Ay124 PS6 Solutions

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1 Galaxy & Black-Hole Mergers, and Dynamical Friction:

a) The collision timescale for individual MW-like galaxy is the mean-free time.

$$\begin{aligned}\tau &= \frac{1}{n\pi b^2\sigma} &(1)\\ &\sim \frac{1}{10^{-2}\mathrm{Mpc}^{-3}\pi(100\mathrm{kpc})^2\cdot 300\mathrm{km/s}} &(2)\\ &\sim \boxed{10^{13}\mathrm{yr}}, &(3)\end{aligned}$$

much greater than the Hubble time.

b) Combining all of the three factors, the number density of galaxies is now,

$$n' = 10(1+z)^3\frac{\int n\left(1+\left(\frac{r}{r_0}\right)^{-1.8}\right)4\pi r^2\,\mathrm{d}r}{V}. \tag{4}$$

For order-of-magnitude purpose, the integration limit can be set from 0 to r_0. Therefore,

$$n' = 10(1+z)^3 n\frac{7}{2}. \tag{5}$$

Thus, at $z\sim 2$,

$$\boxed{\tau' \sim \frac{\tau}{1000} \sim 10^{10}\ \mathrm{yr}}. \tag{6}$$

c) For major merger, $M_p \sim M_s$,

$$t_{\mathrm{fric}} \sim \frac{R}{V} \sim \frac{R_{\mathrm{vir}}^{3/2}}{\sqrt{GM}}, \tag{7}$$

where $R_{\mathrm{vir}} \sim 0.21$ Mpc can be calculated by taking $\rho_{\mathrm{vir}} = 200\rho_c$. Therefore,

$$t_{\mathrm{fric}} \sim 1.4\times 10^9\ \mathrm{yr}. \tag{8}$$

Since there is half probability for an individual galaxy to see a major merger throughout Hubble time, and each major merger lasts for t_{fric}, the fraction of mergers can be seen at a given time is,

$$\boxed{f \sim 0.5\frac{t_{\mathrm{fric}}}{1/H_0} \sim 0.06}. \tag{9}$$

The fraction will go up with increasing redshift, because the density of galaxies is higher.

d) Setting the dynamical friction time equals to the Hubble time,

$$R_{\max} \sim \frac{M_s}{H_0 M_p} V_c, \tag{10}$$

where $M_p \sim M_{\rm SMBH}/0.003 \sim 10^{11} M_\odot$, $V_c \sim \sigma \sim 200$km/s (from M-σ relation). Therefore, for $100 M_\odot$ BH, $R_{\max} \sim \boxed{3 \times 10^{-3}\ \text{pc}}$; for $10^5 M_\odot$ BH, $R_{\max} \sim \boxed{3\ \text{pc}}$; for $3 \times 10^8 M_\odot$ BH, $R_{\max} \sim \boxed{8.5\ \text{kpc}}$.

Now, putting those BHs into bulges, $M_{\rm bulge} \sim M_{\rm BH}/0.003 \sim 3.3 \times 10^4, 3.3 \times 10^7, 10^{11} M_\odot$ correspondingly. Therefore, $R_{\max} \sim \boxed{0.9, 900, 3 \times 10^6 \text{pc}}$.

The virial radius of the primary galaxy is $R \sim GM_V/\sigma^2 \sim 10^5$pc. Therefore, only the galactic SMBH can settle into the center of the center of the galaxy if they have extragalactic origins.

