = \operatorname{cosec}^2\theta - 1$

(9) $\operatorname{cosec}\theta + \cot\theta = \frac{1}{\operatorname{cosec}\theta - \cot\theta}$

(10) Expressing trigonometrical ratio in terms of each other :

	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$
$\sin\theta$	$\sin\theta$	$\sqrt{1-\cos^2\theta}$	$\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\sqrt{1+\cot^2\theta}}$	$\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\frac{1}{\operatorname{cosec}\theta}$
$\cos\theta$	$\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\frac{1}{\sqrt{1+\tan^2\theta}}$	$\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$	$\frac{1}{\sec\theta}$	$\frac{\sqrt{\operatorname{cosec}^2\theta-1}}{\operatorname{cosec}\theta}$
$\tan\theta$	$\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\frac{1}{\cot\theta}$	$\sqrt{\sec^2\theta-1}$	$\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\cot\theta$	$\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\cot\theta$	$\frac{1}{\sqrt{\sec^2\theta-1}}$	$\sqrt{\operatorname{cosec}^2\theta-1}$
$\sec\theta$	$\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\sqrt{1+\tan^2\theta}$	$\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$	$\sec\theta$	$\frac{\operatorname{cosec}\theta}{\sqrt{\operatorname{cosec}^2\theta-1}}$
$\operatorname{cosec}\theta$	$\frac{1}{\sin\theta}$	$\frac{1}{\sqrt{1-\cos^2\theta}}$	$\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\sqrt{1+\cot^2\theta}$	$\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\operatorname{cosec}\theta$

Illustration 2 : If $\sin\theta + \sin^2\theta = 1$, then prove that $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1 = 0$

Solution : Given that $\sin\theta = 1 - \sin^2\theta = \cos^2\theta$

L.H.S. $= \cos^6\theta(\cos^2\theta + 1)^3 - 1 = \sin^3\theta(1 + \sin\theta)^3 - 1 = (\sin\theta + \sin^2\theta)^3 - 1 = 1 - 1 = 0$

Illustration 3 : $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$ is equal to

(A) 0 (B) 1 (C) –2 (D) none of these

Solution : $2[(\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)] - 3[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta] + 1$

$= 2[1 - 3\sin^2\theta\cos^2\theta] - 3[1 - 2\sin^2\theta\cos^2\theta] + 1$

$= 2 - 6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta + 1 = 0$ **Ans.(A)**

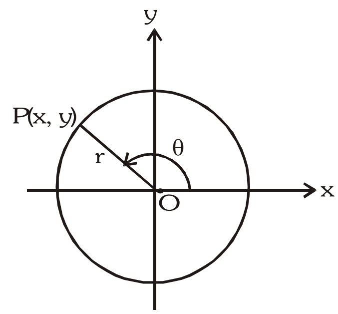
Do yourself - 2 :

(i) If $\cot\theta = \dfrac{4}{3}$, then find the value of sinθ, cosθ and cosecθ in first quadrant.

(ii) If sinθ + cosecθ = 2, then find the value of $\sin^8\theta + \text{cosec}^8\theta$

4. NEW DEFINITION OF T-RATIOS :

By using rectangular coordinates the definitions of trigonometric functions can be extended to angles of any size in the following way (see diagram). A point P is taken with coordinates (x, y). The radius vector OP has length r and the angle θ is taken as the directed angle measured anticlockwise from the x-axis. The three main trigonometric functions are then defined in terms of r and the coordinates x and y.



sinθ = y/r,

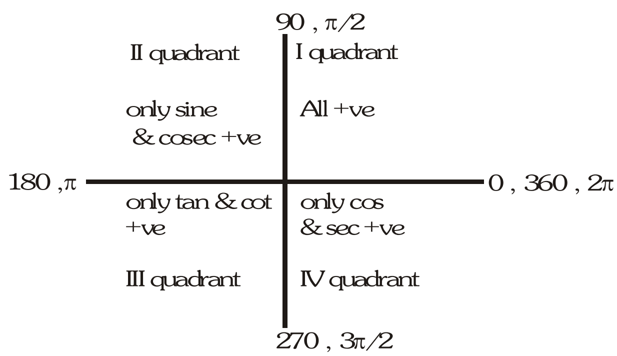
cosθ = x/r

tanθ = y/x,

(The other function are reciprocals of these)

This can give negative values of the trigonometric functions.

5. SIGNS OF TRIGONOMETRIC FUNCTIONS IN DIFFERENT QUADRANTS :



6. TRIGONOMETRIC FUNCTIONS OF ALLIED ANGLES :

(a) sin (2nπ + θ) = sin θ, cos (2nπ + θ) = cos θ, where n ∈ I

(b)

sin (–θ) = – sin θ	cos (–θ) = cos θ
sin(90 – θ) = cosθ	cos(90 – θ) = sinθ
sin(90 + θ) = cosθ	cos(90 + θ) = –sinθ
sin(180 – θ) = sinθ	cos(180 – θ) = –cosθ
sin(180 + θ) = –sinθ	cos(180 + θ) = –cosθ
sin(270 – θ) = –cosθ	cos(270 – θ) = –sinθ
sin(270 + θ) = –cosθ	cos(270 + θ) = sinθ
sin (360 – θ) = –sinθ	cos(360 – θ) = cosθ
sin (360 + θ) = sinθ	cos(360 + θ) = cosθ

7. VALUES OF T-RATIOS OF SOME STANDARD ANGLES :

Angles / T-ratio	0	30	45	60	90	180	270
	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	–1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	–1	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	N.D.	0	N.D.
$\cot\theta$	N.D.	$\sqrt{3}$	1	$1/\sqrt{3}$	0	N.D.	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	N.D.	–1	N.D.
$\text{cosec}\theta$	N.D.	2	$\sqrt{2}$	$2/\sqrt{3}$	1	N.D.	–1

N.D. $\rightarrow$ Not Defined

(a) $\sin n\pi = 0$; $\cos n\pi = (-1)^n$; $\tan n\pi = 0$ where $n \in I$

(b) $\sin(2n+1)\dfrac{\pi}{2} = (-1)^n$; $\cos(2n+1)\dfrac{\pi}{2} = 0$ where $n \in I$

Illustration 4 : If $\sin\theta = -\dfrac{1}{2}$ and $\tan\theta = \dfrac{1}{\sqrt{3}}$ then θ is equal to -

(A) 30 (B) 150 (C) 210 (D) none of these

Solution : Let us first find out θ lying between 0 and 360 .

Since $\sin\theta = -\dfrac{1}{2} \Rightarrow \theta = 210$ or 330 and $\tan\theta = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = 30$ or 210

Hence , $\theta = 210$ or $\dfrac{7\pi}{6}$ is the value satisfying both. **Ans. (C)**

Do yourself - 3 :

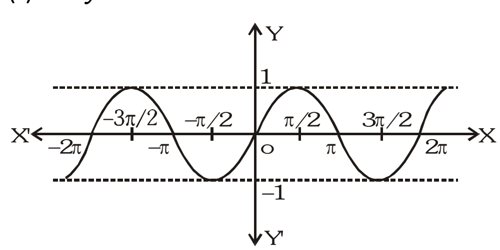
(i) If $\cos\theta = -\dfrac{1}{2}$ and $\pi < \theta < \dfrac{3\pi}{2}$, then find the value of $4\tan^2\theta - 3\text{cosec}^2\theta$.

(ii) Prove that : (a) $\cos 570 \sin 510 + \sin(-330)\cos(-390) = 0$

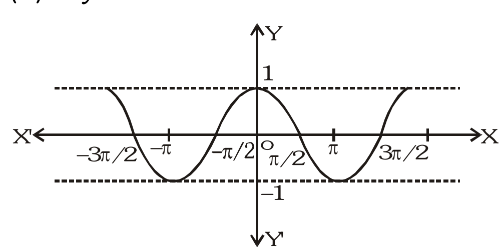
(b) $\tan\dfrac{11\pi}{3} - 2\sin\dfrac{9\pi}{3} - \dfrac{3}{4}\text{cosec}^2\dfrac{\pi}{4} + 4\cos^2\dfrac{17\pi}{6} = \dfrac{3-2\sqrt{3}}{2}$

8. GRAPH OF TRIGONOMETRIC FUNCTIONS :

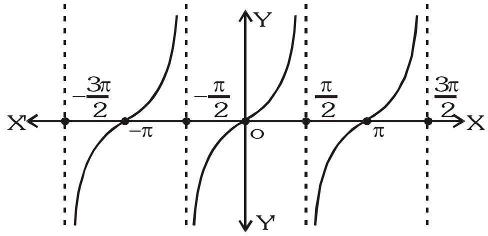
(i) $y = \sin x$



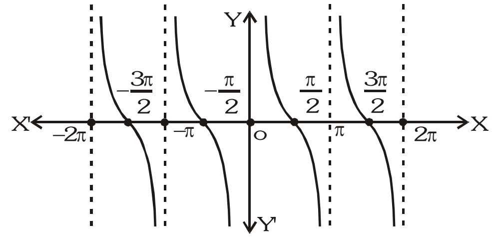
(ii) $y = \cos x$



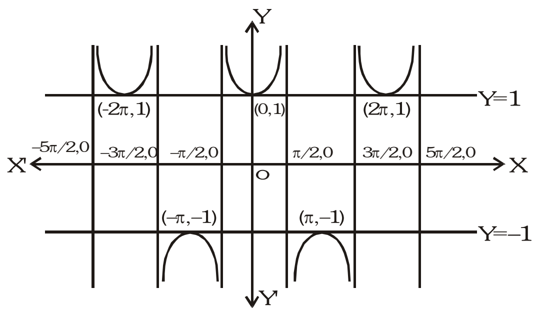
(iii) y = tanx



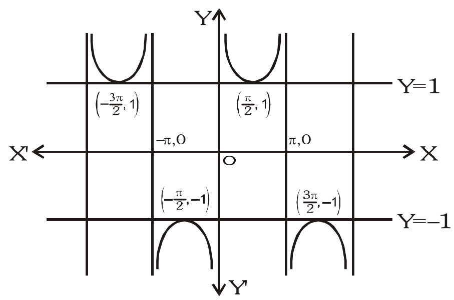
(iv) y = cotx



(v) y = secx



(vi) y = cosecx



9. DOMAINS, RANGES AND PERIODICITY OF TRIGONOMETRIC FUNCTIONS :

T-Ratio	Domain	Range	Period
sin x	R	[–1,1]	2π
cos x	R	[–1,1]	2π
tan x	R–{(2n+1)π/2 ; n∈I}	R	π
cot x	R–{nπ : n ∈ I}	R	π
sec x	R– {(2n+1) π/2 : n ∈ I}	(–∞,–1] ∪[1,∞)	2π
cosec x	R– {nπ : n ∈ I}	(–∞,–1] ∪[1,∞)	2π

10. TRIGONOMETRIC RATIOS OF THE SUM & DIFFERENCE OF TWO ANGLES :

(i) sin (A + B) = sin A cos B + cos A sin B.

(ii) sin (A – B) = sin A cos B – cos A sin B.

(iii) cos (A + B) = cos A cos B – sin A sin B

(iv) cos (A – B) = cos A cos B + sin A sin B

(v) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(vi) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

(vii) $\cot(A+B) = \dfrac{\cot B \cot A - 1}{\cot B + \cot A}$

(viii) $\cot(A-B) = \dfrac{\cot B \cot A + 1}{\cot B - \cot A}$

Some more results :

(i) $\sin^2 A - \sin^2 B = \sin(A+B).\sin(A-B) = \cos^2 B - \cos^2 A$.

