Bandwidth Of Nulls Generated Using An ERS With Serial Search Algorithm

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1 Introduction

An array of axisymmetric circular prime-fed paraboloidal reflector is a general reflector system that is often used in radio astronomy. It involves a circular dish with the required type of feeder at the focus of the paraboloid. In this report, we are dealing with a reflector system having diameter D=18m and focal ratio f/D=0.4 with an electrically short electric dipole as the feeder of the system, operating at 1.5 GHz frequency.

The increase in the Low Earth Orbit (LEO) Satellites creates an interference with the reflector system causing data collected by the system to be highly affected. In order to overcome the interference problem, we are taking an approach that involves implementation of the Electronically Reconfigurable Surface (ERS).

2 Implementation of ERS

2.1 Directivity of the system

As stated above, the reflector system under study is an 18m axisymmetric cirular prime-fed paraboloidal reflector. The angle from the feed to the edge of the dish with respect to z axis is $\theta_0 = 64.01°$. The directivity of the system can be determined by calculating the ratio of maximum power density and power density averaged over all directions.

The power density is related to the scattered electric field as

$$S(r\hat{r}) = \frac{|E^s(r\hat{r})|^2}{2\eta_0}$$

where the scattered electric field at distance s^i from feed to the point on the surface is given by

$$E^s = -j\omega\mu_0 \frac{e^{-jkr}}{4\pi r} \int_{\theta_f=0}^{\theta_0} \int_{\phi=0}^{2\pi} J_0(s^i) e^{-jk\hat{r}\cdot s^i}$$

The feed can be modeled as

$$H^i(s^i) = I_0 \frac{\hat{y} \times \hat{s^i}}{|\hat{y} \times \hat{s^i}|} \frac{e^{-jks^i}}{s^i} (cos\theta_f)^q$$

At q=1.14, the edge illumination is approximately -11 dB, which leads to aperture efficiency of 81.5%. Using the theory of physical optics, we can calculate the equivalent surface current distribution by $J_0 = 2\hat{n} \times H^i$, where $\hat{n}$ is the normal to the surface and H^i is the incident magnetic field.

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For the average power density, we can first calculate the power radiated by the reflector system through a spherical surface of radius R which can be further averaged by dividing over the area of that sphere. Hence,

$$< S(r\hat{r}) >= \frac{P_{rad}}{4\pi r^2} = \frac{1}{4\pi}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} S(R\hat{r}) sin\theta d\theta d\phi$$

The co-polarized component pattern in H-Plane considering the farfield electric field is given in the figure below

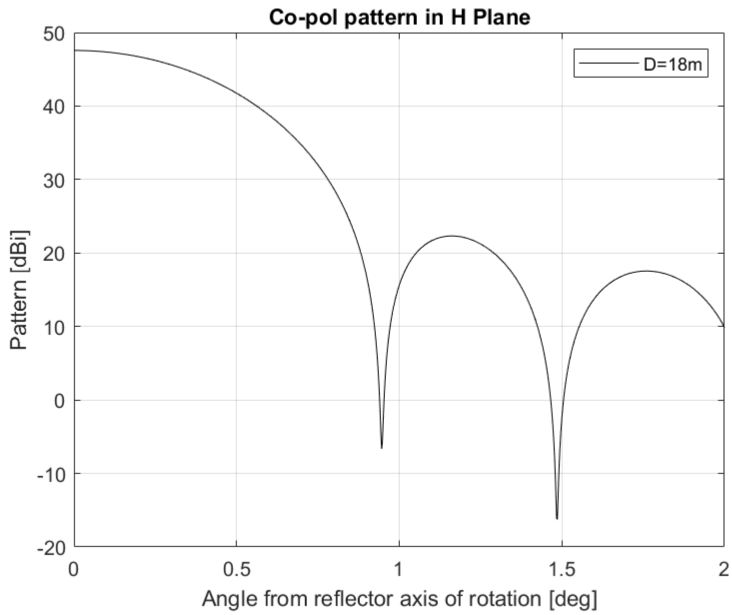

Fig. 1. Co-pol pattern in quiescent state

2.2 ERS : Electronically Reconfigurable Surface

The radiation pattern can be affected by the interference through sidelobes which can be modified by considering the rim of the relfector as unit cells whose phase of scattering can be controlled. These unit cells of subwavelength dimension are assumed to be contiguous.

