AY105 Lab Experiment #4
Interferometry

Purpose

This week, you will work on interferometers. You will build your own Michelson interferometer, examine the output of an interferometer, measure the wavelength of the light source, generate red and white light interference, learn the concept of coherent length, and use the interferometer as a spectrograph to measure the spectrum of white LED.

Background

Interferometry has wide applications in modern astronomy. In 1920, Albert A. Michelson used the 100-inch telescope on Mount Wilson to build a stellar interferometer (Fig. 1). He successfully measured the diameter of Betelgeuse with it. Despite the slow development of interferometry in optical astronomy, radio interferometry saw huge progress in the last few decades. Another application of interferometry in astronomy led to the direct detection of gravitational waves. Using Michelson interferometer (not to be confused with Michelson's stellar interferometer, they have very different optical configurations), Laser Interferometer Gravitational-Wave Observatory (LIGO) can resolve as small as 10^{-19} m change in length caused by gravitational waves. In 2016, LIGO announced the first detection of gravitational wave emitted by two black holes with $36 \pm 4\, M_{\odot}$ and $29 \pm 4\, M_{\odot}$ mergers. Michelson interferometers can also be used as Fourier Transform Spectrographs (FTS), which can measure spectra at broad spectral range and have applications in infrared astronomy.

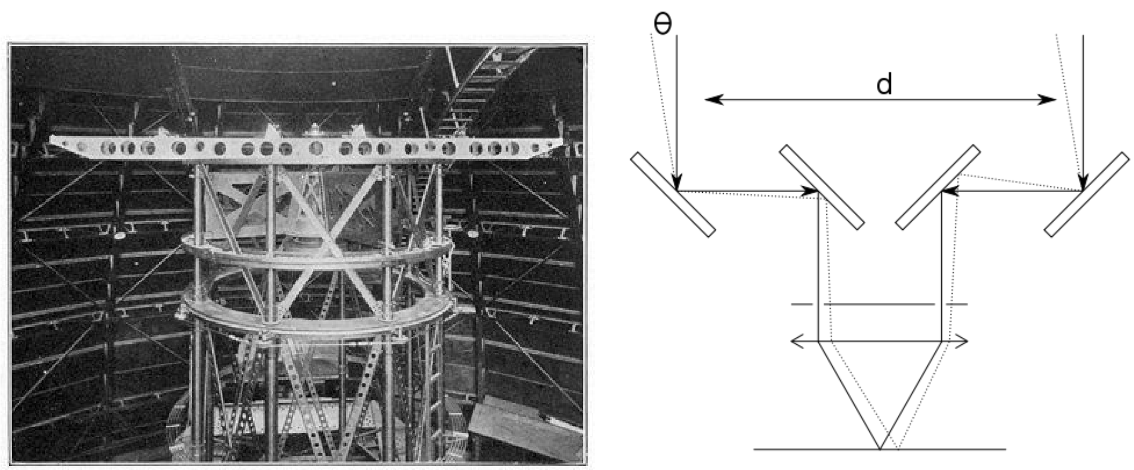

Figure 1: Real historical picture (left) and optical configuration (right) of the Michelson stellar interferometer.

