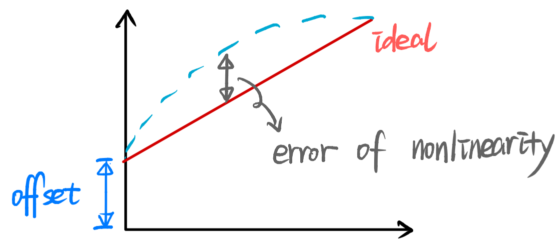
Title : Q1 知識點整理.

△ Sensor Performance



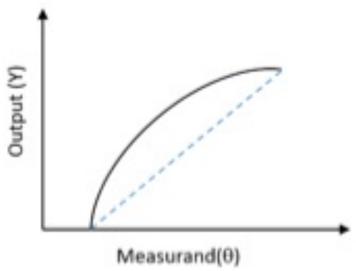
例：(2019-2020 Final)

Q1. A rotary displacement transducer is measured by the equation:

$$Y(\theta) = \theta \times 10^{-2} - \theta^2 \times 10^{-5} - 0.1960\ (V),$$

over the range $\theta = 20° \sim 300°$, boundary condition $Y(20) = 0$.

The output was assumed to be end-point linear with angle θ (Fig. Q1).



a) actual sensitivity

b) assumed sensitivity

c) maximum error

斜率，sensitivity 问题

d). 對 error 种类解释

☆

Title :

☒ sensitivity

$$f'(x) = \frac{d(o/p)}{d(i/p)}$$

> for actual sensitivity,
也就是在該點的導數值.

例: $Y(x) = ax^2 + bx + c$, at $x = 10$

actual sensitivity $= Y'(\theta)\big|_{x=10} = 2ax + b\big|_{x=10} = 20a + b$

> for assumed sensitivity

一般是 linear approximation

$$\Delta = \frac{Y_{(end\ point)} - Y_{(start\ point)}}{input\ span}$$

☒ maximum error

$$\boxed{f(x) - g(x) = D(x),\quad D'(x) = 0,\ 最大.}$$

acturl curve　　straight line.

☒ error type

- systematic error
  - △人為、儀器
  - △可以消除.

→ accuracy : difference {true value / measured value}
精度.

- random error
  - △ unknown, unpredictable
  - △ 不可完全消除.

→ precision : freedom degree from random error.
准度.

- repeatability

maximum difference between two output values at same measurand value approaching to same direction.

- hysteresis error

difference when any point is approached from different direction.

Title :

△ thermocouple

- working principle

two different electrical conducting materials contact,

then contact potential (e.m.f.) will be produced.

{ metals
{ temperature.

- calculation.

A type E thermocouple was used to measure the oil temperature of an engine. Its reference junction was at an ambient temperature of 20ºC. If an output voltage of 2.65mV was recorded, what was the temperature of the oil?

$$V_{ac} = V_{ab} + V_{bc} = 1.19 + 2.65 = 3.84\ mV.$$

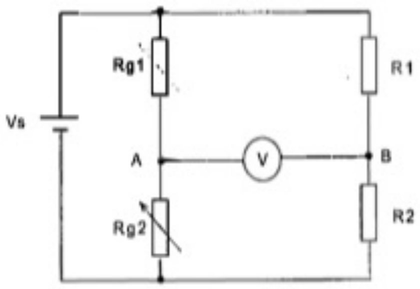
→ 在兩個值中间，建立函數关系

算出對應温度.

