Grid based 2D smoke simulation

HAN QI, University of California at Berkeley, USA

YUHAN YANG, University of California at Berkeley, USA

RUINAN XU, University of California at Berkeley, USA

ANTHONY LING, University of California at Berkeley, USA

In this project, we implemented an interactive 2D smoke simulation with different features. Our approach is manly based on the famous Navier-Stokes equations, and we also utilized the data structure called MAC Grid to simulate smoke in a grid system. To make our simulation more realistic, we simulated the collision between the smoke and a sphere. Different modes which specify the environment of simulated smoke enrich our simulation as well.

CCS Concepts: • **Computer systems organization** → **Embedded systems**; *Redundancy*; Robotics; • **Networks** → Network reliability.

Additional Key Words and Phrases: smoke simulator, Navier-Stokes equations, MAC Grid

1 INTRODUCTION

For our smoke simulator, the fundamental data structure is staggered MAC Grid (Maker-and-Cell Method) view. In this view, we evaluate all the physics based on a constant Eulerian grid which is mac grid. We can use MAC Grid data structure to store the scalar and vector fields of the target object.

The code of this project is written in C++[3]. We first created an interactive interface that users can create sources simply by mouse clicking. Besides that, users can add external forces by dragging on the screen. We also created different modes which specifies the environment of simulated smoke. To make the simulation more realistic, we also simulated the collision between the smoke and a sphere.

Authors' addresses: Han Qi, University of California at Berkeley, USA, han2019@berkeley.edu; Yuhan Yang, University of California at Berkeley, USA, yuhany@berkeley.edu; Ruinan Xu, University of California at Berkeley, USA, ruinanxu@berkeley.edu; Anthony Ling, University of California at Berkeley, USA.

2 APPROACH

2.1 Navier-Stokes equations

Smoke is a type of incompressible fluid whose behaviors can be described and predicted by Navier-Stokes equations. The equations are[1]:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + \frac{1}{\rho} \nabla p = \vec{F} + \nu \nabla \cdot \nabla \vec{u} \tag{1}$$

$$\nabla \cdot \vec{u} = 0 \tag{2}$$

The letter $\vec{u}$ is the velocity of the fluid which has two components in our 2D simulation. ρ is the density of the fluid. p is the pressure. $\vec{F}$ is the external force applied to the fluid. ν is kinematic viscosity, measuring how much the fluid resists deforming while it flows. This equation is nothing but the Newton's second law, conservation of momenutum, $\vec{F} = m\vec{a}$. It basically describes how the fluid flows when internal and external forces are applied on it.

2.2 MAC Grid

Typically, there are two ways to track the movement of a flow, Langrangian and Eulerian. In our simulation, we chose the Eulerian view-points in which fluids are simulated in a grid system, rather than a particle system. Particularly, we used MAC Grid [2] structure to store the scalar and vector fields related to fluids. We discretized the 2D spatial space where the different variables were stored at different locations. All scalar fields, such as pressure, density and temperature, were stored in the center of each cell. Vector field, i.e. the velocity $\vec{u}$, were stored at faces. Specifically, The horizontal u component is sampled at the centers of the vertical cell faces. The vertical $\vec{v}$ component is sampled at the centers of the horizontal cell faces. This data structure benefits us in accurately estimating the derivative of velocity field at the center of cells, see 1.
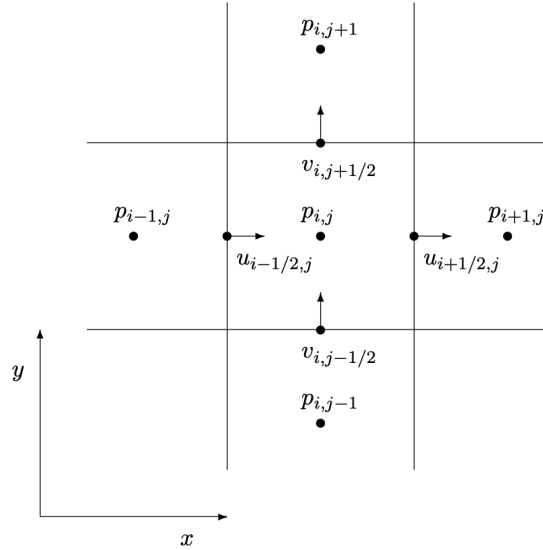

Fig. 1. MAC grid data structure illustration

2.3 Simulation

In our simulation, we firstly track the user input to initialize the density/temperature/velocity field. The user can specify the source of smoke by clicking the mouse on the screen. On a click, the program adds temperature, density and velocity to the cells around the mouse position.

