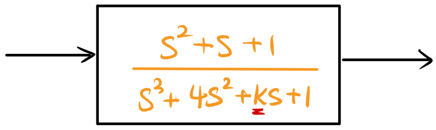
Title : Additional notes : Root locus. (I)

consider :



> TF :

$$T_s = \frac{s^2+s+1}{s^3+4s^2+ks+1}$$

> characteristic equation :

$$\Delta_s = s^3 + 4s^2 + ks + 1 = 0$$

$\triangle$ sth before :

$\because$ Δ_s not a correct form

$\therefore$

(1) group k together

$$(s^3+4s^2+1) + K \cdot s = 0$$

(2) divide by the part without K.

in this example,

$$\frac{s^3+4s^2+1}{s^3+4s^2+1} + \frac{Ks}{s^3+4s^2+1} = 0$$

with a form of

$$1 + K \cdot G(s) = 0$$

$$1 + KG(s) = 0 \Rightarrow \boxed{1 + K\frac{Q(s)}{P(s)} = 0}$$

Title :

for $1 + K \cdot \frac{Q(s)}{P(s)} = 0$

· Role 1 : n loci in a system

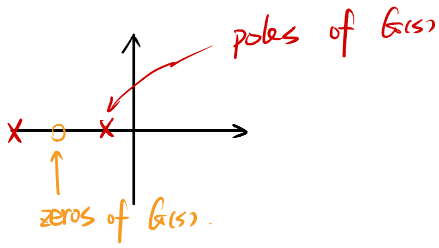
chosen as the higher order in Q and P.

· Role 2 : K increases from 0 to ∞,

roots move from $\boxed{\text{poles}}$ to $\boxed{\text{zeros}}$.

· poles of $G(s)$ when $Q_s = 0$

zeros of $G(s)$ when $P_s = 0$



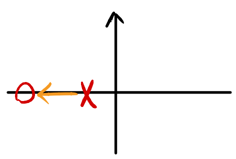
· multiply $P(s)$ in both sides.

$P(s) + KQ(s) = 0$

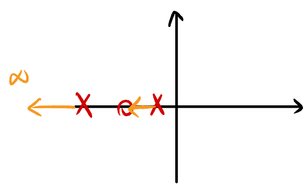
$\Rightarrow$ when $K = 0$, $P(s) = 0$ $\rightarrow$ 極點

$\Rightarrow$ when $K \to \infty$, $P(s) + \infty\, Q(s) = 0$ $\rightarrow$ 零點.

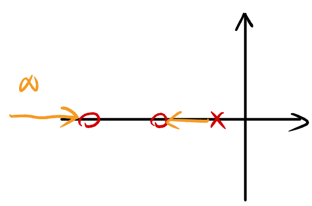
if $P(s) = Q(s)$



if $P(s) > Q(s)$



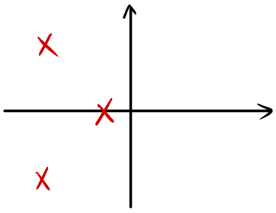
if $P(s) < Q(s)$



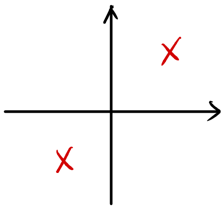
、Role 3

when roots are complex, they occur in conjugate pairs.

(出現)虛根，必對稱).



✓

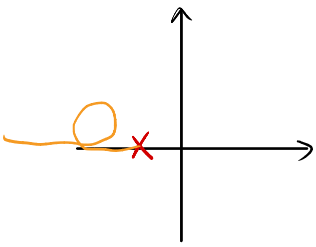


X

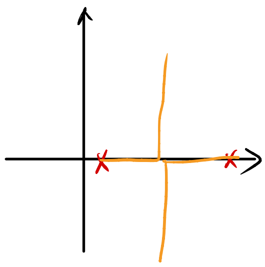
Role

、Role 4

no loci will cross over together (meet twice with themselves).



wrong !



fine

Title : Additional notes : Root locus. (II)

$\boxed{\text{Recap}}$: $1 + KG(s) = 1 + k\dfrac{Q(s)}{P(s)} = 0$

> **Role 1** : chose larger order to be n, which is an amout of loci.

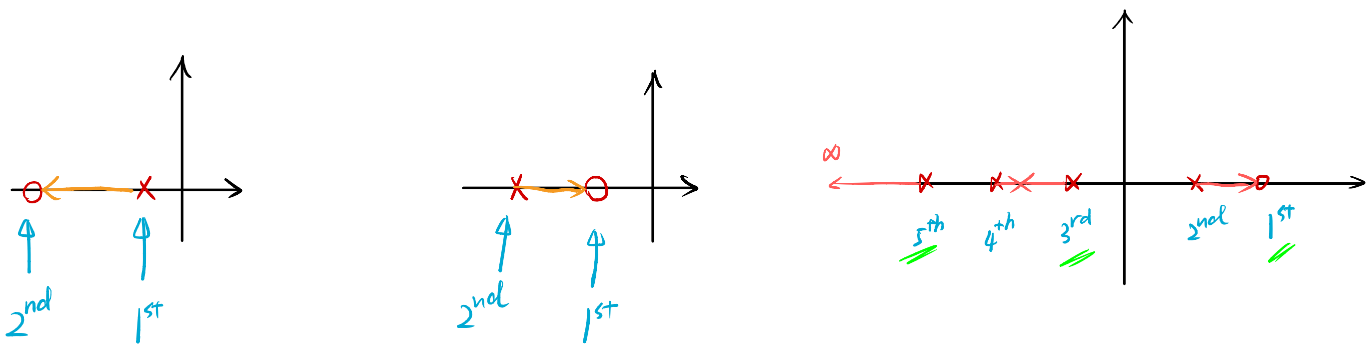
> **Role 2** : poles to zeros.

