# Blindspots in Omni-directional Detectors

Course Project Report PH 821

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# Abstract

Gravitational wave detectors, designed to be nearly omnidirectional, exhibit non-uniform sensitivity across the sky due to their intrinsic directional response. These variations are captured by their antenna patterns, which characterize the detectors' sensitivity to gravitational wave signals as a function of source position and wave polarization. This report investigates the mathematical and physical foundations of these sensitivity patterns and identifies regions of low response, referred to as blindspots. These blindspots arise from the geometric alignment of the detectors' arms with the incoming gravitational wave. They are inherent to the design of interferometric detectors such as LIGO, Virgo, and KAGRA.

By analyzing these detectors' geometry and response functions, we explore how blindspots affect the detection and localization of astrophysical sources, including binary mergers and supernovae. Strategies to mitigate these limitations, such as using a network of detectors, advanced data analysis methods, and next-generation designs, are also discussed. This report highlights the critical role of global detector networks in reducing the impact of blindspots, enabling a more comprehensive and unbiased survey of the gravitational wave sky.

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# Chapter 1

# Introduction

Gravitational wave (GW) detectors have revolutionized astrophysics by enabling the direct observation of cosmic phenomena such as black hole mergers, neutron star collisions, and other extreme astrophysical events. These observations have provided unprecedented insights into the dynamics of the universe and opened a new frontier in multi-messenger astronomy.

However, the sensitivity of these detectors is not isotropic, which means that their ability to detect gravitational waves varies with the direction of the incoming signal. This directional dependence arises due to the geometry of the detector arms and their response to the polarization of gravitational waves. Interferometric detectors, such as those used by LIGO, Virgo, and KAGRA, operate by measuring the tiny distortions in spacetime caused by gravitational waves. Their sensitivity is determined by the relative alignment of the detector arms with the incoming wave's direction and polarization.

This non-uniform sensitivity creates regions in the sky where detection probabilities are significantly reduced, often called '**blind spots**' These blindspots represent a fundamental limitation in the detector's ability to observe events uniformly across the sky. Addressing these blindspots is critical for enhancing the detection rate and improving the localization accuracy of gravitational wave sources.

This report explores the mathematical and physical principles underlying the directional sensitivity of gravitational wave detectors. By analyzing their antenna patterns—mathematical representations of their sensitivity as a function of sky position—we identify the origins of blindspots and evaluate their implications for gravitational wave detection. Furthermore, we discuss strategies to mitigate these limitations, including the use of global detector networks, advancements in interferometer design, and improvements in data analysis techniques.

## 1.1 Mathematical Framework for Detector Response

The sensitivity of a gravitational wave detector is characterized by its response function:

$$h_{\text{det}} = D^{ij} h_{ij} \tag{1.1}$$

# Derivation of $F_+$ and $F_\times$ Including Polarization Angle $\psi$

The detector response is given by:

$$h_{\text{det}} = D^{ij} h_{ij}, \tag{1.2}$$

where $D^{ij}$ is the detector tensor, and $h_{ij}$ is the strain tensor of the gravitational wave. The strain tensor in the transverse-traceless (TT) gauge is:

$$h_{ij} = h_+ e_{ij}^+ + h_\times e_{ij}^\times, \tag{1.3}$$

where $h_+$ and $h_\times$ are the amplitudes of the $+$- and $\times$-polarized waves, and $e_{ij}^+$, $e_{ij}^\times$ are the polarization tensors.

