1 Lattice Surgery

Lattice Surgery [1] is an operation of code deformation, where a code is transformed into another code and then returned to the initial code, resulting in a change in the logical qubit states. In this section, we will first introduce the lattice surgery operation and then describe a CNOT operation implemented using lattice surgery.

1.1 Merging

The Lattice Surgery operation consists of two operations: Merging and Splitting. In this section, we will first describe the merging operation. Briefly, the procedure of the merging operation is shown in Fig. 1.

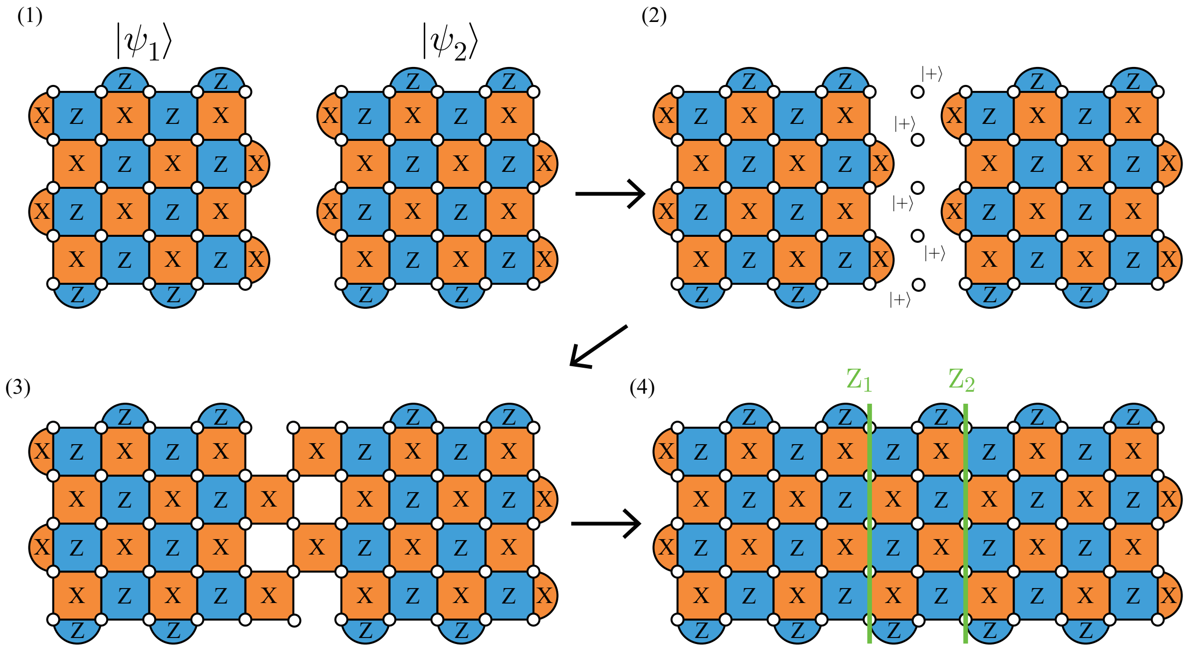

Fig. 1

In the following description of the merging operation, the notations (1), (2), (3), and (4) correspond to (1), (2), (3), and (4) in Fig. 1. In step (1), two arbitrary logical states, $|\psi_1\rangle$ and $|\psi_2\rangle$, encoded by the surface codes, are placed adjacent to each other. In step (2), new data qubits are introduced and initialized in the $|+\rangle$ state between the two logical qubits. By this initialization, 4-weight X stabilizers that connect the two logical states are already established in step (3), so no additional operations are required in step (3). Then, in step (4), we perform the syndrome measurements of Z stabilizers that connect the two logical qubits. The product of all Z stabilizers added in step (4) equals Z_1Z_2, where Z_i is the logical operator of the state $|\psi_i\rangle$. Thus, we can obtain a measurement result $m_{Z_1Z_2}$ for Z_1Z_2. This operation can be written as:

$$O_{\text{merging}} |\psi_1\rangle |\psi_2\rangle = (I + (-1)^{m_{Z_1Z_2}} Z_1Z_2) |\psi_1\rangle |\psi_2\rangle \tag{1}$$

where O_{merging} indicates the merging operation in the equation. We have merged the smooth boundaries of two Surface Codes, but the rough boundaries can be merged in the same way.