△ strain gauge

- calculation

① dummy + active



△ tensile

$R_{g_2} \rightarrow (1+G\varepsilon) R_{g_2}$

[press : $R_{g_2} \rightarrow (1-G\varepsilon) R_{g_2}$]

$$V = V_s \left(\frac{R_{g_2}}{R_{g_1}+R_{g_2}} - \frac{R_2}{R_1+R_2} \right)$$

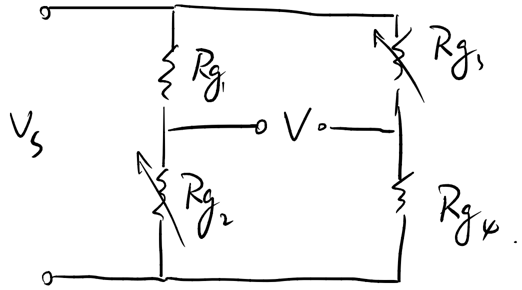
$$= V_s \left(\frac{1+G\varepsilon}{2+G\varepsilon} - \frac{1}{2} \right)$$

$$= V_s \left(\frac{1+G\varepsilon - \frac{1}{2}(2+G\varepsilon)}{2+G\varepsilon} \right)$$

$$= V_s \left(\frac{1+G\varepsilon - 1 - \frac{1}{2}G\varepsilon}{2+G\varepsilon} \right)$$

$$= V_s \frac{G\varepsilon}{4+2G\varepsilon}$$

② double active + dummy



$\triangle$ tensile

$R_{g_2} \longrightarrow (1+G_0 e) R_{g_2}$

[press : $R_{g_2} \longrightarrow (1-G_0 e) R_{g_2}$]

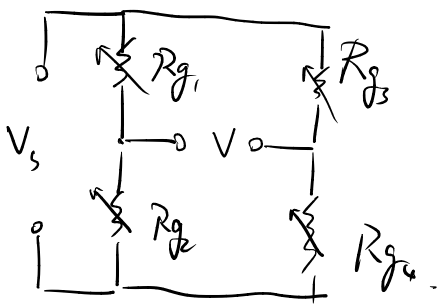
$$V = \left(\frac{R_{g_2}}{R_{g_1}+R_{g_2}} - \frac{R_{g_4}}{R_{g_3}+R_{g_4}} \right) V_s$$

$$= \left(\frac{1+G_0\varepsilon}{2+2G_0\varepsilon} - \frac{1}{2+2G_0\varepsilon} \right) V_s$$

$$= \left(\frac{G_0\varepsilon}{2+2G_0\varepsilon} \right) V_s .$$

$$\frac{G_0\varepsilon}{2} V_s$$

③ four active.



$(1+G\varepsilon)$ R_{g_2}, R_{g_3} tensile

$(1-G\varepsilon)$ R_{g_1}, R_{g_4} compress.

$$V = \left(\frac{R_{g_2}}{R_{g_1}+R_{g_2}} - \frac{R_{g_4}}{R_{g_3}+R_{g_4}} \right) V_s$$

$$= \left(\frac{1+G\varepsilon}{1+G\varepsilon+1-G\varepsilon} - \frac{1-G\varepsilon}{1+G\varepsilon+1-G\varepsilon} \right) V_s$$

$$= \left(\frac{1+G\varepsilon}{2} - \frac{1-G\varepsilon}{2} \right) V_s$$

$$= G\varepsilon \cdot V_s .$$

Title :

△ lowest sampling frequency

$$f_s = 2 \times f_{max}.$$

△ convert to binary code

$$\text{resolution} = \frac{\text{signal range}}{2^{N} - 1}$$

number of bits

△ transmission bandwidth.

$$W_{required} = f_m \times N$$

△ bit finding

$$\text{measure range} \times \text{sensitivity} = \text{full-scale output}$$

$$\text{sensitivity} \times \text{resolution} = \frac{\text{full-scale output}}{2^{n} - 1}$$

$$\boxed{n} //$$

Title: Q_2 知識點整理

parity check code

make number of 1 to be even

- extra bit is attached to the word 0/1 (even parity).

if a single error occurs, the number of 1 will be changed.

$$BW = f_{max} \cdot (N+1)$$

예 :

原碼 : 1011

even parity : 1011 1

若 10011 , number of 1 改變.

Title :

Hamming code.