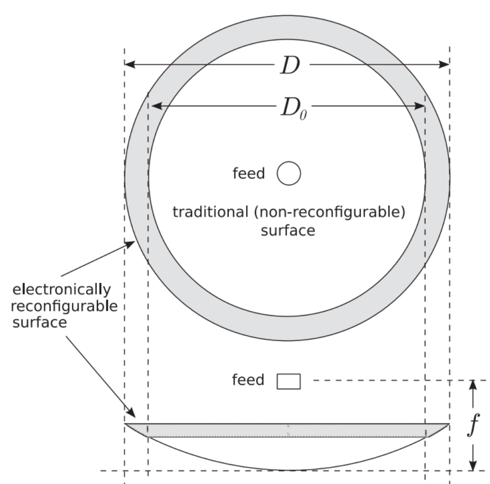

Fig. 2. On-axis(top) and side(bottom) views of an electronically reconfigurable rim scattering system (Image Credit: [1])

For the 18m axisymmetric circular prime-fed paraboloidal dish, we are considering the width of the rim to be 0.5m, leaving the diameter of the dish to be 17m. The logic behind keeping the new diameter to be as less by just 1m is that the main lobe of the pattern should not be affected when modifying the side lobe. The 0.5m rim consists of 2756 contiguous half-wavelength-square flat plates instead of the continous surface. Let us now analyze the change in the main lobe, side lobe level and Half Power Beamwidth (HPBW) when the diameter of the dish is 18m and 17m.

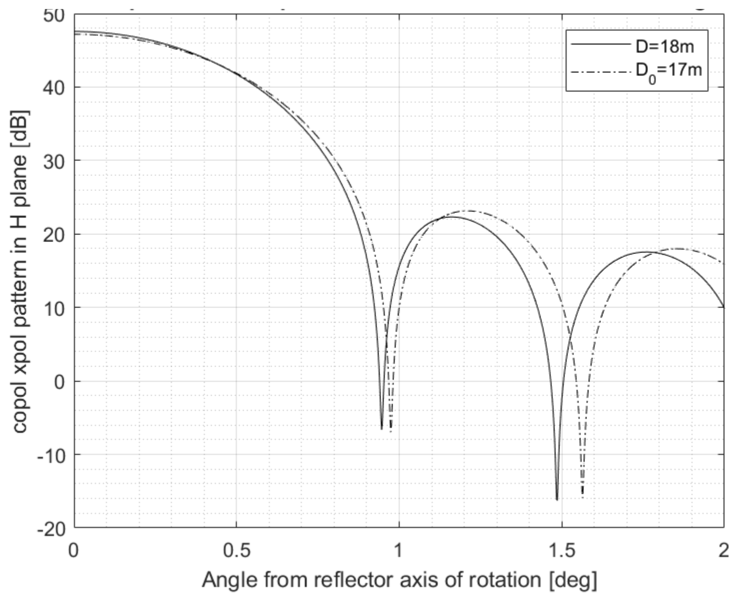

Fig. 3. Comparison of Co-pol pattern for 18m and 17m dish

Diameter	**Gain**	**SLL**	**HPBW**
18m	48.1 dBi	24.9 dB	0.7°
17m	47.7 dBi	24.5 dB	0.7°

Table 1: Comparison of Diameter D=18m and D_0=17m

The statistics show that we can modify the sidelobe without having a dramatic change in the magnitude of co-pol pattern of the main lobe. Thus, we can now consider our system to have 17m diameter dish with 0.5m wide rim consisting of contiguous unit cells.

