ME 41100 SYSTEM MODELING ANALYSIS AND CONTROL

HOMEWORK 4: MARINE PROPELLER RESPONSE IN A DRIVE TRAIN SYSTEM

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Contents

Abstract

In this homework, we will focus on the response of a marine propeller to the input feed of the motor ships. The structure of this system is shown below in figure 1. This system contains a propeller, drive train, engine, and flywheel. The power is provided by the engine. Also there are two shafts which the power will transmitted through. Shaft 1 transmits the power to the flywheel, and shaft 2 transmits the power to the gears. Also, the gear passes the power to the shaft 3, and then to the propeller.

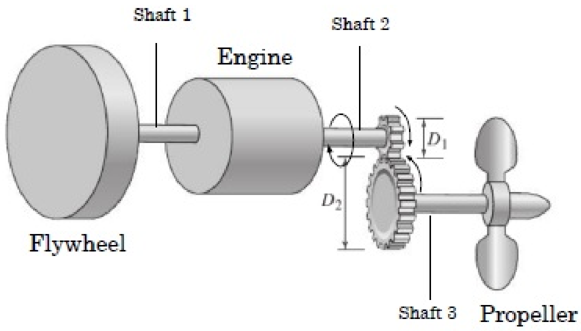

Figure 1: Overall design of the system

There are also many assumptions for this question. First, assume the flywheel is stationary. Second, assume the shaft 1 is elastic and damping owing to the material and friction. In other word, it has a spring constant k_m, and damper constant b_m. Then, the engine can produce input torque T. Also, assume the shaft 2 to be a rigid connector. The gear ratio is n = D_2 / D_1. Moreover, also assume the shaft 3 to be a rigid connector. Lastly, assume the mechanical system is initially at rest.

Governing Equation

a. Free Body Diagram

The overall free body diagram is provided in the assignment sheet, and is also shown in figure 2.

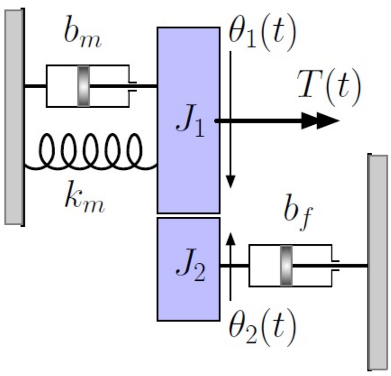

Figure 2: Overall free body diagram

The overall free body diagram could be break down into 2 parts. The first one is the free body diagram of J_1, and the second one is the free body diagram of J_2.

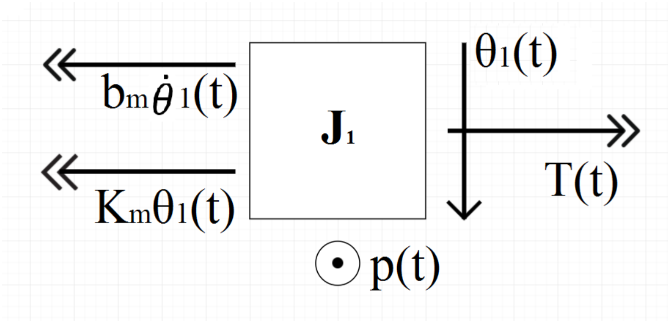

Figure 3: Free body diagram for J_1

As we can see, there are four forces acting on this part. The first one is the torque T(t). The second is the damper. The third one is the spring. The last one is the contact force by the gears. Based on figure 3, we can find the equation to be:

$$T(t) - b_m\dot{\theta}_1(t) - k_m\theta_1(t) - p(t)(\frac{D_1}{2}) = J_1\ddot{\theta}_1(t) \tag{1}$$

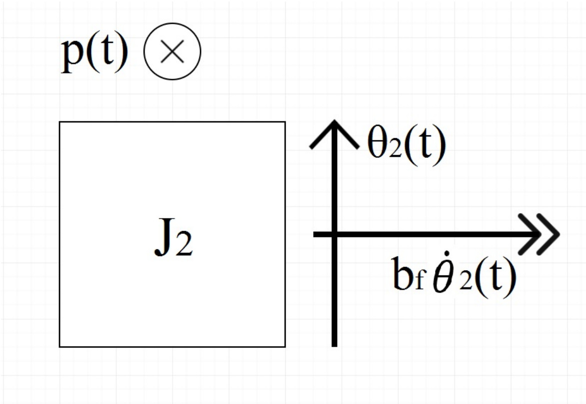

Figure 4: Free body diagram for J_2

For the second part of the free body diagram, there are only 2 forces acting on the system, which is the contact force by the gears, and the drag force, which is treated as a damper. The equation we can get from this figure is:

$$p(t)(\frac{D_2}{2}) - b_f\dot{\theta}_2(t) = J_2\ddot{\theta}_2(t) \tag{2}$$

Moreover, from the gear ratio stated in the assignment sheet, we could derive another equation, which is:

$$\frac{\theta_1(t)}{\theta_2(t)} = \frac{D_2}{D_1} = n \tag{3}$$

b. Governing equation

Based on the free body diagrams, we derived three equations (Equations 1, 2 and 3). We could rearrange the equation to make them in term of torque T, contact force p, and theta 1. The three governing equations can be shown as:

$$\boxed{T(t) = J_1\ddot{\theta}_1(t) + b_m\dot{\theta}_1(t) + k_m\theta_1(t) + p(t)(\frac{D_1}{2})} \tag{4}$$

$$\boxed{p(t)(\frac{D_2}{2}) = J_2\ddot{\theta}_2(t) + b_f\dot{\theta}_2(t)} \tag{5}$$

$$\boxed{\theta_1(t) = \frac{D_2}{D_1}\theta_2(t) = n\theta_2(t)} \tag{6}$$

Also, since the problem states that "Assume the mechanical system is initially at rest," we treat this problem as zero initial-condtion problem.