- Requirement

$$2^p > \underset{\text{data bit}}{n} + \underset{\text{parity bit}}{p}$$

△ General rule.

bit 1	bit 2	bit 3	bit 4	bit 5	bit 6	bit 7	bit 8	bit 9	bit 10	bit 11
p1	p2	d1	p4	d2	d3	d4	p8	d5	d6	d7

×parity code : 奇偶校验位 , bit $= 2^n$

P_1 , check 1 bit , skip 1 bit

bit 1, 3, 5, 7, 9, 11

P_1, d_1, d_2, d_4, d_5, d_7

P_2 , check 2 bits , skip 2 bits

bit 2, 3, 6, 7, 10, 11

P_2, d_1, d_3, d_4, d_6, d_7

P_4 , check 4 bits , skip 4 bits

bit 4, 5, 6, 7

P_4, d_2, d_3, d_4

P_8 , check 8 bits , skip 8 bits

Title: Q_3 知識點整理.

$\mathcal{L}$ 變換.

$$t \cdot f(t) \xleftrightarrow{\mathcal{L}} -\frac{dF}{ds}$$

$$t^k \cdot f(t) \xleftrightarrow{\mathcal{L}} (-1)^k \cdot \frac{d^k F(s)}{d s^k}$$

$$u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s}$$

$$e^{-at} \xleftrightarrow{\mathcal{L}} \frac{1}{s+a}$$

$$\sin\omega t \xleftrightarrow{\mathcal{L}} \frac{\omega}{s^2+\omega^2}$$

$$\cos\omega t \xleftrightarrow{\mathcal{L}} \frac{s}{s^2+\omega^2}$$

dominant pole(s) : poles are nearer to original

Title :

Transfer function

$$T_s = \frac{G_{o_s}}{1 + G_{o_s} H_s}$$

characteristic function

$$|sI - A| = 0$$

- determinate

① 2×2

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

② 3×3

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

state transition matrix

$$\Phi(s) = [sI - A]^{-1}$$

$$\Phi(t) = \mathcal{L}^{-1}\{\Phi(s)\}$$

逆矩陣求法

2x2

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - bc} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

general approach.

、将A與单位矩陣I排成 [A I]

、做行初等变换，化為 [I B] 形式

、$B = A^{-1}$

ss → tf

$$G(s) = \frac{C\,[\,adj\,(sI - A)\,]\,B + |sI - A| \cdot D}{|sI - A|}$$

Title :

tf → ss

$$T(s) = \frac{Y(s)}{U(s)} = \frac{2s^2+8s+6}{s^3+8s^2+16s+6}$$

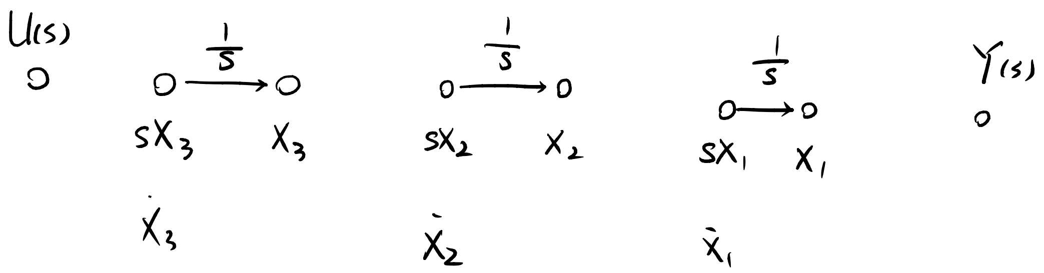
method 1 :

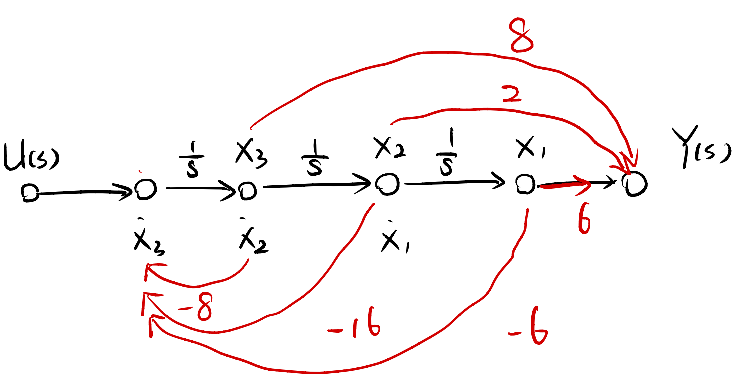
$$\frac{2s^{-1}+8s^{-2}+6s^{-3}}{1+8s^{-1}+16s^{-2}+6s^{-3}}$$

