Ay121 PS2 Solutions

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Based on the solutions from Rachel Theios in 2015.

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1 Seeing Through the Sun

A photon at $\tau_\nu \approx 1$ can escape. From the definition of opacity (κ_ν),

$$\tau_\nu(s) = \int_0^d \alpha_\nu \, \mathrm{d}s = \int_0^d \kappa_\nu \rho \, \mathrm{d}s. \tag{1}$$

Assuming the average density of the Earth's atmosphere at sea level is $\bar{\rho} = 10^{-3}$ g cm^{-3},

$$\kappa_{500} \bar{\rho} d = 1. \tag{2}$$

Solving d gives,

$$\boxed{d = 3.79 \times 10^3 \text{cm} = 37.9\text{m}}. \tag{3}$$

Wow! This is even worse than Beijing! We are so lucky to be living on Earth.

2 Radio Emission from Venus

(a) "Radio waves with >30 cm can easily propagate through the atmosphere of Venus." Thus, we only need to convert $\lambda = 30$cm to ν.

$$\boxed{\nu = \frac{\lambda}{c} = 1 \text{ GHz}}. \tag{4}$$

Rayleigh-Jeans limit applies when $h\nu \ll k_B T$. Plugging in atmospheric temperature (225K, temperature measured in IR),

$$\boxed{\frac{h\nu}{k_B T} \approx 10^{-4} \ll 1}, \tag{5}$$

It is very unlikely that the surface temperature is off by an order of magnitude. Therefore, we are safe in R-J limit.

(b) By definition,

$$\begin{aligned} F_\nu &= \int I_\nu \cos\theta \, \mathrm{d}\Omega \\ &= I_\nu \times 2\pi \times \tfrac{1}{2} \sin\theta \Big|_0^{\theta_V}, \end{aligned} \tag{6}$$