Transfer Function

In this section, we need to find the transfer function for θ_1, θ_2, and T. Moreover, we need to find the transfer function for $G_1(s)$, $G_2(s)$, $G_p(s)$ and $G_f(s)$. These variables are defined as:

$$G_1(s) = \frac{\Theta_1(s)}{T(s)} \tag{7}$$

$$G_2(s) = \frac{\Theta_2(s)}{T(s)} \tag{8}$$

$$G_p(s) = \frac{P(s)}{T(s)} \tag{9}$$

$$G_f(s) = \frac{T_f(s)}{T(s)} \tag{10}$$

a. Laplace transfer function for the governing equations

The first thing we do in this section is to take the Laplace transfer function for the three governing equations we have (equations 4, 5, and 6). Also, since it is zero initial condition problem, all initial displacement and velocity are equal to zero.

$$\mathscr{L}\{T(t) = J_1\ddot{\theta}_1(t) + b_m\dot{\theta}_1(t) + k_m\theta_1(t) + p(t)(\frac{D_1}{2})\} \tag{11}$$

$$\mathscr{L}\{p(t)(\frac{D_2}{2}) = J_2\ddot{\theta}_2(t) + b_f\dot{\theta}_2(t)\} \tag{12}$$

$$\mathscr{L}\{\theta_1(t) = \frac{D_2}{D_1}\theta_2(t) = n\theta_2(t)\} \tag{13}$$

By taking Laplace transfer function, equation 11 could solved as:

$$\mathscr{L}\{T(t) = J_1\ddot{\theta}_1(t) + b_m\dot{\theta}_1(t) + k_m\theta_1(t) + p(t)(\frac{D_1}{2})\} \tag{14}$$

$$T(s) = J_1 s^2\Theta_1(s) + b_m s\Theta_1(s) + k_m\Theta_1(s) + P(s)(\frac{D_1}{2}) \tag{15}$$

$$T(s) = (J_1 s^2 + b_m s + k_m)\Theta_1(s) + (\frac{D_1}{2})P(s) \tag{16}$$

Similarly, equation 12 could solved as:

$$\mathscr{L}\{p(t)(\frac{D_2}{2}) = J_2\ddot{\theta}_2(t) + b_f\dot{\theta}_2(t)\} \tag{17}$$

$$P(s)(\frac{D_2}{2}) = J_2 s^2\Theta_2(s) + b_f s\Theta_2(s) \tag{18}$$

$$P(s)(\frac{D_2}{2}) = (J_2 s^2 + b_f s)\Theta_2(s) \tag{19}$$

Finally, equation 13 could solved as:

$$\mathscr{L}\{\theta_1(t) = \frac{D_2}{D_1}\theta_2(t) = n\theta_2(t)\} \tag{20}$$

$$\Theta_1(s) = \frac{D_2}{D_1}\Theta_2(s) = n\Theta_2(s) \tag{21}$$

b. Summary of the Laplace transfer function for the governing equations

The three governing equations is becoming the following:

$$\boxed{T(s) = (J_1 s^2 + b_m s + k_m)\Theta_1(s) + (\frac{D_1}{2})P(s)} \tag{22}$$

$$\boxed{P(s)(\frac{D_2}{2}) = (J_2 s^2 + b_f s)\Theta_2(s)} \tag{23}$$

$$\boxed{\Theta_1(s) = \frac{D_2}{D_1}\Theta_2(s) = n\Theta_2(s)} \tag{24}$$

c. Laplace transform for $\mathbf{G_1}$, $\mathbf{G_2}$, $\mathbf{G_p}$ and $\mathbf{G_f}$

1. Laplace transform for G_1:

Starting from equation 7, we know G_1 is equal to $\Theta_1(s)\ /\ T(s)$. We could derive it by using equation 22, 23 and 24.

$$T(s) = (J_1 s^2 + b_m s + k_m)\Theta_1(s) + (\frac{D_1}{2})P(s) \tag{25}$$

$$T(s) = (J_1 s^2 + b_m s + k_m)\Theta_1(s) + (\frac{D_1}{2})(\frac{2}{D_2})(J_2 s^2 + b_f s)\Theta_2(s) \tag{26}$$

$$T(s) = (J_1 s^2 + b_m s + k_m)\Theta_1(s) + (\frac{D_1}{D_2})(J_2 s^2 + b_f s)(\frac{D_1}{D_2})\Theta_1(s) \tag{27}$$

$$T(s) = \Theta_1(s)[J_1 s^2 + b_m s + k_m + (\frac{D_1}{D_2})^2 J_2 s^2 + (\frac{D_1}{D_2})^2 b_f s] \tag{28}$$

$$T(s) = \Theta_1(s)[(J_1 + (\frac{D_1}{D_2})^2 J_2)s^2 + (b_m + (\frac{D_1}{D_2})^2 b_f)s + k_m] \tag{29}$$

$$\boxed{G_1(s) = \frac{\Theta_1(s)}{T(s)} = \frac{1}{(J_1 + \frac{1}{n^2}J_2)s^2 + (b_m + \frac{1}{n^2}b_f)s + k_m}} \tag{30}$$

This is a second order system with constant numerator.

2. Laplace transform for G_2:

For this part, we can use chain rule to simplify the calculation.

$$G_2(s) = \frac{\Theta_2(s)}{T(s)} = \frac{\Theta_2(s)}{\Theta_1(s)}\frac{\Theta_1(s)}{T(s)} \tag{31}$$

$$G_2(s) = \frac{1}{n}\frac{1}{(J_1 + \frac{1}{n^2}J_2)s^2 + (b_m + \frac{1}{n^2}b_f)s + k_m} \tag{32}$$

$$\boxed{G_2(s) = \frac{\Theta_2(s)}{T(s)} = \frac{\frac{1}{n}}{(J_1 + \frac{1}{n^2}J_2)s^2 + (b_m + \frac{1}{n^2}b_f)s + k_m}} \tag{33}$$

This is a second order system with constant numerator.

3. Laplace transform for G_s:

$$G_p(s) = \frac{P(s)}{T(s)} \tag{34}$$

$$T(s) = (J_1 s^2 + b_m s + k_m)\Theta_1(s) + (\frac{D_1}{2})P(s) \tag{35}$$

$$T(s) = (\frac{D_1}{2})P(s) + (J_1 s^2 + b_m s + k_m)n\Theta_2(s) \tag{36}$$

$$T(s) = (\frac{D_1}{2})P(s) + (J_1 s^2 + b_m s + k_m)nP(s)(\frac{D_2}{2})(\frac{1}{J_2 s^2 + b_f s}) \tag{37}$$

$$T(s) = (\frac{D_1}{2})P(s) + \frac{(J_1 s^2 + b_m s + k_m)n(\frac{D_2}{2})}{J_2 s^2 + b_f s}P(s) \tag{38}$$

$$T(s) = P(s)\frac{\frac{D_1}{2}(J_2 s^2 + b_f s) + (J_1 s^2 + b_m s + k_m)n(\frac{D_2}{2})}{J_2 s^2 + b_f s} \tag{39}$$

$$T(s) = P(s)\frac{\frac{D_1}{2}\frac{2}{D_2}\frac{1}{n}(J_2 s^2 + b_f s) + (J_1 s^2 + b_m s + k_m)n\frac{D_2}{2}\frac{2}{D_2}\frac{1}{n}}{(J_2 s^2 + b_f s)\frac{2}{D_2}\frac{1}{n}} \tag{40}$$