∵ 3 order

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$





$\dot{x}_3 = -8x_3 - 16x_2 - 6x_1 + u$

$y = 6x_1 + 8x_2 + 2x_3$

method 2 :

$$T_s = \frac{2s^2 + 8s + 6}{s^3 + 8s^2 + 16s + 6}$$

$$Y_{(s)} = (2s^2 + 8s + 6) \cdot Z_s$$

$$U_{(s)} = (s^3 + 8s^2 + 16s + 6) \cdot Z_s$$

$$Y_s = 2\frac{d^2 z}{dt^2} + 8\frac{dz}{dt} + 6z.$$

$$U_s = \frac{d^3 z}{dt^3} + 8\frac{d^2 z}{dt^2} + 16\frac{dz}{dt} + 6z$$

$x_1 = z$

$x_2 = \dot{x}_1 = \dot{z}$

$x_3 = \dot{x}_2 = \ddot{z}$

$\dot{x}_3 = \dddot{z}$

$y = 2x_3 + 8x_2 + 6x_1$

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = u - 8x_3 - 16x_2 - 6x_1$

$$\dot{x}_{(t)} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -16 & -8 \end{bmatrix} x_{(t)} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 6 & 8 & 2 \end{bmatrix} x(t) + 0 \cdot u(t).$$

Steady - state error

$$E(s) = R(s) - Y(s)$$

$$= -Y(s) = -T_{td} \cdot T_d(s)$$

$$e_{ss} = \lim_{s \to 0} s \cdot E(s)$$

system sensitivity

$$S_G^T = \frac{1}{1 + G_c G}$$

definition :

$$S_G^T = \frac{\frac{\Delta T}{T}}{\frac{\Delta G}{G}}$$

$$S_K^T = S_K^G \, S_G^T$$

$$= \frac{\partial G}{\partial K} \cdot \frac{K}{G} \cdot \frac{1}{1+G_c G} \longrightarrow \text{method } 1.$$

$$S_K^T = \frac{\partial T}{\partial K} \cdot \frac{K}{T} \longrightarrow \text{method } 2.$$

Title : Q_6 知識點整理

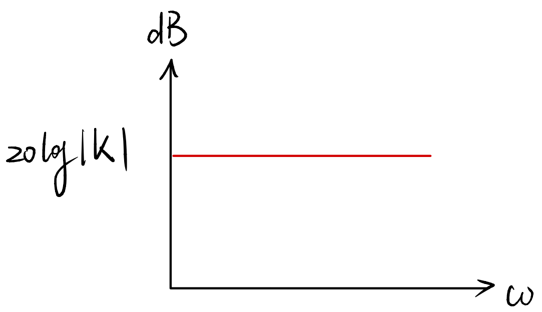
Log. gain

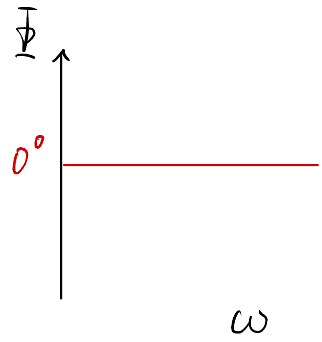
$$Gain = 20 \cdot \log_{10} |L(j\omega)|$$

Bode plot

△ 比例 (Gain)