(ii) $\cos^2 A - \sin^2 B = \cos(A+B).\cos(A-B)$.

Illustration 5 : Prove that $\sqrt{3}$ cosec20 – sec20 = 4.

Solution :

$$\text{L.H.S.} = \frac{\sqrt{3}}{\sin 20^\circ} - \frac{1}{\cos 20^\circ} = \frac{\sqrt{3}\cos 20^\circ - \sin 20^\circ}{\sin 20^\circ.\cos 20^\circ}$$

$$= \frac{4\left(\frac{\sqrt{3}}{2}\cos 20^\circ - \frac{1}{2}\sin 20^\circ\right)}{2\sin 20^\circ \cos 20^\circ} = \frac{4(\sin 60.\cos 20^\circ - \cos 60^\circ.\sin 20^\circ)}{\sin 40^\circ}$$

$$= 4.\frac{\sin(60^\circ - 20^\circ)}{\sin 40^\circ} = 4.\frac{\sin 40^\circ}{\sin 40^\circ} = 4 = \text{R.H.S.}$$

Illustration 6 : Prove that $\tan 70 = \cot 70 + 2\cot 40$.

Solution : L.H.S. = $\tan 70° = \tan(20° + 50°) = \dfrac{\tan 20° + \tan 50°}{1 - \tan 20° \tan 50°}$

or $\tan 70 - \tan 20 \tan 50 \tan 70 = \tan 20 + \tan 50$

or $\tan 70 = \tan 70 \tan 50 \tan 20 + \tan 20 + \tan 50 = 2 \tan 50 + \tan 20$

$= \cot 70 + 2\cot 40 =$ R.H.S.

Do yourself - 4 :

(i) If $\sin A = \dfrac{3}{5}$ and $\cos B = \dfrac{9}{41}$, $0 < A \;\&\; B < \dfrac{\pi}{2}$, then find the value of the following :

(a) $\sin(A + B)$ (b) $\sin(A - B)$ (c) $\cos(A + B)$ (d) $\cos(A - B)$

(ii) If $x + y = 45$, then prove that :

(a) $(1 + \tan x)(1 + \tan y) = 2$ (b) $(\cot x - 1)(\cot y - 1) = 2$

11. FORMULAE TO TRANSFORM THE PRODUCT INTO SUM OR DIFFERENCE :

(i) $2 \sin A \cos B = \sin (A+ B) + \sin (A - B)$.

(ii) $2 \cos A \sin B = \sin (A + B) - \sin (A - B)$.

(iii) $2 \cos A \cos B = \cos (A + B) + \cos (A - B)$

(iv) $2 \sin A \sin B = \cos (A - B) - \cos (A + B)$

Illustration 7 : If $\sin 2A = \lambda \sin 2B$, then prove that $\dfrac{\tan(A+B)}{\tan(A-B)} = \dfrac{\lambda+1}{\lambda-1}$

Solution : Given $\sin 2A = \lambda \sin 2B$

$$\Rightarrow \frac{\sin 2A}{\sin 2B} = \frac{\lambda}{1}$$

Applying componendo & dividendo,

$$\frac{\sin 2A + \sin 2B}{\sin 2B - \sin 2A} = \frac{\lambda+1}{1-\lambda}$$

$$\Rightarrow \frac{2\sin\left(\dfrac{2A+2B}{2}\right)\cos\left(\dfrac{2A-2B}{2}\right)}{2\cos\left(\dfrac{2B+2A}{2}\right)\sin\left(\dfrac{2B-2A}{2}\right)} = \frac{\lambda+1}{1-\lambda}$$

$$\Rightarrow \frac{\sin(A+B)\cos(A-B)}{\cos(A+B)\sin\{-(A-B)\}} = \frac{\lambda+1}{1-\lambda} \qquad \Rightarrow \frac{\sin(A+B)\cos(A-B)}{\cos(A+B)\times -\sin(A-B)} = \frac{\lambda+1}{-(\lambda-1)}$$

$$\Rightarrow \frac{\sin(A+B)\cos(A-B)}{\cos(A+B)\sin(A-B)} = \frac{\lambda+1}{\lambda-1} \qquad \Rightarrow \tan(A+B)\cot(A-B) = \frac{\lambda+1}{\lambda-1}$$

$$\Rightarrow \frac{\tan(A+B)}{\tan(A-B)} = \frac{\lambda+1}{\lambda-1}$$

12. FORMULAE TO TRANSFORM SUM OR DIFFERENCE INTO PRODUCT :

(i) $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(ii) $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

(iii) $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

(iv) $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

Illustration 8 : $\frac{\sin 5\theta + \sin 2\theta - \sin\theta}{\cos 5\theta + 2\cos 3\theta + 2\cos^2\theta + \cos\theta}$ is equal to -

(A) $\tan\theta$ (B) $\cos\theta$ (C) $\cot\theta$ (D) none of these

Solution : L.H.S.= $\frac{2\sin 2\theta\cos 3\theta + \sin 2\theta}{2\cos 3\theta.\cos 2\theta + 2\cos 3\theta + 2\cos^2\theta} = \frac{\sin 2\theta[2\cos 3\theta + 1]}{2\left[\cos 3\theta(\cos 2\theta + 1) + (\cos^2\theta)\right]}$

$$= \frac{\sin 2\theta[2\cos 3\theta + 1]}{2\left[\cos 3\theta(2\cos^2\theta) + \cos^2\theta\right]} = \frac{\sin 2\theta(2\cos 3\theta + 1)}{2\cos^2\theta(2\cos 3\theta + 1)} = \tan\theta$$

Ans. (A)

Illustration 9 : Show that sin12 .sin48 .sin54 = 1/8

Solution : L.H.S. = $\frac{1}{2}[\cos 36° - \cos 60°]\sin 54° = \frac{1}{2}\left[\cos 36°\sin 54° - \frac{1}{2}\sin 54°\right]$

$$= \frac{1}{4}[2\cos 36°\sin 54° - \sin 54°] = \frac{1}{4}[\sin 90° + \sin 18° - \sin 54°]$$

$$= \frac{1}{4}[1 - (\sin 54° - \sin 18°)] = \frac{1}{4}[1 - 2\sin 18°\cos 36°]$$

$$= \frac{1}{4}\left[1 - \frac{2\sin 18°}{\cos 18°}\cos 18°\cos 36°\right] = \frac{1}{4}\left[1 - \frac{\sin 36°\cos 36°}{\cos 18°}\right]$$

$$= \frac{1}{4}\left[1 - \frac{2\sin 36°\cos 36°}{2\cos 18°}\right] = \frac{1}{4}\left[1 - \frac{\sin 72°}{2\sin 72°}\right] = \frac{1}{4}\left[1 - \frac{1}{2}\right] = \frac{1}{8} = \text{R.H.S.}$$

Do yourself - 5 :

(i) Simplify $\frac{\sin 75° - \sin 15°}{\cos 75° + \cos 15°}$

(ii) Prove that

(a) (sin3A + sinA)sinA + (cos3A – cosA)cosA = 0

(b) cos20 cos40 cos60 cos80 $= \frac{1}{16}$

(c) $\frac{\sin 8\theta\cos\theta - \sin 6\theta\cos 3\theta}{\cos 2\theta\cos\theta - \sin 3\theta\sin 4\theta} = \tan 2\theta$

13. TRIGONOMETRIC RATIOS OF SUM OF MORE THAN TWO ANGLES :

(i) sin (A+B+C) = sinAcosBcosC + sinBcosAcosC + sinCcosAcosB − sinAsinBsinC

= ΣsinA cosB cosC − Πsin A

= cosA cosB cosC [tanA + tanB + tanC − tanA tanB tanC]

(ii) cos (A+B+C) = cosA cosB cosC − sinA sinB cosC − sinA cosB sinC − cosA sinB sinC

= Πcos A − Σsin A sin B cos C

= cos A cos B cos C [1 − tan A tan B − tan B tan C − tan C tan A]

(iii) $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A} = \dfrac{S_1 - S_3}{1 - S_2}$

Do yourself - 6 :

Prove the above identities

14. TRIGONOMETRIC RATIOS OF MULTIPLE ANGLES :

(a) Trigonometrical ratios of an angle 2θ in terms of the angle θ :

(i) $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

(ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

(iii) $1 + \cos 2\theta = 2\cos^2\theta$

(iv) $1 - \cos 2\theta = 2\sin^2\theta$

(v) $\tan\theta = \dfrac{1-\cos 2\theta}{\sin 2\theta} = \dfrac{\sin 2\theta}{1+\cos 2\theta}$

(vi) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Illustration 10 : Prove that : $\dfrac{2\cos 2A + 1}{2\cos 2A - 1} = \tan(60° + A)\tan(60° - A)$.

Solution : R.H.S. = tan(60 + A) tan(60 − A)

$$= \left(\frac{\tan 60° + \tan A}{1 - \tan 60° \tan A}\right)\left(\frac{\tan 60° - \tan A}{1 + \tan 60° \tan A}\right) = \left(\frac{\sqrt{3} + \tan A}{1 - \sqrt{3}\tan A}\right)\left(\frac{\sqrt{3} - \tan A}{1 + \sqrt{3}\tan A}\right)$$

$$= \frac{3 - \tan^2 A}{1 - 3\tan^2 A} = \frac{3 - \dfrac{\sin^2 A}{\cos^2 A}}{1 - 3\dfrac{\sin^2 A}{\cos^2 A}} = \frac{3\cos^2 A - \sin^2 A}{\cos^2 A - 3\sin^2 A} = \frac{2\cos^2 A + \cos^2 A - 2\sin^2 A + \sin^2 A}{2\cos^2 A - 2\sin^2 A - \sin^2 A - \cos^2 A}$$

$$= \frac{2(\cos^2 A - \sin^2 A) + \cos^2 A + \sin^2 A}{2(\cos^2 A - \sin^2 A) - (\sin^2 A + \cos^2 A)} = \frac{2\cos 2A + 1}{2\cos 2A - 1} = \text{L.H.S.}$$

Do yourself - 7 :

(i) Prove that :

(a) $\dfrac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$

(b) $\dfrac{1+\sin 2\theta + \cos 2\theta}{1 + \sin 2\theta - \cos 2\theta} = \cot\theta$

(c) $\dfrac{\sec 8\theta - 1}{\sec 4\theta - 1} = \dfrac{\tan 8\theta}{\tan 2\theta}$

(b) Trigonometrical ratios of an angle 3θ in terms of the angle θ :

(i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. (ii) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

(iii) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

Illustration 11 : Prove that : $\tan A + \tan(60° + A) + \tan(120° + A) = 3\tan 3A$