$$T(s) = P(s)\frac{\frac{1}{n^2}(J_2 s^2 + b_f s) + (J_1 s^2 + b_m s + k_m)}{(J_2 s^2 + b_f s)\frac{2}{D_2}\frac{1}{n}} \tag{41}$$

$$T(s) = P(s)\frac{(J_1 + \frac{1}{n^2}J_2)s^2 + (b_m + \frac{1}{n^2}b_f)s + k_m}{\frac{2}{D_2}\frac{1}{n}J_2 s^2 + \frac{2}{D_2}\frac{1}{n}b_f s} \tag{42}$$

$$\boxed{\frac{P(s)}{T(s)} = \frac{\frac{2}{D_2}\frac{1}{n}J_2 s^2 + \frac{2}{D_2}\frac{1}{n}b_f s}{(J_1 + \frac{1}{n^2}J_2)s^2 + (b_m + \frac{1}{n^2}b_f)s + k_m}} \tag{43}$$

This is a second order system with second order numerator.

4. Laplace transform for G_s: For this part, we start from equation 10. Also, we need to find the equation for the torque carried by the propeller in the fluid medium T_f.

$$G_f(s) = \frac{T_f(s)}{T(s)} \tag{44}$$

$$T_f(t) = b_f\dot{\theta}_2(t) \tag{45}$$

$$\mathscr{L}\{b_f\dot{\theta}_2(t)\} = b_f s\Theta_2(s) \tag{46}$$

$$T_f(s) = b_f s\Theta_2(s) \tag{47}$$

$$\frac{T_f(s)}{\Theta_2(s)} = b_f s \tag{48}$$

Then we can use the chain rule to simplify the calculation

$$G_f(s) = \frac{T_f(s)}{T(s)} = \frac{T_f(s)}{\Theta_2(s)}\frac{\Theta_2(s)}{T(s)} \tag{49}$$

$$G_f(s) = \frac{T_f(s)}{T(s)} = (b_f s)\frac{\frac{1}{n}}{(J_1 + \frac{1}{n^2}J_2)s^2 + (b_m + \frac{1}{n^2}b_f)s + k_m} \tag{50}$$

$$\boxed{G_f(s) = \frac{T_f(s)}{T(s)} = \frac{\frac{b_f}{n}s}{(J_1 + \frac{1}{n^2}J_2)s^2 + (b_m + \frac{1}{n^2}b_f)s + k_m}} \tag{51}$$

This is a second order system with first order numerator.

System Parameters

This this section, we need to determine the system order for all cases derived in part b. All equations we derived are listing below:

$$\boxed{\begin{cases} G_1(s) = \frac{\Theta_1(s)}{T(s)} = \frac{1}{(J_1+\frac{1}{n^2}J_2)s^2+(b_m+\frac{1}{n^2}b_f)s+k_m} \\ G_2(s) = \frac{\Theta_2(s)}{T(s)} = \frac{\frac{1}{n}}{(J_1+\frac{1}{n^2}J_2)s^2+(b_m+\frac{1}{n^2}b_f)s+k_m} \\ G_p(s) = \frac{P(s)}{T(s)} = \frac{\frac{2}{D_2}\frac{1}{n}J_2s^2+\frac{2}{D_2}\frac{1}{n}b_f s}{(J_1+\frac{1}{n^2}J_2)s^2+(b_m+\frac{1}{n^2}b_f)s+k_m} \\ G_f(s) = \frac{T_f(s)}{T(s)} = \frac{\frac{b_f}{n}s}{(J_1+\frac{1}{n^2}J_2)s^2+(b_m+\frac{1}{n^2}b_f)s+k_m} \end{cases}} \tag{52}$$

All of them are having a same second order denominator: $(J_1 + \frac{1}{n^2}J_2)s^2 + (b_m + \frac{1}{n^2}b_f)s + k_m$. Then we need to compare with the deferential equation listed below to find the natural frequency, damping ratio, and gains.

$$\frac{d^2c(t)}{dt^2} + 2\zeta\omega_n\frac{dc(t)}{dt} + \omega_n^2 c(t) = \sum_{i=0}^{m}\left[\omega_n^2 K_i \frac{d^i T(t)}{dt^i}\right] \tag{53}$$

Then match the denominator with the equation above. The denominator can be arranged as:

$$s^2 + \frac{b_m + \frac{1}{n^2}b_f}{J_1 + \frac{1}{n^2}J_2}s + \frac{k_m}{J_1 + \frac{1}{n^2}J_2} \tag{54}$$

$$\omega_n^2 = \frac{k_m}{J_1 + \frac{1}{n^2}J_2} \tag{55}$$

$$\boxed{\omega_n = \sqrt{\frac{k_m}{J_1 + \frac{1}{n^2}J_2}}} \tag{56}$$

$$2\zeta\omega_n = \frac{b_m + \frac{1}{n^2}b_f}{J_1 + \frac{1}{n^2}J_2} \tag{57}$$

$$\zeta = \Big(\frac{b_m + \frac{1}{n^2}b_f}{J_1 + \frac{1}{n^2}J_2}\Big)\Big(\frac{1}{2}\Big)\Big(\sqrt{\frac{J_1 + \frac{1}{n^2}J_2}{k_m}}\Big) \tag{58}$$

$$\boxed{\zeta = \frac{b_m + \frac{1}{n^2}b_f}{2\sqrt{k_m(J_1 + \frac{1}{n^2}J_2)}}} \tag{59}$$

To find the gain for p(t), we start from

$$[(\frac{2}{D_2})(\frac{1}{n})J_2 s^2]T(s) + [(\frac{2}{D_2})(\frac{1}{n})b_f]sT(s) = (J_1 + \frac{1}{n^2}J_2)s^2P(s) + (b_m + \frac{1}{n^2}b_f)sP(s) + k_mP(s) \tag{60}$$

$$[(\frac{2}{D_2})(\frac{1}{n})J_2]\frac{d^2T(t)}{dt^2} + [(\frac{2}{D_2})(\frac{1}{n})b_f]\frac{dT(t)}{dt} = (J_1 + \frac{1}{n^2}J_2)\frac{d^2P(t)}{dt^2} + (b_m + \frac{1}{n^2}b_f)\frac{dP(t)}{dt} + k_mP(t) \tag{61}$$

$$\frac{d^2P(t)}{dt^2} + \frac{(b_m + \frac{1}{n^2}b_f)}{(J_1 + \frac{1}{n^2}J_2)}\frac{dP(t)}{dt} + \frac{k_m}{(J_1 + \frac{1}{n^2}J_2)}P(t) = \frac{(\frac{2}{D_2})(\frac{1}{n})J_2}{J_1 + \frac{1}{n^2}J_2}\frac{d^2T(t)}{dt^2} + \frac{(\frac{2}{D_2})(\frac{1}{n})b_f}{J_1 + \frac{1}{n^2}J_2}\frac{dT(t)}{dt} \tag{62}$$

