Topological Data Analysis on Distinguishing Fibonacci Phyllotaxis

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Background

Phyllotaxis is the arrangement of plant organs on a stem, which reveals two families of spirals, also called parastichies, crisscrossing to form lattice-like patterns. Most plants have pairs of Fibonacci numbers of parastichies. Some plants, however, are Quasi-symmetric where the plant displays a similar number of parastichies in each direction. Fibonacci patterns are often more regular then quasi-symmetric ones.

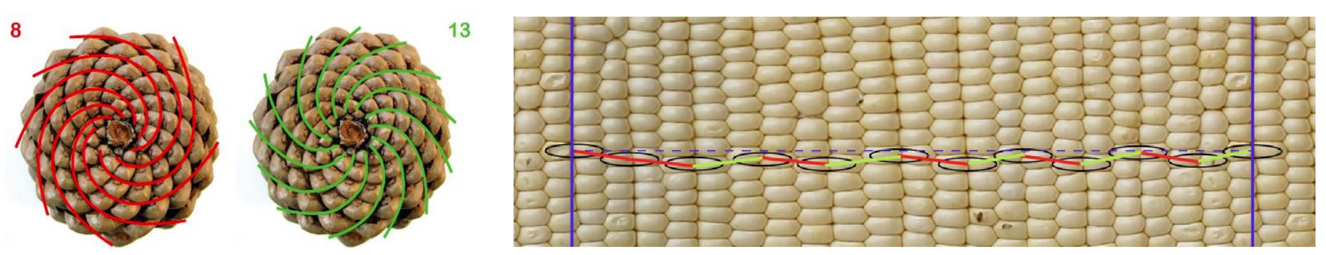

Figure 1. Fibonacci parastichies (left) & Quasi-symmetry parastichies (right)

Objectives

The goal of our research is to statistically distinguish quasi-symmetric from Fibonacci phyllotaxis, and determine which conditions prompt the formation of each.

Methodology

Disk Stacking Model

To analyze both patterns, we use a disk stacking model in python that simulates plant growth on a cylinder. Disks represent the footprint of the primordia on the stem, and they are generated starting from initial conditions represented by points in a hexagon. The model produces a point cloud of the disks, which we use to perform TDA.