3 Phase Manipulation of ERS

Reconfigurable surfaces can be controlled by changing their phase. Each unit cells are assigned when practically implementing, we may need to limit the phase to discrete values. The scattered electric field is now calculated by

$$E_1^s = -j\omega\mu_0 \frac{e^{-jkr}}{4\pi r} \sum_n J_1(s_n^i) e^{jk\hat{r}\cdot s_n^i} \Delta s$$

which is the same as calculating the scattered electric field at a point on the reflector except that the integrand is now limited to the dimensions of the plate i.e. $\Delta s = 0.25\lambda^2$. Here, n is used to index the individual cell or plate. Thus, the total scattered electric field will be $E^s = E_0^s + E_1^s$. Moreover,

$$J_1(s_n^i) = c_n J_0(s_n^i)$$

where c_n is a complex constant whose value is assigned to each plate according to the degree of reconfigurability.

$$c_n = exp\{-j\ arg[J_0(s_n^i)e^{jk\hat{r}\cdot s_n^i}]\}$$

In the quiescent state, the value of $c_n = 1$ for all cells, making the reconfigurable surface a continuous part of the reflector. 1 bit phase quanitization limits the value of c_n to ± 1 whereas 2 bit phase quantization yields the values of c_n to be +1,+j,-1 or -j. The algorithm to decide the value of c_n for each unit cell is designed in a way that reduces the magnitude of the pattern in the quiescent state.

3.1 Serial Search Algorithm

For 1 bit phase quantization, the value of c_n is either +1 or -1, depending on whichever value will reduce the total scattered electric field at that step of the integration. Consider E_0^s to be the value of the scattered electric field for a reflector system with diameter $D_0 = 17m$. Now, since the side lobe peaks at 1.75° in the quiescent state of the system, the serial search algorithm picks the set of values of c_n (± 1) of the cells which reduces the current magnitude of the scattered electric field during the evaluation. This algorithm yields a deep null at 1.75° resulting into the cancellation of the side lobe created when the system is in quiescent state.

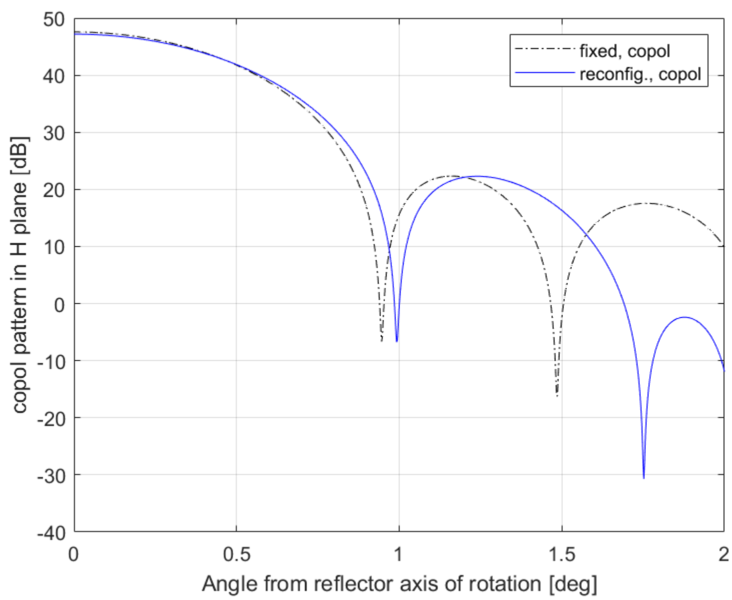

Fig. 4. Deep null obtained at $\theta = 1.75°$ when $c_n = \pm 1$ for the unit cells on the rim

4 Bandwidth of the system

The entire system described in this report operates at a center frequency of 1.5 GHz. However, from application perspective, the system should be capable enough to function if there is some shift in the center frequency. It is interesting to note the behaviour of the depth of the null when we shift the center frequency but leaving the elements of the reflector system unchanged.

