Assignment 05

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1. Modeling of carbon cycle

1.1 **[15 points]** Following equation 1-2 (without the buffer effect), build a two-box model to compute the atmospheric CO_2 level in ppm (parts per million) from 1987 to 2004

The main idea is to use recursion and cycle, each time input the total carbon of the previous year, and use the formula to calculate the increase of carbon in each year.After setting the initial value, the carbon dioxide concentration of each year is obtained by continuously stacking the previous year.

a) First, define the value of the rate of production of CO 2 by fossil-fuel burning (γ) from 1987 to 2004 as "gamma_values".

```
gamma_values = np.array([5.750,5.963,6.094,6.121,6.198,6.136,6.133,6.241,6.374
                         ,6.524,6.624,6.610,6.597,6.763,6.929,6.992,7.405,7.784])
```

b) Set the range and interval of time.

```
years = np.arange(1986, 2005, 1)
```

c) Set the variables "N1" and "N2" for storing carbon in the atmosphere and ocean surface each year.

```
# 初始条件
N1 = np.zeros_like(years, dtype=float)
N2 = np.zeros_like(years, dtype=float)
```

d) Set the initial conditions for the first year to 740 and 900.

```
N1[0] = 740
N2[0] = 900
```

e) According to the formula, use a for loop to calculate the increment of carbon in each year, and then the total amount of carbon for that year is obtained by adding the increment and stored in N1 and N2.

```
for i in range(1, len(years)):
    dN1_dt = -105/740 * N1[i-1] + 102/900 * N2[i-1] + gamma_values[i-1]
    dN2_dt = 105/740 * N1[i-1] - 102/900 * N2[i-1]
    N1[i] = N1[i-1] + dN1_dt
    N2[i] = N2[i-1] + dN2_dt
```

f) Finally, divide the carbon content by 2.13 to get the CO2 concentration.

```
N1_NObuffer = (N1/2.13)[1:19]
```

```
N1_NObuffer
```

```
array([348.70892019, 350.07643066, 351.47370043, 352.85127816,
       354.23910055, 355.57339171, 356.89221479, 358.25147069,
       359.6583235 , 361.11568319, 362.59516617, 364.0429257 ,
       365.46678299, 366.95618434, 368.5032338 , 370.05369386,
       371.77436542, 373.62781873])
```

1.2 **[20 points]** Following equation 3-4 (with the buffer effect), build a two-box model to compute the atmospheric CO_2 level in ppm from 1987 to 2004.

a) The main method is the same as the first problem.

b) Initialize the pre-industrial sea surface carbon dioxide equilibrium value N0, set the initial conditions of N1 and N2.

```
N0 = 821
N1 = np.zeros_like(years, dtype=float)
N2 = np.zeros_like(years, dtype=float)
N1[0] = 740
N2[0] = 900
```

c) The buffer factor xi for each year is calculated according to the formula.

d) Add the buffer effect to the model.

```
for i in range(1, len(years)):

    z = N1[i-1]/2.13
    xi = 3.69 + 1.86e-2 * z - 1.80e-6 * z**2

    dN1_dt = -105/740 * N1[i-1] + 102/900 * (N0 + xi * (N2[i-1] - N0)) + gamma_values[i-1]
    dN2_dt = 105/740 * N1[i-1] - 102/900 * (N0 + xi * (N2[i-1] - N0))

    N1[i] = N1[i-1] + dN1_dt
    N2[i] = N2[i-1] + dN2_dt
```

e) Divide the carbon content by 2.13 to get the CO2 concentration.

```
N1_buffer = (N1/2.13)[1:19]
```

```
N1_buffer
```

```
array([386.2654774 , 379.07215625, 384.81900782, 386.42512765,
       389.38876739, 391.89163945, 394.54149645, 397.19535871,
       399.92394046, 402.71387361, 405.54821489, 408.37416247,
       411.19845888, 414.10277814, 417.07792342, 420.07912161,
       423.27524669, 426.62935219])
```

f) In order to make the result more similar to Figure 2, linear regression is performed on the result.

```
coefficients = np.polyfit(years[1:], N1_buffer, 1)
fit_line = np.poly1d(coefficients)
fit_line
```

```
poly1d([ 2.68253883e+00, -4.95086612e+03])
```

1.3 **[5 points]** Based on your results from **1.1** and **1.2**, reproduce Figure 2 in Tomizuka (2009) as much as you can.

