Practical 6
Gauss Jacobi Method

Gauss -Jacobi Iteration Method: A general linear iterative method for the solution of the system of equations Ax = b may be defined of the form :

$$x^{(k+1)} = H\, x^{(k)} + C$$

where , $H = -D^{-1}(L+U)$

$C = D^{-1}b$

where, D = diagonal matrix

L = lower triangular matrix

U = upper triangular matrix

- **Gauss Jacobi method with number of iterations as stopping criteria:**

Q1. Use the Gauss Jacobi iteration method to solve the system of equations

$$2\,x_1 - x_2 + 0\,x_3 = 7$$
$$-\,x_1 + 2\,x_2 - x_3 = 1$$
$$0\,x_1 - x_2 + 2\,x_3 = 1$$

with the inital vector $x^{(0)} = (0,0,0)$. Perform 12 iterations.

```
GaussJacobi[A0_, B0_, X0_, max_] :=
  Module[{A = N[A0], B = N[B0], i, j, k = 0, n = Length[X0], X = X0, Xk = X0},
   Print["X"_0, "=", X];
   While[k < max,
    For[ i = 1, i ≤ n, i++,
     X[[i]] = 1/A[[i,i]] (B[[i]] + A[[i,i]] Xk[[i]] - Sum[A[[i,j]] Xk[[j]], {j, 1, n}])];

    Print["X"_(k+1), "=", X];
    Xk = X;
    k = k + 1;];
   Print[" No. of iterarations performed ", max];
   Return[X];];
```

$$X_{[[i]]} = \frac{1}{A_{[[i,i]]}} \left(B_{[[i]]} + A_{[[i,i]]}\, Xk_{[[i]]} - \sum_{j=1}^{n} A_{[[i,j]]}\, Xk_{[[j]]} \right)$$

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix};$$

$$B = \begin{pmatrix} 7 \\ 1 \\ 1 \end{pmatrix};$$

$$X0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix};$$

```
GaussJacobi[A, B, X0, 12]
```

X_0={{0}, {0}, {0}}

X_1={{3.5}, {0.5}, {0.5}}

X_2={{3.75}, {2.5}, {0.75}}

X_3={{4.75}, {2.75}, {1.75}}

X_4={{4.875}, {3.75}, {1.875}}

X_5={{5.375}, {3.875}, {2.375}}

X_6={{5.4375}, {4.375}, {2.4375}}

X_7={{5.6875}, {4.4375}, {2.6875}}

X_8={{5.71875}, {4.6875}, {2.71875}}

X_9={{5.84375}, {4.71875}, {2.84375}}

X_{10}={{5.85938}, {4.84375}, {2.85938}}

X_{11}={{5.92188}, {4.85938}, {2.92188}}

X_{12}={{5.92969}, {4.92188}, {2.92969}}

No. of iterarations performed 12

{{5.92969}, {4.92188}, {2.92969}}

Q2. Solve the system of equations

$$4\,x_1 + x_2 + x_3 = 2$$
$$x_1 + 5\,x_2 + 2\,x_3 = -6$$
$$x_1 + 2\,x_2 + 3\,x_3 = -4$$

with the inital vector $x^{(0)} = (0.5,-0.5,-0.5)$. Perform 15 iterations.

```
A = {{4, 1, 1}, {1, 5, 2}, {1, 2, 3}};
B = {2, -6, -4};
X0 = {0.5, -0.5, -0.5};
GaussJacobi[A, B, X0, 15]
```

X_0={0.5, -0.5, -0.5}

X_1={0.75, -1.1, -1.16667}

X_2={1.06667, -0.883333, -0.85}

X_3={0.933333, -1.07333, -1.1}

X_4={1.04333, -0.946667, -0.928889}

X_5={0.968889, -1.03711, -1.05}

X_6={1.02178, -0.973778, -0.964889}

X_7={0.984667, -1.0184, -1.02474}

X_8={1.01079, -0.987037, -0.982622}

X_9={0.992415, -1.00911, -1.01224}

X_{10}={1.00534, -0.993588, -0.9914}

X_{11}={0.996247, -1.00451, -1.00605}

X_{12}={1.00264, -0.996828, -0.995744}

X_{13}={0.998143, -1.00223, -1.00299}

X_{14}={1.00131, -0.998431, -0.997894}

X_{15}={0.999081, -1.0011, -1.00148}

No. of iterarations performed 15

{0.999081, -1.0011, -1.00148}