Compare to the general equation stated in the manual, we can find K_1 and K_2.

$$K_1 = \frac{2b_f}{nD_2k_m} \tag{63}$$

$$K_2 = \frac{2J_2}{nD_2k_m} \tag{64}$$

Repeat the steps above, we can find the gains for all equations. All gains are listed in table 1 below

Table 1: Gains

output	K	K_1	K_2	$K_j(j \geq 3)$
$\theta_1(t)$	$\frac{1}{k_m}$	N/A	N/A	N/A
$\theta_2(t)$	$\frac{1}{nk_m}$	N/A	N/A	N/A
p(t)	N/A	$\frac{2b_f}{nD_2k_m}$	$\frac{2J_2}{nD_2k_m}$	N/A
$T_f(t)$	N/A	$\frac{b_f}{nk_m}$	N/A	N/A

Steady-State Response

Assume the input engine torque is $T(t) = T_s 1(t)$, where 1(t) is the unit step function. Here we need to determine the resultant steady-state responses of $\theta_1(t)$, $\theta_2(t)$, $p(t)$, and $T_f(t)$. Assume T_s is equal to $30kNm = 30000Nm$.

Table 2: Given data

$J_1(kgm^2)$	$J_2(kgm^2)$	$D_1(m)$	$D_2(m)$	$b_m(Nms)$	$b_f(Nms)$	$k_m(Nm)$
400	3000	0.8	1.6	45	220	80

Steady-state response can be referred to the situation when time goes to infinity. Also it could be determined when s = 0 for Laplace transfer function. The steady-response for θ_1, θ_2, p, and T_f are shown below:

$$T(s) = \mathscr{L}\{T(t)\} = T_s\mathscr{L}\{1(t)\} = \frac{T_s}{s} \tag{65}$$

$$\theta_{1s.s.} = \lim_{t=\infty}\theta_1(t) = \lim_{s=0} s\Theta_1(s) = \lim_{s=0} sG_1(s)\frac{T_s}{s} \tag{66}$$

$$\theta_{1s.s.} = \lim_{s=0}\frac{T_s}{(J_1 + \frac{1}{n^2}J_2)s^2 + (b_m + \frac{1}{n^2}b_f)s + k_m} \tag{67}$$

$$\boxed{\theta_{1s.s.} = \frac{T_s}{k_m}} \tag{68}$$

The for θ_2, we can repeat the same steps:

$$\theta_{2s.s.} = \lim_{t=\infty}\theta_2(t) = \lim_{s=0} s\Theta_2(s) = \lim_{s=0} sG_2(s)\frac{T_s}{s} \tag{69}$$

$$\theta_{2s.s.} = \lim_{s=0}\frac{\frac{1}{n}T_s}{(J_1 + \frac{1}{n^2}J_2)s^2 + (b_m + \frac{1}{n^2}b_f)s + k_m} \tag{70}$$

$$\boxed{\theta_{2s.s.} = \frac{T_s}{nk_m}} \tag{71}$$

$$p_{s.s.} = \lim_{t=\infty}p(t) = \lim_{s=0} sP(s) = \lim_{s=0} sG_p(s)\frac{T_s}{s} \tag{72}$$

$$p_{s.s.} = \lim_{s=0} \frac{\frac{2}{D_2}\frac{1}{n}J_2 s^2 + \frac{2}{D_2}\frac{1}{n}b_f s}{(J_1 + \frac{1}{n^2}J_2)s^2 + (b_m + \frac{1}{n^2}b_f)s + k_m} T_s \tag{73}$$

$$\boxed{p_{s.s.} = 0} \tag{74}$$

$$T_{f s.s.} = \lim_{t=\infty} T_f(t) = \lim_{s=0} sT_f(s) = \lim_{s=0} sG_f(s)\frac{T_s}{s} \tag{75}$$

$$T_{f s.s.} = \lim_{s=0} \frac{\frac{b_f}{n}s}{(J_1 + \frac{1}{n^2}J_2)s^2 + (b_m + \frac{1}{n^2}b_f)s + k_m} T_s \tag{76}$$

$$\boxed{T_{f s.s.} = 0} \tag{77}$$

since $n = \frac{D_2}{D_1} = \frac{1.6}{0.8} = 2,\ T_s = 30000Nm$, and $k_m = 80Nm$, we can find the results.

Table 3: Results

$\theta_{1s.s.}$	$\theta_{2s.s.}$	$p_{s.s.}$	$T_{f s.s.}$
375	187.5	0	0

Transient Responses

In this section, we need to use the Matlab to find the responses $\theta_1(t), \theta_2(t), p(t), and T_f(t)$ by using the torques listed below. For all of them, we need to plot the time histories in two separate pictures. One plots the angular displacements $\theta_1(t) and \theta_2(t)$, and another one for $p(t) and T_f(t)$.

Table 4: Inputs

$T_s(kNm)$	$T_d(kNms)$	$T_r(kNm/s)$	$T_p(kNm)$	$T_sin(Nm)$
30	2	0.4	20	25

a. Step Input

For this part, the input $T(t) = T_s 1(t)$.

Table 5: Step Input results

$Response$	Mp Maximum overshoot (%)	t_p peak time (s)	t_s 2% settling time (s)	$c_{s.s.}$ steady state value
θ_1	58.7757	11.6511	87.1113	375
θ_2	58.7757	11.6511	87.1113	188
p	∞	0	86.9381	0
T_f	∞	5.2959	92.5081	0

The figures are listed below:

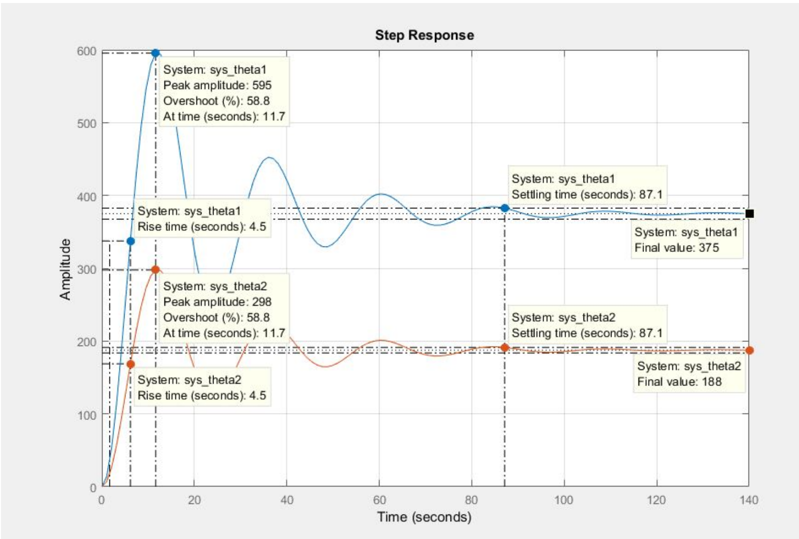

Figure 5: $\theta_1 and \theta_2$

Figure above showed with all information.