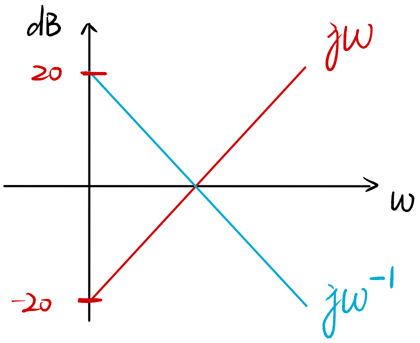
$$G(j\omega) = K$$

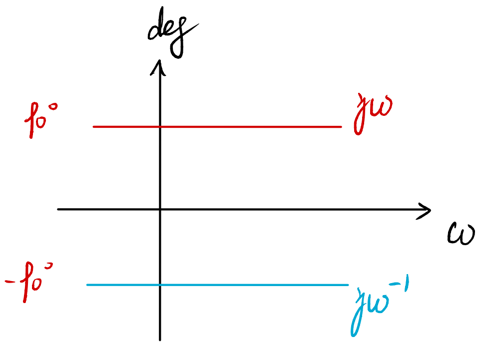




△ 原點處極點

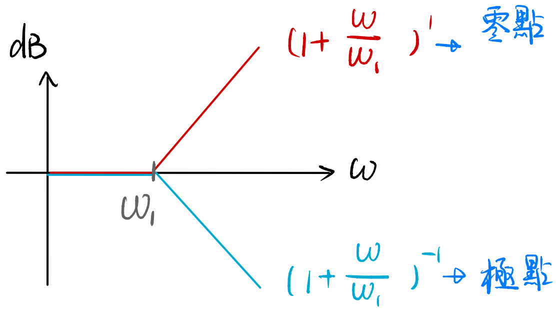
$$G(j\omega) = (j\omega)^{\pm 1}$$

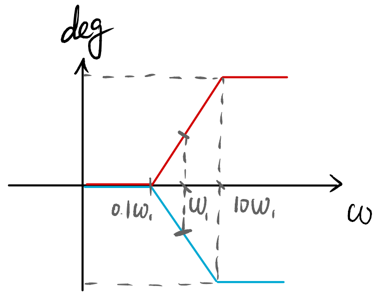




△ 極點、零點

$$G(j\omega) = \left(1+\frac{\omega}{\omega_1}\right)^{\pm 1}$$





△ 複極點 (complex poles)

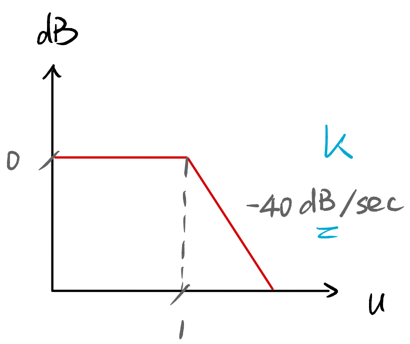
$$G(j\omega) = \left(1 + j2\zeta u - u^2\right)^{-1}$$

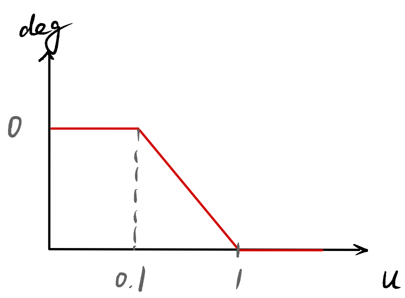
$0.1 < \zeta < 1$, $u = \frac{\omega}{\omega_n} = 1$

when :

$\omega = \omega_n$,

curve will decrease .