Solution : L.H.S. $= \tan A + \tan(60° + A) + \tan(120° + A)$

$= \tan A + \tan(60° + A) + \tan\{180° - (60° - A)\}$

$= \tan A + \tan(60° + A) - \tan(60° - A)$ $[\because \tan(180° - \theta) = -\tan\theta]$

$$= \tan A + \frac{\tan 60° + \tan A}{1 - \tan 60° \tan A} - \frac{\tan 60° - \tan A}{1 + \tan 60° \tan A} = \tan A + \frac{\sqrt{3} + \tan A}{1 - \sqrt{3}\tan A} - \frac{\sqrt{3} - \tan A}{1 + \sqrt{3}\tan A}$$

$$= \tan A + \frac{\sqrt{3} + \tan A + 3\tan A + \sqrt{3}\tan^2 A - \sqrt{3} + \tan A + 3\tan A - \sqrt{3}\tan^2 A}{(1 - \sqrt{3}\tan A)(1 + \sqrt{3}\tan A)}$$

$$= \tan A + \frac{8\tan A}{1 - 3\tan^2 A} = \frac{\tan A - 3\tan^3 A + 8\tan A}{1 - 3\tan^2 A}$$

$$= \frac{9\tan A - 3\tan^3 A}{1 - 3\tan^2 A} = 3\left(\frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}\right) = 3\tan 3A = \text{R.H.S.}$$

Do yourself - 8 :

(i) Prove that :

(a) $\cot\theta \cot(60° - \theta) \cot(60° + \theta) = \cot 3\theta$ (b) $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

(c) $\sin 4\theta = 4\sin\theta\cos^3\theta - 4\cos\theta\sin^3\theta$

15. TRIGONOMETRIC RATIOS OF SUB MULTIPLE ANGLES :

Since the trigonometric relations are true for all values of angle θ, they will be true if instead of θ be substitute $\dfrac{\theta}{2}$

(i) $\sin\theta = 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2} = \dfrac{2\tan\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}$

(ii) $\cos\theta = \cos^2\dfrac{\theta}{2} - \sin^2\dfrac{\theta}{2} = 2\cos^2\dfrac{\theta}{2} - 1 = 1 - 2\sin^2\dfrac{\theta}{2} = \dfrac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}$

(iii) $1 + \cos\theta = 2\cos^2\dfrac{\theta}{2}$

(iv) $1 - \cos\theta = 2\sin^2\dfrac{\theta}{2}$

(v) $\tan\dfrac{\theta}{2} = \dfrac{1 - \cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1 + \cos\theta}$

(vi) $\tan\theta = \dfrac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$

(vii) $\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$

(viii) $\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

(ix) $\tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

(x) $2\sin\dfrac{\theta}{2} = \pm\sqrt{1+\sin\theta} \pm \sqrt{1-\sin\theta}$

(xi) $2\cos\dfrac{\theta}{2} = \pm\sqrt{1+\sin\theta} \mp \sqrt{1-\sin\theta}$

(xii) $\tan\dfrac{\theta}{2} = \dfrac{\pm\sqrt{1+\tan^2\theta}-1}{\tan\theta}$

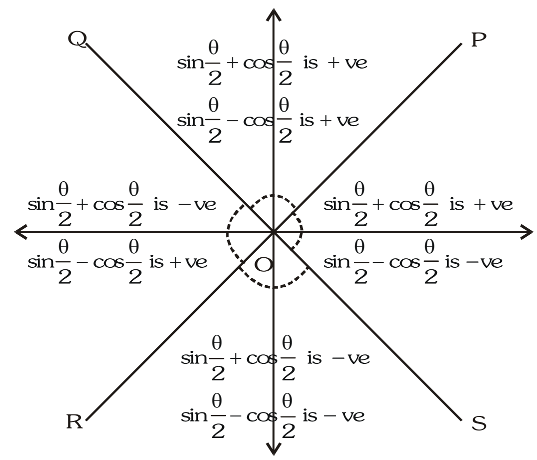


Illustration 12: $\sin 67\frac{1}{2} + \cos 67\frac{1}{2}$ is equal to

(A) $\frac{1}{2}\sqrt{4+2\sqrt{2}}$ (B) $\frac{1}{2}\sqrt{4-2\sqrt{2}}$ (C) $\frac{1}{4}\left(\sqrt{4+2\sqrt{2}}\right)$ (D) $\frac{1}{4}\left(\sqrt{4-2\sqrt{2}}\right)$

Solution : $\sin 67\frac{1}{2} + \cos 67\frac{1}{2} = \sqrt{1+\sin 135^\circ} = \sqrt{1+\frac{1}{\sqrt{2}}}$ (using $\cos A + \sin A = \sqrt{1+\sin 2A}$)

$= \frac{1}{2}\sqrt{4+2\sqrt{2}}$

Ans.(A)

Do yourself - 9 :

(i) Find the value of

(a) $\sin\frac{\pi}{8}$ (b) $\cos\frac{\pi}{8}$ (c) $\tan\frac{\pi}{8}$

16. TRIGONOMETRIC RATIOS OF SOME STANDARD ANGLES :

(i) $\sin 18^\circ = \sin\frac{\pi}{10} = \frac{\sqrt{5}-1}{4} = \cos 72^\circ = \cos\frac{2\pi}{5}$

(ii) $\cos 36^\circ = \cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4} = \sin 54^\circ = \sin\frac{3\pi}{10}$

(iii) $\sin 72^\circ = \sin\frac{2\pi}{5} = \frac{\sqrt{10+2\sqrt{5}}}{4} = \cos 18^\circ = \cos\frac{\pi}{10}$

(iv) $\sin 36^\circ = \sin\frac{\pi}{5} = \frac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54^\circ = \cos\frac{3\pi}{10}$

(v) $\sin 15^\circ = \sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ = \cos\frac{5\pi}{12}$

(vi) $\cos 15^\circ = \cos\frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \sin 75^\circ = \sin\frac{5\pi}{12}$

(vii) $\tan 15^\circ = \tan\frac{\pi}{12} = 2-\sqrt{3} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \cot 75^\circ = \cot\frac{5\pi}{12}$

(viii) $\tan 75^\circ = \tan\frac{5\pi}{12} = 2+\sqrt{3} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \cot 15^\circ = \cot\frac{\pi}{12}$

(ix) $\tan(22.5^\circ) = \tan\frac{\pi}{8} = \sqrt{2}-1 = \cot(67.5^\circ) = \cot\frac{3\pi}{8}$

(x) $\tan(67.5^\circ) = \tan\frac{3\pi}{8} = \sqrt{2}+1 = \cot(22.5^\circ) = \cot\frac{\pi}{8}$

Illustration 13 : Evaluate sin78 – sin66 – sin42 + sin6 .

Solution : The expression = (sin78 – sin42) – (sin66 – sin6) = 2cos(60) sin(18) – 2cos36 . sin30

$$= \sin 18 - \cos 36 = \left(\frac{\sqrt{5}-1}{4}\right) - \left(\frac{\sqrt{5}+1}{4}\right) = -\frac{1}{2}$$

Do yourself - 10 :

(i) Find the value of

(a) $\sin\frac{\pi}{10} + \sin\frac{13\pi}{10}$ (b) $\cos^2 48° - \sin^2 12°$

17. CONDITIONAL TRIGONOMETRIC IDENTITIES :

If A + B + C = 180 , then

(i) tan A + tan B + tan C = tan A tan B tan C

(ii) cot A cot B + cot B cot C + cot C cot A = 1

(iii) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

(iv) $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$

(v) sin 2A + sin 2B + sin 2C = 4 sinA sinB sinC

(vi) cos 2A + cos 2B + cos 2C =–1–4 cosA cosB cosC

(vii) sin A + sin B + sin C = 4 $\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(viii) cos A + cos B + cos C = 1 + 4 $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Illustration 14 : In any triangle ABC, sin A – cos B = cos C, then angle B is

(A) $\pi/2$ (B) $\pi/3$ (C) $\pi/4$ (D) $\pi/6$

Solution : We have , sin A – cos B = cos C

sin A = cos B + cos C

$$\Rightarrow \quad 2\sin\frac{A}{2}\cos\frac{A}{2} = 2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)$$

$$\Rightarrow \quad 2\sin\frac{A}{2}\cos\frac{A}{2} = 2\cos\left(\frac{\pi-A}{2}\right)\cos\left(\frac{B-C}{2}\right) \quad \because \ A + B + C = \pi$$

$$\Rightarrow \quad 2\sin\frac{A}{2}\cos\frac{A}{2} = 2\sin\frac{A}{2}\cos\left(\frac{B-C}{2}\right)$$

$$\Rightarrow \quad \cos\frac{A}{2} = \cos\frac{B-C}{2} \ \text{ or } \ A = B - C \ ; \ \text{But } A + B + C = \pi$$

Therefore 2B = π $\Rightarrow$ B = $\pi/2$ **Ans.(A)**

Illustration 15 : If A + B + C = $\frac{3\pi}{2}$, then cos 2A + cos 2B + cos2C is equal to-

(A) 1 – 4cosA cosB cosC (B) 4 sinA sin B sinC

(C) 1 + 2cosA cosB cosC (D) 1 – 4 sinA sinB sinC

Solution : cos 2A + cos 2B + cos 2C = 2 cos (A + B) cos (A – B) + cos 2C

$$= 2\cos\left(\frac{3\pi}{2} - C\right)\cos(A-B) + \cos 2C \quad \because \ A + B + C = \frac{3\pi}{2}$$

$$= -2\sin C\cos(A-B) + 1 - 2\sin^2 C = 1 - 2\sin C\,[\cos(A-B) + \sin C\,)$$

$$= 1 - 2\sin C\,[\cos(A-B) + \sin\left(\frac{3\pi}{2} - (A+B)\right)]$$

$$= 1 - 2\sin C\,[\cos(A-B) - \cos(A+B)\,] = 1 - 4\sin A\sin B\sin C$$ **Ans.(D)**

Do yourself - 11 :

(i) If ABCD is a cyclic quadrilateral, then find the value of sinA + sinB − sinC − sinD

(ii) If A + B + C = $\frac{\pi}{2}$, then find the value of tanA tanB + tanBtanC + tanC tanA

18. MAXIMUM & MINIMUM VALUES OF TRIGONOMETRIC EXPRESSIONS :

(i) $a\cos\theta + b\sin\theta$ will always lie in the interval $[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$ i.e. the maximum and minimum values are $\sqrt{a^2+b^2}$, $-\sqrt{a^2+b^2}$ respectively.

(ii) Minimum value of $a^2\tan^2\theta + b^2\cot^2\theta = 2ab$ where a, b > 0

(iii) $-\sqrt{a^2+b^2+2ab\cos(\alpha-\beta)} \le a\cos(\alpha+\theta) + b\cos(\beta+\theta) \le \sqrt{a^2+b^2+2ab\cos(\alpha-\beta)}$ where α and β are known angles.

(iv) If $\alpha, \beta, \in \left(0, \frac{\pi}{2}\right)$ and $\alpha + \beta = \sigma$ (constant) then

(i) Maximum value of the expression $\cos\alpha\cos\beta$, $\cos\alpha + \cos\beta$, $\sin\alpha\sin\beta$ or $\sin\alpha + \sin\beta$ occurs when $\alpha = \beta = \sigma/2$

(ii) Minimum value of $\sec\alpha + \sec\beta$, $\tan\alpha + \tan\beta$, $\text{cosec}\,\alpha + \text{cosec}\,\beta$ occurs when $\alpha = \beta = \sigma/2$

(v) If A, B, C are the angles of a triangle then maximum value of sin A + sin B + sin C and sin A sin B sin C occurs when A= B =C = 60

(vi) In case a quadratic in $\sin\theta$ & $\cos\theta$ is given then the maximum or minimum values can be obtained by making perfect square.

Illustration 16 : Prove that : $-4 \le 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3 \le 10$, for all values of θ.