The detector response can be expressed as:

$$h_{\text{det}} = h_+ D^{ij} e_{ij}^+ + h_\times D^{ij} e_{ij}^\times. \tag{1.4}$$

Define the antenna patterns as:

$$F_+ = D^{ij} e_{ij}^+, \quad F_\times = D^{ij} e_{ij}^\times. \tag{1.5}$$

To transform the polarization tensors $e_{ij}^+$ and $e_{ij}^\times$ from the wave frame to the detector frame, we use Euler rotations. The rotation matrix is:

$$R = R_z(\psi) R_y(\theta) R_z(\phi), \tag{1.6}$$

where:

1. Rotation about the $z$-axis:

$$R_z(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad R_z(\psi) = \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{1.7}$$

2. Rotation about the $y$-axis:

$$R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}. \tag{1.8}$$

The full rotation matrix $R$ is:

$$R = \begin{pmatrix} \cos\psi\cos\theta\cos\phi - \sin\psi\sin\phi & -\cos\psi\cos\theta\sin\phi - \sin\psi\cos\phi & \cos\psi\sin\theta \\ \sin\psi\cos\theta\cos\phi + \cos\psi\sin\phi & -\sin\psi\cos\theta\sin\phi + \cos\psi\cos\phi & \sin\psi\sin\theta \\ -\sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{pmatrix}. \tag{1.9}$$

In the wave frame, the polarization tensors are:

$$(e_{ij}^+)_{\text{wave}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (e_{ij}^\times)_{\text{wave}} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{1.10}$$

Transform these into the detector frame:

$$(e_{ij}^{+})_{\text{detector}} = R_{ik}R_{jl}(e_{kl}^{+})_{\text{wave}}, \tag{1.11}$$

$$(e_{ij}^{\times})_{\text{detector}} = R_{ik}R_{jl}(e_{kl}^{\times})_{\text{wave}}. \tag{1.12}$$

For an interferometric detector, the detector tensor is:

$$D^{ij} = \frac{1}{2}\left(\hat{u}^i\hat{u}^j - \hat{v}^i\hat{v}^j\right), \tag{1.13}$$

where $\hat{u}$ and $\hat{v}$ are unit vectors along the detector arms. For simplicity, assume that the arms are aligned with the x- and y-axes of the detector frame. Then:

$$\hat{u}^i = \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \hat{v}^i = \begin{pmatrix}0\\1\\0\end{pmatrix}. \tag{1.14}$$

The explicit form of $D^{ij}$ is:

$$D^{ij} = \frac{1}{2}\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix}. \tag{1.15}$$

Substitute the transformed polarization tensors into the detector response:

$$F_+ = D^{ij}R_{ik}R_{jl}(e_{kl}^{+})_{\text{wave}}, \quad F_\times = D^{ij}R_{ik}R_{jl}(e_{kl}^{\times})_{\text{wave}}. \tag{1.16}$$

After simplifying the matrix multiplications, the final forms of the antenna patterns are:

$$F_+ = \frac{1}{2}(1+\cos^2\theta)\cos 2\phi\cos 2\psi - \cos\theta\sin 2\phi\sin 2\psi, \tag{1.17}$$

$$F_\times = \frac{1}{2}(1+\cos^2\theta)\cos 2\phi\sin 2\psi + \cos\theta\sin 2\phi\cos 2\psi. \tag{1.18}$$

### 1.1.1 Antenna Patterns and Sensitivity

The antenna pattern quantifies the directional sensitivity of a detector. For a linearly polarized wave, the response functions for the + and × polarizations are given by:

$$F_+ = \frac{1}{2}\left(1+\cos^2\theta\right)\cos 2\phi\cos 2\psi - \cos\theta\sin 2\phi\sin 2\psi, \tag{1.19}$$

$$F_\times = \frac{1}{2}\left(1+\cos^2\theta\right)\cos 2\phi\sin 2\psi + \cos\theta\sin 2\phi\cos 2\psi. \tag{1.20}$$

where $\theta$ and $\phi$ define the source location, and $\psi$ is the polarization angle[2]. If we put $\phi = 45$ we will get $F_+$ and $F_\times$ equals to zero.

### 1.1.2 Blindspots in Sensitivity

The combined sensitivity is:

$$S(\theta, \phi) = \sqrt{F_+^2 + F_\times^2} \tag{1.21}$$

Blindspots occur at specific angular configurations where $S(\theta, \phi) \approx 0$, typically due to geometric misalignment between the incoming wave and detector arms.

