Ay121 PS8 Solutions

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Based on the solutions from Rachel Theios in 2015.

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1 Dispersion

(a) The difference in arrival time between two pulses with frequencies ν_1 and ν_2 is

$$\Delta t_{\mathrm{A}} = 4.15 \text{ ms} \times \mathrm{DM} \times \left[\left(\frac{\nu_1}{\mathrm{GHz}} \right)^{-2} - \left(\frac{\nu_2}{\mathrm{GHz}} \right)^{-2} \right]. \tag{1}$$

Plugging in $\Delta t_{\mathrm{A}} = 1$ s, $\nu_1 = 1000$ MHz, and $\nu_2 = 2000$ MHz, we can solve for the dispersion measure:

$$\boxed{\mathrm{DM} = 320 \text{ pc cm}^{-3}}. \tag{2}$$

(b) Recall that the dispersion measure is the integrated free electron column density along the line of sight:

$$\mathrm{DM} \equiv \int_0^d n_e \, \mathrm{d}s. \tag{3}$$

If the pulsar is known to be at $d = 4000$ pc from Earth, the average electron density along the line of sight is

$$\langle n_e \rangle = \frac{\mathrm{DM}}{d} = \frac{230 \text{ pc cm}^{-3}}{4000 \text{ pc}} \tag{4}$$

$$\boxed{\langle n_e \rangle = 0.08 \text{ cm}^{-3}} \tag{5}$$

(c) The minimum frequency at which radiation can propagate through a plasma is the plasma frequency, ω_{p}:

$$\omega_{\mathrm{p}}^2 = \frac{4\pi e^2 n_e}{m_e} \tag{6}$$

or

$$\nu_{\mathrm{p}} \approx 9000 \; n_e^{1/2} \text{ Hz}. \tag{7}$$

With the average electron density found in part (b), the minimum frequency at which radiation can propagate through an ISM of this density is

$$\boxed{\nu_{\mathrm{p}} \sim 2.5 \text{ kHz}}. \tag{8}$$

(d) The optical depth to Thomson scattering along the line of sight to the pulsar is

$$\tau = \int_0^d \sigma_\mathrm{T} n_e \,\mathrm{d}s. \tag{9}$$

With the definition of the dispersion measure (Equation 3), and the value found in part (a), we find

$$\boxed{\tau = \sigma_\mathrm{T} \times \mathrm{DM} = 6 \times 10^{-4}}. \tag{10}$$

2 Faraday Rotation

Recall that the arrival time of a pulse is given by

$$t_p = \frac{d}{c} + \frac{2\pi e^2}{m_e c}\frac{1}{\omega^2}\int n_e \,\mathrm{d}s, \tag{11}$$

so that

$$\frac{\mathrm{d}t_p}{\mathrm{d}\omega} = -\frac{4\pi e^2}{m_e c}\frac{1}{\omega^3}\int n_e \,\mathrm{d}s. \tag{12}$$

The change in polarization angle due to Faraday rotation is given by

$$\Delta\theta = \frac{2\pi e^3}{m_e^2 c^2}\frac{1}{\omega^2}\int n_e B_\parallel \,\mathrm{d}s, \tag{13}$$

so that

$$\frac{\mathrm{d}\Delta\theta}{\mathrm{d}\omega} = -\frac{4\pi e^3}{m_e^2 c^2}\frac{1}{\omega^3}\int n_e B_\parallel \,\mathrm{d}s. \tag{14}$$

Therefore with the given numbers

$$\boxed{\langle B_\parallel \rangle = \frac{\int n_e B_\parallel \,\mathrm{d}s}{\int n_e \,\mathrm{d}s} = \frac{m_e c}{e}\frac{\mathrm{d}\Delta\theta/\,\mathrm{d}\omega}{\mathrm{d}t_p/\,\mathrm{d}\omega} = 0.98\ \mu\mathrm{G}}. \tag{15}$$