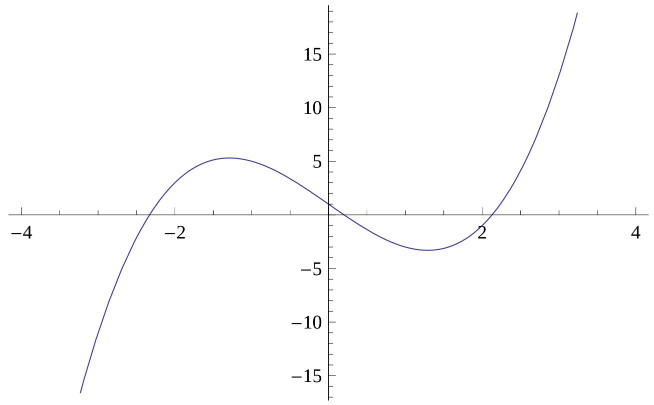
Secant Method

To find a root of an equation using secant method in given number of iterations.

(1) Find a real root of the equation $f(x) = x^3 - 5x + 1 = 0$ using secant method in six iterations.

```
Secant[x0_, x1_, n_, f_] :=
 Module[{xk, xk1, xk2}, xk = N[x0]; xk1 = N[x1]; i = 0;
  Output = { };
  While[i < n,
   xk2 = (xk * f[xk1] - xk1 * f[xk]) / (f[xk1] - f[xk]);
   interval = "[" <> ToString[NumberForm[xk, 12]] <>
     "," <> ToString[NumberForm[xk1, 12]] <> "]";
   xk = xk1; xk1 = xk2; i++;
   Output = Append[Output,
     {i, interval, xk2, f[xk2]}];];
  Print[NumberForm[TableForm[Output, TableHeadings →
      {None, {"i", "interval", "xi", "f[xi]"}}], 8]];
  Print[" Root after ", n, " iterrations ",
   NumberForm[xk2, 8]];
  Print[" Function value at approximated
    root,f[xi]= ", NumberForm[f[xk2], 8]];]
g[x_] := x^3 - 5 x + 1;
Plot[g[x], {x, -4, 4}]
Secant[0, 1, 6, g]
```



i	interval	xi	f[xi]
1	[0.,1.]	0.25	-0.234375
2	[1.,0.25]	0.18644068	0.074277312
3	[0.25,0.186440677966]	0.20173626	-0.00047111617
4	[0.186440677966,0.201736256179]	0.20163985	-8.642293×10^{-7}
5	[0.201736256179,0.201639852891]	0.20163968	1.0352719×10^{-11}
6	[0.201639852891,0.201639675721]	0.20163968	-2.220446×10^{-16}

Root after 6 iterrations 0.20163968

Function value at approximated root,f[xi]= -2.220446×10^{-16}

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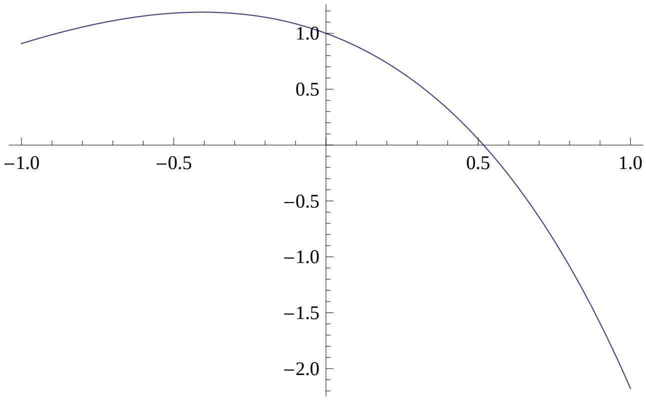
(2) Find a real root of the equation $f(x) = \text{Cos}x - xe^{x}$ using secant method in eight iterations

```
f[x_] := Cos[x] - x Exp[x];
Plot[f[x], {x, -1, 1}]
Secant[0, 1, 8, f]
```



i	interval	xi	f[xi]
1	[0.,1.]	0.31466534	0.51987117
2	[1.,0.314665337801]	0.44672814	0.20354478
3	[0.314665337801,0.446728144591]	0.53170586	-0.042931093
4	[0.446728144591,0.531705860645]	0.51690447	0.0025927631
5	[0.531705860645,0.516904467567]	0.51774747	0.000030111941
6	[0.516904467567,0.517747465271]	0.51775737	-2.1513164×10^{-8}
7	[0.517747465271,0.517757370754]	0.51775736	1.7841284×10^{-13}
8	[0.517757370754,0.517757363682]	0.51775736	-3.3306691×10^{-16}