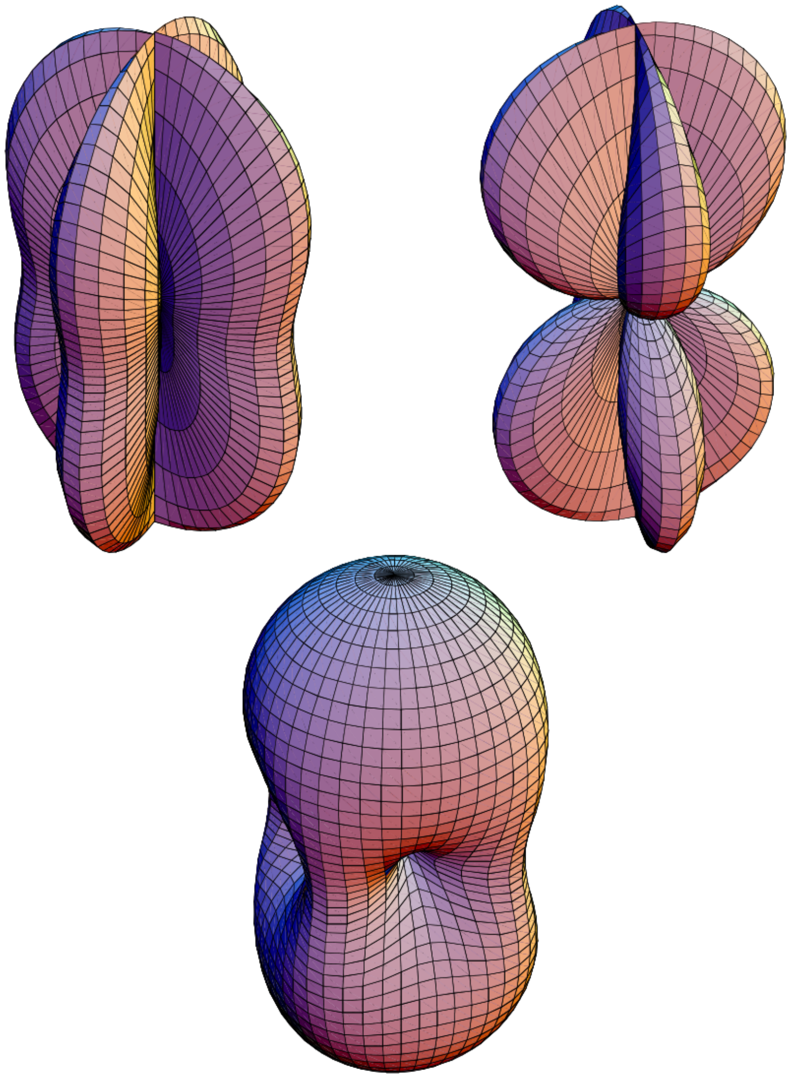


Figure 1.1: Illustration of the antenna pattern of a gravitational wave detector[1].

## 1.2 Blindspots and Detection Efficiency

Blindspots can significantly impact the efficiency of detecting gravitational waves from certain astrophysical sources. Events occurring in low-sensitivity regions might only be detected if their signal strength exceeds the noise threshold. The presence of blindspots can also bias statistical analyses of event rates and source properties, as regions of reduced sensitivity might lead to under-sampling of specific parts of the sky.

### 1.2.1 Impacts on Multi-messenger Astronomy

Blindspots not only affect gravitational wave detections but also have implications for multi-messenger astronomy. The ability to localize a source on the sky depends on precise measurements of the arrival times and amplitudes at different detectors. Poor coverage in certain sky regions can hinder the identification of electromagnetic or neutrino counterparts.

