Practical 7
SOR Method

- **SOR method with number of iterations as stopping criteria:**

Q. Use the SOR iteration method to solve the system of equations in 7 iterations

$$4\,x_1 - 2\,x_2 + 0\,x_3 = 8$$
$$-2\,x_1 + 6\,x_2 - 5\,x_3 = -29$$
$$0\,x_1 - 5\,x_2 + 11\,x_3 = 43$$

with the inital vector $x^{(0)} = (0,0,0)$.

In[1]:=
```
w = 1.2;
SOR[A0_, B0_, X0_, max_] :=
  Module[{A = N[A0], B = N[B0], i, j, k = 0, n = Length[X0], X = X0, Xk = X0},
   Print["X"_0, "=", X];
   While[k < max,
    For[ i = 1, i ≤ n, i++,
     X[[i]] = (1 - w) X[[i]] + w/A[[i,i]] (B[[i]] + A[[i,i]] X[[i]] - Sum[A[[i,j]] X[[j]], {j, 1, n}])];
    Print["X"_(k+1), "=", X];
    Xk = X;
    k = k + 1;];
   Print["No. of iterations performed ", k];
   Return[X];];
A = {{4, -2, 0}, {-2, 6, -5}, {0, -5, 11}};
B = {8, -29, 43};
X0 = {0, 0, 0};
SOR[A, B, X0, 7]
```

X_0 = {0, 0, 0}

X_1 = {2.4, -4.84, 2.05091}

X_2 = {-0.984, -3.17469, 2.54908}

X_3 = {0.691985, -2.33919, 2.90517}

X_4 = {0.858089, -2.08375, 2.97328}

X_5 = {0.978129, -2.01872, 2.99514}

X_6 = {0.993145, -2.00386, 2.99887}

X_7 = {0.999053, -2.00074, 2.99982}

No. of iterations performed 7

Out[6]= {0.999053, -2.00074, 2.99982}

Q. Use the SOR iteration method to solve the system of equations in 9 iterations

$$3x_1 - x_2 + x_3 = -1$$
$$-x_1 + 3x_2 - x_3 = 7$$
$$x_1 - x_2 + 3x_3 = -7$$

with the inital vector $x^{(0)} = (0, 0, 0)$.

In[13]:=
```
w = 1.25;
SOR[A0_, B0_, X0_, max_] :=
  Module[{A = N[A0], B = N[B0], i, j, k = 0, n = Length[X0], X = X0, Xk = X0},
   Print["X"_0, "=", X];
   While[k < max,
    For[ i = 1, i ≤ n, i++,
     X[[i]] = (1 - w) X[[i]] + w/A[[i,i]] (B[[i]] + A[[i,i]] X[[i]] - Sum[A[[i,j]] X[[j]], {j, 1, n}])];
    Print["X"_(k+1), "=", X];
    Xk = X;
    k = k + 1;];
   Print["No. of iterations performed ", k];
   Return[X];];
A = {{3, -1, 1}, {-1, 3, -1}, {1, -1, 3}};
B = {-1, 7, -7};
X0 = {0, 0, 0};
SOR[A, B, X0, 9]
```

X_0={0, 0, 0}

X_1={-0.416667, 2.74306, -1.60012}

X_2={1.49715, 2.188, -2.22878}

X_3={1.04937, 1.87824, -2.01411}

X_4={0.942803, 2.00073, -1.97234}

X_5={1.00308, 2.01263, -2.00294}

X_6={1.00572, 1.998, -2.00248}

X_7={0.998772, 1.99895, -1.9993}

X_8={0.999581, 2.00038, -1.99984}

X_9={1.0002, 2.00005, -2.0001}

No. of iterations performed 9

Out[18]= {1.0002, 2.00005, -2.0001}