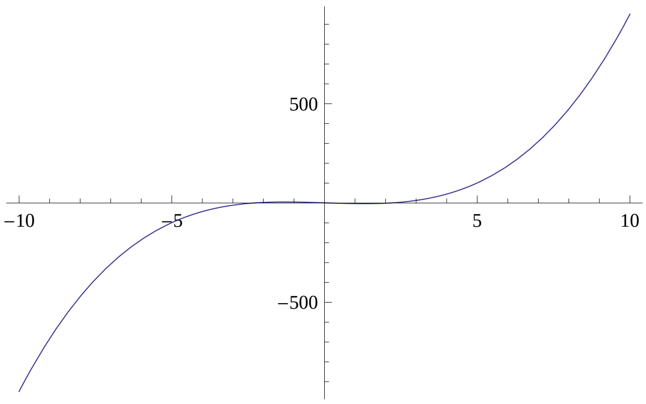
Root after 8 iterrations 0.51775736

Function value at approximated root,f[xi]= -3.3306691×10^{-16}

Regula-Falsi Method

(1) Find out the root of the function $g(x) = x^3 - 5x + 1$ after 10 iterations of the Regula Falsi method.

```
RegulaFalsi[x0_, x1_, n_, f_] := Module[{xk, xk1, xk2},
  xk = N[x0]; xk1 = N[x1]; If[f[xk] * f[xk1] > 0,
   Print["We cannot continue with Regula Falsi
     method as function values are not
     opposite sign at end points of interval"];
   Return[]]; i = 1; Output = { };
  While[i ≤ n, xk2 = (xk * f[xk1] - xk1 * f[xk]) /
     (f[xk1] - f[xk]);
   interval = "[" <> ToString[NumberForm[xk, 12]] <>
     "," <> ToString[NumberForm[xk1, 12]] <> "]";
   Output = Append[Output, {i, interval, xk2, f[xk2]}];
   If[Sign[f[xk1]] == Sign[f[xk2]],
    xk1 = xk2, xk = xk2]; i++;];
  Print[NumberForm[TableForm[Output, TableHeadings →
      {None, {"i", "interval", "xi", "f[xi]"}}], 8]];
  Print[" Root after ", n, " iterations ",
   NumberForm[xk2, 8]];
  Print[" Function value at approximated
     root,f[xi]= ", NumberForm[f[xk2], 8]];];
g[x_] := x^3 - 5 x + 1;
Plot[g[x], {x, -10, 10}]
RegulaFalsi[-1, 1, 10, g]
```



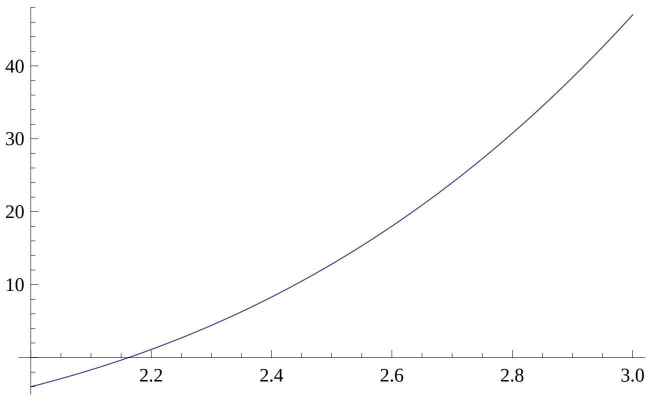
i	interval	xi	f[xi]
1	[-1.,1.]	0.25	-0.234375
2	[-1.,0.25]	0.19402985	0.037155501
3	[0.194029850746,0.25]	0.20168865	-0.00023892045
4	[0.194029850746,0.201688654959]	0.20163972	$-2.2244344 \times 10^{-7}$
5	[0.194029850746,0.201639721325]	0.20163968	$-2.0708324 \times 10^{-10}$
6	[0.194029850746,0.201639675766]	0.20163968	$-1.9273472 \times 10^{-13}$
7	[0.194029850746,0.201639675723]	0.20163968	$-4.4408921 \times 10^{-16}$
8	[0.194029850746,0.201639675723]	0.20163968	1.110223×10^{-16}
9	[0.201639675723,0.201639675723]	0.20163968	1.110223×10^{-16}
10	[0.201639675723,0.201639675723]	0.20163968	$-2.220446 \times 10^{-16}$

Root after 10 iterations 0.20163968

Function value at approximated root,f[xi]= $-2.220446 \times 10^{-16}$

(2) **Find out the root of the function $f(x) = x^4 - 3x^2 + x - 10$ over the interval [2, 3] after 7 iterations of the Regula Falsi method.**

```
f[x_] := x^4 - 3 x^2 + x - 10;
Plot[f[x], {x, 2, 3}]
RegulaFalsi[2, 3, 7, f]
```



i	interval	xi	f[xi]
1	[2.,3.]	2.0784314	-2.2198625
2	[2.07843137255,3.]	2.119995	-1.1637008
3	[2.11999499205,3.]	2.1412571	-0.59162874
4	[2.14125711528,3.]	2.1519325	-0.29607559
5	[2.15193245843,3.]	2.1572414	-0.1469951
6	[2.15724139986,3.]	2.159869	-0.072691406
7	[2.15986895617,3.]	2.1611663	-0.035876602

Root after 7 iterations 2.1611663

Function value at approximated root,f[xi]= -0.035876602