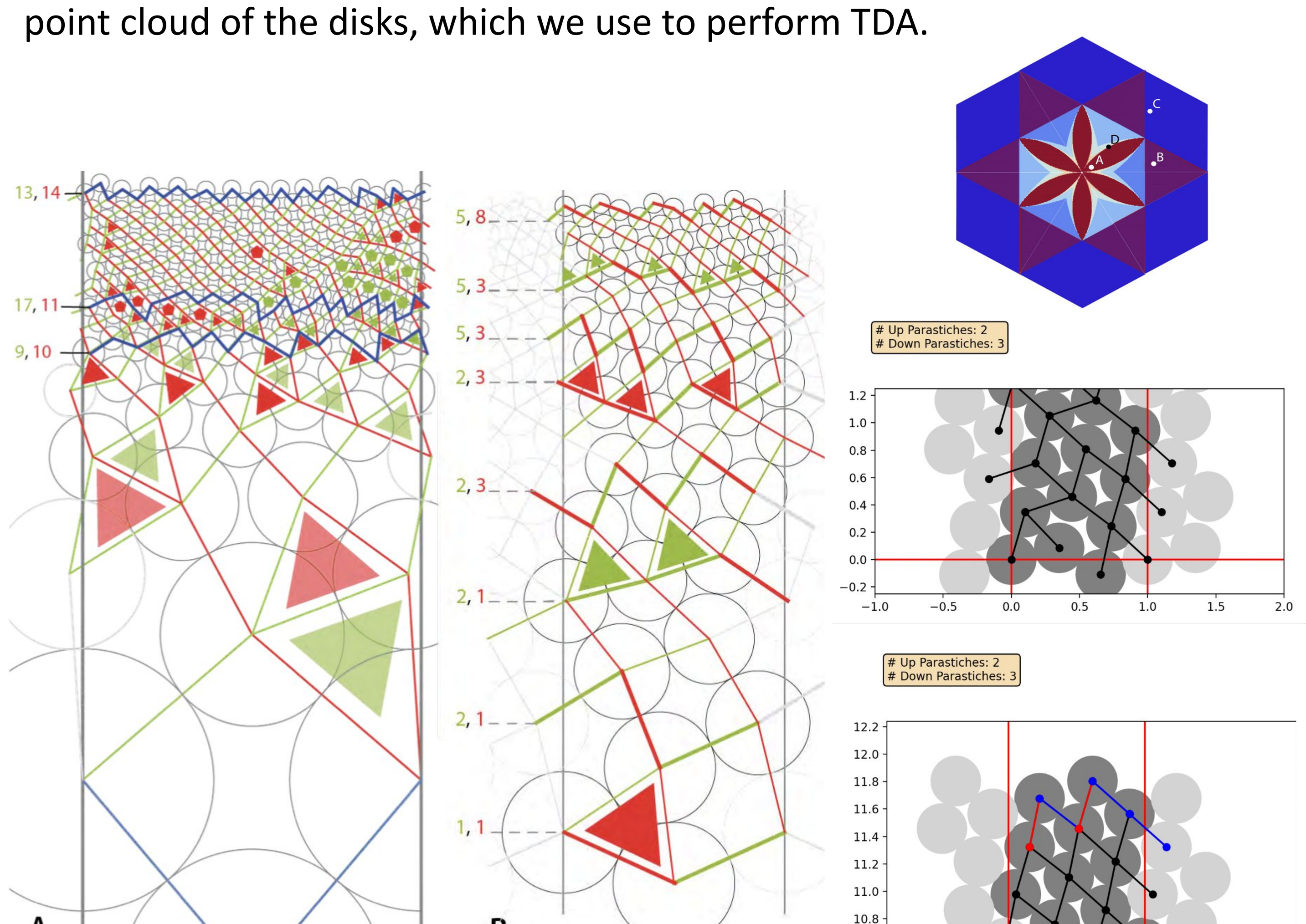

Figure 2. DIsk Stacking Model Visualization (A) Quasi-symmetry transition (B) FIbonacci transition

Methodology

TDA Approaches

We first apply a Vietoris-Rips filtration on the point cloud and a Bravais Lattice generated from the point cloud. Then we compute persistence diagrams that track the persistence of topological features within the filtrations. To compare the point-could to the Bravais lattice, we calculate the Wasserstein-2 distance between the persistence diagrams. This measures the regularity of the point cloud.

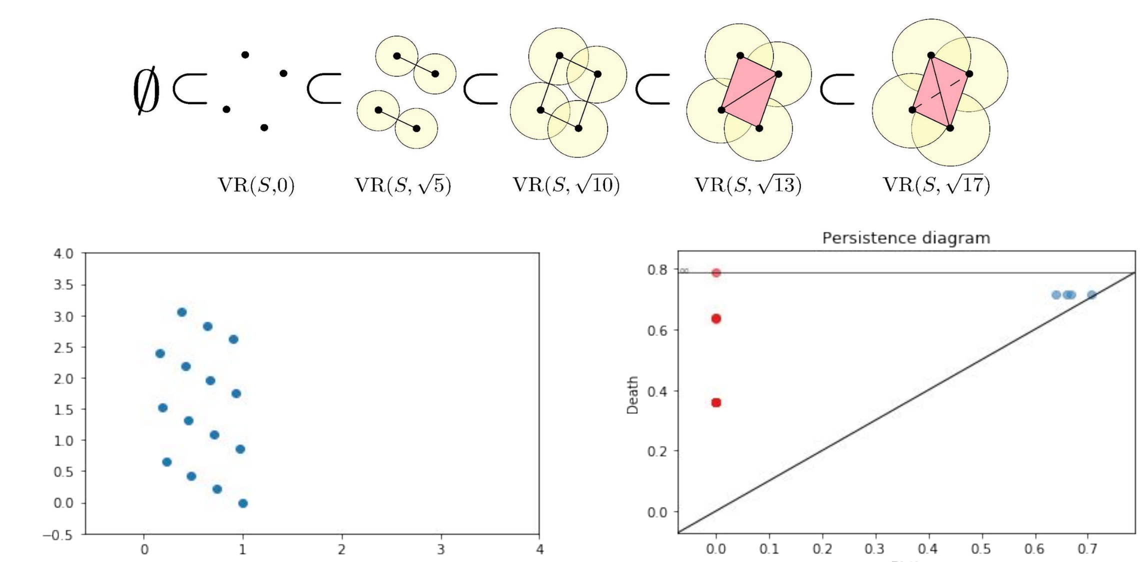

Figure 3. Vietoris-Rips Filtration Visualization (top), Bravais Lattice & Persistence Diagram Example (bottom)