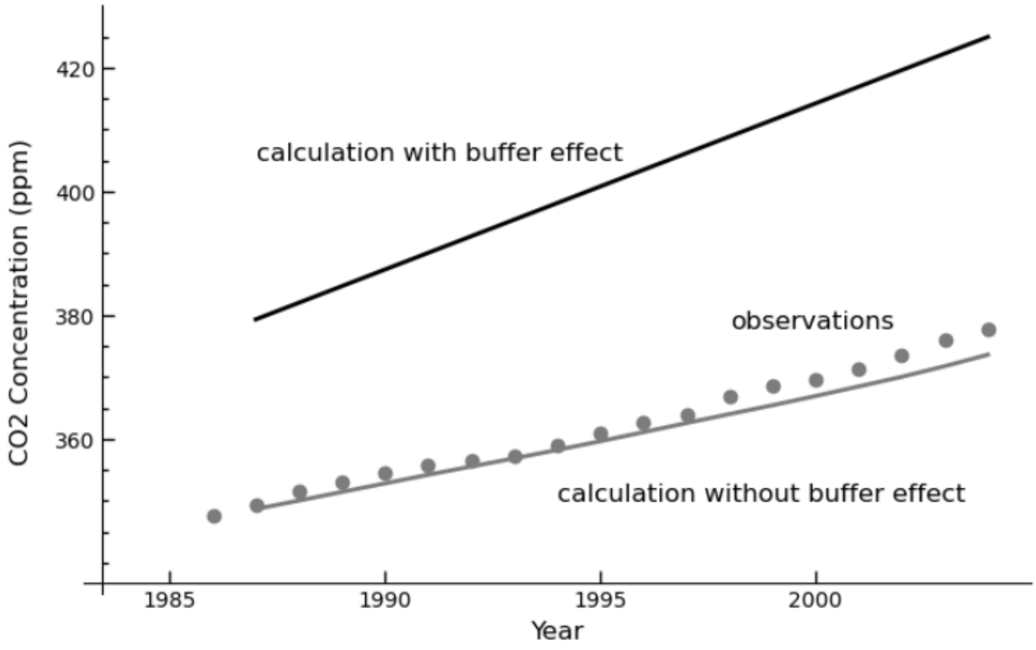
a) Import the observations of volcano.

```
N1_ob = pd.read_csv("unc.csv")["mean"].values
N1_ob
```

```
array([315.98, 316.91, 317.64, 318.45, 318.99, 319.62, 320.04, 321.37,
       322.18, 323.05, 324.62, 325.68, 326.32, 327.46, 329.68, 330.19,
       331.13, 332.03, 333.84, 335.41, 336.84, 338.76, 340.12, 341.48,
       343.15, 344.87, 346.35, 347.61, 349.31, 351.69, 353.2 , 354.45,
       355.7 , 356.54, 357.21, 358.96, 360.97, 362.74, 363.88, 366.84,
       368.54, 369.71, 371.32, 373.45, 375.98, 377.7 ])
```

b) Plot the simulated values for both models, and plot the observed data, adjusting the graph parameters to look more like Figure 2.

```
fig = plt.figure(figsize=(8, 5))
ax = fig.add_subplot(1, 1, 1)
ax.plot(years[1:19], N1_NObuffer, label='Model 1', linewidth=2.0,c = "#7c7c7c")
ax.plot(years[1:19], fit_line(years[1:]), label='Model 2', linewidth=2.0,c = "#020202")
ax.plot(years[0:19], N1_ob[27:] ,'o',c = "#7c7c7c",label='Observation')
font = {'size': 12}
ax.set_xlabel('Year',fontdict=font)
ax.set_ylabel('CO2 Concentration (ppm)',labelpad=10,fontdict=font)
ax.set_xlim(1983, 2005)
ax.set_ylim(335,430)
ax.set_xticks([1985, 1990, 1995, 2000])
ax.set_yticks([360, 380, 400, 420])
ax.set_yticks(np.arange(340, 430, 5), minor=True)
ax.tick_params(which='both', direction='in', length=6, width=1)
ax.tick_params(which='minor', direction='in', length=3, width=1)
ax.text(1987, 405, 'calculation with buffer effect',fontdict=font)
ax.text(1998, 378, 'observations',fontdict=font)
ax.text(1994, 350, 'calculation without buffer effect',fontdict=font)
ax.spines['top'].set_visible(False)
ax.spines['right'].set_visible(False)
ax.spines.left.set_position(('axes', 0.02))
ax.spines.bottom.set_position(('axes', 0.02))
plt.show()
```



[Bonus] [15 points] Following equation 5-13, compute the atmospheric CO_2 level in ppm and reproduce Figure 4 in Tomizuka (2009).

a) Co2 emissions are caused by land use change and were obtained by linear interpolation before 1850.

```
import numpy as np

start_year = 1750
end_year = 1850
start_value = 0.2
end_value = 0.5
years = np.arange(start_year, end_year + 1)
values = start_value + (end_value - start_value) / (end_year - start_year) * (years - start_year)
```

b) Import the data from 1850 to 2000 and combine the values before 1850.

```
LUCE = pd.read_csv("global_carbon.csv")['luce'].to_numpy()[0:151]
delte = np.concatenate((values, LUCE))
```

c) Import the value of the rate of production of CO 2 by fossil-fuel burning (γ) from 1750to 2000.

```
gamma_values = pd.read_csv('global_1751_2014.csv')['C'].to_numpy()/1000
```

d) Define a function named "seven_box_model" with a parameter beta (the fertilization factor β)

```
def seven_box_model(beta):
```

