Ay124 PS3 Solutions

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1 The Hubble Flow

[3]

From Hubble's law,

$$v = H_0 D = 67 \text{ km/s/Mpc} \times 1.4 \times 10^{-16} \text{ Mpc} = \boxed{0.30 \text{ mm/yr}}, \tag{1}$$

which is two-order-magnitude slower than the plate motion. Such motion is not happening because earth is bond by gravity.

2 Comoving Distance

[3]

3 Age of the Universe

[3*6]

a) Taking cosmological parameters from Planck (Ade et al., 2015), $H_0 = 68$ km/s, $\Omega_m = 0.31$, $\Omega_\Lambda = 0.69$, the lookback time can be expressed as,

$$t_L = \int_0^z \frac{\mathrm{d}z'}{(1+z')E(z')}, \tag{2}$$

where $t_H = 1/H_0$, and $E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$. $t_L(z)$ can be plotted using *mathematica* (Fig.1).

b) Changing the upper and lower limits of the integration in Eq. 2, it gives,

$$t_L(\infty) - t_L(30) = 9.7 \times 10^7 \text{ yr}, \tag{3}$$
$$t_L(\infty) - t_L(6) = 9.0 \times 10^8 \text{ yr}, \tag{4}$$
$$t_L(\infty) - t_L(2) = 3.2 \times 10^9 \text{ yr}, \tag{5}$$
$$t_L(\infty) - t_L(1) = 5.7 \times 10^9 \text{ yr}, \tag{6}$$
$$t_L(\infty) - t_L(0) = 1.3 \times 10^{10} \text{ yr}. \tag{7}$$
$$\tag{8}$$

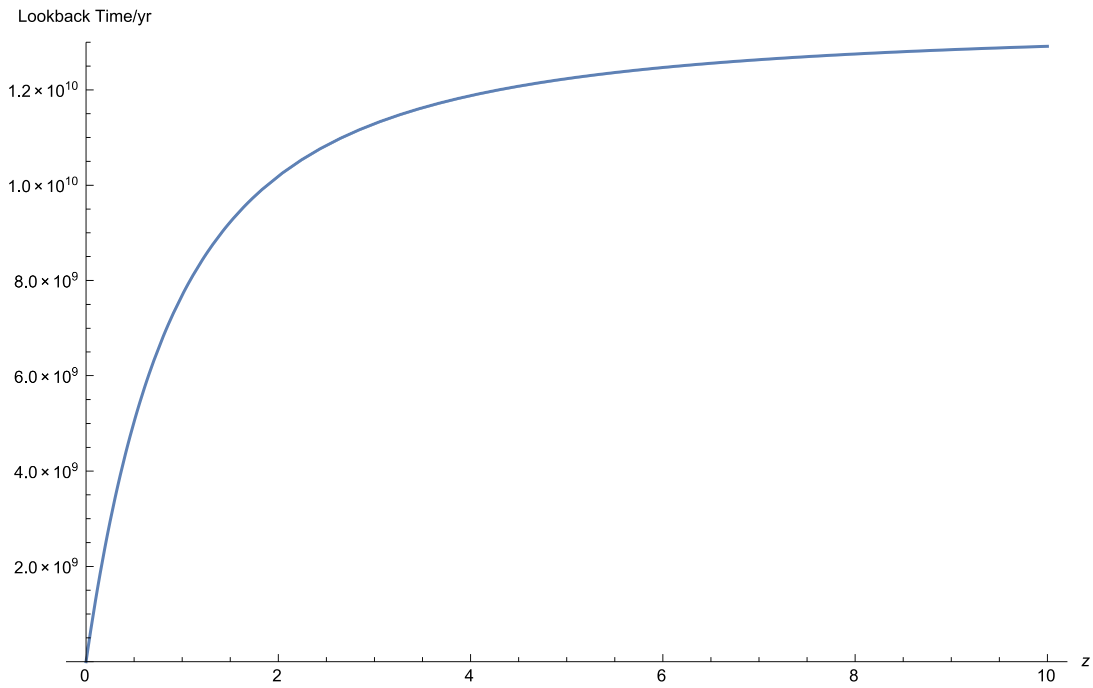

Figure 1: Lookback time as a function of redshift

c) Solving Eq. 2 with $t_L = 4\text{Gyr}$, $z = 0.37$.

d) Converting z to t_L is not linear. Specifically, there is more physical time in $z < 1$ than $z > 1$. Thus, even if the star formation peaks at $z \sim 2.5$, it does not last for a long time.

e) Solving equation,

$$t_L\infty - t_L(z) \sim 500 \text{ Myr}, \tag{9}$$

gives $\boxed{z \sim 9}$. The dust appearance at $z > 10$ can be reconciled by assuming Pop-III stars are more massive and evolve faster than the stars in low-z. Meanwhile, since they are more massive, they can end up in SNe explosions which also contribute to the cosmic metallicity.

f) The time difference between $z \sim 6$ and $z \sim 30$ is $\Delta t_L = 0.81\text{Gyr}$. Plugging in the BH mass equation,

$$M_{\text{BH}} = M_0 \exp(t/t_S), \tag{10}$$

with $M_0 = 10M_\odot$, $\boxed{M_{\text{BH}} = 5.2 \times 10^8 M_\odot}$.