Next, we advect and update the velocity. We use Langrangian method to do an Eulerian calculation (so-called semi-Langrangian advection). We go through each cell in the grid and traceback to fetch the velocity that will be at our current location next step $\vec{u}_P^{n+1}$. It should be equal to the velocity at the old location at the current step $\vec{u}_G^n$. In other words, $\vec{u}_P^{n+1} = \vec{u}_G^n$. The current location $\vec{x}_P$ and the old location $\vec{x}_G$ have the relationship $\vec{x}_G = \vec{x}_P - dt * \vec{u}_P$.

Then we add external forces to the smoke to drive the smoke motion. The external forces contains three portions:

(1) We allow users to input an external force by dragging the mouse along a specific direction.

(2) Thermal Buoyancy force which drives the smoke flowing upwards caused by fluid pressure.

$$F_{buoy} = (\alpha d - \beta(T - T_{amb})) * g \tag{3}$$

where g is the gravity. The constant α and β were properly chose. T is the temperature at the current cell. T_{amb} is the average temperature of the fluid grid. Thie Buoyancy force is along y direction to push the smoke moving upwards.

(3) Vorticity force which is used to capture the swirly motion characteristic of turbulence. The vorticity force can be calculated as follows:

$$F_{vort} = \vec{N} \times \vec{\omega} \tag{4}$$

where $\vec{\omega} = \nabla \times \vec{u}$ and $\vec{u}$ is the velocity. $\vec{N}$ is the vector pointing to the vortex center $\vec{N} = \frac{\nabla|\vec{w}|}{|\nabla|\vec{w}||}$

Once we computed all forces, we can update the velocity using Newton's second law:

$$Force = mass * acceleration \tag{5}$$

$$\vec{u}_{n+1} = \vec{u}_n + dt * acceleration \tag{6}$$

Next, we projected our velocity field onto a divergence-free space to satisfy eq (2). Specifically, we compute and update pressure so that when we apply them to the field the divergence-free property still holds. The values of THE fluid at the next time step can be described in terms of our current (divergent, unstable) velocity field and the pressure:

$$\vec{u}_{n+1} = \vec{u}_* - dt \cdot \frac{1}{\rho}\nabla p \tag{7}$$

The divergence-free velocity field must satisfy

$$\nabla \cdot \vec{u} = \frac{du}{dx} + \frac{dv}{dy} = 0 \tag{8}$$

We can get the pressure with the above two equations.

Lastly, we advect the temperature and density just as what we did for velocity advection.

2.4 Collision with the sphere

1. Totally reflecting sphere. We imitate the collision of the smoke with a totally reflecting sphere. In the function of advectRenderingParticles, we will tell whether the particle's next position is inside the sphere or not. If its next position is inside the sphere, we calculated a correction position by pushing it back to the surface of the sphere along the line of sphere center and the old next position. In the function of getVelocity, we recalculated the velocity of the particle if it

Table 1. Pseudo code

Smoke simulation pseudo code
1. **for** each frame **do**:
2.   get smoke source from user. Initialize velocity, density and temperature.
3.   Compute the velocity:
Advect the velocity.
Compute the external force
Projection.
4.   Advect the temperature and density.

inside or on the sphere. We make the new velocity to be tangent to the sphere surface. 2. Totally absorptive sphere. We imitate the collision of the smoke with a totally absorptive sphere. This means that all particles hitting the sphere will be absorbed.

3 IMPLEMENTATION

We created a simulation of smoke based on physical properties like velocity, temperature and density.