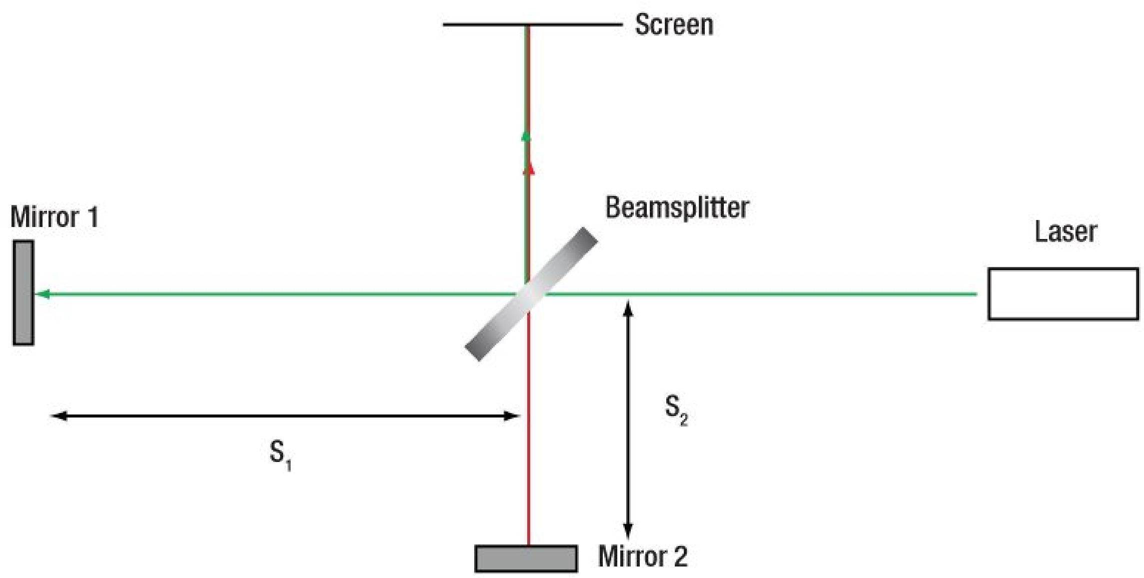

Figure 2: Sketch of a Michelson interferometer. The laser is aimed at the beamsplitter which divides the beam into two partial beams. These are reflected by the respective mirrors. An interference pattern can be observed on the screen.

Fig. 2 is a sketch of a Michelson interferometer. Limiting ourselves to examining an incident plane wave along the optical axis,

$$\vec{E}_i = \vec{E}_0 \cos(\omega t - kx).$$

The amplitude of the partial wave of one interferometer arm at the location of the screen can be expressed as:

$$|\vec{E}_1| = \sqrt{RT} E_0 \cos(\omega t + \phi_1),$$

where R and T are the reflection and transmission capacity of the beamsplitter. Similarly, for the other arm,

$$|\vec{E}_2| = \sqrt{RT} E_0 \cos(\omega t + \phi_2).$$

The intensity on the screen is then determined by,

$$I = c\varepsilon_0 |\vec{E}_1 + \vec{E}_2|^2 .$$

In reality, the observable physical property is time averaged intensity. Show that

$$\bar{I} = \tfrac{1}{4} c\varepsilon_0 E_0{}^2 (1 + \cos\Delta\phi),$$

where the phase difference of the two partial waves translates directly into the path length difference Δs between them:

$$\Delta\phi = \tfrac{2\pi}{\lambda} \Delta s .$$

Plot $\bar{I}$ as functions of $\Delta\phi$ and Δx in your note.

To measure the wavelength of a given monochromatic light source, one can shift the mirror by a defined path (Δx) and count the number (N) of the maximums (or minimums). Write down the equation relating λ, Δx and N.

Coherence in the largest sense describes the capacity of light to create interference. The maximum time span Δt_c, during which the phase differences of random partial waves at a point change by less than 2π ,is called the coherence time. If the change falls below 2π,one says that the partial waves are temporally coherent. What this means is best described with Fig. 3.

