A new Approximate Bayesian Inference algorithm for Bayesian Lasso: A local approximation correction approach

Yuhao Li

Supervisor: A/Prof. John Ormerod

A thesis submitted in partial fulfillment of
the requirements for the degree of
Bachelor of Science(Honour)(Data Science)

Mathematics and Statistics

THE UNIVERSITY OF SYDNEY

June 2023

Statement of originality

This is to certify that to the best of my knowledge, the content of this thesis is my own work. This thesis has not been submitted for any degree or other purposes.

I certify that the intellectual content of this thesis is the product of my own work and that all the assistance received in preparing this thesis and sources have been acknowledged.

Yuhao Li

Abstract

Variational Approximation: as a deterministic approximation algorithm for Approximate Bayesian Inference(ABI) of intractable posterior distribution, has been used prevalently among the Bayesian Statistical community for fast Approximate Bayesian Inference(ABI). It is also a faster alternative to Monte Carlo methods such as Markov Chain and Monte Carlo(MCMC). Nevertheless, the Variational Approximation algorithm suffers from the loss of approximation accuracy under some special circumstances, such as underestimating the variance when the correlation of variables becomes large.

In addition, the Lasso penalized regression could be estimated in the Bayesian paradigm to facilitate standard error estimates of coefficients and credible intervals of estimated coefficients.

In this thesis, we develop two fast and more accurate variational approximation algorithms for the Bayesian Lasso regression problem. The main idea behind these methods involves using the information of local parameter estimates by accommodating the univariate and multivariate lasso distribution, which results in more accurate global parameter approximation via Gaussian Variational Approximation. Our experimental results on numerous real-world datasets suggest their high Variational Approximation accuracy with a descent time efficiency, compared with the traditional Monte Carlo methods and Mean-Field Variational Bayes(MFVB).

Acknowledgements

Thanks to Supervisor and family

Finally, I would like to thank my friends my family: my mother, father, brother, grandparents, and cousins. The last couple years has been full of adversity and I could not have overcome it without your support behind the scenes.

Contents

Chapter 1

Introduction

1.1 Motivation

Bayesian and Frequentist Frequentist Bayesian

Bayesian Inferenece

Variational Inference

Stochastic Sampling based Determinstic Approach

Variational Approximation: an optimimization based technique for approximate bayesian inference, obtain interval estimate and error variance. A class of techniques which try to approximate the itnractable posterior distribution with a tractable distribuition. Generally the paraemters of tracatble approximation are chosen to minimize some measure of its distance from the true posterior(KL-divergence)

Bayesian Lasso Problem

1.2 Contribution

Our main contribution could be concluded as the following part:

- Design of a new posterior parameter correction approach based on the posterior estimate of Mean-Field Variation Bayes parameter

1.3 Thesis Organization

This paper will be divided up into 6 chapters. Chapter 1 will briefly illustrate the motivation and application of variational approximation. Section 2 will briefly introduce basic definition and methodlogy in previous work such as MCMC(Monte Carlo Method) and Mean-Field Variational Bayes(MFVB). We will present our main methodlogy of variational correction algorithm in Chapter 3, followed by a comprehensive experiment for testing the effectiveness of algorithm in Chapter 4. In Chapter 5 and Chapter 6, we will briefly discuss and explain our result and potential improvement in the future.

Chapter 2

Definition and Literature Review

2.1 Least Absolute Shrinkage and Selection Operator(LASSO) penalized regression

2.1.1 Lasso penalty formulation

2.1.2 Bayesian Lasso regression

Univariate Lasso Distribution

Multivariate Lasso Distribution

2.2 Bayesian Paradigm

2.3 Variational Inference

2.3.1 Mean Field Variational Bayes

2.4 Expectation Maximization

2.4.1 Bayesian Expectation Maximization

2.5 Markov Chain Monte Carlo(MCMC)

2.5.1 Gibbs Sampler

Chapter 3

Methodlogy

3.1 Lasso distribution

3.1.1 Univariate Lasso Distribution

Basic Property

Derivation

3.1.2 Multivariate Lasso Distribution

Basic Property

Derivation

3.2 local-Global Algorithm

Chapter 4

Experiment Result and Analysis

4.1 Experimental Setting

4.1.1 Parameter selection

4.1.2 Evaluation metric

L1 accuracy

(MORE) Matrix norm Posterior cov and estimated Cov

TP/FP Rate after variable selection(Soft thresholding operator)

4.1.3 Experimental datasets

toy dataset 3-4 datasets

4.2 Experimental Result

Chapter 5

Dicussion and Conclusion

Bibliography