Solution : We have, $5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) = 5\cos\theta + 3\cos\theta\cos\frac{\pi}{3} - 3\sin\theta\sin\frac{\pi}{3} = \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta$

Since, $-\sqrt{\left(\frac{13}{2}\right)^2 + \left(-\frac{3\sqrt{3}}{2}\right)^2} \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta \le \sqrt{\left(\frac{13}{2}\right)^2 + \left(-\frac{3\sqrt{3}}{2}\right)^2}$

$\Rightarrow \quad -7 \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta \le 7$

$\Rightarrow \quad -7 \le 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) \le 7$ for all θ.

$\Rightarrow \quad -7 + 3 \le 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3 \le 7 + 3$ for all θ.

$\Rightarrow \quad -4 \le 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3 \le 10$ for all θ.

Illustration 17 : Find the maximum value of $1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2\cos\left(\frac{\pi}{4} - \theta\right)$ -

(A) 1 (B) 2 (C) 3 (D) 4

Solution : We have $1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2\cos\left(\frac{\pi}{4} - \theta\right)$

$= 1 + \frac{1}{\sqrt{2}}(\cos\theta + \sin\theta) + \sqrt{2}(\cos\theta + \sin\theta) = 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)(\cos\theta + \sin\theta)$

$= 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right) . \sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right)$

$\therefore$ maximum value $= 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right) . \sqrt{2} = 4$ **Ans. (D)**

Do yourself - 12 :

(i) Find maximum and minimum value of $5\cos\theta + 3\sin\left(\theta+\frac{\pi}{6}\right)$ for all real values of θ.

(ii) Find the minimum value of $\cos\theta + \cos2\theta$ for all real values of θ.

(iii) Find maximum and minimum value of $\cos^2\theta - 6\sin\theta\cos\theta + 3\sin^2\theta + 2$.

19. IMPORTANT RESULTS :

(i) $\sin\theta \sin(60° - \theta)\sin(60° + \theta) = \frac{1}{4}\sin 3\theta$

(ii) $\cos\theta.\cos(60° - \theta)\cos(60° + \theta) = \frac{1}{4}\cos 3\theta$

(iii) $\tan\theta \tan(60° - \theta)\tan(60° + \theta) = \tan 3\theta$

(iv) $\cot\theta \cot(60° - \theta)\cot(60° + \theta) = \cot 3\theta$

(v) (a) $\sin^2\theta + \sin^2(60° + \theta) + \sin^2(60° - \theta) = \frac{3}{2}$ (b) $\cos^2\theta + \cos^2(60° + \theta) + \cos^2(60° - \theta) = \frac{3}{2}$

(vi) (a) If $\tan A + \tan B + \tan C = \tan A \tan B \tan C$, then $A + B + C = n\pi,\ n \in I$

(b) If $\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$, then $A + B + C = (2n + 1)\frac{\pi}{2}, n \in I$

(vii) $\cos\theta\cos2\theta\cos4\theta \ldots \cos(2^{n-1}\theta) = \frac{\sin(2^n\theta)}{2^n\sin\theta}$

(viii) (a) $\cot A - \tan A = 2\cot 2A$
(b) $\cot A + \tan A = 2\operatorname{cosec} 2A$

(ix) $\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots \sin(\alpha + \overline{n-1}\,\beta) = \dfrac{\sin\left\{\alpha+\left(\frac{n-1}{2}\right)\beta\right\}\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$

(x) $\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \ldots + \cos(\alpha + \overline{n-1}\,\beta) = \dfrac{\cos\left\{\alpha+\left(\frac{n-1}{2}\right)\beta\right\}\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$

Illustration 18 : Prove that $\tan A + 2\tan 2A + 4\tan 4A + 8\cot 8A = \cot A$.

Solution : $8\cot 8A = \cot A - \tan A - 2\tan 2A - 4\tan 4A$

$= 2\cot 2A - 2\tan 2A - 4\tan 4A$ (using viii (a) in above results)

$= 4\cot 4A - 4\tan 4A$ (using viii (a) in above results)

$= 8\cot 8A.$

Aliter Method : L.H.S. $= \tan A + 2\tan 2A + 4\tan 4A + 8\left(\frac{1-\tan^2 4A}{2\tan 4A}\right)$

$= \tan A + 2\tan 2A + \left(\frac{4\tan^2 4A + 4 - 4\tan^2 4A}{\tan 4A}\right)$

$= \tan A + 2\tan 2A + 4\cot 4A = \tan A + 2\tan 2A + 4\left(\frac{1-\tan^2 2A}{2\tan 2A}\right)$

$= \tan A + \left[\frac{2\tan^2 2A + 2 - 2\tan^2 2A}{\tan 2A}\right] = \tan A + 2\cot 2A$

$= \tan A + 2\left(\frac{1-\tan^2 A}{2\tan A}\right) = \frac{\tan^2 A + 1 - \tan^2 A}{\tan A} = \cot A = $ R.H.S.

Illustration 19 : Evaluate $\sum_{r=1}^{n-1}\cos^2\left(\frac{r\pi}{n}\right)$; $n \ge 2$

Solution : Sum = $\frac{1}{2}\sum_{r=1}^{n-1}\left(1+\cos\frac{2r\pi}{n}\right) = \frac{1}{2}(n-1)+\frac{1}{2}\left\{\cos\frac{2\pi}{n}+\cos\frac{4\pi}{n}+\ldots\ldots+\cos\frac{(2n-2)\pi}{n}\right\}$

$$= \frac{1}{2}(n-1)+\frac{1}{2}\left\{\frac{\sin(n-1)\frac{2\pi}{2n}}{\sin\frac{2\pi}{n.2}}.\cos\left\{\frac{2\left(\frac{2\pi}{n}\right)+(n-2)\frac{2\pi}{n}}{2}\right\}\right\}$$

$$\left\{\text{Using}, \cos\alpha+\cos(\alpha+\beta)+\cos(\alpha+2\beta)+\ldots\ldots.+\cos(\alpha+(n-1)\beta)=\frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}.\cos\left\{\frac{2\alpha+(n-1)\beta}{2}\right\}\right\}$$

$$=\frac{1}{2}(n-1)+\frac{1}{2}\left\{\frac{\sin\frac{(n-1)\pi}{n}.\cos\pi}{\sin\left(\frac{\pi}{n}\right)}\right\} = \frac{1}{2}(n-1)-\frac{1}{2}=\frac{n}{2}-1$$

$\therefore \quad \sum_{r=1}^{n-1}\cos^2\left(\frac{r\pi}{n}\right)=\frac{n-2}{2}$ **Ans.**

Illustration 20 : Prove that : $(1 + \sec2\theta)(1 + \sec2^2\theta)(1 + \sec2^3\theta)\ldots\ldots(1 + \sec2^n\theta) = \tan2^n\theta.\cot\theta$.

Solution : L.H.S. $=\left(1+\frac{1}{\cos2\theta}\right)\left(1+\frac{1}{\cos2^2\theta}\right)\left(1+\frac{1}{\cos2^3\theta}\right)\ldots.\left(1+\frac{1}{\cos2^n\theta}\right)$

$$= \left(\frac{1+\cos2\theta}{\cos2\theta}\right)\left(\frac{1+\cos2^2\theta}{\cos2^2\theta}\right)\left(\frac{1+\cos2^3\theta}{\cos2^3\theta}\right)\ldots.\left(\frac{1+\cos2^n\theta}{\cos2^n\theta}\right)$$

$$= \frac{2\cos^2\theta.2\cos^2 2\theta.2\cos^2 2^2\theta\ldots.2\cos^2 2^{n-1}\theta}{\cos2\theta.\cos2^2\theta.\cos2^3\theta\ldots.\cos2^n\theta}$$

$$= \cos\theta(2\cos\theta)(2\cos2\theta)(2\cos2^2\theta)\ldots(2\cos2^{n-1}\theta).\ \frac{1}{\cos2^n\theta}$$

$$= \frac{\cos\theta}{\sin\theta}(2\sin\theta\ \cos\theta)(2\cos2\theta)(2\cos2^2\theta)\ldots(2\cos2^{n-1}\theta).\ \frac{1}{\cos2^n\theta}$$

$$= \frac{\cos\theta}{\sin\theta}(2\sin2\theta.\ \cos2\theta)(2\cos2^2\theta)\ldots(2\cos2^{n-1}\theta).\frac{1}{\cos2^n\theta}$$

$$= \frac{\cos\theta}{\sin\theta}(2\sin2^{n-1}\theta.\cos2^{n-1}\theta).\frac{1}{\cos2^n\theta} = \frac{\cos\theta}{\sin\theta}.\sin2^n\theta.\frac{1}{\cos2^n\theta} = \tan2^n\theta.\cot\theta = \text{R.H.S.}$$

Do yourself - 13 :

(i) Evaluate $\sin\frac{\pi}{n}+\sin\frac{3\pi}{n}+\sin\frac{5\pi}{n}+\ldots\ldots...$ to n terms

(ii) If $(2^n + 1)\theta = \pi$, then find the value of $2^n\cos\theta\cos2\theta\cos2^2\theta\ldots\ldots...\cos2^{n-1}\theta$.

Miscellaneous Illustration :

Illustration 21 : Prove that

$\tan\alpha + 2\tan2\alpha + 2^2\tan^2\alpha + \ldots\ldots + 2^{n-1}\tan2^{n-1}\alpha + 2^n\cot2^n\alpha = \cot\alpha$

Solution : We know $\tan\theta = \cot\theta - 2\cot2\theta$ .....(i)

Putting $\theta = \alpha, 2\alpha, 2^2\alpha$, ...in (i), we get

$\tan\alpha = (\cot\alpha - 2\cot2\alpha)$

$2(\tan2\alpha) = 2(\cot2\alpha - 2\cot2^2\alpha)$

$2^2(\tan2^2\alpha) = 2^2(\cot2^2\alpha - 2\cot2^3\alpha)$

....

$2^{n-1}(\tan2^{n-1}\alpha) = 2^{n-1}(\cot2^{n-1}\alpha - 2\cot2^n\alpha)$

Adding,

$\tan\alpha + 2\tan 2\alpha + 2^2\tan^2\alpha + \ldots\ldots + 2^{n-1}\tan 2^{n-1}\alpha = \cot\alpha - 2^n\cot 2^n\alpha$

$\therefore$ $\tan\alpha + 2\tan 2\alpha + 2^2\tan^2\alpha + \ldots\ldots + 2^{n-1}\tan 2^{n-1}\alpha + 2^n\cot 2^n\alpha = \cot\alpha$

Illustration 22 : If A,B,C and D are angles of a quadrilateral and $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\sin\frac{D}{2} = \frac{1}{4}$, prove that $A = B = C = D = \pi/2$.