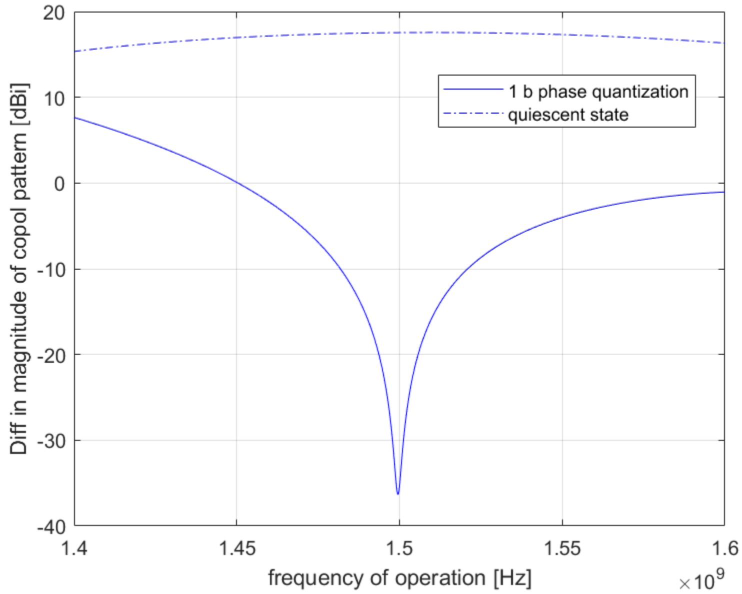

Fig. 5. Magnitude of the co-pol pattern at $\theta = 1.75°$ as a functionof frequency

The solid line represents the magnitude of co-pol at $\theta = 1.75°$ when we implement 1 bit phase quantization. The dotted line denotes the magnitude of co-pol in the quiescent state ($c_n = 1$). In order to evaluate the effect of implementing 1 bit phase quanitized unit cells on the rim of the reflector, we can check the difference in the magnitude of the co-pol pattern at $\theta = 1.75°$ generated in both cases.

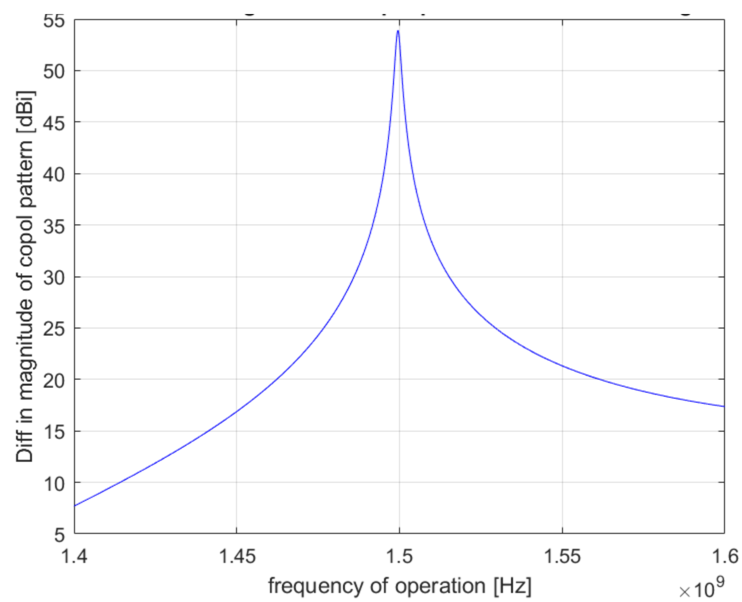

Fig. 6. Magnitude of the difference in co-pol pattern (dBi)

It is evident from the figure since the elements were designed for the center frequency of 1.5GHz, the difference in the magnitude of co-pol pattern is also observed to be highest there. But, we can still obtain a null at the desired angle (in our case $\theta = 1.75°$) when deviating from the intended center frequency. To determine the bandwidth of the system, we can fix the desirable magnitude of the difference in co-pol pattern of quiescent state and the 1 bit phase quantization state.