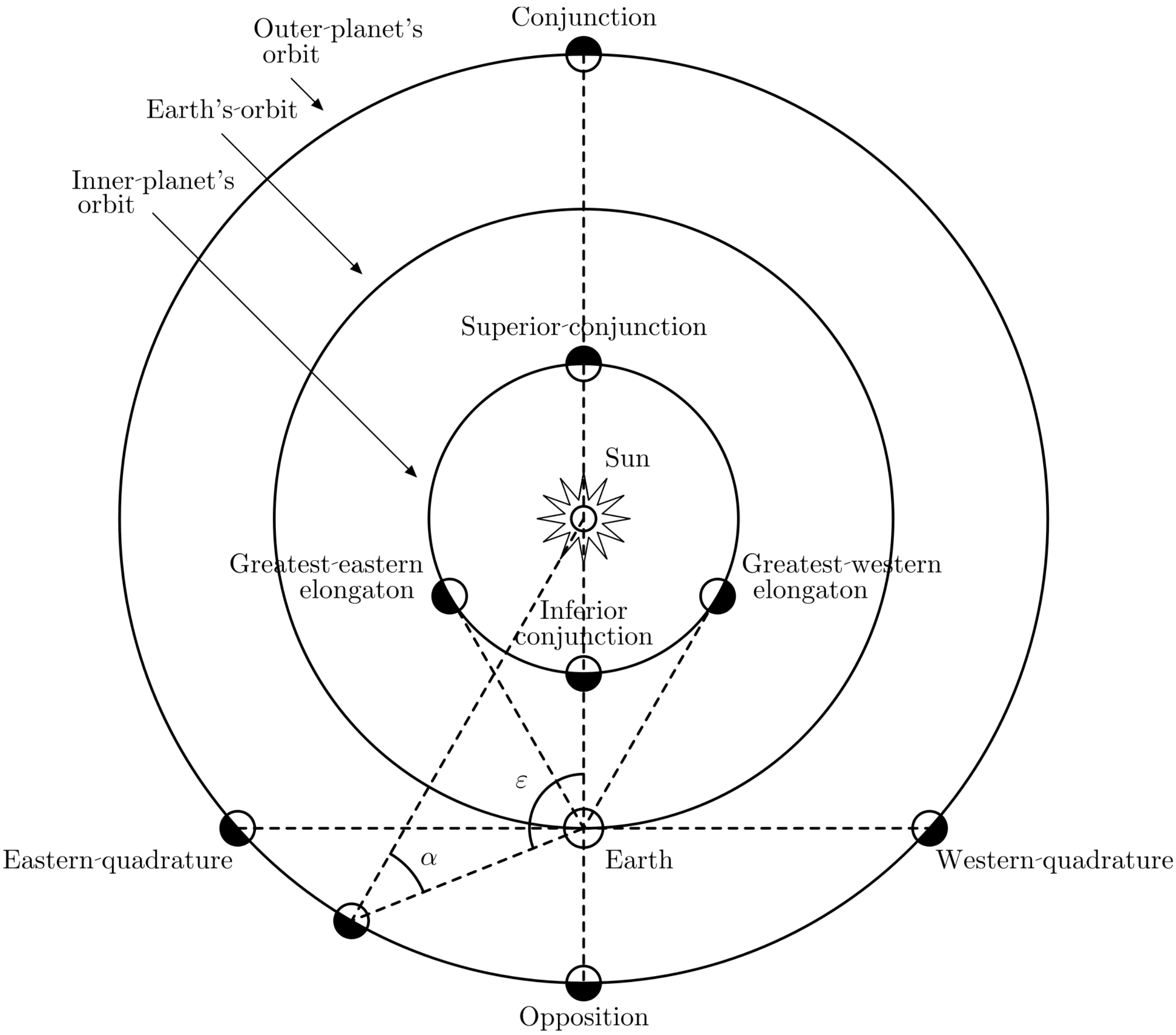

Figure 1: Nomenclature of planet positions. [Wikipedia]

where θ_V is the angular diameter of Venus as viewed from Earth. Since Venus is at the greatest elongation (Fig. 1),

$$d = \sqrt{a_{\text{Earth}}^2 - a_{\text{Venus}}^2} = \sqrt{(1\text{AU})^2 - (0.72\text{AU})^2} \tag{7}$$

where a_{Earth} and a_{Venus} are the average orbital radii. Therefore, with $R_V = 6052\text{km}$ is the radius of Venus,

$$\theta_V \approx \frac{R_V}{d}. \tag{8}$$

$$I_\nu = \frac{F_\nu}{\pi \theta_V^2} = \frac{2\nu^2}{c^2} k_B T_b. \tag{9}$$

Assuming the radio emission is thermal, and we are in R-J regime, $\boxed{T = T_b \approx 700\text{K}}$.

(c) Total power,

$$P = \pi (\frac{D}{2})^2 F, \tag{10}$$

where the Flux (F) is obtained by integrating F_ν over the bandwidth ($\Delta\nu$),

$$\begin{aligned} F &= \textstyle\int_{\nu-\Delta\nu/2}^{\nu+\Delta\nu/2} F_\nu \,\mathrm{d}\nu \\ &= \pi\theta_V^2 \times \frac{2k_B T_b}{c^2} \textstyle\int_{\nu-\Delta\nu/2}^{\nu+\Delta\nu/2} \nu^2 \,\mathrm{d}\nu. \end{aligned} \tag{11}$$

Therefore, $\boxed{P = 5.6 \times 10^{-11} \text{ erg s}^{-1}}$.

Since $\Delta\nu \ll \nu$, it is okay to let $F \approx F_\nu \times \Delta\nu$.

3 Eddington Limit

(a) Assuming the luminous object emits radiation isotropically. The radiation force on cloud per unit mass is,

$$f_{\text{rad}} = \frac{\kappa F}{c} = \frac{\kappa L}{4\pi r^2 c}. \tag{12}$$

If it is larger than the gravitational force per unit mass, the cloud is ejected.

$$f_{\text{grav}} < f_{\text{rad}} \tag{13}$$

$$\frac{GM}{r^2} < \frac{\kappa L}{4\pi r^2 c} \tag{14}$$

$$\boxed{\frac{M}{L} < \frac{\kappa}{4\pi G c}}. \tag{15}$$

(b) Since the radiation force has r^{-2} proportionality, it acts as inverse gravitational force.

$$G_{\text{eff}} = G - \frac{\kappa L}{4\pi M c} \tag{16}$$

If the shell is ejected, $G_{\text{eff}} < 0$. Because of the conservation of energy,

$$-\frac{G_{\text{eff}} M}{R} = \frac{1}{2} v^2. \tag{17}$$

Therefore,

$$\boxed{v^2 = \frac{2}{R}(\frac{\kappa L}{4\pi c} - GM)}. \tag{18}$$

(c) Maximum luminosity happens when Eq. 13 becomes equality.

$$\begin{aligned} L_{\text{Edd}} &= \tfrac{4\pi G M c}{\kappa} \\ &= \boxed{\frac{4\pi G M c m_H}{\sigma_T}} \\ &= \boxed{1.25 \times 10^{38} \text{ erg s}^{-1} (M/M_\odot)} \end{aligned} \tag{19}$$