拓扑习题-26

1. Armstrong 的书的第六章的习题 20, 习题 22.

2. Munkres 的 Topology 的第 73 节的习题 1，习题 2:

Exercises

1. Find spaces whose fundamental groups are isomorphic to the following groups. (Here $\mathbb{Z}/n$ denotes the additive group of integers modulo n.)
   (a) $\mathbb{Z}/n \times \mathbb{Z}/m$.
   (b) $\mathbb{Z}/n_1 \times \mathbb{Z}/n_2 \times \cdots \times \mathbb{Z}/n_k$.
   (c) $\mathbb{Z}/n * \mathbb{Z}/m$. (See Exercise 2 of §71.)
   (d) $\mathbb{Z}/n_1 * \mathbb{Z}/n_2 * \cdots * \mathbb{Z}/n_k$.

2. Prove the following:
   Theorem. *If G is a finitely presented group, then there is a compact Hausdorff space X whose fundamental group is isomorphic to G.*
   Proof. Suppose G has a presentation consisting of n generators and m relations. Let A be the wedge of n circles; form an adjunction space X from the union of A and m copies $B_1, \ldots, B_m$ of the unit ball by means of a continuous map $f : \bigcup \text{Bd } B_i \to A$.
   (a) Show that X is Hausdorff.
   (b) Prove the theorem in the case $m = 1$.
   (c) Proceed by induction on m, using the algebraic result stated in the following exercise.

   The construction outlined in this exercise is a standard one in algebraic topology; the space X is called a two-dimensional **CW complex**.