1.2 Splitting

In the previous section, we introduced the merging operation of lattice surgery. In this section, we will introduce the splitting operation, which is the opposite of the merging operation.

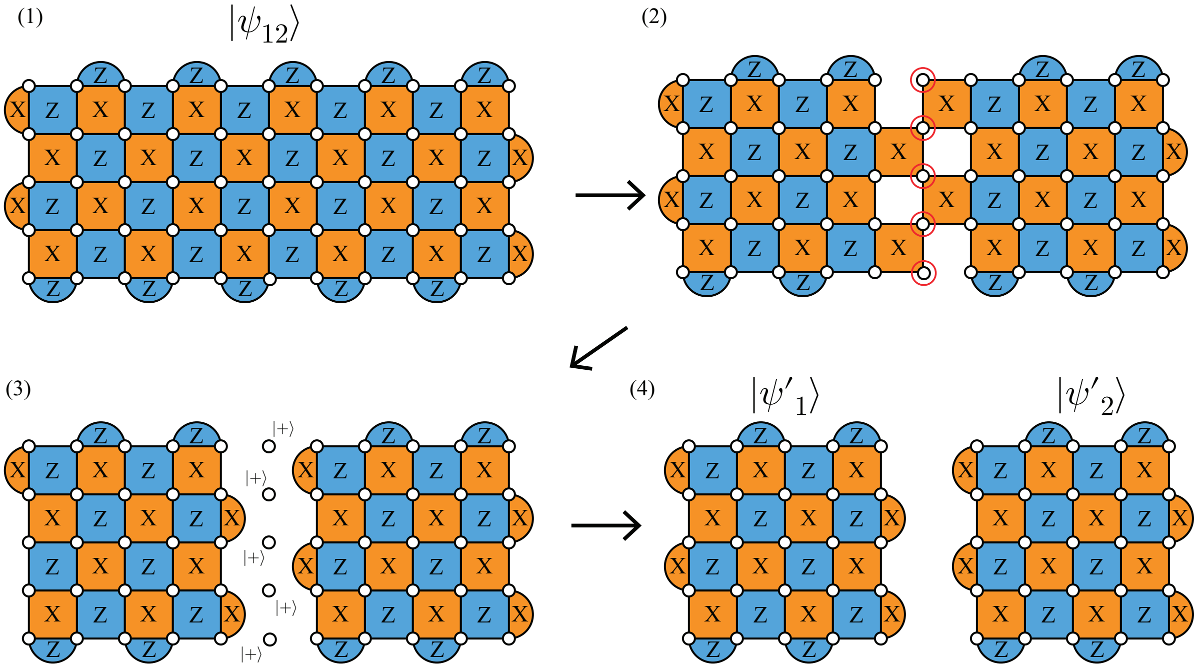

Fig. 2

In the following description of the splitting operation, the notations (1), (2), (3), and (4) correspond to (1), (2), (3), and (4) in Fig. 2. In step (1), we start with a state $|\psi_{12}\rangle$. In step (2), the middle column of qubits is measured in the Pauli-X basis. As a result, some Z stabilizers in the middle of the qubits are lost in step (3). By eliminating the data qubits measured in the Pauli-X basis, we obtain two logical states, $|\psi_1'\rangle$ and $|\psi_2'\rangle$, in step (4). While the measurement results will be used for error correction after the splitting operation, we avoid delving into that aspect here.

1.3 Logical CNOT Gate

In this section, we introduce a logical CNOT gate using the lattice surgery operation described in the previous sections. In quantum error correction theory, a logical CNOT gate is often implemented using measurement-based quantum computation. Within the lattice surgery framework, we can leverage this approach. The CNOT gate implemented by local measurements is shown in Fig. 3.

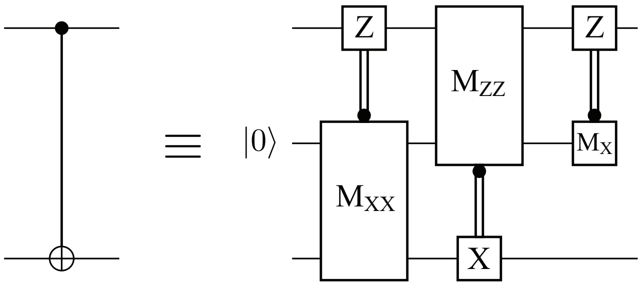

Fig. 3

In the previous sections, we have seen the measurement operation $Z_1 Z_2$ $(X_1 X_2)$ for logical qubits. Ignoring the Pauli correction based on the measurement results, as long as a 2-weight Pauli measurement on the logical qubits can be performed, we can achieve a CNOT gate.