Variations of TDA

- TDA based on Periodicity

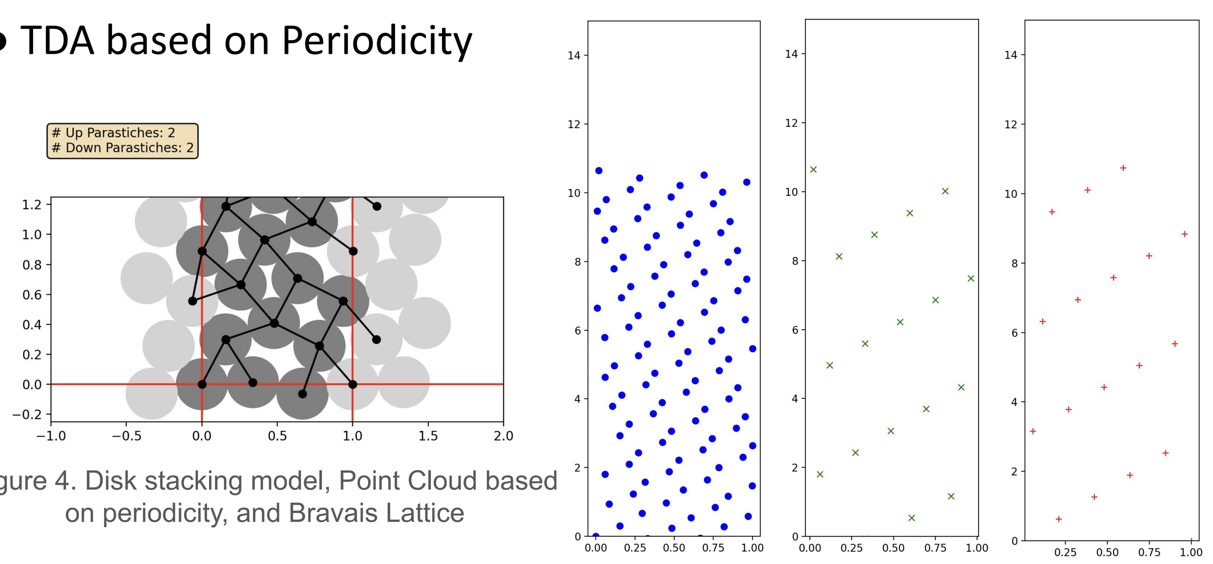

Figure 4. Disk stacking model, Point Cloud based on periodicity, and Bravais Lattice

- TDA based on Megatile

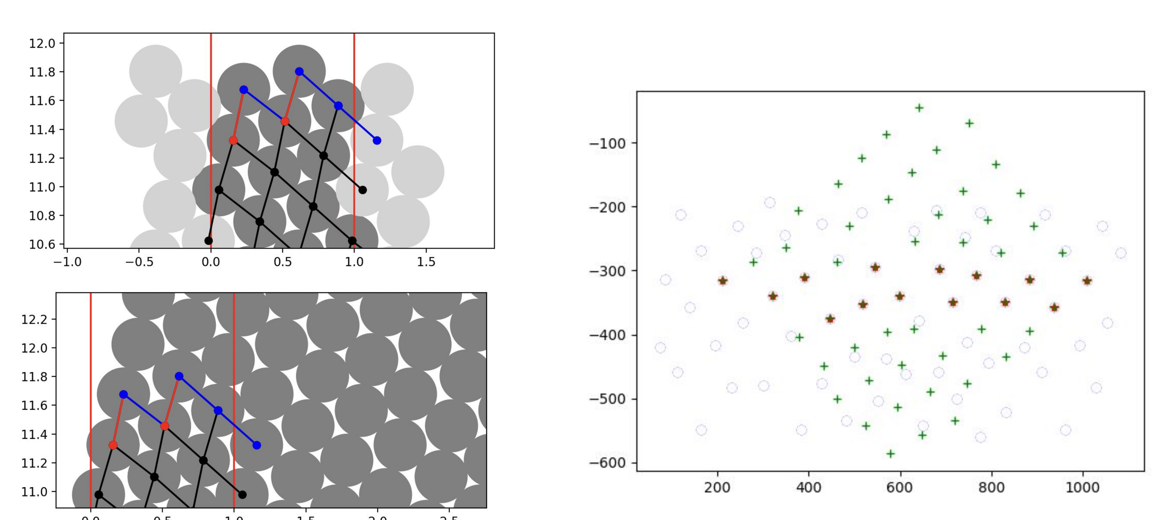

Figure 5. Megatile Generation from disk stacking model (left) & real plant species (right)