Solution :

$$\left(2\sin\frac{A}{2}\sin\frac{B}{2}\right)\left(2\sin\frac{C}{2}\sin\frac{D}{2}\right) = 1$$

$$\Rightarrow \left\{\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right\}\left\{\cos\left(\frac{C-D}{2}\right) - \cos\left(\frac{C+D}{2}\right)\right\} = 1$$

Since, $A + B = 2\pi - (C + D)$, the above equation becomes,

$$\Rightarrow \left\{\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right\}\left\{\cos\left(\frac{C-D}{2}\right) + \cos\left(\frac{A+B}{2}\right)\right\} = 1$$

$$\Rightarrow \cos^2\left(\frac{A+B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\left\{\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{C-D}{2}\right)\right\} + 1 - \cos\left(\frac{A-B}{2}\right)\cos\left(\frac{C-D}{2}\right) = 0$$

This is a quadratic equation in $\cos\left(\frac{A+B}{2}\right)$ which has real roots.

$$\Rightarrow \left\{\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{C-D}{2}\right)\right\}^2 - 4\left\{1 - \cos\left(\frac{A-B}{2}\right).\cos\left(\frac{C-D}{2}\right)\right\} \geq 0$$

$$\left(\cos\frac{A-B}{2} + \cos\frac{C-D}{2}\right)^2 \geq 4$$

$\Rightarrow$ $\cos\frac{A-B}{2} + \cos\frac{C-D}{2} \geq 2$, Now both $\cos\frac{A-B}{2}$ and $\cos\frac{C-D}{2} \leq 1$

$$\Rightarrow \cos\frac{A-B}{2} = 1 \ \&\ \cos\frac{C-D}{2} = 1$$

$$\Rightarrow \frac{A-B}{2} = 0 = \frac{C-D}{2}$$

$\Rightarrow$ $A = B,\ C = D.$

Similarly $A = C,\ B = D \Rightarrow A = B = C = D = \pi/2$

ANSWERS FOR DO YOURSELF

1 : (i) 144, 160^g, $\left(\frac{4\pi}{5}\right)^c$ (ii) 10π cm

2 : (i) $\frac{3}{5}, \frac{4}{5}, \frac{5}{3}$ (ii) 2

3 : (i) 8

4 : (i) (a) $\frac{187}{205}$ (b) $\frac{-133}{205}$ (c) $\frac{-84}{205}$ (d) $\frac{156}{205}$

5 : (i) $\frac{1}{\sqrt{3}}$

9: (i) (a) $\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$ (b) $\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$ (c) $\sqrt{2}-1$

10 : (i) (a) $-\frac{1}{2}$ (b) $\frac{\sqrt{5}+1}{8}$

11 : (i) 0 (ii) 1

12 : (i) 7 & –7 (ii) $-\frac{9}{8}$ (iii) $4+\sqrt{10}$ & $4-\sqrt{10}$

13 : (i) 0 (ii) 1

EXERCISE - 01 CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. The expression $\dfrac{\tan\left(x-\dfrac{\pi}{2}\right).\cos\left(\dfrac{3\pi}{2}+x\right)-\sin^3\left(\dfrac{7\pi}{2}-x\right)}{\cos\left(x-\dfrac{\pi}{2}\right).\tan\left(\dfrac{3\pi}{2}+x\right)}$ simplifies to -

(A) $(1+\cos^2x)$ (B) $\sin^2x$ (C) $-(1+\cos^2x)$ (D) $\cos^2x$

2. Exact value of $\cos^2 73 + \cos^2 47 - \sin^2 43 + \sin^2 107$ is equal to -

(A) 1/2 (B) 3/4 (C) 1 (D) none

3. The expression $\dfrac{\sin 22^\circ\cos 8^\circ+\cos 158^\circ\cos 98^\circ}{\sin 23^\circ\cos 7^\circ+\cos 157^\circ\cos 97^\circ}$ when simplified reduces to -

(A) 1 (B) – 1 (C) 2 (D) none

4. The two legs of right triangle are $\sin\theta+\sin\left(\dfrac{3\pi}{2}-\theta\right)$ and $\cos\theta-\cos\left(\dfrac{3\pi}{2}-\theta\right)$. The length of its hypotenuse is

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) some function of θ

5. If $\tan\theta=\sqrt{\dfrac{a}{b}}$ where a, b are positive reals then the value of $\sin\theta\sec^7\theta+\cos\theta\,\text{cosec}^7\theta$ is -

(A) $\dfrac{(a+b)^3(a^4+b^4)}{(ab)^{7/2}}$ (B) $\dfrac{(a+b)^3(a^4-b^4)}{(ab)^{7/2}}$ (C) $\dfrac{(a+b)^3(b^4-a^4)}{(ab)^{7/2}}$ (D) $-\dfrac{(a+b)^3(a^4+b^4)}{(ab)^{7/2}}$

6. The expression $\dfrac{\sin(\alpha+\theta)-\sin(\alpha-\theta)}{\cos(\beta-\theta)-\cos(\beta+\theta)}$ is –

(A) independent of α (B) independent of β (C) independent of θ (D) independent of α and β

7. The tangents of two acute angles are 3 and 2. The sine of twice their difference is -

(A) 7/24 (B) 7/48 (C) 7/50 (D) 7/25

8. If $\dfrac{\sin 2\alpha-\sin 3\alpha+\sin 4\alpha}{\cos 2\alpha-\cos 3\alpha+\cos 4\alpha}=\tan k\alpha$ is an identity then the value of k is equal to -

(A) 2 (B) 3 (C) 4 (D) 6

9. Exact value of $\cos 20 + 2\sin^2 55 - \sqrt{2}\sin 65$ is -

(A) 1 (B) $\dfrac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) zero

10. If $\cos(\theta+\phi)=m\cos(\theta-\phi)$, then $\tan\theta$ is equal to -

(A) $\left(\dfrac{1+m}{1-m}\right)\tan\phi$ (B) $\left(\dfrac{1-m}{1+m}\right)\tan\phi$ (C) $\left(\dfrac{1-m}{1+m}\right)\cot\phi$ (D) $\left(\dfrac{1+m}{1-m}\right)\cot\phi$

11. If $\sin\theta+\text{cosec}\,\theta=2$, then the value of $\sin^8\theta+\text{cosec}^8\theta$ is equal to -

(A) 2 (B) 2^8 (C) 2^4 (D) none of these

12. If the expression $4\sin 5\alpha\cos 3\alpha\cos 2\alpha$ is expressed as the sum of three sines then two of them are $\sin 4\alpha$ and $\sin 10\alpha$. The third one is -

(A) $\sin 8\alpha$ (B) $\sin 6\alpha$ (C) $\sin 5\alpha$ (D) $\sin 12\alpha$

13. The expression, $3\left[\sin^4\left(\dfrac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\dfrac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$ when simplified is equal to -

(A) 0 (B) 1 (C) 3 (D) $\sin 4\alpha+\cos 6\alpha$

14. If $\cos\theta=\dfrac{1}{2}\left(a+\dfrac{1}{a}\right)$ then $\cos 3\theta$ in terms of 'a' =

(A) $\dfrac{1}{4}\left(a^3+\dfrac{1}{a^3}\right)$ (B) $4\left(a^3+\dfrac{1}{a^3}\right)$ (C) $\dfrac{1}{2}\left(a^3+\dfrac{1}{a^3}\right)$ (D) none

15. $\dfrac{1}{\cos 290^\circ} + \dfrac{1}{\sqrt{3}\sin 250^\circ} =$

(A) $\dfrac{2\sqrt{3}}{3}$ (B) $\dfrac{4\sqrt{3}}{3}$ (C) $\sqrt{3}$ (D) none

16. The product cot 123 . cot 133 . cot 137 . cot 147 , when simplified is equal to -

(A) – 1 (B) tan 37 (C) cot 33 (D) 1

17. Given $\sin B = \dfrac{1}{5}\sin(2A+B)$ then, $\tan(A+B) = k\tan A$, where k has the value equal to -

(A) 1 (B) 2 (C) 2/3 (D) 3/2

18. If $A + B + C = \pi$ & $\sin\left(A+\dfrac{C}{2}\right) = k\sin\dfrac{C}{2}$, then $\tan\dfrac{A}{2}\tan\dfrac{B}{2} =$

(A) $\dfrac{k-1}{k+1}$ (B) $\dfrac{k+1}{k-1}$ (C) $\dfrac{k}{k+1}$ (D) $\dfrac{k+1}{k}$

19. The value of the expression $\dfrac{1-4\sin 10^\circ \sin 70^\circ}{2\sin 10^\circ}$ is –

(A) 1/2 (B) 1 (C) 2 (D) none of these

20. Which of the following number (s) is / are rational ?

(A) sin15 (B) cos15 (C) sin15 cos15 (D) sin15 cos75

21. If α and β are two positive acute angles satisfying $\alpha - \beta = 15$ and $\sin\alpha = \cos 2\beta$ then the value of $\alpha + \beta$ is equal to-

(A) 35 (B) 55 (C) 65 (D) 85

22. If $\alpha + \beta + \gamma = 2\pi$, then -

(A) $\tan\dfrac{\alpha}{2}+\tan\dfrac{\beta}{2}+\tan\dfrac{\gamma}{2}=\tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2}\tan\dfrac{\gamma}{2}$

(B) $\tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2}+\tan\dfrac{\beta}{2}\tan\dfrac{\gamma}{2}+\tan\dfrac{\gamma}{2}\tan\dfrac{\alpha}{2}=1$

(C) $\tan\dfrac{\alpha}{2}+\tan\dfrac{\beta}{2}+\tan\dfrac{\gamma}{2}=-\tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2}\tan\dfrac{\gamma}{2}$

(D) $\tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2}+\tan\dfrac{\beta}{2}\tan\dfrac{\gamma}{2}+\tan\dfrac{\gamma}{2}\tan\dfrac{\alpha}{2}=0$

23. The value of sin10 + sin20 + sin30 +......+ sin360 is -

(A) 1 (B) 0 (C) –1 (D) none of these

24. If A and C are two angles such that $A + C = \dfrac{3\pi}{4}$, then (1+cotA)(1+cotC) equals -

(A) 1 (B) 2 (C) -1 (D) -2

25. $\log_{t_1}(4\sin 9^\circ\cos 9^\circ)$; where t_1= 4sin63 cos63 , equals -

(A) $\dfrac{\sqrt{5}+1}{4}$ (B) $\dfrac{\sqrt{5}-1}{4}$ (C) 1 (D) none of these

26. $l = \left(\dfrac{\cot^2 x.\cos^2 x}{\cot^2 x - \cos^2 x}\right)^2$ and $m = a^{\log_{\sqrt{a}}\left[2\cos\frac{y}{2}\right]}$, at $y = 4\pi$, then $l^2 + m^2$ is equal to -

(A) 4 (B) 16 (C) 17 (D) none of these

27. If $(a + b)\tan(\theta - \phi) = (a - b)\tan(\theta + \phi)$, then $\dfrac{\sin(2\theta)}{\sin(2\phi)}$ is equal to -

(A) ab (B) $\dfrac{a}{b}$ (C) $\dfrac{b}{a}$ (D) a^2b^2

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

28. If θ is internal angle of n sided regular polygon, then $\sin\theta$ is equal to -

(A) $\sin\dfrac{\pi}{n}$ (B) $\sin\dfrac{2\pi}{n}$ (C) $\sin\dfrac{\pi}{2n}$ (D) $\sin\dfrac{n}{\pi}$

29. If $\sin\theta+\sqrt{\sin\theta+\sqrt{\sin\theta+\sqrt{\sin\theta+......\infty}}}=\sec^4\alpha$, then sinθ is equal to -

(A) $\sec^2\alpha\ \tan^2\alpha$ (B) $2\dfrac{(1-\cos2\alpha)}{(1+\cos2\alpha)^2}$ (C) $2\dfrac{(1+\cos2\alpha)}{(1-\cos2\alpha)^2}$ (D) $\cot^2\alpha\ \text{cosec}^2\alpha$