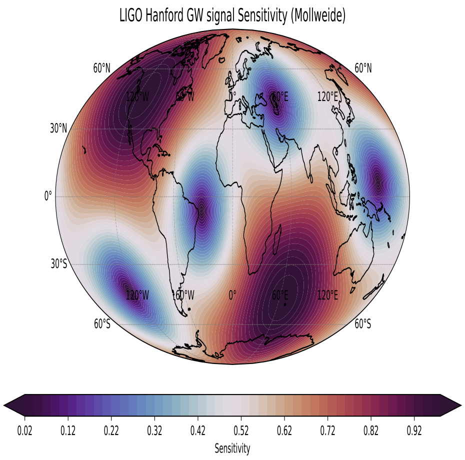
# Chapter 2

# Case Study: LIGO, Virgo and KAGRA Sensitivity Patterns

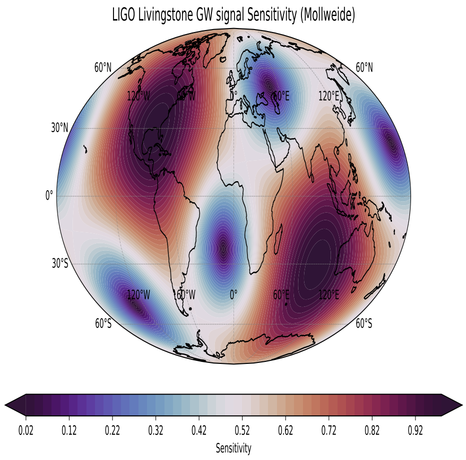
The LIGO, Virgo and KAGRA detectors demonstrate complementary sensitivity patterns. While LIGO's two detectors (Hanford and Livingston) share similar orientations, Virgo and KAGRA provide distinct sensitivity patterns due to their geographic location and orientation. This complementary nature highlights the importance of international collaboration in gravitational wave astronomy. In here I have checked the sensitivity pattern for a fixed UTC of 1732431926 sec after the epoch. The sensitivity pattern changes according to time.

## 2.1 Results

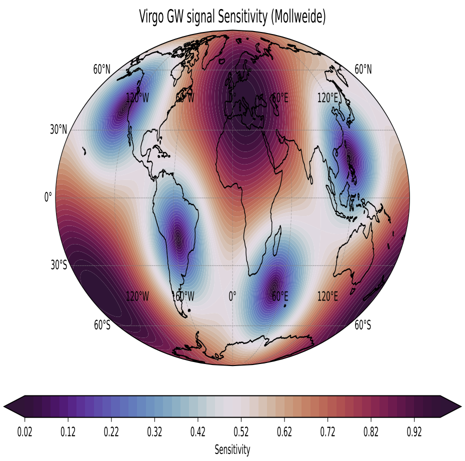
The blue region corresponds to the area of least sensitivity i.e. the blind spots. The advent of using a combination of multiple detectors increases the effective area of higher sensitivity. Also, the lower band of sensitivity increases as we use the combination of detectors. The sensitivity patterns are plotted on the earth's map using **Mollweide** projection.



