Multi-modal contrastive learning adapts to *intrinsic dimension* of shared latent variables

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Growing availability of multi-modal data

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How can one efficiently integrate data from multi-modalities?

Multi-modal Contrastive Learning

maximize $\mathrm{sim}(f(X_i), g(Y_i))$, minimize $\mathrm{sim}(f(X_i), g(Y_j))$, $i \neq j$

- Contrastive language-image pre-training (CLIP)[2] has been the SOTA pipeline for multi-modal learning
- infoNCE loss $\mathcal{L}^N(f, g, \tau)$ with temperature optimization

$$\mathcal{L}^N(f,g,\tau) = -\frac{1}{N}\sum_{i\in[N]} \log \frac{\exp\left(\frac{\sigma(f(X_i),g(Y_i))}{\tau}\right)}{N^{-1}\sum_{j\in[N]}\exp\left(\frac{\sigma(f(X_i),g(Y_j))}{\tau}\right)} + \text{symmetric term}$$

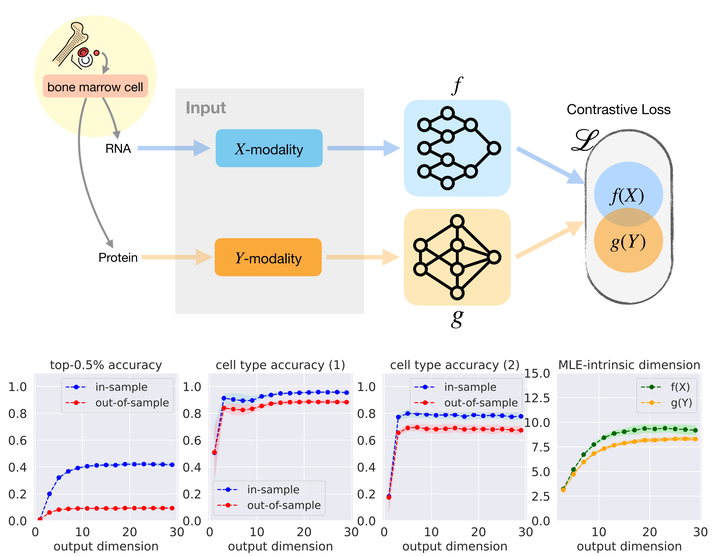


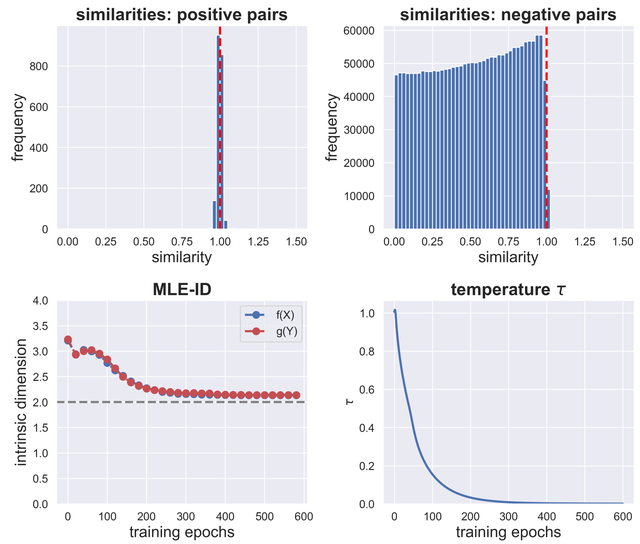
Figure 1. Results with `CITE-seq` dataset.

Question

- What representations does CLIP learn?
- What is the role of temperature τ?