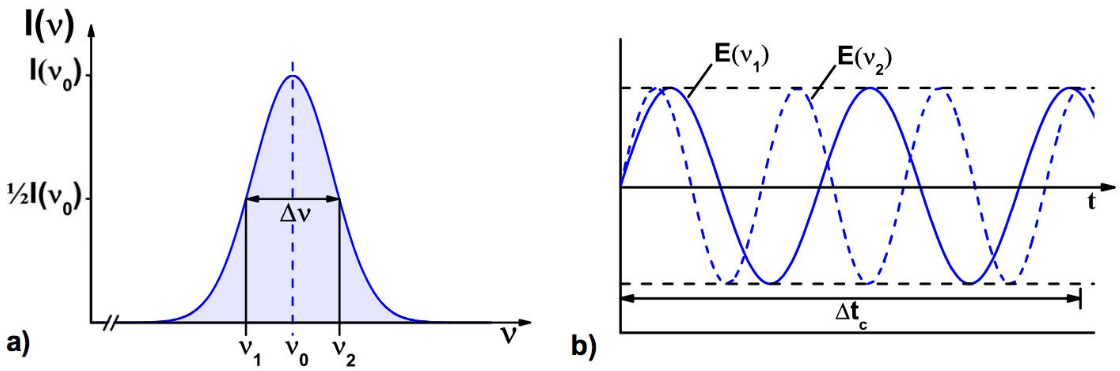

Figure 3: (a) Pulse with spectral width $\Delta\nu$ (FWHM) and (b) the partial waves with frequencies ν_1 and ν_2 after the coherence time Δt_c have a phase offset of 2π.

We imagine a light source with a spectrum as shown in Fig. 3(a); its spectral width is denoted by $\Delta\nu$ .The source therefore emits light that we can view as the overlapping of various partial waves with frequencies in the interval ($\nu_1 = \nu_0 - 0.5\Delta\nu$, $\nu_2 = \nu_0 + 0.5\Delta\nu$). If their phase difference at time $t = 0$ is zero, the maximum phase offset of two partial waves is given by:

$$\Delta\phi(t) = 2\pi(\nu_2 - \nu_1)t .$$

If the time span has increased to $1/\Delta\nu$, the phase offset is 2π. This results in the coherence time as:

$$\Delta t_c = \frac{1}{\Delta\nu} .$$

Linked to the coherence time, the coherence length is ΔL_c. This is the path the light can travel within the coherence time, that is:

$$\Delta L_c = c \cdot \Delta t_c .$$

Converting $\Delta\nu$ to $\Delta\lambda$, show that,

$$\Delta L_c \simeq \frac{\lambda_0^{\ 2}}{\Delta\lambda} .$$

For a quantitative evaluation of the coherence, one would have to measure the contrast function of the interference pattern. In this experiment, the coherence length is estimated by shifting one of the mirrors in the interferometer and observing the disappearance of the interference pattern.

From the previous equations, we can form a sense that Δs and $\Delta\nu$ are closely related. In fact, they form a Fourier transform pair, indicating that $I(s)$ is the Fourier transformation of $I(\nu)$.

$$I(\Delta s) = \int_0^\infty I(\sigma)(1 + \cos(2\pi\sigma\Delta s))\, d\sigma,$$

where $\sigma = 1/\lambda = \nu/c$.

Equipment

We will mostly be using the ThorLabs Michelson Interferometer Educational Kit, which includes the following equipments:

- Three Light Sources: Laser, Red LED, White LED
- Mirrors and Lenses
- Beam Splitter
- Aluminum Mirror and Aluminum Post
- Digital Thermometer
- Foil Heater
- Viewing Screen
- Necessary Mounts and Holders
- Rotation Platform with Plexiglas Plate

The full list of ThorLabs Michelson Interferometer Educational Kit components are listed in the following table.

Description	EDU-MINT1 Item #	EDU-MINT1/M Item #	Qty.
Basic Interferometer Components			
Steel Breadboard, 12" x 18" (30 cm x 45 cm)	B1218FE	B3045AE	1
Rubber Damping Feet, Set of 4	RDF1		1
Collimated Laser Diode Module, 532 nm, Class II	CPS532-C2		1
5 VDC Regulated Power Supply	LDS5	LDS5-EC	1
Ø1" Unthreaded Adapter for Ø11 mm Components	AD11NT		1
Ø1" Kinematic Mirror Mount	KM100		2
Ø1" Protected Silver Mirror	PF10-03-P01		2