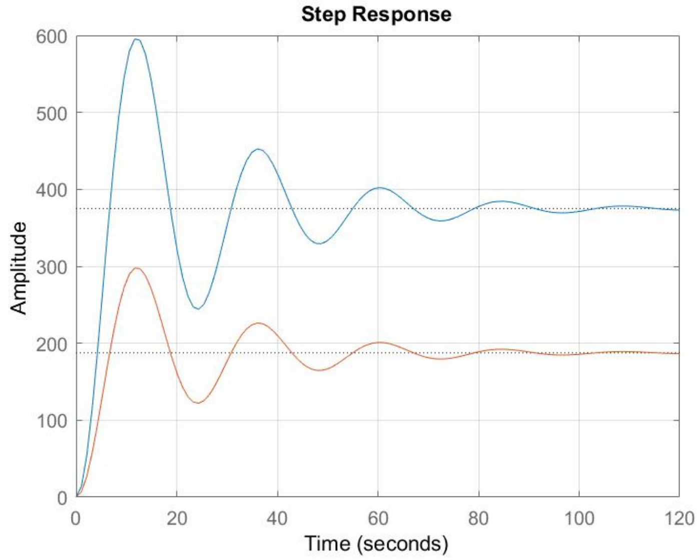

Figure 6: $\theta_1 and \theta_2$

Figure above showed the plot without information.

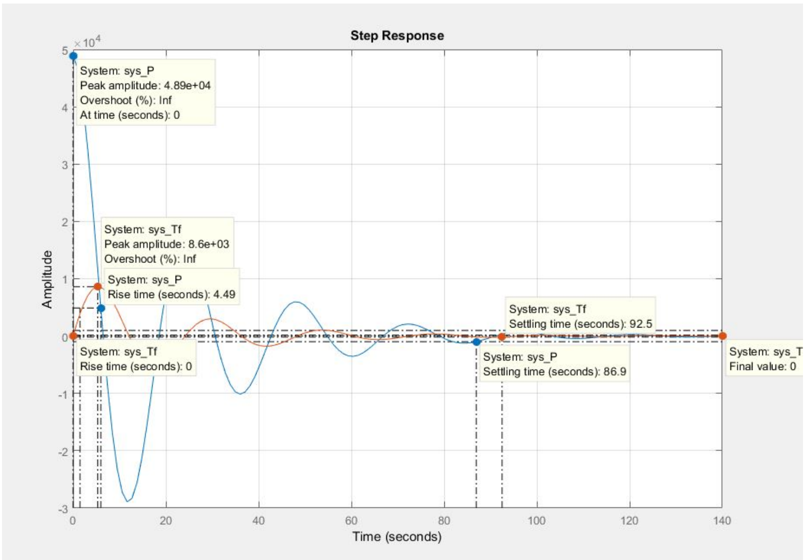

Figure 7: p and T_f

The figure above showed the information for the p and T_f.

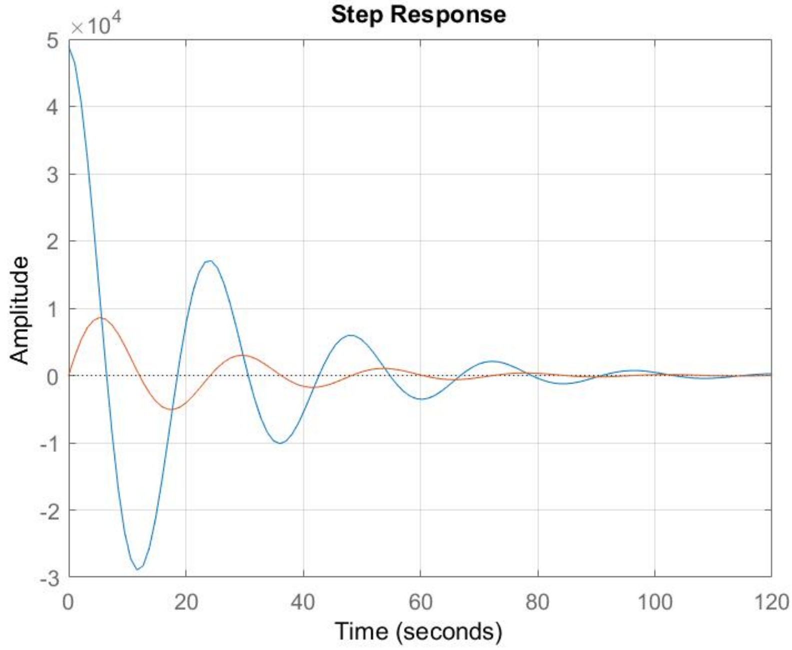

Figure 8: p and T_f

Figure above showed the plot without information.

b. Impulse Input

For this part, the input $T(t) = T_d\delta(t)$ where $\delta(t)$ is the unit impulse function.

Table 6: Impulse Input results

Response	t_p peak time (s)	t_s 2% settling time (s)
θ_1	5.3	92.5
θ_2	5.3	92.5
p	5.3	92.3
T_f	0	86.1

The figures are listed below:

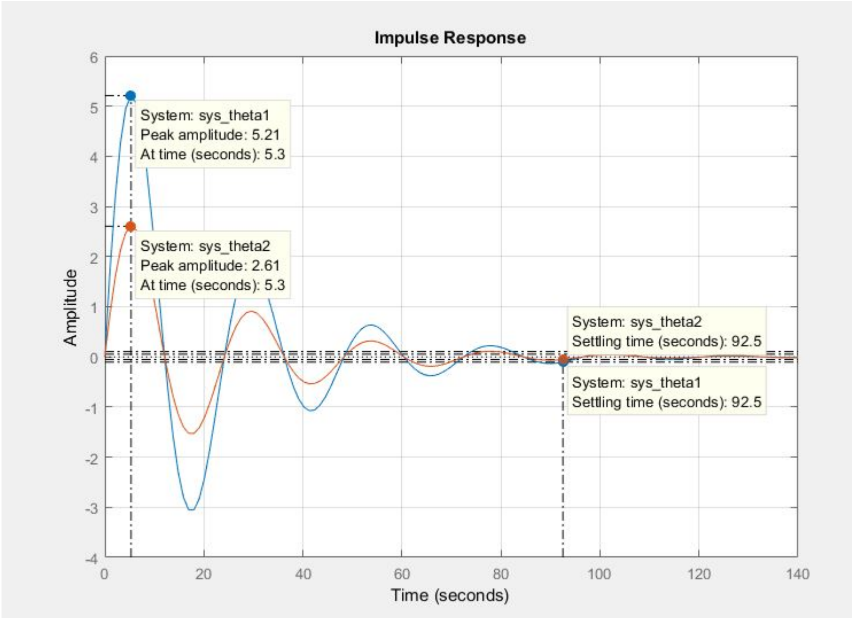

Figure 9: $\theta_1 and \theta_2$

Figure above showed with all information.

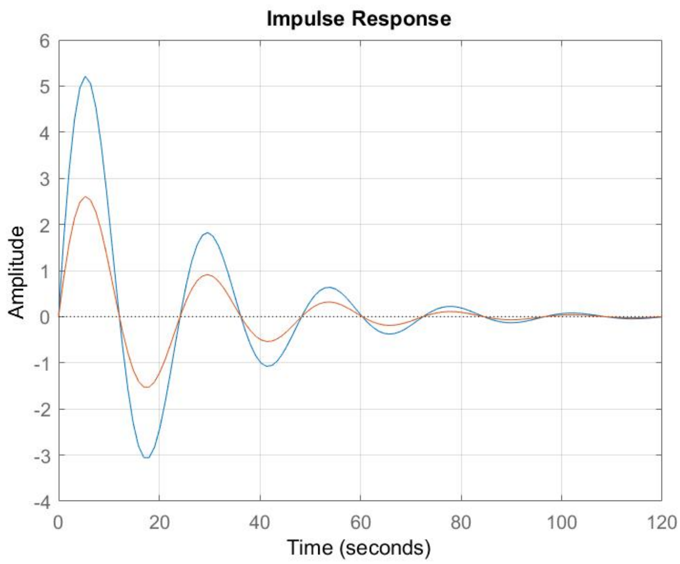

Figure 10: $\theta_1 and \theta_2$

Figure above showed the plot without information.

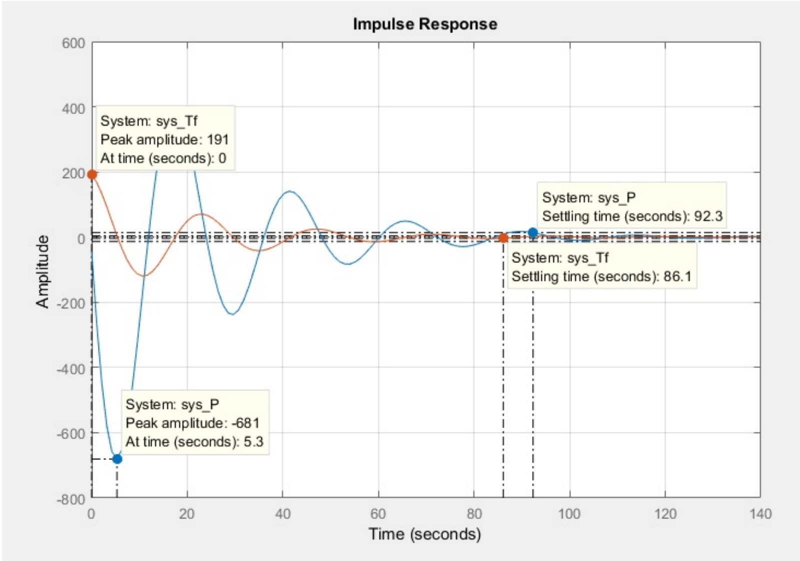

Figure 11: p and T_f

The figure above showed the information for the p and T_f.

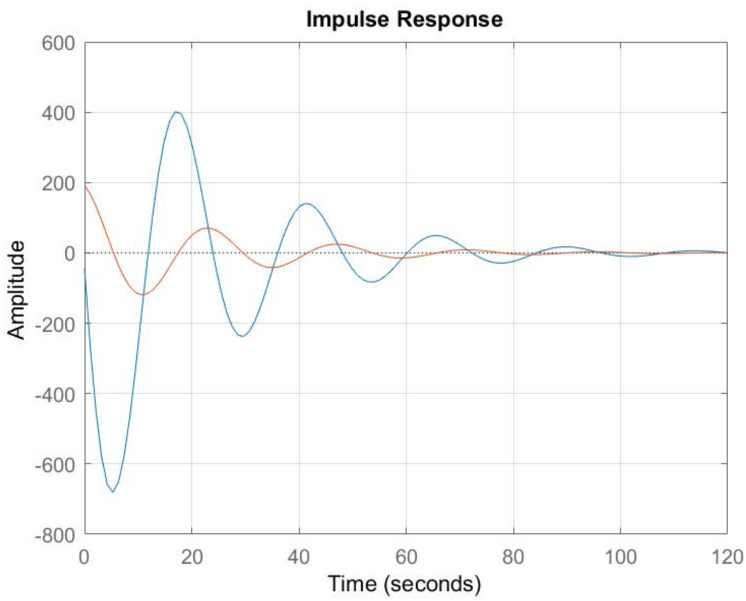

Figure 12: p and T_f

Figure above showed the plot without information.

c. Ramp Input

For this part, the input $T(t) = T_r t$.

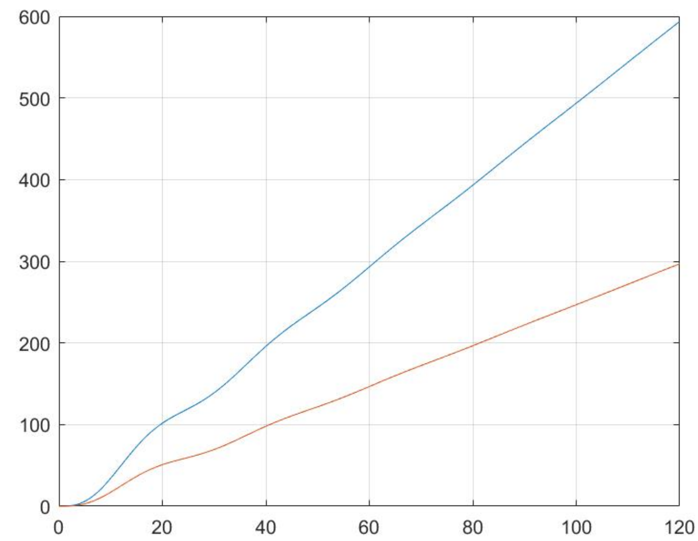

Figure 13: $\theta_1 and \theta_2$

The figure above showed the plot for $\theta_1 and \theta_2$.

