Ay124 PS2 Solutions

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1 Galaxy Masses

[3+3+4]

a) Given that,

$$\frac{\mathrm{d}N}{\mathrm{d}M_{\mathrm{halo}}} \propto M_{\mathrm{halo}}^{-2}, \tag{1}$$

and

$$\frac{\mathrm{d}N}{\mathrm{d}M_*} \propto M_*^{-1}, \tag{2}$$

divide the two equations and solve the differential equation,

$$\boxed{M_* = A\exp\left(-\frac{M_0}{M_{\mathrm{halo}}}\right)}. \tag{3}$$

Note: There was an intention of simplify this question by setting $\mathrm{d}N/\,\mathrm{d}M_{\mathrm{halo}} \propto M_{\mathrm{halo}}^{-2}$. However, that actually caused more troubles later in this PS.

b) Plugging in two data points,

$$\text{MW: } M_{\mathrm{halo}} = 10^{12}M_\odot, M_* = 10^{11}M_\odot$$

$$\text{LMC: } M_{\mathrm{halo}} = 9\times 10^{9}M_\odot, M_* = 3\times 10^{9}M_\odot,$$

we get

$$\boxed{A = 1.2\times 10^{11}M_\odot, M_0 = 3\times 10^10 M_\odot}. \tag{4}$$

Therefore, the fraction of baryonic mass that turns into stars is,

$$\boxed{f_* = \frac{M_*}{f_b M_{\mathrm{halo}}} = \frac{7.5\times 10^{11}M_\odot}{M_{\mathrm{halo}}}\exp\left(-\frac{3\times 10^10 M_\odot}{M_{\mathrm{halo}}}\right)}. \tag{5}$$

This function decreases exponentially as M_{halo} approaches 0.

Note: 1. Aside from MW, the other data point shall be less massive than the "knee" of the Press-Schechter formalism, which happens at $\sim$ MW. 2. The final result is highly sensitive to the other data point. Thus, the TA only cares about the qualitative description, instead of the physical numbers of the coefficients.

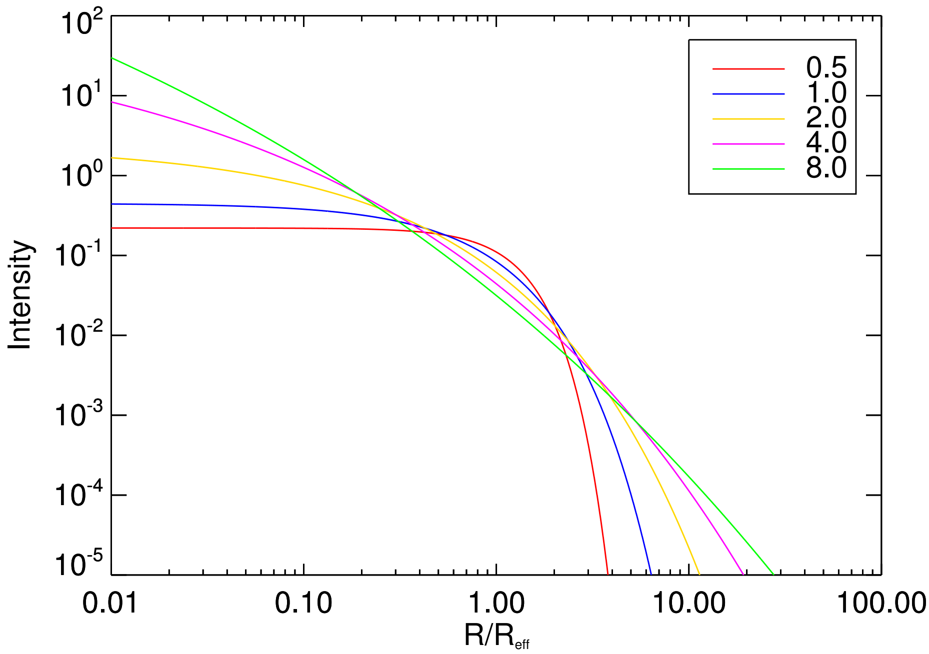

Figure 1: The Sersic profile with different Sersic indices.

c) Similar as the previous questions, solve the differential equation,

$$M_* = \left(\frac{C}{A + 1/M_{\text{halo}}} \right)^{1/7}. \tag{6}$$

Therefore,

$$f_* = \frac{M_*}{f_b M_{\text{halo}}} = \frac{1}{f_b M_{\text{halo}}} \left(\frac{C}{A + 1/M_{\text{halo}}} \right)^{1/7}. \tag{7}$$

When M_{halo} is large, this function is proportional to $\boxed{M_{\text{halo}}^{-6/7}}$, which is dropping as M_{halo} increases.

2 Galaxy Mass Profiles

[10]

The easiest way to solve this question is to do it numerically with a little bit programming. You can choose an arbitrary length scale that is related to R_0 as the unit of length. Then integrate the Sersic functions, renormalize the functions, and figure out where the R_{eff} are. You will figure out that since the unit of radius changed, the integration result also changed. Thus, you need to write the program recursively. Fortunately, the program

A short IDL program has been uploaded to the course website. (Sorry, the TA is very old-fashioned.) The plot is shown in Fig.1.

