# Ay124 PS5 Solutions

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## 1 Giant Molecular Cloud and Turbulence

[4+1+2+4+3+2]

Table 1: Physical Properties of GMCs

| | Large | Medium | Small |
|---|---|---|---|
| $M/M_\odot$ | $10^6$ | $10^4$ | $10^2$ |
| $R/\mathrm{pc}$ | 31.6 | 3.16 | 0.316 |
| $\Sigma/(M_\odot/\mathrm{pc}^2)$ | 318 | 318 | 318 |
| $\Sigma/(\mathrm{g/cm}^2)$ | 0.0667 | 0.0667 | 0.0667 |
| $\rho/(M_\odot/\mathrm{pc}^3)$ | 7.55 | 75.5 | 755 |
| $\rho/(\mathrm{g/cm}^3)$ | $5.12 \times 10^{-22}$ | $5.12 \times 10^{-21}$ | $5.12 \times 10^{-2}$ |
| $t_{\mathrm{ff}}/\mathrm{yr}$ | $2.9 \times 10^6$ | $9.2 \times 10^5$ | $2.9 \times 10^5$ |

Adopted from the previous PS: for large, medium, and small GMCs, their mass($M$), radius($R$), surface density ($\Sigma$), density ($\rho$) and free-fall time ($t_{\mathrm{ff}}$) are listed in Table 1.

a) Plugging in $c_s \sim \sqrt{\frac{kT}{2m_H}}$ for molecular hydrogen,

$$\begin{aligned} \lambda_J &\sim \sqrt{\frac{kT}{2m_H G\rho}} & (1) \\ &= 3.48 \times 10^{18}, 1.10 \times 10^{18}, 3.48 \times 10^{17} \mathrm{g/cm}^3 & (2) \\ &= \boxed{1.12, 0.35, 0.11 \text{ pc}}. & (3) \end{aligned}$$

All of the clouds are unstable since $R > \lambda_J$. If all of the fragmented gas turns into a star, their masses will be,

$$\begin{aligned} M_* &= \frac{4\pi}{3}\lambda_J^3 \rho & (4) \\ &= \boxed{45, 14, 4.5 M_\odot}. & (5) \end{aligned}$$

b) The characteristic timescales are the dynamical timescales, which are $\boxed{t_{\mathrm{dyn}} \sim 2.9, 0.92, 0.29 \mathrm{Myr}}$ for large, medium and small GMCs.

c)

$$N_* \sim \frac{0.01 M_{\mathrm{cloud}}}{10 M_\odot} \sim \boxed{1000, 10, 0.1}. \quad (6)$$

Since the average number of high-mass stars in low mass cloud is 0.1, the high-mass stars are generally insignificant for those clouds. If by chance, they do form a high-mass star, the cloud will be easily destroyed.

d) Under photoionization equilibrium (rate of ionization equals to rate of recombination)[1],

$$Q = \frac{4\pi}{3} R_S^3 \alpha_B n(H^+) n_e, \tag{7}$$

where $\alpha_B$ is the case B recombination coefficient, and it approximately is $\alpha_B \approx 2.56 \times 10^{-13} \left(\frac{T}{10^4\ \mathrm{K}}\right)^{-0.83}$ $\mathrm{cm}^3/\mathrm{s}$. Thus,

$$\begin{aligned} R_S &= \left(\frac{3Q}{4\pi n_H^2 \alpha_B}\right)^{1/3} & (8) \\ &= 4.6 \times 10^{18}, 1.0 \times 10^{18}\ \mathrm{cm} & (9) \\ &= \boxed{1.5, 0.32\ \mathrm{pc}}, & (10) \end{aligned}$$

for large and medium mass clouds correspondingly.

The thermal energy for each Stromgren sphere is,

$$\begin{aligned} E_{\mathrm{th}} &\sim 2N_H kT & (11) \\ &\sim \frac{8\pi}{3} R_S^3 n_H kT & (12) \\ &\sim \boxed{3.5 \times 10^{47}, 3.5 \times 10^{46}\ \mathrm{erg}}. & (13) \end{aligned}$$

The potential energy is,

$$\begin{aligned} \Phi &\sim \frac{GM_S^2}{R_S} & (14) \\ &\sim \boxed{6.5 \times 10^{44}, 3.0 \times 10^{43}\ \mathrm{erg}}. & (15) \end{aligned}$$

Therefore, the thermal energies are much larger than the potential energies of GMCs by a factor of $10^3$. It will cause the gas in the Stromgren sphere to expand.