- Delaunay Triangulation

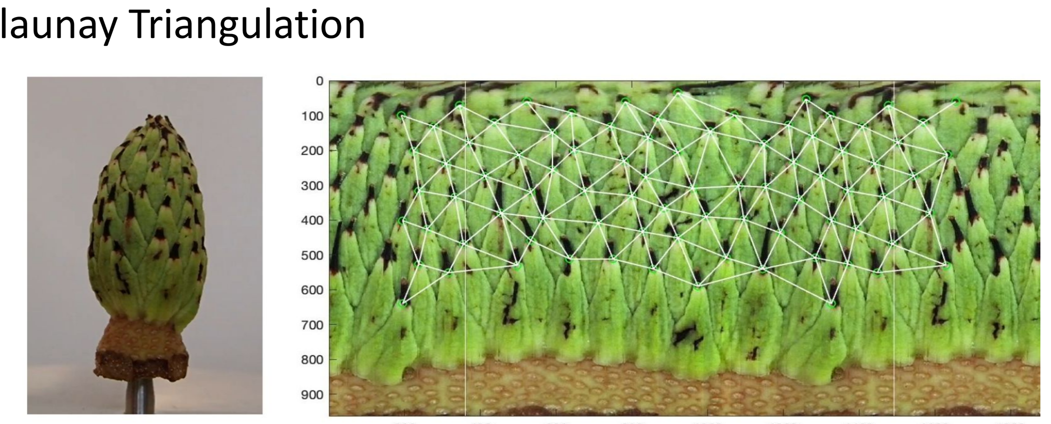

Figure 6. Example of plant species (Magnolia) & its unrolled data generated by Delaunay Triangulation