30. If $\tan\dfrac{\theta}{2}=\text{cosec}\theta-\sin\theta$, then -

(A) $\sin^2\dfrac{\theta}{2}=2\sin^2 18°$ (B) $\cos2\theta+2\cos\theta+1=0$

(C) $\sin^2\dfrac{\theta}{2}=4\sin^2 18°$ (D) $\cos2\theta+2\cos\theta-1=0$

31. If $\cos(A-B)=\dfrac{3}{5}$ & $\tan A\tan B=2$, then -

(A) $\cos A\cos B=\dfrac{1}{5}$ (B) $\sin A\sin B=-\dfrac{2}{5}$ (C) $\cos(A+B)=-\dfrac{1}{5}$ (D) $\sin A\sin B=\dfrac{2}{5}$

32. Factors of $\cos4\theta-\cos4\phi$ are -

(A) $(\cos\theta+\cos\phi)$ (B) $(\cos\theta-\cos\phi)$ (C) $(\cos\theta+\sin\phi)$ (D) $(\cos\theta-\sin\phi)$

33. For the equation $\sin3\theta+\cos3\theta=1-\sin2\theta$ -

(A) $\tan\theta=1$ is possible (B) $\cos\theta=0$ is possible (C) $\tan\dfrac{\theta}{2}=-1$ is possible (D) $\cos\dfrac{\theta}{2}=0$ is possible

34. If 2tan10 + tan50 = 2x, tan20 + tan50 = 2y, 2tan10 + tan70 = 2w and tan20 + tan70 = 2z, then which of the following is/are true -

(A) z > w > y > x (B) w = x + y (C) 2y = z (D) z + x = w + y

35. If $(3-4\sin^2 1)(3-4\sin^2 3)(3-4\sin^2 3^2)\ ......\ (3-4\sin^2(3^{n-1}))=\sin a/\sin b$, where $n\in N$ & a, b are integers in radian, then the digit at the unit place of (a + b) may be-

(A) 4 (B) 0 (C) 8 (D) 2

CHECK YOUR GRASP — ANSWER KEY — EXERCISE-1

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	C	A	B	A	C	D	B	A	C	A	B	B	C	B
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	D	D	A	B	C	C	A	B	B	D	C	B	B	A,B	A,D
Que.	31	32	33	34	35										
Ans.	A,C,D	A,B,C,D	A,B,C	A,B,C,D	A,B,C,D										

EXERCISE - 02

BRAIN TEASERS

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. Let m = tan 3 and n = sec 6, then which of the following statement(s) does/do not hold good ?

(A) m & n both are positive(B) m & n both are negative

(C) m is positive and n is negative (D) m is negative and n is positive

2. If $\sqrt{\frac{1-\sin A}{1+\sin A}}+\frac{\sin A}{\cos A}=\frac{1}{\cos A}$, for all permissible values of A, then A belongs to -

(A) first quadrant (B) second quadrant (C) third quadrant (D) fourth quadrant

3. If $\pi < 2\theta < \frac{3\pi}{2}$, then $\sqrt{2+\sqrt{2+2\cos 4\theta}}$ equals -

(A) $-2\cos\theta$ (B) $-2\sin\theta$ (C) $2\cos\theta$ (D) $2\sin\theta$

4. $\frac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta}-\frac{\cos\theta}{\sqrt{1+\cot^2\theta}}-2\tan\theta\cot\theta=-1$ if -

(A) $\theta \in \left(0,\frac{\pi}{2}\right)$ (B) $\theta \in \left(\frac{\pi}{2},\pi\right)$ (C) $\theta \in \left(\pi,\frac{3\pi}{2}\right)$ (D) $\theta \in \left(\frac{3\pi}{2},2\pi\right)$

5. If sec A = $\frac{17}{8}$ and cosecB = $\frac{5}{4}$ then sec (A + B) can have the value equal to -

(A) $\frac{85}{36}$ (B) $-\frac{85}{36}$ (C) $-\frac{85}{84}$ (D) $\frac{85}{84}$

6. Which of the following when simplified reduces to unity ?

(A) $\frac{1-2\sin^2\alpha}{2\cot\left(\frac{\pi}{4}+\alpha\right)\cos^2\left(\frac{\pi}{4}-\alpha\right)}$ (B) $\frac{\sin(\pi-\alpha)}{\sin\alpha-\cos\alpha\tan\frac{\alpha}{2}}+\cos(\pi-\alpha)$

(C) $\frac{1}{4\sin^2\alpha\cos^2\alpha}+\frac{(1-\tan^2\alpha)^2}{4\tan^2\alpha}$ (D) $\frac{1+\sin 2\alpha}{(\sin\alpha+\cos\alpha)^2}$

7. It is known that $\sin\beta = \frac{4}{5}$ & $0 < \beta < \pi$ then the value of $\frac{\sqrt{3}\sin(\alpha+\beta)-\frac{2}{\cos\frac{\pi}{6}}\cos(\alpha+\beta)}{\sin\alpha}$ is -

(A) independent of α for all β in $(0, \pi)$ (B) $\frac{5}{\sqrt{3}}$ for $\tan\beta < 0$

(C) $\frac{\sqrt{3}(7+24\cot\alpha)}{15}$ for $\tan\beta > 0$ (D) none

8. In a triangle ABC, angle A is greater than angle B. If the measures of angles A and B satisfy the equation $2\tan x - k(1+\tan^2 x) = 0$, where $k \in (0, 1)$, then the measure of the angle C is -

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (D) $\frac{5\pi}{12}$ (D) $\frac{\pi}{2}$

9. If $\frac{\sin 3\theta}{\sin\theta}=\frac{11}{25}$ then $\tan\frac{\theta}{2}$ can have the value equal to -

(A) 2 (B) 1/2 (C) – 2 (D) – 1/2

10. The expression $\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^m + \left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^m$ where m $\in$ N, has the value -

(A) $2\cot^m\left(\frac{A-B}{2}\right)$, if m is odd
(B) 0, if m is odd
(C) $2\cot^m\left(\frac{A-B}{2}\right)$, if m is even
(D) 0, if m is even

11. If cos(A – B) = 3/5, and tanA tanB = 2, then -

(A) $\cos A\cos B=\frac{1}{5}$
(B) $\sin A\sin B=\frac{-2}{5}$
(C) $\cos(A+B)=\frac{-1}{5}$
(D) none of these

12. If $A+B=\frac{\pi}{3}$ and cosA + cosB = 1, then -

(A) cos(A – B) = 1/3
(B) $|\cos A-\cos B|=\sqrt{\frac{2}{3}}$
(C) $\cos(A-B)=-\frac{1}{3}$
(D) $|\cos A-\cos B|=\frac{1}{2\sqrt{3}}$

13. If A and B are acute positive angles satisfying the equations $3\sin^2A + 2\sin^2B = 1$ and $3\sin2A - 2\sin2B = 0$ then A + 2B is-

(A) $\frac{\pi}{3}$
(B) $\frac{\pi}{2}$
(C) $\frac{2\pi}{3}$
(D) none

14. If A + B – C = 3π, then sinA + sinB – sinC is equal to -

(A) $4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$
(B) $-4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$
(C) $4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$
(D) $-4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

15. 2 sin11 15' is equal to –

(A) $\sqrt{2-\sqrt{2+\sqrt{2}}}$
(B) $\sqrt{2-\sqrt{2-\sqrt{2}}}$
(C) $\sqrt{\frac{2+\sqrt{2-\sqrt{2}}}{2}}$
(D) $\sqrt{\frac{2+\sqrt{2+\sqrt{2}}}{2}}$

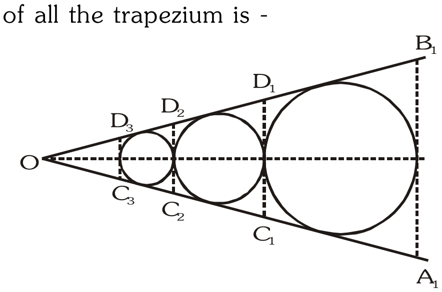
16. If $\tan^3\theta+\cot^3\theta=52$, then the value of $\tan^2\theta+\cot^2\theta$ is equal to -

(A) 14
(B) 15
(C) 16
(D) 17

17. If 60 + α & 60 – α are the roots of $\sin^2x + b\sin x + c = 0$, then -

(A) $4b^2 + 3 = 12c$
(B) $4b + 3 = 12c$
(C) $4b^2 - 3 = -12c$
(D) $4b^2 - 3 = 12c$

18. If $\angle B_1OA_1$ = 60 & radius of biggest circle is r. According to figure trapezium $A_1B_1D_1C_1$, $C_1D_1D_2C_2$, $C_2D_2D_3C_3$........ and so on are obtained. Sum of areas of all the trapezium is -



(A) $\frac{r^2}{2\sqrt{3}}$
(B) $\frac{9r^2}{2\sqrt{3}}$
(C) $\frac{9r^2}{\sqrt{3}}$
(D) $\frac{r^2}{9\sqrt{3}}$

19. If θ & φ are acute angles & $\sin\theta=\frac{1}{2}$, $\cos\phi=\frac{1}{3}$, then the value of θ + φ belongs to the interval -

(A) $\left(\frac{\pi}{3},\frac{\pi}{2}\right]$
(B) $\left(\frac{\pi}{2},\frac{2\pi}{3}\right]$
(C) $\left(\frac{2\pi}{3},\frac{5\pi}{6}\right]$
(D) $\left(\frac{5\pi}{6},\pi\right]$

20. The maximum value of $\log_{20}(3\sin x - 4\cos x + 15)$ -

(A) 1 (B) 2 (C) 3 (D) 4

21. If $x^2 + y^2 = 9$ & $4a^2 + 9b^2 = 16$, then maximum value of $4a^2x^2 + 9b^2y^2 - 12abxy$ is -

(A) 81 (B) 100 (C) 121 (D) 144

22. Let A,B,C are 3 angles such that $\cos A + \cos B + \cos C = 0$ and if $\cos A \cos B \cos C = \lambda(\cos 3A + \cos 3B + \cos 3C)$, then λ is equal to -

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{9}$ (D) $\frac{1}{12}$

23. $f(x) = \frac{\sin x}{\sqrt{1+\tan^2 x}} + \frac{\cos x}{\sqrt{1+\cot^2 x}}$ is constant in which of following interval -

(A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(\frac{\pi}{2}, \pi\right)$ (C) $\left(\pi, \frac{3\pi}{2}\right)$ (D) $\left(\frac{3\pi}{2}, 2\pi\right)$

24. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$, for every value of θ, then -

(A) $b_0 = 1,\ b_1 = 3$ (B) $b_0 = 0,\ b_1 = n$

(C) $b_0 = -1,\ b_1 = n$ (D) $b_0 = 0,\ b_1 = n^2 - 3n + 3$

25. For a positive integer n, let $f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)\ldots.(1+\sec 2^n\theta)$. Then **[JEE 99, 3M]**

(A) $f_2\left(\frac{\pi}{16}\right) = 1$ (B) $f_3\left(\frac{\pi}{32}\right) = 1$ (C) $f_4\left(\frac{\pi}{64}\right) = 1$ (D) $f_5\left(\frac{\pi}{128}\right) = 1$

BRAIN TEASERS — ANSWER KEY — EXERCISE-2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A,B,C	A,D	D	B	A,B,C,D	A,B,D	D	D	A,B,C,D	B,C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A,C	B,C	B	D	A	A	D	C	B	A
Que.	21	22	23	24	25					
Ans.	D	D	B,D	B	A,B,C,D					

EXERCISE - 03

MISCELLANEOUS TYPE QUESTIONS

TRUE / FALSE

1. If A + B + C = π, then cos2A + cos2B + cos2C + 4cosA cosB cosC is positive.
2. (tan20 tan40 tan80)^2 is a prime number.
3. ....$\sin^8\theta \le \sin^6\theta \le \sin^4\theta \le \sin^2\theta \le 1$ also ....$\cos^8\theta \le \cos^6\theta \le \cos^4\theta \le \cos^2\theta \le 1$.