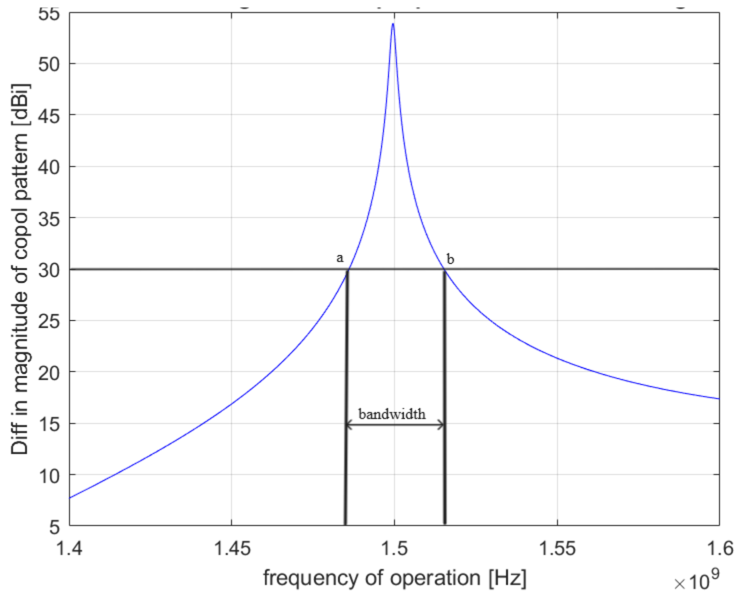

Fig. 7. Frequency covered between point a and b is considered as the bandwidth of the system.

In this case, let us consider the magnitude of difference in the co-pol pattern to be greater than 30dBi. Thus, the bandwidth that reduces the magnitude of the side lobe by 30dBi or more will be considered as the bandwidth of the system. The frequencies involved between point a and b is 14.86GHz to 15.15GHz.

$$\therefore Bandwidth(B.W.) = 0.29GHz$$

5 Example

Let us implement this model for a different possible setting with the center frequency at 2 GHz. The parameters will now be

1. D and D_0 as defined in the given problem statement
2. v=2 GHz
3. λ=0.15m
4. θ_0 as defined in the given problem statement
5. β=0.419

Along with these parameters, the "set" of values of c_n will also be changed according to the direction at which null is to be driven. It should be noted that the algorithm to choose the values of c_n remains the same, however, the set of values will differ in each case as we introduce a new reflector system.

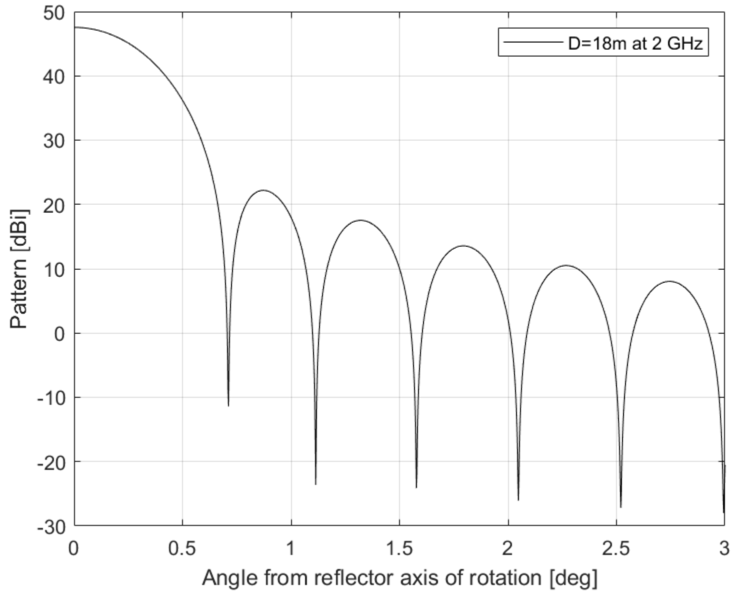

Fig. 8. Co-pol pattern in quiescent state at 2 GHz

The pattern is viewed from 0° to 3° in the H-Plane. The second side lobe level in this case appears to be at $\theta = 1.3°$. Using the ERS with serial search algorithm, we can get a deep null at $\theta = 1.3°$.