If the first stars do not form until $z = 10$, $\Delta t_L = 0.44\text{Gyr}$, which gives $\boxed{M_{\text{BH}} = 1.8 \times 10^5 M_\odot}$.

If the blackholes are only half-sufficient when accreting, t_S is doubled. Therefore, for the case when the first stars formed at $z = 30$, $\boxed{M_{\text{BH}} = 8.1 \times 10^4 M_\odot}$.

The result from SDSS means that only if the early BHs accreting at very high efficiency, can they match with the most massive BHs observed in early universe. Thus, it means that the BH mergers are vital to BH growth.

4 Lensing and the Galaxy Luminosity Function

[5]

The flux of a galaxy is magnified by lens by a factor of $A = 1 + \epsilon$. Therefore, after lens, the true minimum luminosity becomes $L_{\rm min} = \frac{L^{\rm app}_{\rm min}}{1+\epsilon}$. The luminosity distribution function takes the form,

$$\frac{\mathrm{d}N}{\mathrm{d}L\,\mathrm{d}V} \propto L^{-\alpha}. \tag{11}$$

Thus,

$$\mathrm{d}N/\,\mathrm{d}V = \int_{L_{min}}^{\infty} L^{-\alpha}\,\mathrm{d}L = \frac{1+(\alpha-1)\epsilon}{1-\alpha} L_{min,app}^{-\alpha+1}. \tag{12}$$

Now let's consider the volume change. The magnification of flux comes from reducing the physical area by a factor of $(1+\epsilon)$. Meanwhile, the physical depth increases by a factor of $(1+\epsilon)^{1/2}$. Therefore,

$$\mathrm{d}N = \frac{1+(\alpha-1)\epsilon}{1-\alpha} L^{-\alpha+1}(1+\epsilon)^{-1/2}\,\mathrm{d}V_0. \tag{13}$$

Therefore, the ratio between the numbers of objects that can be seen from survey with lens and without lens is,

$$\frac{\mathrm{d}N(\epsilon)}{\mathrm{d}N(\epsilon=0)} = \frac{(\alpha-1)\epsilon+1}{(1+\epsilon)^{1/2}} > 1. \tag{14}$$

Taking $\epsilon \ll 1$, this equation can be simplified as,

$$2(\alpha-1) > 1, \tag{15}$$

$$\boxed{\alpha > \frac{3}{2}}. \tag{16}$$

5 Cosmic Backgrounds

[5]

a) For isotropic emission,

$$u = \frac{4\pi}{c}\int I_\nu\,\mathrm{d}\nu = 1.26\times10^{-17}\mathrm{J\ m^{-3}}. \tag{17}$$

Three orders of magnitude weaker than CMB.

b) The luminosity distance is,

$$d_L = \frac{c}{H_0}(1+z)\int_0^z \frac{\mathrm{d}z'}{E(z')} = 15.5\ \mathrm{Gpc}, \tag{18}$$

for $z = 2$. Therefore, the observed flux for each quasar is,

$$F = \frac{0.3L_{\rm bol}}{4\pi d_L^2} = 4.2\times10^{-17}\mathrm{W\ m^{-2}}. \tag{19}$$

Given that $u = F/c$ for isotopic emission, we need $\boxed{\sim 10^8}$ quasars to create CXB. Similarly, for $z = 0.5$ Seyferts, $d_L = 2.83$ Gpc, $F = 1.3 \times 10^{-16}$ W m^{-2}. Therefore, it needs $\boxed{\sim 3 \times 10^7}$ Seyferts to create CXB.

6 Feedback Again

[5]

Taking $M_{\rm BH} = 10^8 M_\odot$, and $M_{\rm galaxy} = 5 \times 10^{10} M_\odot$. Energy radiated by BH is,

$$E_{\rm rad} = \frac{\epsilon}{1-\epsilon} M_{\rm BH} c^2 = 2 \times 10^{61} \text{ erg.} \tag{20}$$

The gravitational binding energy of baryons is

$$\phi \sim f_b M_{\rm gal} V_{\rm vir}^2 \sim 4 \times 10^{58} \text{ erg.} \tag{21}$$

The energy emitted by BH is much larger than the potential energy.