# Rayleigh-Taylor Instability in Stellar Interior

## PH-819 Course Project

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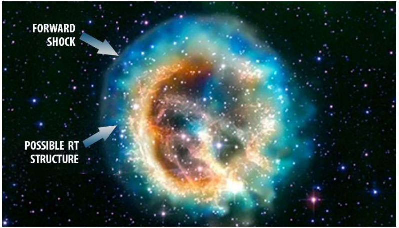
# Overview

1. Rayleigh-Taylor Instability (RTI)
2. RTI with Magnetic Field
3. Results

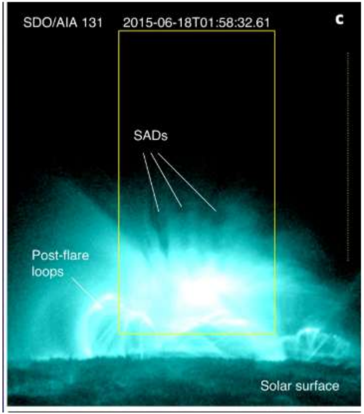
# Goals

- Why Rayleigh-Taylor Instability occurs in stellar interior?
- What we can infer by studying Rayleigh-Taylor Instability?
- What are factors that affect Rayleigh-Taylor Instability?
- What are the affects of Magnetic Field on the instability and observations?

# RTI in astrophysical systems




- Supernova Explosions
- Stellar Convection Zones
- Accretion Disks



# Fluid Instability

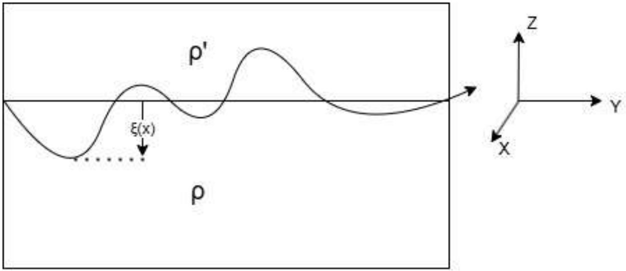



Figure: Perturbation in fluids [3]

**Assumptions:**

- irrotational fluid: $\nabla \times \mathbf{v} = 0 \Rightarrow \boxed{\mathbf{v} = \nabla\psi}$
- first order (linearized) correction
- incompressible fluid: $\nabla^2\psi = 0 \Rightarrow \psi = f(z)cos(kx - \omega t)$

# Propagation, Mixing or Instability??

Dispersion Relation:

$$\boxed{\omega^2 = \frac{kg(\rho - \rho')}{\rho \coth(kh) + \rho' \coth(kh')}}$$

$\rho' = 0$:

$\omega^2 = gk \tanh(kh)$

- Deep-water waves $(kh \gg 1) : \omega^2 \approx gk$
- Shallow-water waves $(kh \ll 1) : \omega^2 \approx gk^2 h$

$\rho' \neq 0$:

- $kh \gg 1, kh' \gg 1 : \omega^2 = -kgA$
- $kh \ll 1, kh' \ll 1 : \omega^2 = \frac{k^2 g(\rho - \rho')hh'}{\rho h' + \rho' h}$
- $kh \gtrsim 1, kh' \ll 1 : \omega^2 = \frac{k^2 gh'(\rho - \rho')}{\rho}$

- Equilibrium is unstable if $\rho < \rho'$, i.e., *when heavier fluid is residing on top of the lighter fluid.*

- In general, for $d\rho/dz \times g_{eff} < 0 \Rightarrow$ **Rayleigh-Taylor Instability [1]**.

# RTI with Magnetic field

**Why do we study magnetized-RTI?**

- Stellar interiors, supernova remnants, and accretion disks often involve magnetized plasmas.
- RTI is a main concern in inertial confinement fusion (ICF).
- Magnetic fields can suppress or stabilize the instability.