Q21 :

$$G(j\omega) = \frac{5(1+j0.1\omega)}{j\omega(1+j0.5\omega)[1+j0.6(\frac{1}{50}\omega)+(j\frac{1}{50}\omega)^2}$$

- constant gain : $K = 5$ $20 \cdot \log|5| = 13.9794$

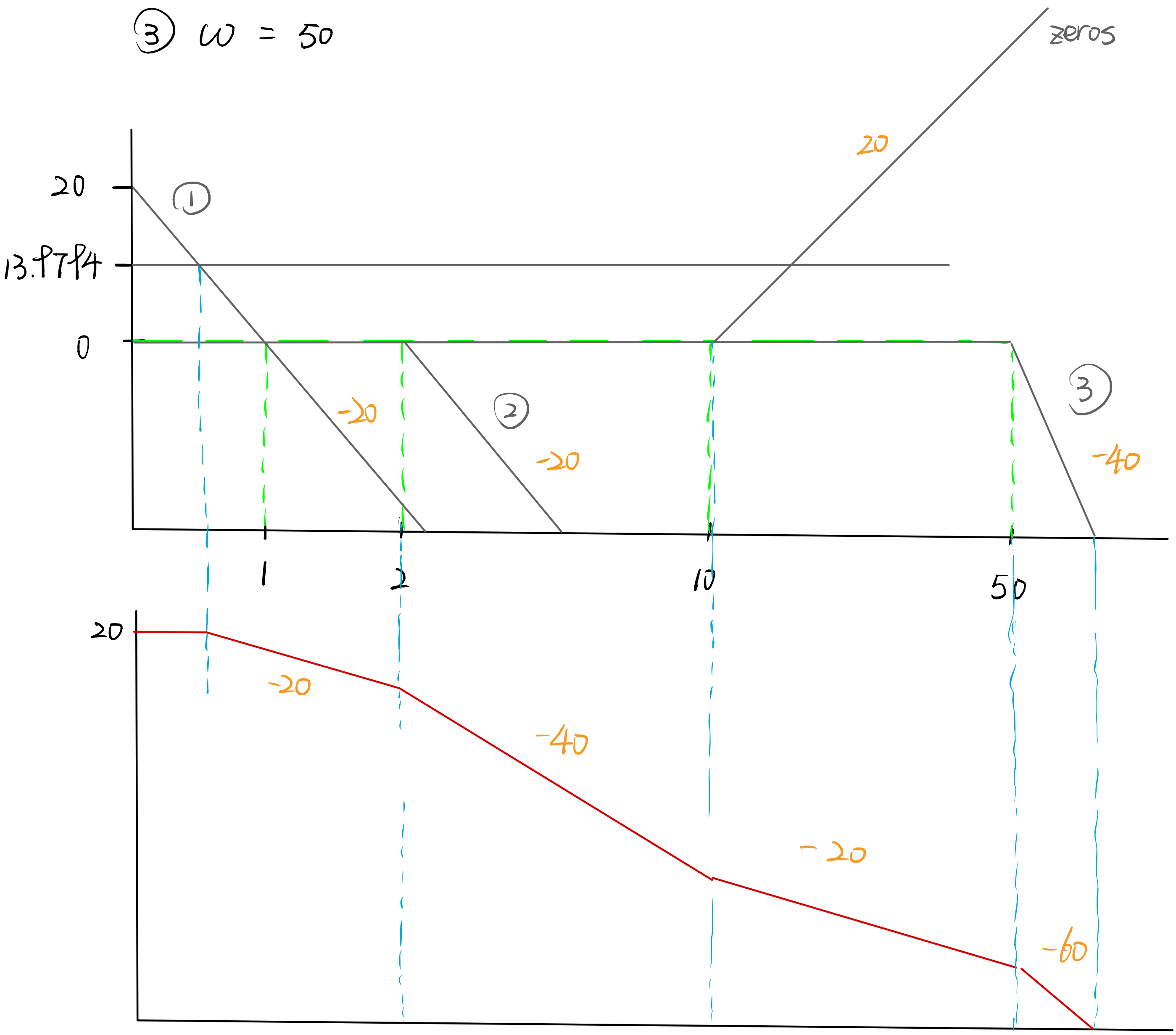
- zeros : $\omega = 10$

- poles :

  ① $\omega = 0$

  ② $\omega = 2$

  ③ $\omega = 50$



gain margin (GM)