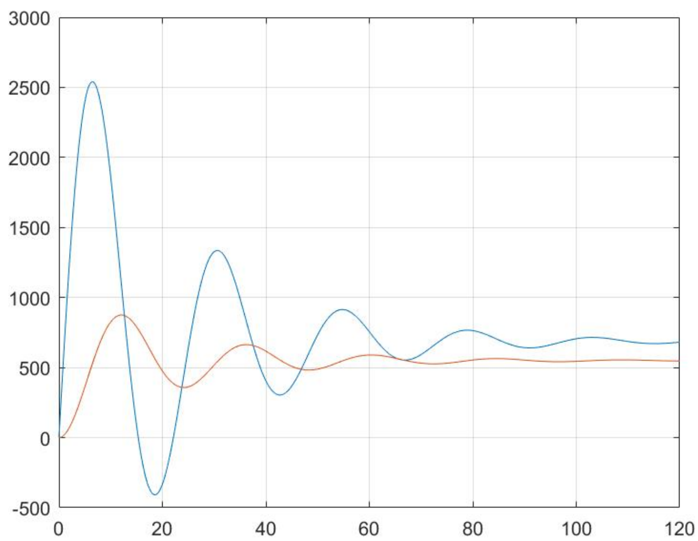

Figure 14: p and T_f

The figure above showed the plot for p and T_f

d. pulse Input

This this part, $T_p = 20000Nm$ when time is between 0 and 20. After 20 seconds, the $T_p = 0$.

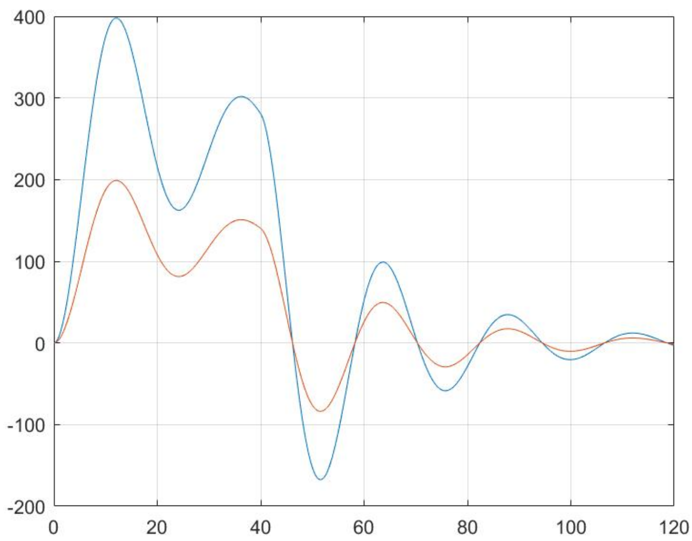

Figure 15: $\theta_1 and \theta_2$

The figure above showed the plot for $\theta_1 and \theta_2$.

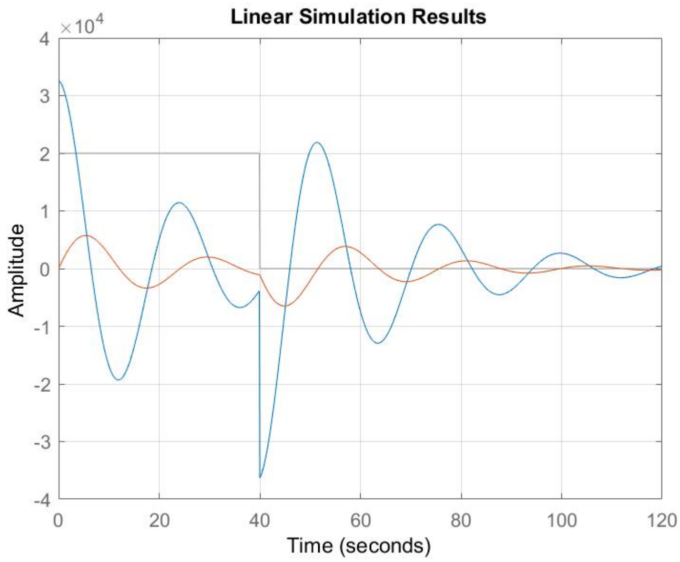

Figure 16: p and T_f

The figure above showed the plot for p and T_f

e. Sinusoidal Input

for this part, $T = T_{sin} \mid sin(0.8t) \mid$ when time is between 0 and 50 seconds. After 50 seconds, the $T = 0$.

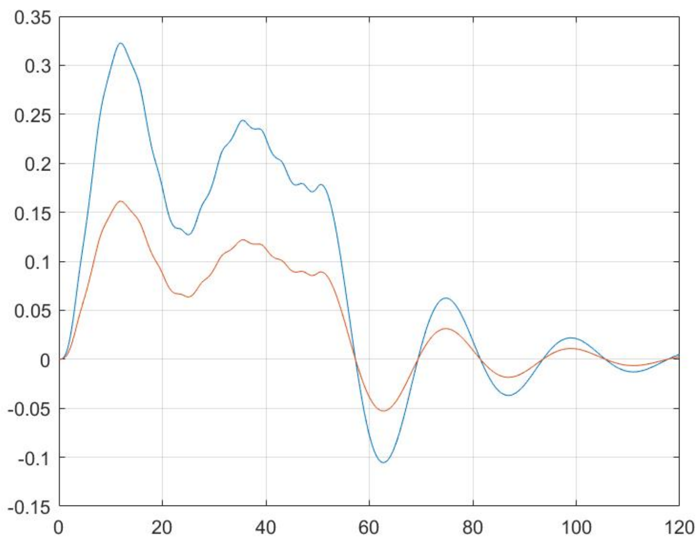

Figure 17: $\theta_1 and \theta_2$

The figure above showed the plot for $\theta_1 and \theta_2$.