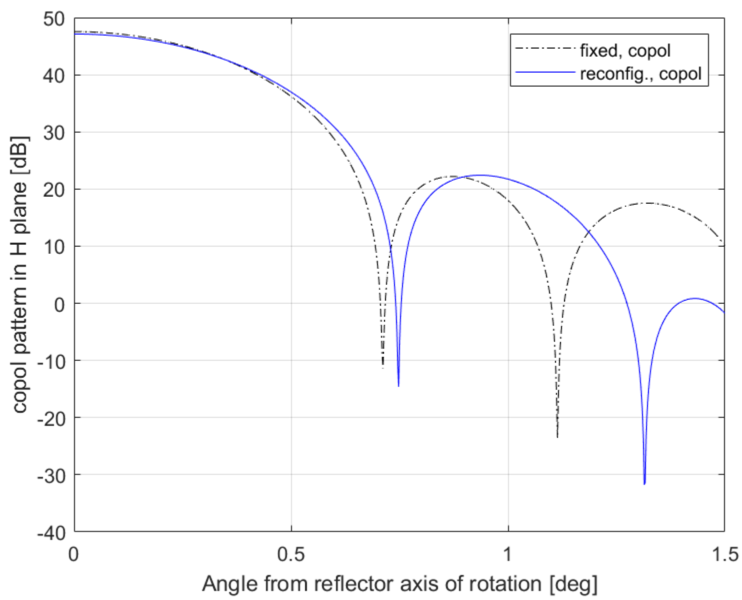

Fig. 9. Deep null obtained at $\theta = 1.3°$ canceling the side lobe

At $\theta = 1.3°$, we are interested in evaluating the bandwidth of the system having the capacity of canceling the side lobe by 30dBi or more.

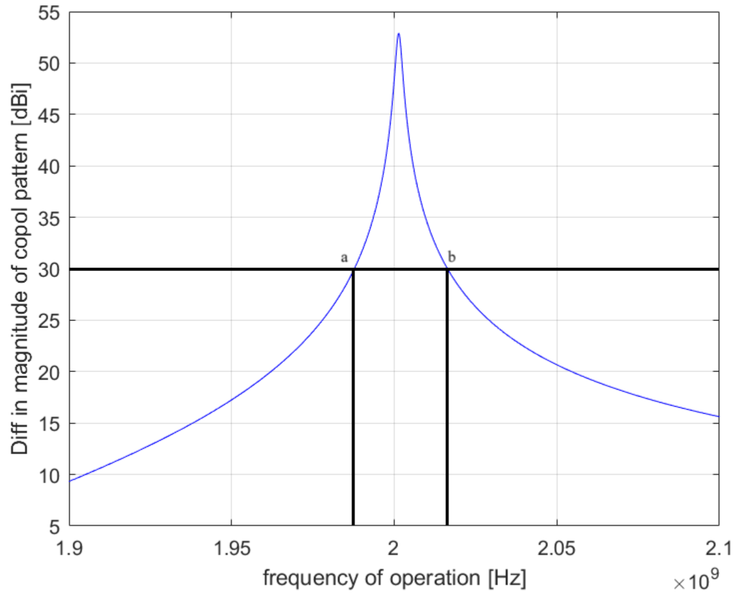

Fig. 10. Points a and b defined at 30 dBi to determine the bandwidth of the system

The bandwidth covered from point a to point b is

$$B.W. = 20.16 - 19.87 = 0.29GHz$$

Thus, it is apparent that for a system having the capacity to cancel the side lobe by a magnitude of 30 dBi or more, the bandwidth will be 0.29 GHz.