e) Set the year and initial condition N0 to N7.

```python
years = np.arange(1750, 2001, 1)
# 初始条件
N0 = 842
N1 = np.zeros_like(years, dtype=float)
N2 = np.zeros_like(years, dtype=float)
N3 = np.zeros_like(years, dtype=float)
N4 = np.zeros_like(years, dtype=float)
N5 = np.zeros_like(years, dtype=float)
N6 = np.zeros_like(years, dtype=float)
N7 = np.zeros_like(years, dtype=float)
N1[0] = 615
N2[0] = 842
N3[0] = 9744
N4[0] = 26280
N5[0] = 90000000
N6[0] = 731
N7[0] = 1238
```

f) The annual increment from N1 to N7 is calculated according to the formula, and finally the atmospheric carbon content N1 is set as the return value.

```python
for i in range(1, len(years)):

    z = N1[i-1]/2.13
    xi = 3.69 + 1.86e-2 * z - 1.80e-6 * z**2
    f = 62*(1+(beta*math.log(z/289)))

    dN1_dt = -60/615 * N1[i-1] + 60/842 * (N0 + xi * (N2[i-1] - N0)) + gamma_values[i-1]-f + delte[i-1]
    + 0.2/90000000*N5[i-1] + 62/1238*N7[i-1]
    dN2_dt = 60/615 * N1[i-1] - 60/842 * (N0 + xi * (N2[i-1] - N0)) - 9/842*N2[i-1] + 52/9744*N3[i-1] -
    43/842*N2[i-1]
    dN3_dt =  9/842*N2[i-1] - 52/9744*N3[i-1] - 162/9744*N3[i-1] + 205/26280*N4[i-1]
    dN4_dt =  162/9744*N3[i-1] - 205/26280*N4[i-1] + 43/842*N2[i-1] - 0.2/26280*N4[i-1]
    dN5_dt = 0.2/26280*N4[i-1] - 0.2/90000000*N5[i-1]
    dN6_dt = f - 62/731*N6[i-1]-2*delte[i-1]
    dN7_dt = 62/731*N6[i-1] - 62/1238*N7[i-1] + delte[i-1]

    N1[i] = N1[i-1] + dN1_dt
    N2[i] = N2[i-1] + dN2_dt
    N3[i] = N3[i-1] + dN3_dt
    N4[i] = N4[i-1] + dN4_dt
    N5[i] = N5[i-1] + dN5_dt
    N6[i] = N6[i-1] + dN6_dt
    N7[i] = N7[i-1] + dN7_dt
return N1
```

g) Using the “seven_box_model” function, pass in the 0.38 and 0.5 arguments and divide by 2.13 to get the CO2 concentration.

```python
N1_038 = seven_box_model(0.38)/2.13
N1_050 = seven_box_model(0.5)/2.13
```

h) Import the observations from 1750 to 1960.

```
ice_ture = pd.read_csv("icecores.csv")['ic'].values
ice_ture
```

```
array([277. , 277.2, 277.6, 278. , 278.6, 279.3, 280.1, 280.8, 281.6,
       282.3, 282.9, 283.4, 283.8, 284. , 284.2, 284.3, 284.4, 284.5,
       284.6, 284.8, 285.2, 285.7, 286.3, 287.2, 288.2, 289.4, 290.8,
       292.3, 293.7, 295.2, 296.7, 298.2, 299.9, 301.5, 303.2, 304.9,
       306.5, 308. , 309.3, 310.5, 312. , 314.1, 316.9, 320.5, 324.7,
       329.4])
```

i) Plot the simulated values for $\beta = 0.38$ and $\beta = 0.5$, and plot the observed data, adjusting the graph parameters to look more like Figure 4.

```
fig = plt.figure(figsize=(8, 5))
ax = fig.add_subplot(1, 1, 1)
years = np.arange(1750, 2001, 1)
ax.plot(years[0:251], N1_038, linewidth=2.0,c = "red")
ax.plot(years[0:251], N1_050,  linewidth=2.0,c = "blue")
ax.plot(years[211:251], N1_ob[1:41] ,'o',c = "black",markersize=3)
ax.plot(years[0:210:5], ice_ture[0:42] ,'o',c = "black",markersize=3)
font = {'size': 12}
ax.set_xlabel('Year',fontdict=font)
ax.set_ylabel('CO2 Concentration (ppm)',labelpad=10,fontdict=font)
ax.set_xlim(1745, 2000)
ax.set_ylim(270,380)
ax.set_xticks([1800, 1850, 1900,1950, 2000])
ax.set_yticks([280, 300, 320, 340,360])
ax.set_yticks(np.arange(280, 380, 5), minor=True)
ax.set_xticks(np.arange(1750, 2000,10), minor=True)
ax.tick_params(which='both', direction='in', length=6, width=1)
ax.tick_params(which='minor', direction='in', length=3, width=1)
ax.text(1850, 305, 'calculation',fontdict=font)
ax.text(1900, 285, 'observations',fontdict=font)
ax.text(1960, 370, 'β = 0.38',fontdict=font,color='red')
ax.text(1994, 350, 'β = 0.5',fontdict=font,color='blue')
ax.spines['top'].set_visible(False)
ax.spines['right'].set_visible(False)
ax.spines.left.set_position(('axes', 0.02))
ax.spines.bottom.set_position(('axes', 0.02))
plt.show()
```