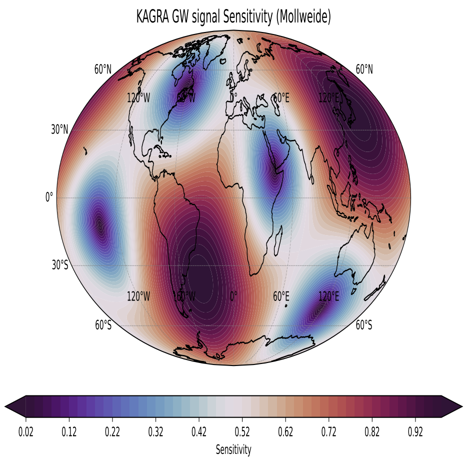

(a) **LIGO Hanford**



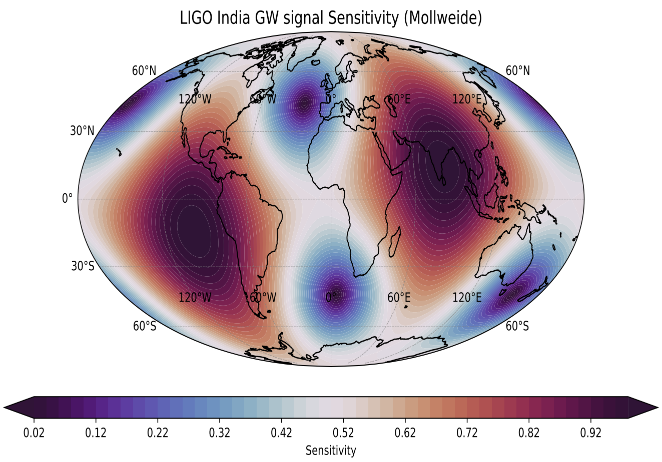

(b) **LIGO Livingstone**




(c) **VIRGO**

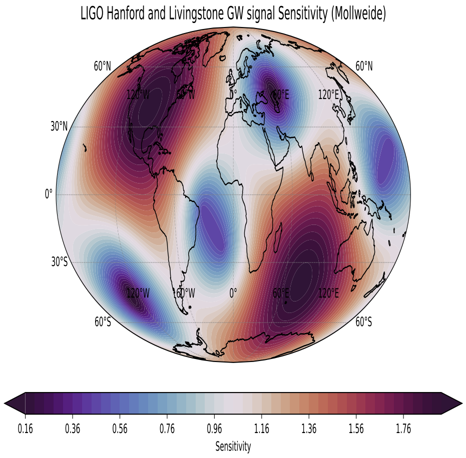



(d) **KAGRA**

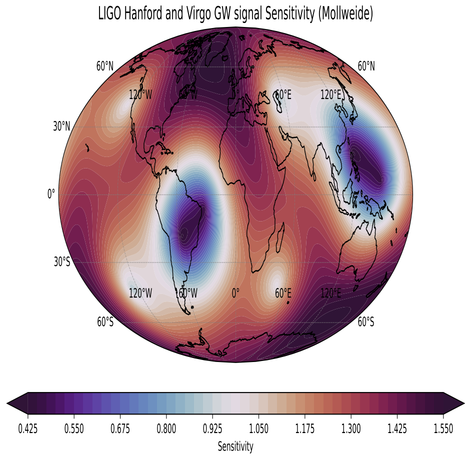



(e) **LIGO India**

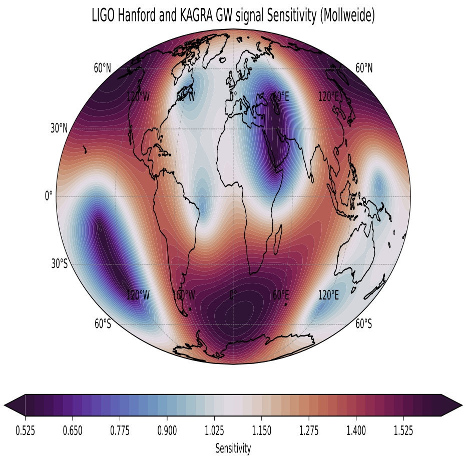
Figure 2.1: Sensitivity maps for standalone detectors currently in operation