A toy example

$$Y_i \overset{i.i.d.}{\sim} \mathcal{N}(0, \mathbf{I}_{20}), \quad \xi_i \overset{i.i.d.}{\sim} \mathcal{N}(0, \mathbf{I}_{20-k^*}), \quad X_i = (Y_{i1}, \cdots, Y_{ik^*}, \xi_i^\top)^\top$$



- **Similarity concentration**: For positive pairs, similarities concentrates around 1, while negative pairs are capped by 1
- **Intrinsic dimension adaptation**: Although output $d = 3$, representations with intrinsic dimension $k^* = 2$ are preferred
- **Temperature convergence**: The optimized temperature $\tau \to 0$

Intrinsic dimension:

$$\mathtt{ID}(f) = \min\left\{k \in \mathbb{N}: \ f = \phi \circ h, \ h: \mathbb{R}^{d_x} \to \mathbb{R}^d, \ \phi: R(h) \to \mathbb{R}^d \text{ injective}\right\}$$

Ideal Representations

- Alignment: with $m_\sigma(f,g) = \operatorname{ess\,sup}_{X \perp\!\!\!\perp \widetilde{Y}} \sigma(f(X), g(\widetilde{Y}))$

$$\mathcal{A}(\mathcal{H}) = \left\{(f,g) \in \mathcal{H}: \ \frac{f(X)}{\mathbb{E}\|f(X)\|} = \frac{g(Y)}{\mathbb{E}\|g(Y)\|}, \quad \sigma(f(X), g(Y)) = m_\sigma(f,g) \ \text{ a.s.}\right\}$$

- Mutual information maximization: $I_M^*(\mathcal{H}) = \sup_{\mathcal{H}} I(f_M(X); g_M(Y))$

$$\mathcal{W}(\mathcal{H}) = \left\{(f,g) \in \mathcal{H}: \ \liminf_{M\to+\infty} \left(I(f_M(X); g_M(Y)) - I_M^*(\mathcal{H})\right) \geq 0\right\}$$

$$\mathcal{V}(\mathcal{H}) = \mathcal{A}(\mathcal{H}) \cap \mathcal{W}(\mathcal{H})$$

- Intrinsic dimension adaptation:

Lemma. Suppose $\mathcal{V}(\mathcal{H}) \neq \varnothing$. Then, for all $(f,g) \in \mathcal{V}(\mathcal{H})$, we have $\mathtt{ID}(f) = \mathtt{ID}(g) = k^*$, i.e., maps in $\mathcal{V}(\mathcal{H})$ have the same intrinsic dimension $k^\star$

Is any (approximate) minimizer of CLIP ideal?

$$O_{\mathcal{L},\eta}(\mathcal{H}) = \left\{(f,g) \in \mathcal{H}: \ \exists\, \tau \geq \varepsilon(\eta), \ \limsup_{M\to+\infty} \left(\mathcal{L}(f_M, g_M, \tau) + 2I_M^*(\mathcal{H})\right) \leq 2\eta\right\}$$

Main results [1]

$$\mathcal{V}(\mathcal{H}) \neq \varnothing \implies \bigcap_{\eta\geq 0} \mathcal{O}_{\mathcal{L},\eta}(\mathcal{H}) \neq \varnothing.$$

In addition, for any $(f,g) \in \bigcap_{\eta\geq 0} \mathcal{O}_{\mathcal{L},\eta}(\mathcal{H})$, with $\sigma(f(X), g(Y)) = \frac{\langle f(X), g(Y)\rangle}{\mathbb{E}\|f(X)\|\cdot\mathbb{E}\|g(Y)\|}$,

- (similarity maximization) $\sigma(f(X), g(Y)) = m_\sigma(f,g)$ almost surely
- (intrinsic dimension adaptation) $\mathtt{ID}(f) = \mathtt{ID}(g) = k^*$
- (monotonicity in temperature) $\mathcal{L}(f,g,\tau)$ is increasing in τ
- (mutual information maximization) $(f,g) \in \mathcal{W}(\mathcal{H})$

References

[1] Gui, Y., Ma, C., and Ma, Z. (2025). Multi-modal contrastive learning adapts to intrinsic dimensions of shared latent variables. *NeurIPS*.

[2] Radford, A., Kim, J. W., Hallacy, C., Ramesh, A., Goh, G., Agarwal, S., Sastry, G., Askell, A., Mishkin, P., Clark, J., et al. (2021). Learning transferable visual models from natural language supervision. In *International conference on machine learning*, pages 8748–8763. PMLR.