It can be seen from the plot that with high Sersic index, the light is more concentrated at the center, and the drop-off at the outskirt is smoother. With low Sersic index, the light is more extended at around $R = R_{\rm eff}$, but drops sharply outside.

3 Galaxy Angular Momentum and Scaling Relations

[4+3+4+2]

a) Rearange the definition of λ,

$$|L_{\rm vir}| = \frac{\lambda G M_{\rm vir}^{5/2}}{|E_{\rm vir}|^{1/2}}. \tag{8}$$

Plugging in $|E_{\rm vir}| \sim M_{\rm vir} V_{\rm vir}^2 \sim \frac{GM_{\rm vir}^2}{R_{\rm vir}}$,

$$|L_{\rm vir}| \sim \lambda \sqrt{G M_{\rm vir}^3 R_{\rm vir}}. \tag{9}$$

Since the specific angular momentum is conserved,

$$|L_b| = f_b |L_{\rm vir}|. \tag{10}$$

Meanwhile, the rotational velocity of a galaxy is relatively flat, meaning that for baryons, $V_b \sim V_{\rm vir}$. Thus,

$$|L_b| = f_b M_{\rm vir} R_{\rm eff} V_{\rm vir}. \tag{11}$$

Equating Eq.10 and 11 and plugging in values,

$$\boxed{R_{\rm eff} \sim \lambda R_{\rm vir} \sim \lambda M_{\rm vir}^{1/3} \rho^{-1/3}}. \tag{12}$$

Plugging in Eq.3, we have,

$$\boxed{R_{\rm eff} \sim \lambda \rho^{-1/3} \left(\frac{M_0}{\ln(A/M_*)} \right)^{1/3}}. \tag{13}$$

Note: Due to the complication introduced in Q1, it is also okay to use the scaling

$$\left(\frac{M_*}{10^{11} M_\odot} \right) \sim \left(\frac{M_{\rm halo}}{10^{12} M_\odot} \right)^2, \tag{14}$$

instead of Eq.3.

b) Plug in $M_* \sim 10^{11} M_\odot$, $\lambda \sim 0.033$, and $\rho \sim 200\rho_c \sim 2 \times 10^{-27}$ g cm^{-3}. We get,

$$\boxed{R_{\rm eff} \sim 5.9 \text{ kpc}}. \tag{15}$$

(Using Eq.14, $R_{\rm eff} \sim 6.6$ kpc.)

c) Let's use Eq.14 in this question since Eq.3 gives a very nasty logarithm.

Case I:

$$V_{\rm rot} = \sqrt{\frac{GM_{\rm halo}}{R_{\rm vir}}} \propto M_{\rm halo}^{1/3} \propto M_*^{1/6}. \tag{16}$$

Case II:

$$V_{\mathrm{rot}} = \sqrt{\frac{GM_*}{R_{\mathrm{eff}}}} \propto \sqrt{M_*/M_*^{1/6}} \propto M_*^{5/12}. \tag{17}$$

The TF relation gives $V_{\mathrm{rot}} \propto M_*^{0.23}$. Therefore, most of the galaxies are more close to DM potential.

d) From Barkana & Loeb, $R_{\mathrm{vir}} \propto (1+z)^{-1}$. Therefore, $\boxed{R_* = \lambda R_{\mathrm{vir}} \propto (1+z)^{-1}}$. Early galaxies were formed smaller.

4 Matter Fluctuations and the Non-Linear Power Spectrum

[5]

Based on the definition of correlation function,

$$\begin{aligned} \mathrm{d}P &= n\left[1+\left(\tfrac{r}{r_0}\right)^{\gamma}\right]\mathrm{d}V \\ &= n\left[1+\left(\tfrac{r}{r_0}\right)^{\gamma}\right]\cdot 2\pi r^2\,\mathrm{d}r. \end{aligned} \tag{18}$$

Therefore,

$$\begin{aligned} P &= \int n\left[1+\left(\tfrac{r}{r_0}\right)^{\gamma}\right]\cdot 2\pi r^2\,\mathrm{d}r \\ &= nV + \tfrac{4\pi n}{(\gamma+3)r_0^{\gamma}}R^{\gamma+3} \end{aligned} \tag{19}$$

If there is no correlation, $\xi(r) = 0$. Therefore, the excess correlation is,

$$\boxed{P/P_0 = 1 + \frac{3}{(\gamma+3)r_0^{\gamma}}R^{\gamma}}. \tag{20}$$

Plug in numbers, $r_0 = 5h^{-1}$ Mpc $= 7.14$Mpc and $\gamma = -1.8$. For $R = 100$ kpc, $P/P_0 = 5428$. For $R = 1$Mpc, $P/P_0 = 87$.

Larger objects are likely to be located at the high-biased region, where there is more likely to find another galaxy.

5 Alternative Dark Matter Models

[5]