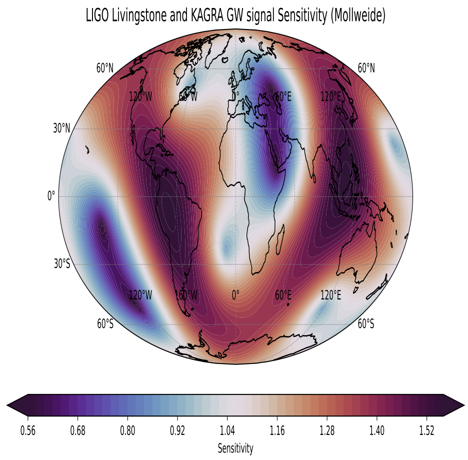

(a) **LIGO Hanford** and **Livingstone**



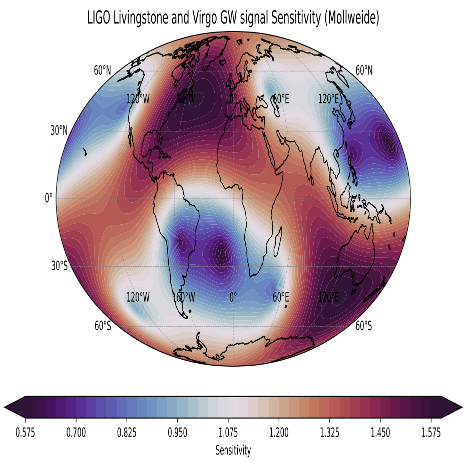

(b) **LIGO Hanford** and **VIRGO**



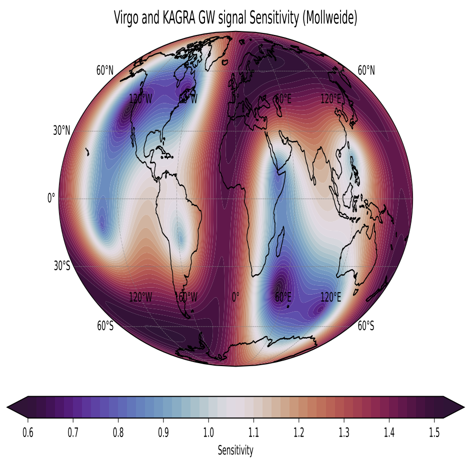

(c) **LIGO Hanford** and **KAGRA**




(d) **LIGO Livingstone** and **KAGRA**

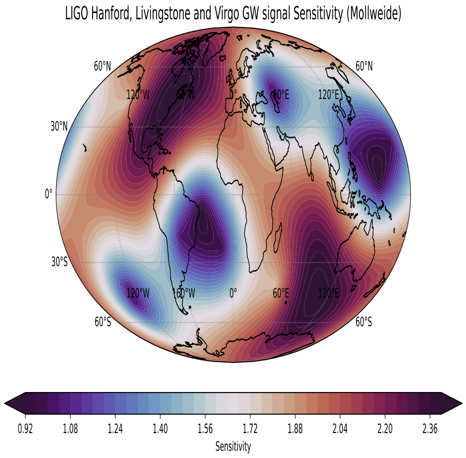



(e) **LIGO Livingstone** and **VIRGO**

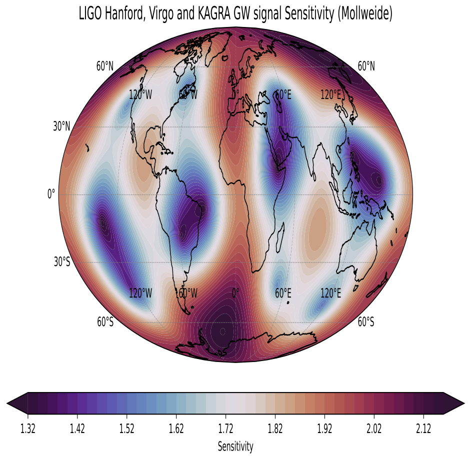



(f) **VIRGO** and **KAGRA**

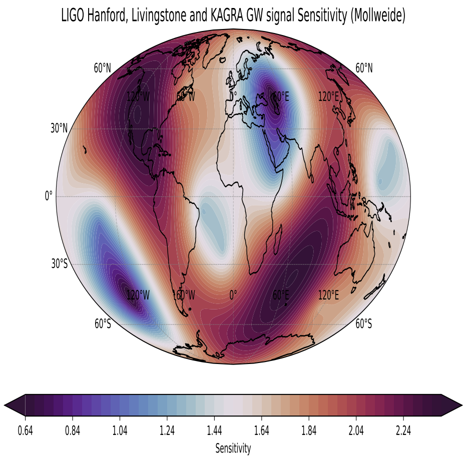
Figure 2.2: Sensitivity maps for the combination of two detectors currently in operation