> **Role 3** : if complex roots occur,

must be mirrored mirrored in both location and track.

> **Role 4** : cannot cross over with themselves

Role 5 : count from far right, and fill tracks between odd numbers of poles and zeros.

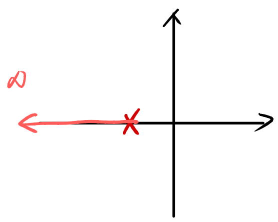


Role 6 : line leaves and enters the real axis at 90°.

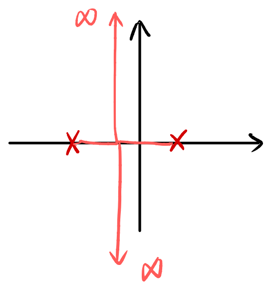
Role 7 : there are not enough poles and zeros to make pairs

extra lines go to / come from infinity.

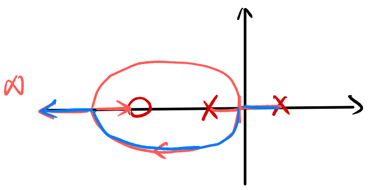
1 extra pole.



2 extra poles



1 extra pole



(1) start from odd.

Rule 8 : lines go to infinity along asymptotes.

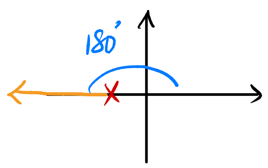
> asymptotes angle, $\phi_A = \dfrac{2q+1}{n-m} \times 180^\circ$ integer limited to $(n-m-1)$

> asymtotes centroid : $\dfrac{\Sigma_{poles} - \Sigma_{zeros}}{n-m}$

if 1 line $\rightarrow \infty$ (1 pole)

$$\phi_A = \frac{2\times(1-1)+1}{1} \times 180^\circ$$

$$= 180^\circ$$

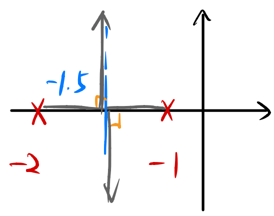


if 2 lines $\rightarrow \infty$ (2 poles)

$$\phi_{A_0} = \frac{2\times 0+1}{2} \times 180^\circ = 90^\circ$$

$$\phi_{A_1} = \frac{2\times 1+1}{2} \times 180^\circ = 270^\circ = -90^\circ$$

Centroid : $\dfrac{-2-1}{2} = -1.5$



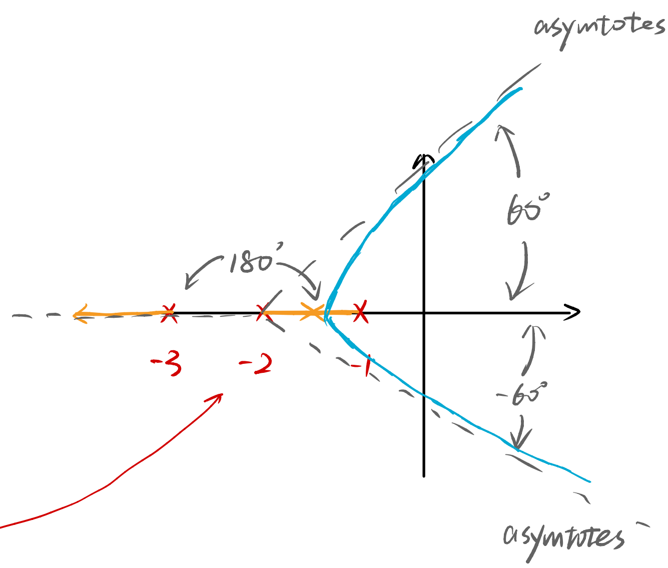
if 3 lines $\rightarrow \infty$

$$\phi_{A_0} = \frac{2\times 0+1}{3} \times 180^\circ = 60^\circ$$

$$\phi_{A_1} = \frac{2\times 1+1}{3} \times 180^\circ = 180^\circ$$

$$\phi_{A_2} = \frac{2\times 2+1}{3} \times 180^\circ = 300^\circ = -60^\circ$$

centroid $= \dfrac{-3-2-1}{3} = \underline{-2}$



Role 9 : K going from 0 to negative infinity can be drawn by reversing role 5 and adding 180° to the asymptote angles.

△ draw from left