Results

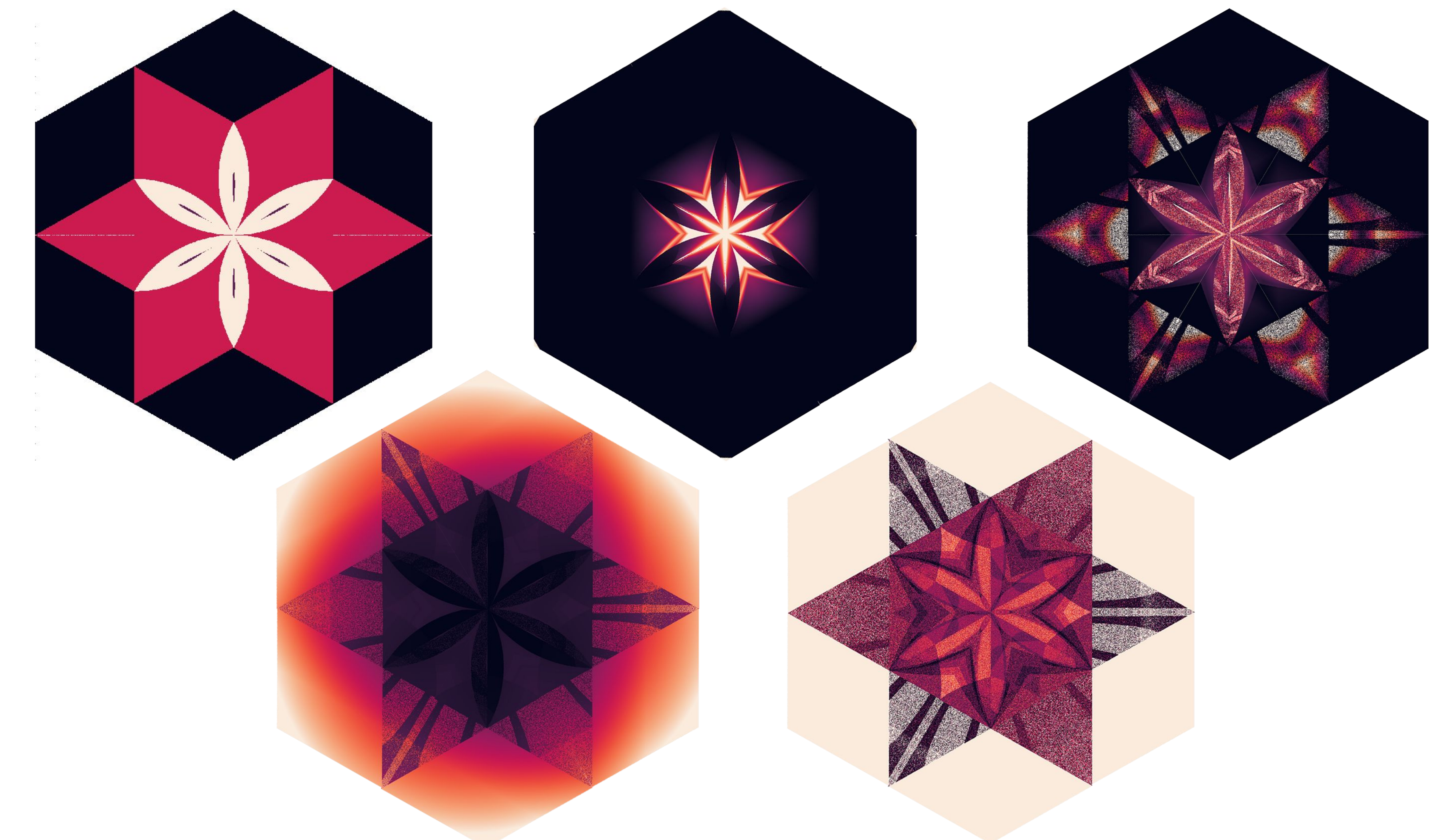

Figure 7. Heatmap Visualizations. From left to right: Periodicity results; TDA based on Periodicity results; TDA based on Megatile results; Delaunay Triangulation Length & Angle Results. Color maps varies from one graph to another but in general, lighter colors signify larger values.

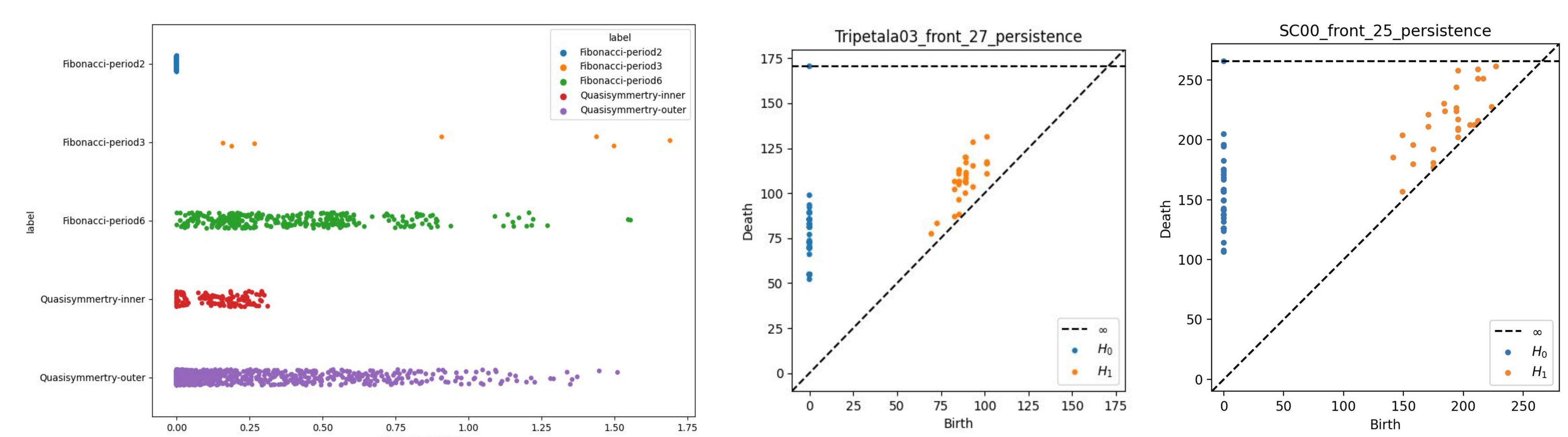

Figure 8. TDA based on megatile distribution for disk models with different periodicity (left); Persistence Diagram comparison between Tripetala and Skunk Cabbage (right)

Conclusion

Our study of Phyllotaxis uses mathematical models to understand properties of Fibonacci and quasi-symmetric patterns. By applying TDA, we can successfully gain quantitative measurements on the regularity of both types of phyllotaxis from three perspectives. For data from the disk stacking model, we can gather quantitative and qualitative information about the regularity distribution for Fibonacci and quasi-symmetric disk stacking patterns. For real plant data, we can quantitatively compare visually different phyllotaxis across plant species.

Acknowledgement

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