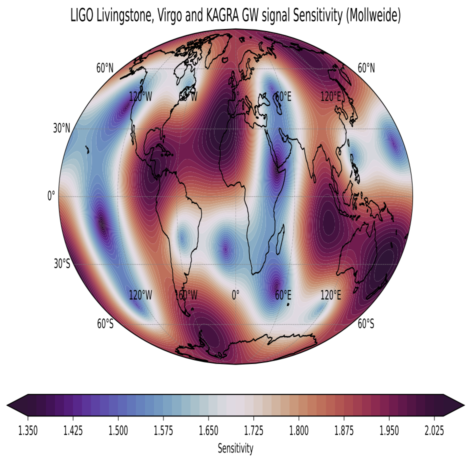

(a) **LIGO H & L** and **VIRGO**




(b) **LIGO Hanford, VIRGO** and **KAGRA**

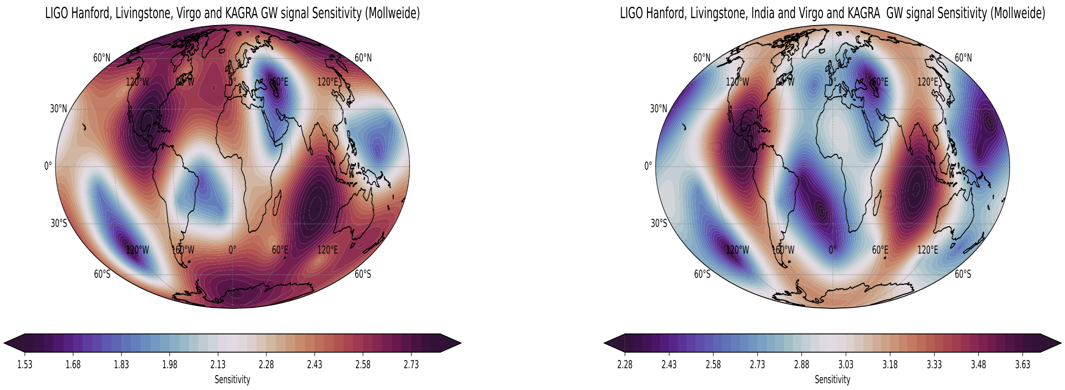



(c) **LIGO H & L** and **KAGRA**




(d) **LIGO Livingstone VIRGO** and **KAGRA**

Figure 2.3: Sensitivity maps for the combination of three detectors currently in operation




(a) Sensitivity maps for all four operational detectors **LIGO Hanford, Livingstone, VIRGO and KAGRA**

(b) Sensitivity maps for the combination of detectors **LIGO Hanford, Livingstone, India, VIRGO and KAGRA**

Figure 2.4: Sensitivity maps for the combination of all detectors

## 2.2 Conclusion

A single detector has nearly zero sensitivity for a GW signal arriving at 45 deg ,because it stretches both arms equally, nullifying the interference effect. This report highlights the directional limitations of current gravitational wave detectors and their implications for astrophysical observations. By leveraging multi-detector networks and advanced analytical techniques, the impact of blindspots can be mitigated, paving the way for more comprehensive gravitational wave astronomy. With the advent of next-generation detectors and emerging technologies, the field is poised to achieve unprecedented levels of sensitivity and coverage.

## 2.3 Mitigation Strategies

Blindspots can be minimized by:

1. **Networked Detectors:** Combining data from multiple detectors with varied orientations and locations improves sky coverage.
2. **Advanced Data Analysis:** Bayesian inference and coherent data analysis techniques can improve results.
3. **Next-generation Detectors:** Optimizing arm orientations and incorporating additional polarization modes can reduce blind spots.