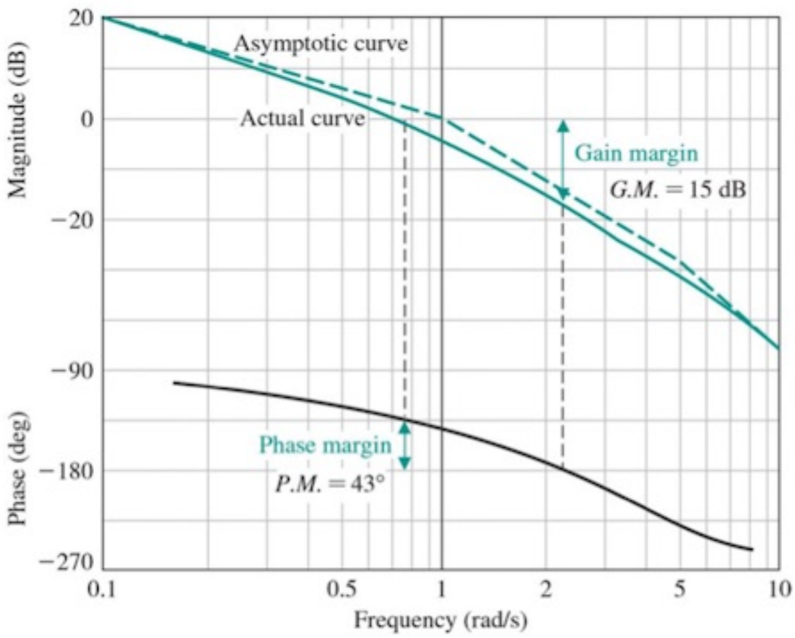
當 phase = ±180° 時，對應 dB 圖中的 gain，

距離 gain = 0 的差值.

phase margin (PM)

當 gain = 0 時，對應相位圖中的 phase，

距離 phase = ±180° 的差值.



Q21 : $L(s) = \frac{300(s+100)}{s(s+10)(s+40)}$

calculate : 1). phase angle Φ_ω when $\omega = 28.3$ rad/s.

2). logarithmic gain

$\Phi(j\omega) = \angle(s+100) - \angle(s) - \angle(s+10) - \angle(s+40)$

$= \angle 15.8 - \angle 90 - \angle 70.53878765 - \angle 35.2784$

$= 180°$

$20\log_{10}|L(s)| = -2.5$ dB

Title: Gain margin and phase margin

MANUAL CALCULATION !!

例:

$$G \cdot H = \frac{40}{s(s^2+2s+25)}$$

step 1:

$$G(j\omega)\cdot H(j\omega) = \frac{40}{j\omega \cdot (25-\omega^2+j2\omega)}$$

step 2:

$$|G\cdot H| = \frac{1}{\omega} \cdot \frac{40}{\sqrt{(25-\omega^2)^2+4\omega^2}}$$

$$\angle G\cdot H = 0 - 90^\circ - \tan^{-1}\left(\frac{2\omega}{25-\omega^2}\right)$$

step 3:

phase margin

$\triangle$ $|G\cdot H| = 1$, $\omega_\Phi = 1.82$, $\angle G\cdot H|_{\omega=1.82} = \underline{80.5^\circ}$

$\triangle$ $\angle G\cdot H = -180^\circ$, $\omega_c = 5$, $amp = \frac{1}{|G\cdot H|_{\omega=5}} = 1.25$,

$20\log_{10} 1.25 = \underline{1.94\ dB}$ → gain margin step 1 :

写出 $G(j\omega)\cdot H(j\omega)$

step 2 :

$|G\cdot H|$ ：實部模相乘

$\angle G\cdot H$ = 相位角相加 ， $\arctan\left(\frac{虛部}{實部}\right)$.

step 3 :

△ $|G\cdot H| = 1$ ，得 ω_1 ，代入 $\angle G\cdot H$ → phase margin

△ $\angle G\cdot H = -180°$ ，得 ω_2 ，代入 $\frac{1}{|G\cdot H|}$

$20\cdot\log_{10}(ANS)$ → gain margin