Z-Axis Translation Mount	SM1ZP	SM1ZP/M	1
Height Spacer, 2" x 3" x 0.25" (50 mm x 75 mm x 6.25 mm)	BA2S5	BA2S5/M	1
Base, 2" x 3" x 3/8" (50 mm x 75 mm x 10 mm)	BA2	BA2/M	2
Plastic Viewing Screen	EDU-VS1	EDU-VS1/M	1
Ø1" Lens Mount	LMR1	LMR1/M	1
Ø1" N-BK7 Bi-Convex Lens, f = 50.0 mm	LB1471		1
Non-Polarizing Beamsplitter Cube, 400 - 700 nm	CCM1-BS013	CCM1-BS013/M	1
1.5" (30 mm) Long Universal Post Holder	UPH1.5	UPH30/M	4
Ø1/2" Post, 1.5" Long	TR1.5	N/Aa	4
Components to Observe 2nd Interferometer Output			
Ø1" Economy Beamsplitter	EBS1		1
Ø1" Lens Mount	LMR1	LMR1/M	1
Ø1/2" (Ø12.7 mm) Post, 1.5" (40 mm) Long	TR1.5	TR40/M	1

Refractive Index Measurement			
High-Precision Rotation Mount	PR01	PR01/M	1
General Purpose Plate Holder	FP01		1
Plexiglas Plate, 8 mm Thick	-		1
Plexiglas Plate, 12 mm Thick	-		1
Interference with LEDs			
Red LED with USB Connector, 635 nma	-		2
White LED with USB Connectorb	-		2
LED Mount	LEDMF		2
1.5" (40 mm) Long Universal Post Holder	UPH1.5	UPH40/M	2
Ø1/2" (Ø12.7 mm) Post, 1.5" (40 mm) Long	TR1.5	TR40/M	2
5 VDC Battery Pack, 10 000 mAh	CPS1		1
Ruler, 12" (30 cm)	-		1
Thermal Expansion Setup			
Mirror Holder for Ø1" Optics	MH25		1

Ø1" Aluminum Mirror	ME1-G01		1
Aluminum Post, Ø12.7 mm x 90 mm Long	-		1
Right-Angle Post Clamp	RA90	RA90/M	1
Ø1/2" (Ø12.7 mm) Post, 2" (50 mm) Long	TR2	TR50/M	1
1.5" (40 mm) Long Post Holder	PH1.5	PH40/M	1
Base, 1" x 2.3" x 3/8" (25 mm x 58 mm x 10 mm)	BA1S	BA1S/M	1
Digital Thermometer	-		1
Sensor	-		1
Foil Heater with 10 kΩ Thermistor	HT10K		1
Crocodile Clip to Banana Plug Cable	-		1
Electrical Tape	-		1

Lab activities

This lab will let you get familiar with how interferometry works and its basic concepts. We will learn how to setup the Michelson interferometer, get familiar with the concept of coherence, mimic the light-path difference introduced by gravitational waves by measuring the refraction index of a certain material, and measure the spectrum of an LED (bonus part). **Be careful with the laser beam.**

Part I: Preliminary Tests

First, setup the individual equipments based on Fig. 4-6. The small additional optical table has rubber feet which can help reduce the disturbance. Use the 1.5" posts and post holders to further reduce the disturbance.

