Ay124 PS4 Solutions

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1 More Backgrounds & Relic Population

[1+3+2+3]

a) The energy density of CMB is $\approx 4 \times 10^{-13}$ erg/cm^3, much greater than CIRB.

b) The luminosity distance to $z \sim 3$ is,

$$d_L = \frac{c}{H_0}(1+z)\int \frac{\mathrm{d}z'}{E(z')} = 25 \text{ Gpc}. \tag{1}$$

Therefore, the radiation energy density contributed by each ULIRG received in current redshift is,

$$u = F/c = \frac{L}{4\pi d_L^2 c} = 1.7 \times 10^{-24} \text{ erg/cm}^3. \tag{2}$$

Thus, it requires,

$$\boxed{N = \frac{7 \times 10^{-15}}{1.7 \times 10^{-24}} = 4 \times 10^9} \tag{3}$$

ULIRGs to make up the CIRB today.

c)

$$M_* = \text{SFR} \times \Delta t = 200 M_\odot/\text{yr} \times 3 \times 10^7 \text{yr} = 6 \times 10^9 M_\odot. \tag{4}$$

For a typical galaxy (MW), the stellar mass is $\sim 10^{11} M_\odot$.

d) At $z = 3$, the energy density of CIRB is $u_{\text{CIRB}} * (1+z)^4$. Therefore, the number density of ULIRGs per volume at $z = 3$ is,

$$n' \sim \frac{u_{\text{CIRB}}(1+z)^4}{L\Delta t}. \tag{5}$$

Converting it to comoving volume,

$$\boxed{n \sim \frac{u_{\text{CIRB}}(1+z)^4}{L\Delta t}(1+z)^{-3} \sim 0.22 \text{ Mpc}^{-3}}. \tag{6}$$

This is much larger than the galaxy number density today, which is in the middle between 10^{-3} and 10^{-2} Mpc^{-3}. The reason is that we assumed all ULIRGs were ignited at once. This significantly under estimated the total volume.

2 Feedback

[2+2+3]

a) The total metallicity librated by SNe,

$$M_{z,\mathrm{SNe}} \sim \frac{1.5M_\odot}{100M_\odot} \times 10^{11} M_\odot \sim 1.5 \times 10^9 M_\odot. \tag{7}$$

The total metallicity in stars is,

$$M_{z,*} \sim (0.014 \to 0.02) 10^{11} M_\odot \sim (1.4 \to 2) \times 10^9 M_\odot, \tag{8}$$

which is similar to the metallicity created by SNe. Thus, most of the metals are still in MW.

b) For dwarf galaxies, the total mass of metals ejected is,

$$M_{z,\mathrm{SNe}} \sim \frac{1.5M_\odot}{100M_\odot} \times 10^5 M_\odot \sim 1.5 \times 10^3 M_\odot. \tag{9}$$

The total metallicity in stars is,

$$M_{z,*} \sim 0.003 \times 0.02 \times 10^5 M_\odot \sim 6M_\odot. \tag{10}$$

The metal mass in gas is,

$$M_{z,\mathrm{gas}} \sim 0.003 \times 0.02 \times 2 \times 10^5 M_\odot \sim 12M_\odot. \tag{11}$$

By all means, the metal produced by SNe is much more than the metals preserved in galaxy. Therefore, most of the metals are ejected in IGM.

c) The gravitational binding energy is,

$$U \sim \frac{GM_*^2}{R^2} \sim M_* V_{\mathrm{rot}}^2. \tag{12}$$

For MW, $U(\mathrm{MW}) \sim 10^{58}$ erg, for dwarf galaxies, $U(\mathrm{dwarf}) \sim 10^{51}$ erg.

The energy released by SNe is,

$$E_k \sim \frac{10^{51}\ \mathrm{erg}}{100M_\odot} M_*. \tag{13}$$

For MW, that is $E_k(\mathrm{MW}) \sim 10^{60}$ erg, for dwarf galaxies, that is $E_k(\mathrm{dwarf}) \sim 10^{54}$.

Ptentially, for both MW and dwarf galaxies, SNe feedback release enough energy that can blow up the entire galaxy. However, that does not happen to MW because not all of the SNe energy can be effectively converted into heat. But based on the order-of-magnitude difference, the feedback becomes more of a problem as approaching to dwarf galaxies.

3 Giant Molecular Clouds, the ISM, and Turbulence

[2+4+3+4]

a) The mass fraction can be calculated from integrating the distribution function $\frac{\mathrm{d}N}{\mathrm{d}M}$,

$$f = \frac{\int_{M_1}^{M_2} M^{-1.6} M \,\mathrm{d}M}{\int_{10M_\odot}^{10^7 M_\odot} M^{-1.6} M \,\mathrm{d}M}. \tag{14}$$

For large, medium and small clouds, the results are $\boxed{f = 0.84, 0.13, 0.02}$.

b) By plugging the masses in equations given in the question, we can get the results in Table 1.

Table 1: Answers for Q3b

	Large	Medium	Small
$M/M_\odot$	10^6	10^4	10^2
R/pc	31.6	3.16	0.316
$\Sigma/(M_\odot/\mathrm{pc}^2)$	318	318	318
$\Sigma/(\mathrm{g/cm}^2)$	0.0667	0.0667	0.0667
$\rho/(M_\odot/\mathrm{pc}^3)$	7.55	75.5	755
$\rho/(\mathrm{g/cm}^3)$	5.12×10^{-22}	5.12×10^{-21}	5.12×10^{-2}
t_{ff}/yr	2.9×10^6	9.2×10^5	2.9×10^5

c) The ratio between gravitation energy and kinetic energy is,

$$f = \frac{GM^2/R}{M\sigma^2/2}. \tag{15}$$

If $f = 2$, the cloud is in Virial equilibrium. For large, medium and small clouds, $\boxed{f = 1.9, 1.7, 2.7}$. Thus, they are close to equilibrium.

d) The gas temperature in a GMS is $T \sim 10\mathrm{K}$, which gives the sound speed,

$$c_s = \sqrt{\frac{kT}{m_{H_2}}} \sim 2 \times 10^4 \mathrm{cm/s}. \tag{16}$$

Thus, the Mach numbers are $\mathcal{M} \sim 60, 20, 5$ for large, medium and small clouds correspondingly. The standard deviation is,

$$S^{1/2} = (\ln[1 + \mathcal{M}^2])^{1/2} = 2.86, 1.45, 1.81. \tag{17}$$

Since $\ln[\rho/\langle\rho\rangle]$ follows Gaussian distribution, the lower limit of the mass of the densest 5% turbulence is at $1.64 S^{1/2}$. Thus,

$$\ln(\rho/\langle\rho\rangle) = S/2 + 1.64 S^{1/2} = 8.8, 7.0, 4.6, \tag{18}$$

which correspond to,

$$\rho/\langle\rho\rangle = 6600, 1110, 98. \tag{19}$$

Therefore, $\rho = 4.9 \times 10^4, 8.4 \times 10^4, 7.4 \times 10^4 M_\odot/\mathrm{pc}^3$ These are very large numbers considering that a randomly generated turbulence can be 5% denser than that.