Firstly, in our simulation, we enable users to create multiple sources with mouse clicking on the screen. This gives users the flexibility of simulation and they can simulate the flowing smoke starting from anywhere on the screen. Also, based on this function, one improvement possible for the future is that we can simulate the collision of two flow of smokes.

Secondly, we add the features that users can drag on the screen to add external forces to smoke and control the direction of the forces. This feature provides the ability to simulate the smoke in the real world. For example, the users can add the force of wind to the smoke and see how the smoke flows under the external forces.

Thirdly, we create two mode simulating the smoke movement in two different situations: smoggy winter morning and smoke of fire in a small room. In a winter morning, the temperature of smoke is low and usually the smog is not very dense. So we set low temperature value and low density value for the smoke in this situation. However, for the smoke of fire in a small room, the temperature is high and the smoke is very dense since the space is limited. So we set high temperature value and high density value for the smoke in this situation.

Finally, we make the simulation of the collision between the smoke and a sphere. When the smoke hits the sphere, it cannot enter the sphere since the sphere is solid. Also, its velocity will be redirected because of the collision. So the effect is that the smoke will flow around the sphere, which is in the space of simulation. We can set the mode with absorptive sphere. So if the smoke hit the sphere, the smoke will be absorptive.

In conclusion, we make a simulation of smoke based on physical properties and formulas. This simulation is a simplified but realistic imitation about how the smoke would flow in real world. We add different functions to diversify the simulation scenes, like different temperatures and collision with a sphere.

4 RESULTS

As seen in Figure 2, these are the two main results to present, featuring two distinct smoke styles. The smoke on the left features smoke in a smoggy winter morning, while the smoke on the right features the smoke of a small fire.
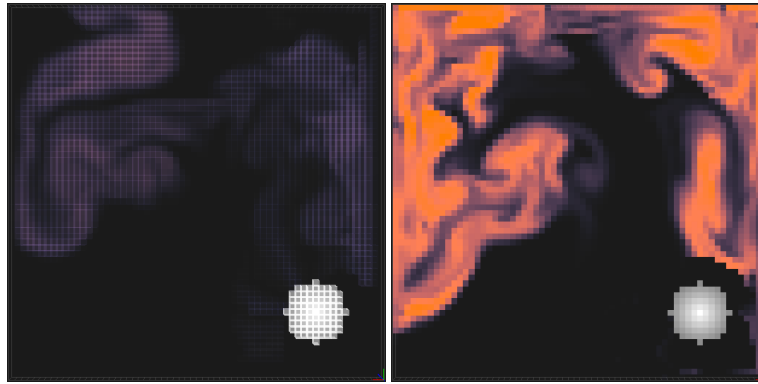
Fig. 2. Both cold and hot temperature smoke examples

5 FURTHER WORK

From the results above, we can see that the smoke collides with the sphere and doesn't interact. There is a bug smoke doesn't appear to be rendering at certain parts of the image, and the smoke may have a chance to diverge wildly.

Using the Navier-Stokes equations, we are able to produce an accurate smoke simulator with smoke-object interaction and with different smoke temperatures. For the rest of the project, we would like to add more features to the smoke simulator, such as dynamic object interaction and different types of smoke features such as density and viscosity. Our current implementation also only has one sphere to simulate interaction, but we plan to add more such as blocks and complex shapes.

Our current implementation also makes use of GLUT's GUI and 3D representation, however we place an emphasis on only displaying 2D simulations. For performance benefits, we're thinking of transferring most of the code base into a 2D graphical rendering model.

6 CONTRIBUTIONS

(1) **Han Qi**: implemented the collision between a sphere and the smoke, implemented two simulation modes: smoggy winter morning and smoke of fire, implemented reflective sphere and absorptive sphere, wrote the result part in the paper, make the video.

(2) **Yuhan Yang**: implemented user interactive input, allow users to add smoke source and change the direction of the external force in the interface, wrote the approach part in in the paper, make the slides

(3) **Ruinan Xu**: wrote the abstraction and introduction part in the paper.

(4) **Anthony Ling**: wrote the results and further work part in the paper. Searched for alternatives to execute on Windows.

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