# Analytical Methodology

- Start from ideal MHD equations with gravity and incompressibility
- Linearize about equilibrium: $(\rho_0, p_0, \mathbf{B}_0)$ + small perturbations
- Apply normal-mode ansatz $e^{i(kx-\omega t)}$ to reduce to ODEs in $z$
- Impose continuity boundary conditions at interfaces
- Derive dispersion relations $\gamma^2(k)$ for three cases.
- Introduce Atwood number $A_t$.
- Plot growth rate $\gamma^2$ vs. $k$ to identify stability thresholds

# RTI growth rate with wave number [2]

Frequency of perturbations:

$$\omega = \omega_r + i\,\gamma$$

For **Unstable** equilibrium case,

$$\omega_r = 0$$

**Three cases:**

1. $\rho(z) \propto z^3$ and $B(z) \propto e^z$
2. Densities are constant and $B(z) \propto e^z$.
3. Densities and magnetic field both are constant.

## 1. Densities $(\rho_1, \rho_2)$ are constant and $B(z) \propto e^z$

- $\gamma^2 = A_t\, g\, k - \frac{B^2}{\mu_0(\rho_1+\rho_2)}\, k^2$

  where $A_t$ is 'Atwood number',

  $$A_t = \frac{\rho_2-\rho_1}{\rho_2+\rho_1}$$

- Minimum dimensionless magnetic field frequency to suppress RTI:

  $$\omega_{\mathrm{th}} = \sqrt{\frac{\rho_2-\rho_1}{\rho_2+\rho_1}} = \sqrt{A_t}$$

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  $$\omega_{\text{th}} = \sqrt{\frac{\rho_2-\rho_1}{\rho_2+\rho_1}} = \sqrt{A_t}$$

## 2. Densities $(\rho_1, \rho_2)$ and magnetic field $(B)$ both are constant [4]

- $\gamma^2 = A_t\, gk - \frac{2(k\,B)^2}{\mu_0(\rho_1+\rho_2)}$

- Critical wavelength $(\lambda_c)$, below which perturbations are stable:

  $$\lambda_c = \frac{4\pi B^2}{\mu_0 A_t(\rho_1+\rho_2)g}$$

- Maximum growth rate $(\gamma_m)$:

  $$\gamma_m = \sqrt{\frac{g\,k\,A_t}{2}}$$

  for $\lambda_m = \frac{8\pi B_0^2}{\mu_0 A_t(\rho_1+\rho_2)g} = 2\lambda_c$

# Visualization of stability

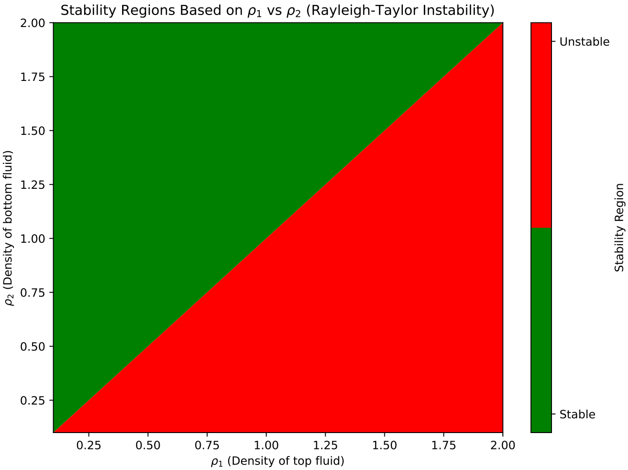


Figure: Plot of $\rho_1$ vs $\rho_2$ to display stable and unstable regions

# Impact of constant Magnetic field on growth rate

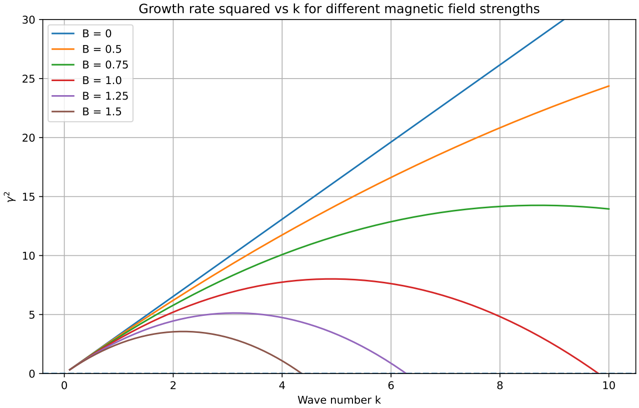


Figure: *Plot of $\gamma^2$ vs k shows how horizontal magnetic impacts the normalized growth rate*