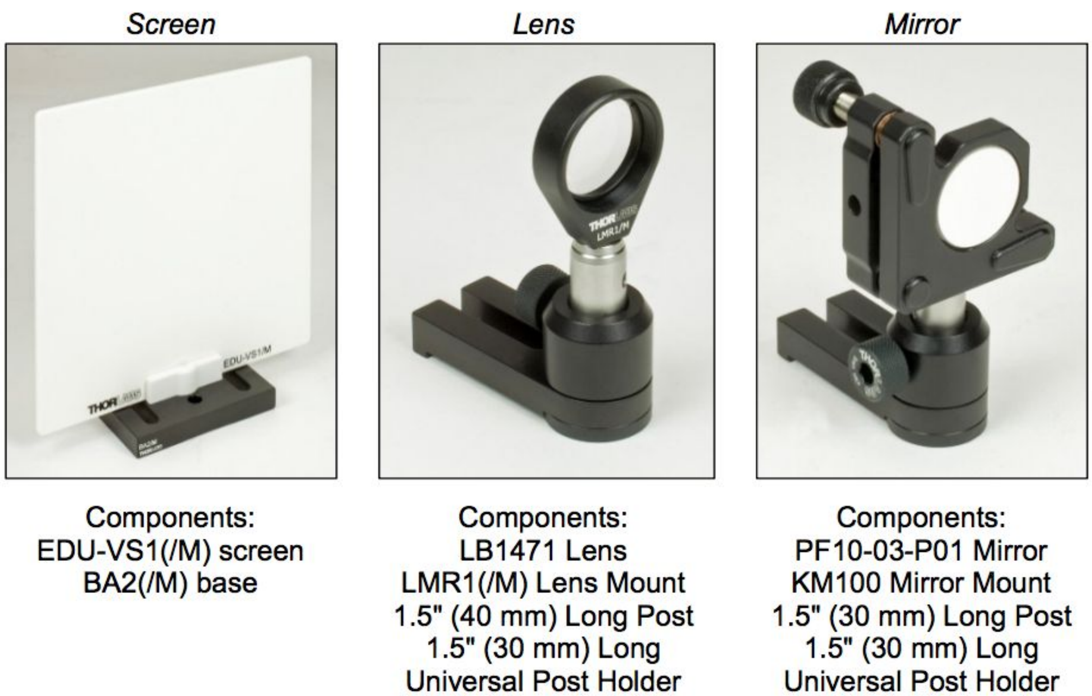

Figure 4: Setup of the individual equipments.

Laser

Components:
Laser
KM100 Mount
AD11NT Adapter
1.5" (30 mm) Long Post
1.5" (30 mm) Long
Universal Post Holder

Beamsplitter

Components:
Beamsplitter Cube
1.5" (40 mm) Long Post
1.5" (30 mm) Long Universal
Post Holder

Movable Mirror

Components:
PF10-03-P01 Mirror
SM1ZP(/M) Stage
BA2(/M) Base
BA2S5(/M) Spacer

Figure 5: -- Continued.

Rotation Platform

Components:
Rotation Stage
FP01 Universal Mount
Plexiglas Plate

LED Mount

Components:
LED Holder
LED
1.5" (40 mm) Long Post
1.5" (40 mm) Long
Universal Post Holder
CPS1 Battery Pack

Figure 6: -- Continued.

Assemble the instruments based on Fig. 1. We will be using the laser as the light source first. Ensure the mirrors are perpendicular to the beam, and the beam is split at a 90° angle. Also, make sure that the distance between the beamsplitter and the mirrors is about the same along both interferometer arms (think about why).

After the initial setup, the two laser beams are very unlikely to overlap. Tip and tilt the mirrors and the beam splitter to make them overlap. You may already see interference rings. Then, place a lens between the laser and the beamsplitter to enlarge the interference pattern. If not, turn the screws on the adjustment mirror and try to create interference. Describe the interference pattern and discuss why it looks like that. Change the length of and interferometer arm. What is the effect on the interference pattern? (Hint: the light is no longer collimated with the lens.) Verify that a Michelson interferometer has two outputs by placing another beamsplitter in front of the original beamsplitter. Find a way to project the other output to the screen. Compare both patterns.

Part II: Measuring the Wavelength of Laser

Adjust the interferometer so that the interference pattern is neither too large nor too small. Determine the wavelength of the laser through translation of the mirror. Don't forget to estimate errors for all the values you measure. Measure the number of maximums or minimums changed while shifting the movable mirror to a certain Δs. Calculate the wavelength and its uncertainty based on your data.

Part III: Determining the Refractive Index

Measuring the change of optical path due to thermal expansion is very similar to what scientists do in LIGO.

Insert the rotation platform with the Plexiglas plate in one arm of the interferometer. Establish an interference pattern. Adjust the plate so that it stands perpendicular to the beam. Determine the refractive index of Plexiglas by rotating the thin plate. Note that when the plate is rotated, the light path is shown in Fig. 7. Therefore, the optical depth can be calculated from the following equation.

$$n = \frac{(\frac{N\lambda}{2t}+\cos\alpha-1)^2+\sin^2\alpha}{2(-\frac{N\lambda}{2t}-\cos\alpha+1)} \; .$$