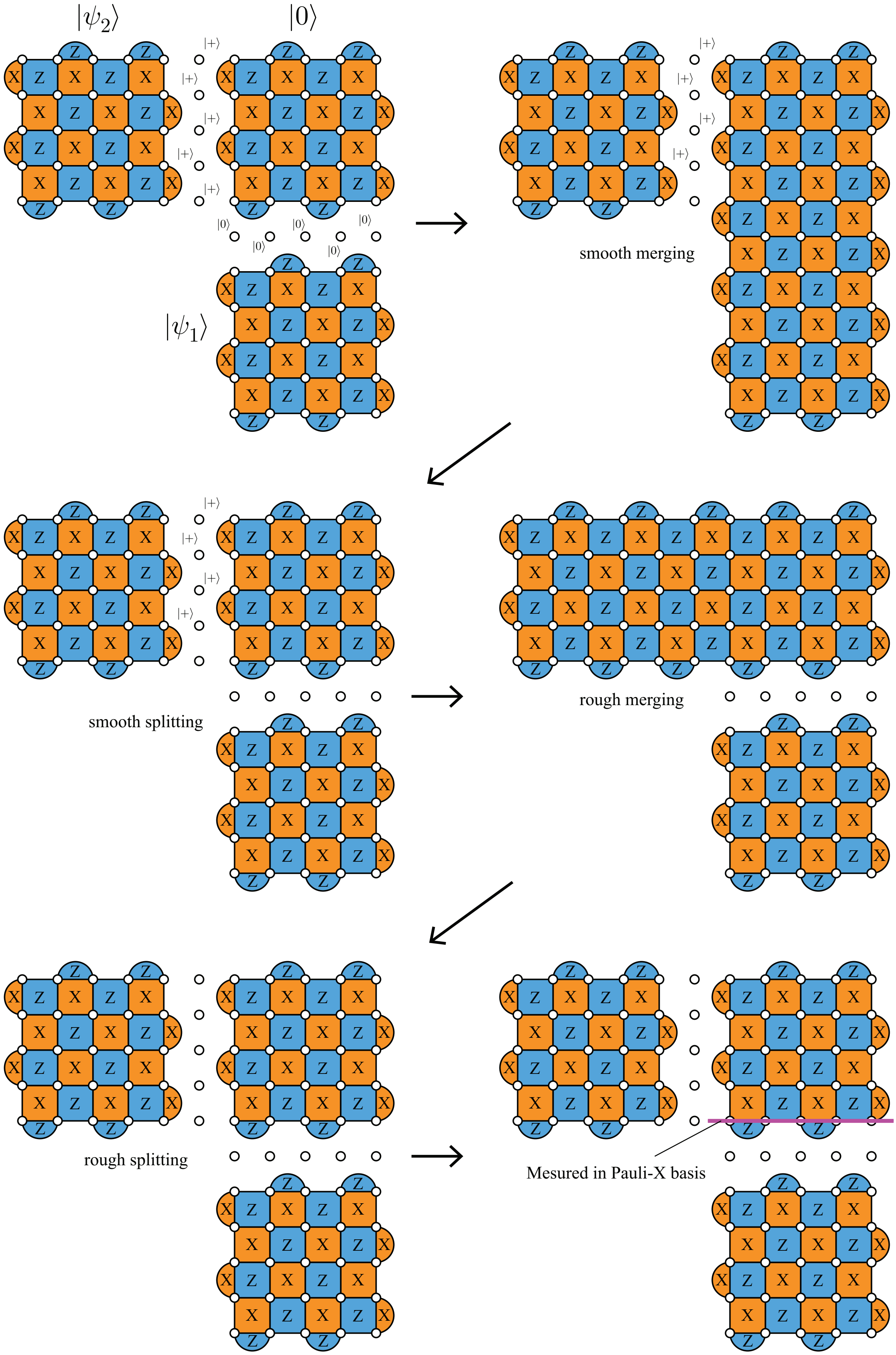
|ψ2⟩
|0⟩
|ψ1⟩
smooth merging
smooth splitting
rough merging
rough splitting
Mesured in Pauli-X basis

Fig. 4