FILL IN THE BLANKS

1. If tan α = 2 and $\alpha \in \left(\pi, \frac{3\pi}{2}\right)$ then the value of the expression $\frac{\cos\alpha}{\sin^3\alpha + \cos^3\alpha}$ is equal to ....
2. The expression $\frac{\sin^4 t + \cos^4 t - 1}{\sin^6 t + \cos^6 t - 1}$ when simplified reduces to ....
3. Exact value of tan 200 (cot 10 − tan 10) is ....
4. $96\sqrt{3}\ \sin\frac{\pi}{48}\ \cos\frac{\pi}{48}\ \cos\frac{\pi}{24}\ \cos\frac{\pi}{12}\ \cos\frac{\pi}{6}$ has the value = ...
5. If $[1 - \sin(\pi + \alpha) + \cos(\pi + \alpha)]^2 + \left[1 - \sin\left(\frac{3\pi}{2} + \alpha\right) + \cos\left(\frac{3\pi}{2} - \alpha\right)\right]^2 = a + b\sin 2\alpha$ then the value of 'a' & 'b' are.................. & .... respectively.
6. The least value of the expression $\frac{\cot 2x - \tan 2x}{1 + \sin\left(\frac{5\pi}{2} - 8x\right)}$ for $0 < x < \frac{\pi}{8}$ is..................

MATCH THE COLUMN

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE** statement in **Column-II**.

1.

	Column-I		Column-II
(A)	cosec 10 − $\sqrt{3}$ sec 10 =	(p)	$-\frac{1}{2}$
(B)	4 cos 20 − $\sqrt{3}$ cot 20 =	(q)	−1
(C)	$\frac{2\cos 40° - \cos 20°}{\sin 20°}$ =	(r)	$\sqrt{3}$
(D)	$2\sqrt{2}\ \sin 10°\left[\frac{\sec 5°}{2} + \frac{\cos 40°}{\sin 5°} - 2\sin 35°\right]$ =	(s)	4

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r, s and t. Any given statement in **Column-I** can have correct matching with **ONE OR MORE** statement(s) in **Column-II**.

2. If maximum and minimum values of expression are λ and μ respectively then match the columns :

	Column-I		Column-II
(A)	$\sin^6\theta + \cos^6\theta$ for all θ	(p)	$\lambda + \mu = 2$
(B)	$\log_{\sqrt{5}}\left[\sqrt{2}(\sin\theta - \cos\theta) + 3\right]$ for all θ	(q)	$\lambda + \mu = 6$
(C)	$\frac{7 + 6\tan\theta - \tan^2\theta}{(1 + \tan^2\theta)}$ for all real values of $\theta \sim \frac{\pi}{2}$	(r)	$\lambda - \mu = 10$
(D)	$5\cos\theta + 3\cos(\theta + \frac{\pi}{3}) + 3$ for all real	(s)	$\lambda - \mu = 14$
	values of θ	(t)	$\lambda + \mu = \frac{5}{4}$

ASSERTION & REASON

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement-I** : $\tan 5\theta - \tan 3\theta - \tan 2\theta = \tan 5\theta \tan 3\theta \tan 2\theta$

Because

Statement-II : $x = y + z \Rightarrow \tan x - \tan y - \tan z = \tan x \tan y \tan z$.

(A) A (B) B (C) C (D) D

2. **Statement-I** : If $\sin\theta + \mathrm{cosec}\theta = 2$, then $\sin^n\theta + \mathrm{cosec}^n\theta = 2^n$.

Because

Statement-II : If $a + b = 2$, $ab = 1$, then $a = b = 1$

(A) A (B) B (C) C (D) D

3. **Statement-I** : $\sec^2\theta = \dfrac{4xy}{(x+y)^2}$ is positive for all real values of x and y only when $x = y$

Because

Statement-II : $t^2 \geq 0 \ \forall \ t \in R$

(A) A (B) B (C) C (D) D

4. **Statement-I** : If A is obtuse angle in ΔABC, then $\tan B \tan C < 1$

Because

Statement-II : In ΔABC, $\tan A = \dfrac{\tan B + \tan C}{\tan B \tan C - 1}$

(A) A (B) B (C) C (D) D

5. **Statement-I** : $\cos^3\alpha + \cos^3\left(\alpha + \dfrac{2\pi}{3}\right) + \cos^3\left(\alpha + \dfrac{4\pi}{3}\right) = 3\cos\alpha \cos\left(\alpha + \dfrac{2\pi}{3}\right)\cos\left(\alpha + \dfrac{4\pi}{3}\right)$

Because

Statement-II : If $a + b + c = 0 \Leftrightarrow a^3 + b^3 + c^3 = 3abc$

(A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1

Continued product $\cos\alpha \cos 2\alpha \cos 2^2\alpha \ \ldots\ldots\ldots \ \cos 2^{n-1}\alpha$

$$= \begin{cases} \dfrac{\sin 2^n\alpha}{2^n \sin\alpha}, & \text{if } \alpha \neq n\pi \\ \dfrac{1}{2^n}, & \text{if } \alpha = \dfrac{\pi}{2^n+1} \quad \text{i.e. } 2^n\alpha = \pi - \alpha \\ -\dfrac{1}{2^n}, & \text{if } \alpha = \dfrac{\pi}{2^n-1} \quad \text{i.e. } 2^n\alpha = \pi + \alpha \end{cases}$$

Where, $n \in I$ (Integer)

On the basis of above information, answer the following questions :

1. The value of $\cos\dfrac{2\pi}{7}\cos\dfrac{4\pi}{7}\cos\dfrac{6\pi}{7}$ is -

(A) $-1/2$ (B) $1/2$ (C) $1/4$ (D) $1/8$

2. If $\alpha = \dfrac{\pi}{15}$, then the value of $\prod_{r=1}^{7}\cos r\alpha$ is -

(A) $\dfrac{1}{128}$ (B) $-\dfrac{1}{128}$ (C) $\dfrac{1}{64}$ (D) $\dfrac{1}{32}$

3. The value of $\sin\left(\frac{\pi}{14}\right)\sin\left(\frac{3\pi}{14}\right)\sin\left(\frac{5\pi}{14}\right)\sin\left(\frac{7\pi}{14}\right)\sin\left(\frac{9\pi}{14}\right)\sin\left(\frac{11\pi}{14}\right)\sin\left(\frac{13\pi}{14}\right)$ is -

(A) 1 (B) $\frac{1}{8}$ (C) $\frac{1}{32}$ (D) $\frac{1}{64}$

Comprehension # 2

The measure of an angle in degrees, grades and radians be D, G and C respectively, then the relation between them

$$\frac{D}{90}=\frac{G}{100}=\frac{2C}{\pi} \text{ but } 1^c=\left(\frac{180}{\pi}\right)^\circ$$

$$\simeq 57°,17', 44.8''$$

and sum of interior angles of a n-sided regular polygon is $(2n-4)\pi/2$

On the basis of above information, answer the following questions :

1. Which of the following are correct -

(A) $\sin 1° < \sin 1$ (B) $\cos 1° > \cos 1$ (C) $\cos 1° < \cos 1$ (D) $\sin 1° < \frac{\pi}{180}\sin 1$

2. The angles between the hour hand and minute hand of a clock at half past three is -

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$

3. The number of sides of two regular polygon are as 5 : 4 and the difference between their angles is $\frac{\pi}{20}$, then the number of sides in the polygons respectively are-

(A) 25, 20 (B) 20, 16 (C) 15, 12 (D) 10, 8

4. One angle of a triangle is $\frac{4x}{3}$ grades and another is 3x degrees, while the third is $\frac{2\pi x}{75}$ radians. Then the angles in degrees are-

(A) 20 , 60 , 100 (B) 24 , 60 , 96 (C) 36 , 60 , 84 (D) 20 , 40 , 120

MISCELLANEOUS TYPE QUESTION **ANSWER KEY** EXERCISE -3

- ***True / False***

1. F 2. T 3. T

- ***Fill in the Blanks***

1. $\frac{5}{9}$ 2. $\frac{2}{3}$ 3. 2 4. 9 5. a = 4 & b = – 2 6. 2

- ***Match the Column***

1. (A)→(s), (B)→(q), (C)→(r), (D)→(s) 2. (A)→(t), (B)→(p), (C)→(q,r), (D)→(q,s)

- ***Assertion & Reason***

1. A 2. D 3. B 4. A 5. C

- ***Comprehension Based Questions***

Comprehension # 1 : 1. D 2. A 3. D

Comprehension # 2 : 1. A,B 2. C 3. D 4. B

EXERCISE - 04 [A]   CONCEPTUAL SUBJECTIVE EXERCISE

1. If $\cos(y-z)+\cos(z-x)+\cos(x-y)=-\frac{3}{2}$, prove that $\cos x+\cos y+\cos z=0=\sin x+\sin y+\sin z$.

2. Prove that, $\cos 2\alpha = 2\sin\beta + 4\cos(\alpha+\beta)\sin\alpha\ \sin\beta + \cos 2(\alpha+\beta)$

3. For all values of α, β, γ prove that :

$$\cos\alpha+\cos\beta+\cos\gamma+\cos(\alpha+\beta+\gamma)=4\cos\frac{\alpha+\beta}{2}.\cos\frac{\beta+\gamma}{2}.\cos\frac{\gamma+\alpha}{2}.$$

4. If $\cos(\alpha+\beta)=\frac{4}{5}$; $\sin(\alpha-\beta)=\frac{5}{13}$ & α, β lie between 0 & $\frac{\pi}{4}$, then find the value of $\tan 2\alpha$

5. Prove that :

(a) $\sin^4\frac{\pi}{16}+\sin^4\frac{3\pi}{16}+\sin^4\frac{5\pi}{16}+\sin^4\frac{7\pi}{16}=\frac{3}{2}$

(b) $\sin 6\ .\sin 42\ .\sin 66\ .\sin 78 = \cos 6\ .\cos 42\ .\cos 66\ .\cos 78 = \frac{1}{16}$

6. If $\cos\theta=\frac{\cos\alpha-e}{1-e\cos\alpha}$, prove that $\tan\frac{\theta}{2}=\pm\sqrt{\frac{1+e}{1-e}}\ \tan\frac{\alpha}{2}$.

7. Prove that, $\cot 7\frac{1^\circ}{2}$ or $\tan 82\frac{1^\circ}{2}=\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{2}+1\right)$ or $\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

8. Prove that : $\operatorname{cosec}\theta+\operatorname{cosec}2\theta+\operatorname{cosec}2^2\theta+\ldots\ldots+\operatorname{cosec}2^{n-1}\theta=\cot(\theta/2)-\cot 2^{n-1}\theta$

9. If $\alpha+\beta=c$ where $\alpha, \beta>0$ each lying between 0 and $\pi/2$ and c is a constant, find the maximum or minimum value of –

(a) $\sin\alpha+\sin\beta$ (b) $\sin\alpha\sin\beta$ (c) $\tan\alpha+\tan\beta$

10. (a) Find the maximum & minimum values of $27^{\cos 2x}.81^{\sin 2x}$.