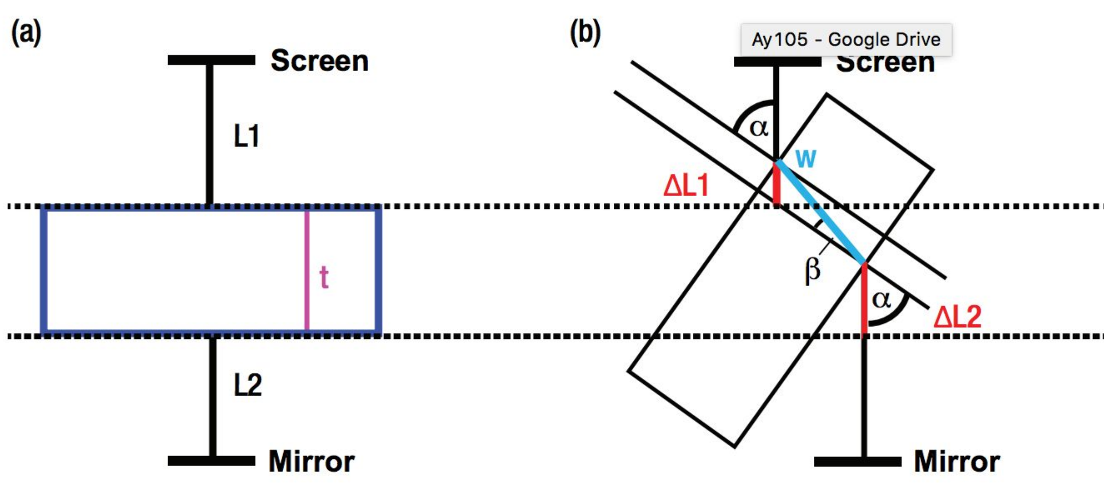

Figure 7: (a) Placing the plate in an interferometer arm (b) Rotation and change in the (optical) path.

Measure the amount of the maximums or minimums changed when rotating the plate to different angles. Calculate n and its uncertainty based on your measurements.

Part IV: Coherence

In order to see interference with LEDs, one has to adjust the interferometer so that both arms have almost the same length. You can achieve this by making the interference patterns as large as possible. (This is very important since LEDs have much broader spectral profiles.) Replace the laser with the red LED. To increase the intensity of the light on the screen, where do you want to place the LED? Move the mirror until you see an interference pattern. Measure the approximate coherence length of the red LED by shifting the movable mirror and find when the interference disappear on both of the directions.

Adjust the mirror back to the position where both arms have almost the same length. Replace the red LED with the white one. Adjust the mirror to find the interference. Measure the approximate coherence length of the white light LED and compare it to the red LED.

Part V: Measuring the Spectrum of the White LED (Bonus)

From the background, we know that $I(\nu)$ is the Fourier transformation of $I(\Delta s)$. Thus, we can measure the intensity at the center of the interference pattern to measure the spectrum of a certain light source.

Keep the white LED as the light source. (Why?) Place the Si photodiode at the center of the interference. Place an iris to reduce the scattering light if necessary. Sample the steps of Δs uniformly and adequately. Start at point slightly beyond one of the edges when you can barely see any interference. Measure the flux on the photodiode for each step while moving along to the other edge. Repeat your measurements. **Note that the measurements can be very tricky, given that any small perturbations can cause huge fluctuations.** You will need to get rid of any ambient light and let people stay still. After each step of shifting the mirror, you will need to wait for a few seconds to let the vibrations settle.

Determine the point when $\Delta s = 0$ in your data. Use Python, IDL, Mathematica, Matlib, or whatever programing language that you are comfortable with, convert $I(\Delta s)$ to $I(\nu)$. Note that you will need to subtract $I(\Delta s)$ with $I(\Delta s = \infty)$ to get rid of the continuum, which contributes to the 0th order signal. What determines your measured spectral range? What determines your measured spectral resolution? Does the spectrum match with you expectation?

Discussion

1) Discuss the advantages and disadvantages of FTS.
2) The Michelson's stellar interferometer is a term that can be easily confused with the Michelson interferometer. Do some research online about that and discuss how it can measure the diameter of stars.
3) Discuss why interferometer arrays have wide applications in radio astronomy, not in optical/infrared astronomy. What is the bottleneck?