The timescales to evaporate the GMCs are,

$$t_{\mathrm{evaporate}} \sim \frac{R_{\mathrm{cloud}}}{c_s} \sim \boxed{3.5, 0.35 \mathrm{Myr}}. \tag{16}$$

*Note: now the gas is ionized, so the number density is calculated from the mass of each hydrogen atom.*

e) For $M > 10^6 M_\odot$ clouds, the timescale for ionized gas to push out is too long to finish before SNe explosion.

To put in enough momentum to unbind the cloud, the timescale $\Delta t$ must satisfy,

$$\frac{2N_* L}{c} \Delta t \sim M_{\mathrm{cloud}} \sqrt{\frac{GM_{\mathrm{cloud}}}{R_{\mathrm{cloud}}}}. \tag{17}$$

Therefore, $\boxed{\Delta t \sim 2.9, 0.94\ \mathrm{Myr}}$, similar to $t_{\mathrm{dyn}}$.

f) Since most of the GMC is destroyed by the time SNe explodes, the SNe mostly affects the outside ISM. Since the typical ISM is $10^{-4}$ more diffuse than GMC, the radius of each bubble in ISM is $\sim 22$ times larger than the bubble in GMC. Thus, the probability of overlapping bubbles is significantly increased, causing the formation of galactic outflows.

# 2 Feedback

[1+3+3+2+2]

a) The energy injection rate is,

$$\boxed{P \sim \frac{1}{2} \times 10^{51}\ \mathrm{erg} \left(\frac{\dot{M}_*}{100 M_\odot}\right) \sim 5 \times 10^{48}\ \mathrm{erg/s} \left(\frac{\dot{M}_*}{1 M_\odot/\mathrm{s}}\right)}. \tag{18}$$

b) Given energy conservation,

$$P = \frac{1}{2} \dot{M}_{\mathrm{wind}} v_{\mathrm{esc}}^2 \sim 5 \times 10^{48}\ \mathrm{erg/s} \left(\frac{\dot{M}_*}{1 M_\odot/\mathrm{s}}\right). \tag{19}$$

Therefore, the mass loading factor,

$$\eta = \frac{\dot{M}_{\mathrm{wind}}}{\dot{M}_*} \sim \boxed{10^{49}\ \mathrm{erg}/M_\odot \frac{1}{v_{\mathrm{esc}}^2}}. \tag{20}$$

Plugging in $v_{\mathrm{esc}}$, we get,

$$\boxed{\eta \sim 2.0 \left(\frac{10^{12} M_\odot}{M_{\mathrm{habo}}}\right)^{2/3}}. \tag{21}$$

c)

$$M_{b,\mathrm{gal}} = \frac{f_b M_{\mathrm{halo}}}{1+\eta} \sim \frac{f_b M_{\mathrm{halo}}}{\eta}, \tag{22}$$

if $\eta \gg 1$. Therefore,

$$M_{b,\mathrm{gal}} \sim \frac{f_b}{2} \frac{M_{\mathrm{halo}}^{5/3}}{(10^{12} M_\odot)^{2/3}}. \tag{23}$$

For momentum-driven wind, $M_{b,\mathrm{gal}} \propto M_{\mathrm{halo}}^{4/3}$. Therefore, the gas fraction and baryon "conversion" and "retention" efficiencies are expected to have stronger dependence on mass.

d) For a typical SNe, the energy ejection rate is $\sim 2\%$, which is $\sim 25$ times as the energy ejection rate of photoinozation star light. Therefore, the mass loading factor $\eta$ becomes $\sim 25$ times larger, and at a given halo mass, the mass of the galaxy is $\sim 25$ times smaller.

## References

[1] Bruce T Draine. *Physics of the interstellar and intergalactic medium.* Princeton University Press, 2010.