2 Toomre's Q

a) Assuming flat rotation curve, $\kappa = \sqrt{2}\Omega$. The surface density follows $n = 1$ Sersic profile,

$$\Sigma = \Sigma_0 \exp\left(-\frac{R}{R_0}\right), \tag{11}$$

where $\Sigma_0 \sim 10^{11} M_\odot/(\pi R_0^2) \sim 0.74 \text{g/cm}^2$. Therefore,

$$Q = \frac{\sigma\kappa}{3.36 G\Sigma} \tag{12}$$

$$\sim \frac{\sqrt{2}\sigma v_{\rm rot}}{3.36 G\Sigma_0 R} \exp\left(\frac{R}{R_0}\right) < 1. \tag{13}$$

Plugging in $\sigma \sim 10$ km/s, $v_{\rm rot} \sim 200$ km/s, the solution is $R > 0.027$ kpc and $R < 19.7$ kpc. The inner solution is too small and flat rotational curve clearly does not apply there. Thus, inside $\boxed{R \sim 19.7 \text{kpc}}$, the disk is unstable.

The most unstable mode,

$$\lambda = \frac{\sigma^2}{\pi G \Sigma_0} \sim \boxed{2.1\ \text{pc}}. \tag{14}$$

b) The instability will pump up σ until $Q \sim 1$. Now, set $Q = 1$,

$$\sigma \sim \frac{3.36 G\Sigma_0 R}{\sqrt{2} v_{\rm rot}} \exp\left(-\frac{R}{R_0}\right) \tag{15}$$

$$\sim \boxed{\frac{R}{5.4 \times 10^6 \text{yr}} \exp\left(-\frac{R}{3\text{kpc}}\right)}. \tag{16}$$

c)

$$\frac{\sigma}{v_c} = \frac{3.36RG\Sigma_{\rm disk}}{\sqrt{2}v_c^2} \tag{17}$$

$$= \frac{3.36M_{\rm disk}(< R)}{2\sqrt{2}\pi M_{\rm enc}(< R)} \tag{18}$$

$$\sim \boxed{\frac{M_{\rm disk}}{M_{\rm enc}}}. \tag{19}$$

d)

$$\frac{h}{R} \sim \frac{\sigma}{R\Omega} \tag{20}$$

$$\sim \frac{\sigma}{v_c} \tag{21}$$

$$\sim \boxed{\frac{M_{\rm disk}}{M_{\rm enc}}}. \tag{22}$$

e) Taking $R_0 = 6\text{kpc}, T \sim 10^4\text{K}$,

$$c_s \sim \sqrt{\frac{kT}{2m_H}} \sim 6.4\text{km/s} \tag{23}$$

$$\Sigma_0 \sim \frac{0.1M}{\pi R_0^2} \sim 0.018\text{g/cm}^2. \tag{24}$$

Therefore,

$$Q = \frac{c_s\kappa}{\pi G\Sigma} \tag{25}$$

$$\sim \frac{\sqrt{2}c_s v_{\rm rot}}{\pi GR\Sigma_0}\exp\left(\frac{R}{R_0}\right) < 1. \tag{26}$$

Its solution is $\boxed{R < 12.8\text{kpc}}$

f) With $Q_{\rm turb} = 1$,

$$v_{\rm turb} \sim \frac{\pi G\Sigma_0 R}{\sqrt{2}v_{\rm rot}}\exp\left(-\frac{R}{R_0}\right). \tag{27}$$

Setting $R_0 \sim 6\text{kpc}$, $\Sigma_0 \sim M/(\pi R_0^2) \sim 0.18\text{g/cm}^2$. Therefore,

$$\boxed{v_{\rm turb} \sim \frac{R}{2.3\times 10^7\text{yr}}\exp\left(-\frac{R}{6\text{kpc}}\right)}. \tag{28}$$

g)

$$v_{\rm turb} > c_s \sim 6.4\text{km/s}. \tag{29}$$

The solution is $\boxed{R < 32\text{kpc}}$.

If the gas temperature is 10K,

$$\frac{v_{\rm turb}}{c_s} \sim \frac{R}{0.0046{\rm kpc}} \exp\left(-\frac{R}{6{\rm kpc}}\right), \tag{30}$$

which is ~ 500 for $R \sim R_0$.

h) The turbulence velocity dispersion is more localized in small regions where there will be collapsing and star-formation. The stellar velocity dispersion is a smooth quantity.