## 2.4 Future Prospects

Next-generation detectors, such as the Einstein Telescope (ET) and Cosmic Explorer (CE), promise to address the limitations posed by blindspots. These detectors will have improved sensitivity and cover a broader frequency range, enabling the detection of weaker signals from a wider variety of sources. Additionally, space-based detectors like LISA will complement ground-based observatories by targeting lower-frequency gravitational waves, thereby providing nearly all-sky sensitivity.

## 2.5 Emerging Technologies

Emerging technologies, such as quantum-enhanced interferometry and advanced noise reduction techniques, are expected to enhance the sensitivity further and reduce the impact of blind spots. These advancements will play a crucial role in ensuring the comprehensive observation of the gravitational wave universe.

# Bibliography

[1] Drew Garvin Keppel. *Signatures and Dynamics of Compact Binary Coalescences and a Search in LIGO's S5 Data*. Ph.d. dissertation, California Institute of Technology, 2009. URL `https://resolver.caltech.edu/CaltechETD:etd-05202009-115750`.

[2] B. S. Sathyaprakash and Bernard F. Schutz. Physics, astrophysics and cosmology with gravitational waves. *Living Reviews in Relativity*, 12(1), March 2009. ISSN 1433-8351. doi: 10.12942/lrr-2009-2. URL `http://dx.doi.org/10.12942/lrr-2009-2`.

[3] PyCBC Team. Detector response. `https://pycbc.org/pycbc/latest/html/detector.html`, n.d. Accessed: 2024-11-24.

# Appendix[3]

## 2.6 Code

```
from datetime import datetime, timedelta
utc = datetime.utcnow()
sec = time.mktime(utc.timetuple()) + utc.microsecond / 1e6
print("Current UTC time in seconds:", int(sec))

import numpy as np
import matplotlib.pyplot as plt
import cartopy.crs as ccrs
import time
import lal
from pycbc.detector import Detector
from joblib import Parallel, delayed

ra = np.linspace(0, 2 * np.pi, 360)
dec = np.linspace(-np.pi/2, np.pi/2, 180)
# t = lal.LIGOTimeGPS(time.time())
t=1e10 + 8000

def calc_sens(det_name, ra, dec, t):
    det = Detector(det_name)
    sens = np.zeros((len(dec), len(ra)))
    for i, r in enumerate(ra):
        for j, d in enumerate(dec):
            Fp, Fc = det.antenna_pattern(r, d, 0, t)
            sens[j, i] = np.sqrt(Fp**2 + Fc**2)
    return sens

det_names = ['H1','L1','V1','K1']
res = Parallel(n_jobs=-1)(delayed(calc_sens)(name, ra, dec, t) for name in det_names)
sens_map = np.sum(res, axis=0)
lats = np.degrees(dec)
longs = np.degrees(ra)

fig = plt.figure(figsize=(12, 6))
ax = plt.axes(projection=ccrs.Mollweide())
lg, lt = np.meshgrid(longs, lats)
ctr = ax.contourf(lg, lt, sens_map, levels=50, cmap='twilight_shifted',
transform=ccrs.PlateCarree(), extend='both')
```

```
ax.coastlines()
gl = ax.gridlines(draw_labels=True, linewidth=0.5, color='gray', alpha=0.7,
linestyle='--')
gl.top_labels = False
gl.right_labels = False
cbar = plt.colorbar(ctr, orientation='horizontal', pad=0.07, aspect=40,
label='Sensitivity')
plt.title("LIGO Hanford, Livingstone,Virgo and KAGRA GW signal Sensitivity (Mollweide)",
fontsize=14)
plt.savefig('H1_L1_V1_K1_current.pdf', dpi=300)
plt.show()
```

## 2.7 Link

For the output and the jupyter notebook file please refer to the following git repository link: PH821_Project