6 Bandwidth of the nulls

For a paraboloidal reflector of 17m diameter and 1m wide ERS rim, The copol pattern with side lobes is given as below

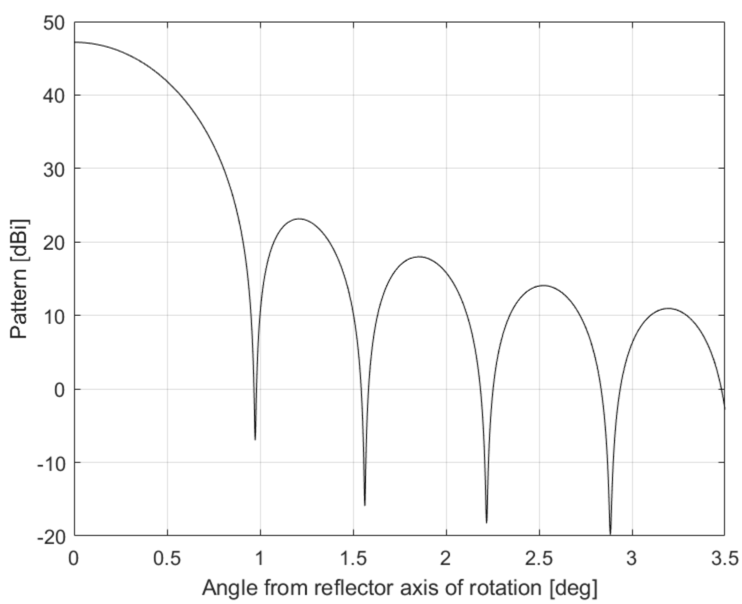

Fig. 11. Co-pol pattern when $D = 18m$, $D_0 = 17m$

When cancelling the side lobes, we have to consider the direction of the null starting from the first side lobe. For simplification, only the first three side lobes are considered to be canceled. Thus, the direction of the nulls will be from 1° to 3°.

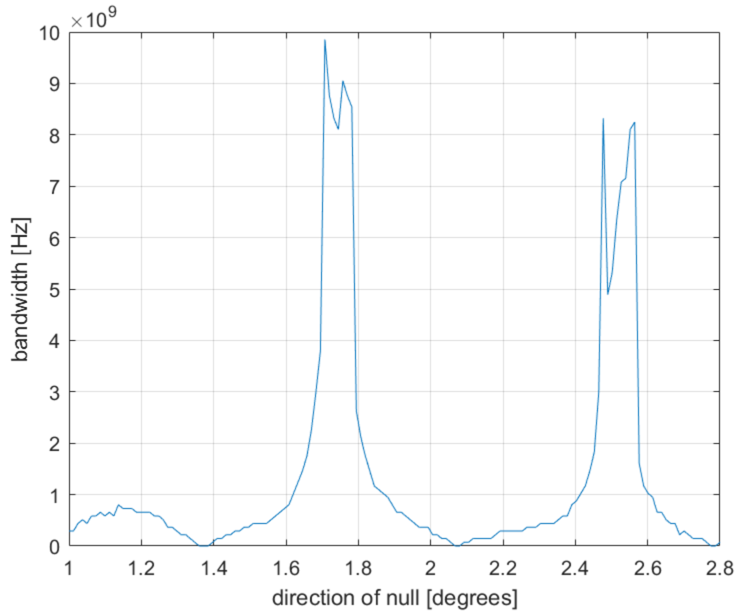

Fig. 12. Bandwidth of the nulls from $\theta = 1°$ to $3°$

The bandwidth is maximum at $\theta = 1.75°$ and $2.5°$ which also happens to be where the second and the third side lobe are formed. Thus, the system is highly effective for canceling the secondary side lobes.

7 References

[1] Steven Ellingson and Ramonika Sengupta, "Sidelobe Modification for Reflector Antennas by Electronically Reconfigurable Rim Scattering," *IEEE Trans. Antennas Propag.*, vol.20,no. 6, pp. 1, June 2021