(b) Find the smallest positive values of x & y satisfying, $x-y=\frac{\pi}{4}$, $\cot x+\cot y=2$

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CONCEPTUAL SUBJECTIVE EXERCISE   **ANSWER KEY**   EXERCISE-4(A)

4. $\frac{56}{33}$

9. (a) max.= $2\sin c/2$ (b) max.= $\sin^2 c/2$ (c) min.= $2\tan c/2$

10. (a) Minimum Value = 3^{-5} ; Maximum Value = 3^5 (b) $x=\frac{5\pi}{12}$, $y=\frac{\pi}{6}$

EXERCISE - 04 [B]

BRAIN STORMING SUBJECTIVE EXERCISE

1. Prove that :

(a) In an acute angled triangle ABC, the least values of $\Sigma\sec A$ and $\Sigma\tan^2 A$ are 6 and 9 respectively.

(b) In triangle ABC, the least values of $\Sigma\text{cosec}\left(\frac{A}{2}\right)$ and $\Sigma\sec^2\left(\frac{A}{2}\right)$ are 6 and 4 respectively.

2. Prove that ; $\text{cosec}\, x \,.\, \text{cosec}\, 2x \,.\, \sin 4x \,.\, \cos 6x \,.\, \text{cosec}\, 10x$

$$= \frac{\cos 3x}{\sin 2x \sin 4x} + \frac{\cos 5x}{\sin 4x \sin 6x} + \frac{\cos 7x}{\sin 6x \sin 8x} + \frac{\cos 9x}{\sin 8x \sin 10x}\ .$$

3. If $\tan\alpha = p/q$ where $\alpha = 6\beta$, α being an acute angle, prove that ;

$$\frac{1}{2}(p\ \text{cosec}2\beta - q\ \sec 2\beta) = \sqrt{p^2+q^2}$$

4. If $\tan\left(\frac{\pi}{4}+\frac{y}{2}\right) = \tan^3\left(\frac{\pi}{4}+\frac{x}{2}\right)$ prove that $\sin y = \sin x\left[\frac{3+\sin^2 x}{1+3\sin^2 x}\right]$

5. Prove that from the equality $\frac{\sin^4\alpha}{a} + \frac{\cos^4\alpha}{b} = \frac{1}{a+b}$ follows the relation ;

$$\frac{\sin^8\alpha}{a^3} + \frac{\cos^8\alpha}{b^3} = \frac{1}{(a+b)^3}$$

6. If $P = \cos\frac{\pi}{19} + \cos\frac{3\pi}{19} + \cos\frac{5\pi}{19} + \ldots\ldots + \cos\frac{17\pi}{19}$ and

$Q = \cos\frac{2\pi}{21} + \cos\frac{4\pi}{21} + \cos\frac{6\pi}{21} + \ldots\ldots + \cos\frac{20\pi}{21}$, then find P – Q

7. Prove that : $4\sin 27 = \left(5+\sqrt{5}\right)^{1/2} - \left(3-\sqrt{5}\right)^{1/2}$.

8. If $A+B+C = \pi$; prove that $\tan\frac{A}{2} + \tan\frac{B}{2} + \tan\frac{C}{2} \geq 1$.

9. If $\alpha + \beta = \gamma$, prove that $\cos\alpha + \cos\beta + \cos\gamma = 1 + 2\cos\alpha\cos\beta\cos\gamma$.

10. Prove that the triangle ABC is equilateral iff , $\cot A + \cot B + \cot C = \sqrt{3}$.

BRAIN STORMING SUBJECTIVE EXERCISE	ANSWER KEY	EXERCISE-4(B)
6. 1		

EXERCISE - 05 [A]

JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

1. Period of $f(x) = \sin^4 x + \cos^4 x$ is - [AIEEE-2002]

(1) π (2) $\frac{\pi}{2}$ (3) 2π (4) None of these

2. Period of $\sin^2\theta$ is- [AIEEE-2002]

(1) π^2 (2) π (3) 2π (4) $\frac{\pi}{2}$

3. If $y = \sec^2\theta + \cos^2\theta$, $\theta \neq 0$, then- [AIEEE-2002]

(1) $y = 0$ (2) $y \leq 2$ (3) $y \geq -2$ (4) $y > 2$.

4. The value of $\frac{1-\tan^2 15°}{1+\tan^2 15°}$ = [AIEEE-2002]

(1) 1 (2) $\sqrt{3}$ (3) $\frac{\sqrt{3}}{2}$ (4) 2

5. If α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$, $\frac{\pi}{2} < \alpha < \pi$, then $\sin 2\alpha$ = [AIEEE-2002]

(1) $\frac{24}{25}$ (2) $-\frac{24}{25}$ (3) $\frac{13}{18}$ (4) $-\frac{13}{18}$

6. If $\sin(\alpha + \beta) = 1$, $\sin(\alpha-\beta) = \frac{1}{2}$, then $\tan(\alpha+ 2\beta)\tan(2\alpha + \beta)$ = [AIEEE-2002]

(1) 1 (2) –1 (3) zero (4) None of these

7. If $\tan\theta = -\frac{4}{3}$, then $\sin\theta$ is- [AIEEE-2002]

(1) $-\frac{4}{5}$ but not $\frac{4}{5}$ (2) $-\frac{4}{5}$ or $\frac{4}{5}$ (3) $\frac{4}{5}$ but not $-\frac{4}{5}$ (4) None of these

8. $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is true if and only if - [AIEEE-2003]

(1) $x + y \neq 0$ (2) $x = y, x \neq 0$ (3) $x = y$ (4) $x \neq 0, y \neq 0$

9. If $u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$ then the difference between the maximum and minimum values of u^2 is given by- [AIEEE-2004]

(1) $2(a^2 + b^2)$ (2) $2\sqrt{a^2+b^2}$ (3) $(a + b)^2$ (4) $(a - b)^2$

10. Let α,β be such that $\pi < \alpha-\beta < 3\pi$.

If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ is- [AIEEE-2004]

(1) $-\frac{3}{\sqrt{130}}$ (2) $\frac{3}{\sqrt{130}}$ (3) $\frac{6}{65}0$ (4) $\frac{-6}{65}$

11. If $0 < x < \pi$, and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is- [AIEEE-2006]

(1) $(4-\sqrt{7})/3$ (2) $-(4+\sqrt{7})/3$ (3) $(1+\sqrt{7})/4$ (4) $(1-\sqrt{7})/4$

12. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \leq \alpha, \beta \leq \frac{\pi}{4}$. Then $\tan 2\alpha$ = [AIEEE-2010]

(1) $\frac{25}{16}$ (2) $\frac{56}{33}$ (3) $\frac{19}{12}$ (4) $\frac{20}{7}$

13. If $A = \sin^2 x + \cos^4 x$, then for all real x :- [AIEEE-2011]

(1) $1 \leq A \leq 2$ (2) $\frac{3}{4} \leq A \leq \frac{13}{16}$ (3) $\frac{3}{4} \leq A \leq 1$ (4) $\frac{13}{16} \leq A \leq 1$

14. In a ΔPQR, if $3 \sin P + 4 \cos Q = 6$ and $4 \sin Q + 3 \cos P = 1$, then the angle R is equal to : [AIEEE-2012]

(1) $\frac{3\pi}{4}$ (2) $\frac{5\pi}{6}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$

PREVIOUS YEARS QUESTIONS **ANSWER KEY** EXERCISE-5 [A]

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	2	4	3	2	1	2	2	4	1
Que.	11	12	13	14						
Ans.	2	2	3	3						

EXERCISE - 05 [B]

JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

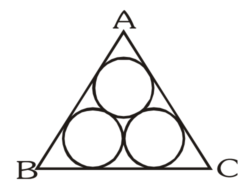
1. If $\alpha+\beta=\frac{\pi}{2}$ and $\beta+\gamma=\alpha$ then tan α equals - [JEE 2001 Screening, 1M out of 35M]

(A) $2(\tan\beta+\tan\gamma)$ (B) $\tan\beta+\tan\gamma$ (C) $\tan\beta+2\tan\gamma$ (D) $2\tan\beta+\tan\gamma$

2. If θ and ϕ are acute angles satisfying $\sin\theta=\frac{1}{2}$, $\cos\phi=\frac{1}{3}$, then $\theta+\phi\in$ [JEE 2004 Screening]

(A) $\left(\frac{\pi}{3},\frac{\pi}{2}\right]$ (B) $\left(\frac{\pi}{2},\frac{2\pi}{3}\right)$ (C) $\left(\frac{2\pi}{3},\frac{5\pi}{6}\right)$ (D) $\left(\frac{5\pi}{6},\pi\right)$

3. In an equilateral triangle, 3 coins of radii 1 unit each are kept so that they touch each other and also the sides of the triangle. Area of the triangle is -



(A) $4+2\sqrt{3}$ (B) $6+4\sqrt{3}$ [JEE 2005 Screening]

(C) $12+\frac{7\sqrt{3}}{4}$ (D) $3+\frac{7\sqrt{3}}{4}$

4. Let $\theta\in\left(0,\frac{\pi}{4}\right)$ and $t_1=(\tan\theta)^{\tan\theta}, t_2=(\tan\theta)^{\cot\theta}, t_3=(\cot\theta)^{\tan\theta}, t_4=(\cot\theta)^{\cot\theta}$, then -

[JEE 06,3M,–1M]

(A) $t_1>t_2>t_3>t_4$ (B) $t_4>t_3>t_1>t_2$ (C) $t_3>t_1>t_2>t_4$ (D) $t_2>t_3>t_1>t_4$

One or more than one is/are correct : [Q.5(a) & (b)]

5.(a) If $\frac{\sin^4 x}{2}+\frac{\cos^4 x}{3}=\frac{1}{5}$, then [JEE 2009, 4 + 4]

(A) $\tan^2 x=\frac{2}{3}$ (B) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{1}{125}$

(C) $\tan^2 x=\frac{1}{3}$ (D) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{2}{125}$

(b) For $0<\theta<\frac{\pi}{2}$, the solution(s) of $\sum_{m=1}^{6}\operatorname{cosec}\left(\theta+\frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta+\frac{m\pi}{4}\right)=4\sqrt{2}$ is (are) -

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

6.(a) The maximum value of the expression $\frac{1}{\sin^2\theta+3\sin\theta\cos\theta+5\cos^2\theta}$ is

(b) Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$ where k > 0, then the value of [k] is -

[**Note** : [k] denotes the largest integer less than or equal to k] [JEE 2010,3+3]

7. Let $P=\left\{\theta:\sin\theta-\cos\theta=\sqrt{2}\cos\theta\right\}$ and $Q=\left\{\theta:\sin\theta+\cos\theta=\sqrt{2}\sin\theta\right\}$ be two sets. Then

(A) $P\subset Q$ and $Q-P\neq\varnothing$ (B) $Q\not\subset P$

(C) $P\not\subset Q$ (D) P = Q [JEE 2011,3]

PREVIOUS YEARS QUESTIONS	ANSWER KEY	EXERCISE-5 [B]

1.	2.	3.	4.	5.	6.	7.
C	B	B	B	(a) A,B; (b) C,D	(a) 2; (b) k = 3	D