# Impact of constant Magnetic field on growth rate

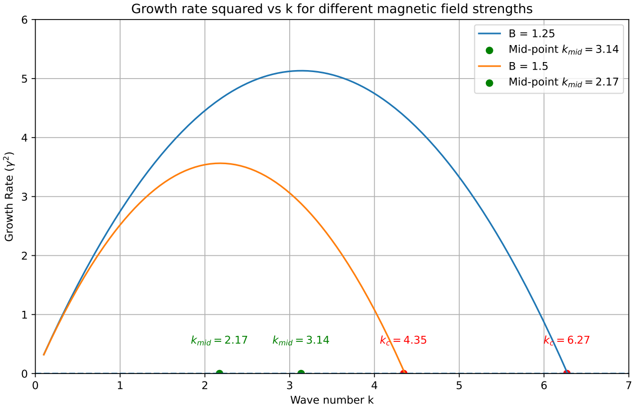


Figure: *Plot between $\gamma^2$ vs $k$ shows critical wavenumber $k_c$ and fastest growing wavenumber $k_{mid}$*

# Impact of exponential Magnetic field on growth rate

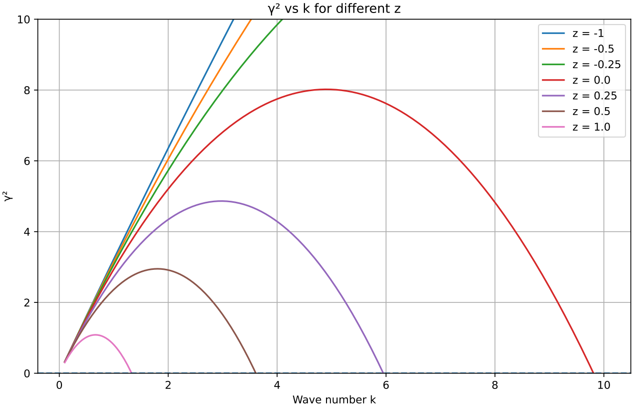


Figure: *Plot between $\gamma^2$ vs $k$ for exponential magnetic field $B_0 e^z$ showing growth decays faster than that of constant magnetic field*

# Conclusion

- Growth rate ($\gamma$) increase as Atwood number ($A_t$) increases.
- Application of a constant horizontal magnetic field can invert the instability in the system. As the strength of the magnetic field increases, the growth rate decreases for the same given perturbation, i.e. k.
- Application of a horizontal exponential magnetic field can increase or decrease the instability in the system, depending upon the value/sign of the exponent.

# References


[1] S. Chandrasekhar. "Hydrodynamic and Hydromagnetic Stability". In: *Hydrodynamic and Hydromagnetic Stability*. Chapter X: The Stability of Superposed Fluids – The Rayleigh-Taylor Instability. Oxford: Oxford University Press, 1961. Chap. 10, pp. 428–478.

[2] Mohammad-Ali Masoumparast Katek-Lahijani and Soheil Khoshbinfar. "Stabilization of magneto-Rayleigh-Taylor instability with non-uniform density and magnetic field profiles in Cartesian Geometry". In: *Chinese Journal of Physics* 91 (2024), pp. 479–493.

[3] Thanu Padmanabhan. "Theoretical Astrophysics: Volume I – Astrophysical Processes". In: *Theoretical Astrophysics: Volume I – Astrophysical Processes*. Chapter 8: Fluid Instability. Cambridge: Cambridge University Press, 2000. Chap. 8, pp. 361–427.

[4] Yang Zhang, Pakorn Wongwaitayakornkul, and Paul M Bellan. "Magnetic Rayleigh–Taylor instability in an experiment simulating a solar loop". In: *The Astrophysical Journal Letters* 889.2 (2020), p. L32.

**Thank you.**