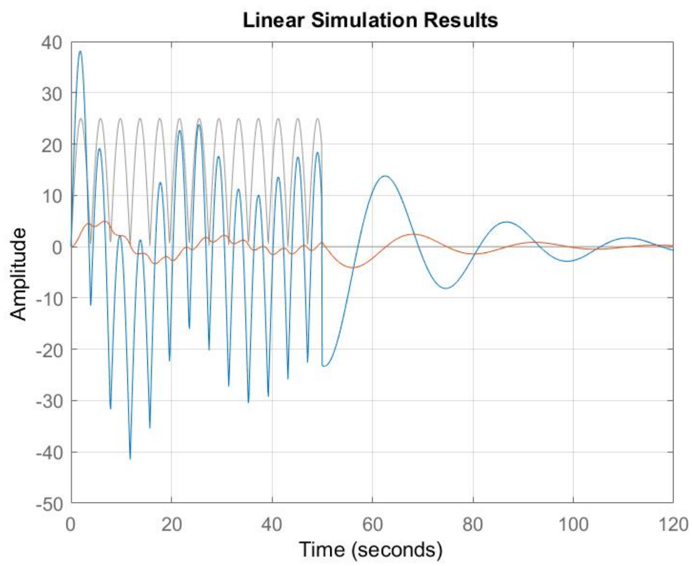

Figure 18: p and T_f

The figure above showed the plot for p and T_f

Appendix

a. Matlab for section e.1

```
J1 = 400;
J2 = 3000;
D1 = 0.8;
D2 = 1.6;
bm = 45;
bf = 220;
km = 80;
n = D2/D1;
%% For theta1 and theta2
den = [J1+(1/(n^2))*J2 bm+(1/(n^2))*bf km];

theta1_num_step = [1];
theta2_num_step = [1/n];
sys_theta1 = tf(30000 * theta1_num_step,den);
sys_theta2 = tf(30000 * theta2_num_step,den);

figure
step(sys_theta1);
hold on
step(sys_theta2);
xlim([0,120])

disp('Theta 1: ');
stepinfo(sys_theta1)
disp('Theta 2: ');
stepinfo(sys_theta2)

%% For P and Tf

P_num_step = [(2/D2)*(1/n)*J2 (2/D2)*(1/n)*bf 0];
Tf_num_step = [bf/n 0];

sys_P = tf(30000 * P_num_step,den);
sys_Tf = tf(30000 * Tf_num_step,den);

figure
step(sys_P);
hold on
step(sys_Tf);
xlim([0,120])

disp('P: ');
stepinfo(sys_P)
disp('Tf: ');
stepinfo(sys_Tf)
```

b. Matlab for section e.2

```
J1 = 400;
J2 = 3000;
```

```
D1 = 0.8;
D2 = 1.6;
bm = 45;
bf = 220;
km = 80;
n = D2/D1;
%% For theta1 and theta2
den = [J1+(1/(n^2))*J2 bm+(1/(n^2))*bf km];

theta1_num_impulse = [1];
theta2_num_impulse = [1/n];
sys_theta1 = tf(2000 * theta1_num_impulse,den);
sys_theta2 = tf(2000 * theta2_num_impulse,den);

figure
impulse(sys_theta1);
hold on
impulse(sys_theta2);
xlim([0,120])

%% For P and Tf

P_num_impulse = [(2/D2)*(1/n)*J2 (2/D2)*(1/n)*bf 0];
Tf_num_impulse = [bf/n 0];

sys_P = tf(2000 * P_num_impulse,den);
sys_Tf = tf(2000 * Tf_num_impulse,den);

figure
impulse(sys_P);
hold on
impulse(sys_Tf);
xlim([0,120])
```

c. Matlab for section e.3

```
J1 = 400;
J2 = 3000;
D1 = 0.8;
D2 = 1.6;
bm = 45;
bf = 220;
km = 80;
n = D2/D1;
%% For theta1 and theta2
den = [(J1+(1/(n^2))*J2) (bm+(1/(n^2))*bf) (km)];

theta1_num_ramp = [1];
theta2_num_ramp = [1/n];

t=0:0.1:120;
ramp=400*t;

sys_theta1 = tf(theta1_num_ramp,den);
sys_theta2 = tf(theta2_num_ramp,den);

```

```

thetalgraph = lsim(sys_theta1,ramp,t);
theta2graph = lsim(sys_theta2,ramp,t);

figure
plot(t,thetalgraph);
hold on
plot(t,theta2graph);
grid on
%% For P and Tf
P_num_ramp = [((2/D2)*(1/n)*J2) ((2/D2)*(1/n)*bf) (0)];
Tf_num_ramp = [(bf/n) (0)];

sys_p = tf(P_num_ramp,den);
sys_Tf = tf(Tf_num_ramp,den);


pgraph = lsim(sys_p,ramp,t);
Tfgraph = lsim(sys_Tf,ramp,t);

figure
plot(t,pgraph);
hold on
plot(t,Tfgraph);
grid on
```

d. Matlab for section e.4

```
J1 = 400;
J2 = 3000;
D1 = 0.8;
D2 = 1.6;
bm = 45;
bf = 220;
km = 80;
n = D2/D1;
%% For theta1 and theta2
den = [(J1+(1/(n^2))*J2) (bm+(1/(n^2))*bf) (km)];
theta1_num_pulse = [1];
theta2_num_pulse = [1/n];

t = 0 : 0.1 : 120;
pulse(1:400+1) = 20000;
pulse(400+1:1200+1) = 0;

sys_theta1 = tf(theta1_num_pulse,den);
sys_theta2 = tf(theta2_num_pulse,den);

thetalgraph = lsim(sys_theta1,pulse,t);
theta2graph = lsim(sys_theta2,pulse,t);

figure
plot(t,thetalgraph);
hold on
plot(t,theta2graph);
grid on
```

```
%% For P and Tf
P_num_pulse = [((2/D2)*(1/n)*J2) ((2/D2)*(1/n)*bf) (0)];
Tf_num_pulse = [(bf/n) (0)];

sys_p = tf(P_num_pulse,den);
sys_Tf = tf(Tf_num_pulse,den);

figure
lsim(sys_p,pulse,t);
hold on
lsim(sys_Tf,pulse,t);
grid on
```

e. Matlab for section e.5

```
J1 = 400;
J2 = 3000;
D1 = 0.8;
D2 = 1.6;
bm = 45;
bf = 220;
km = 80;
n = D2/D1;
%% For theta1 and theta2
den = [(J1+(1/(n^2))*J2) (bm+(1/(n^2))*bf) (km)];

theta1_num_sin = [(1)];
theta2_num_sin = [(1/n)];

t = 0 : 0.1 : 120;
sin(1:500+1)=25*abs(sin(0.8*t(1:500+1)));
sin(500+1:1200+1)= 0;

sys_theta1 = tf(theta1_num_sin,den);
sys_theta2 = tf(theta2_num_sin,den);

theta1graph = lsim(sys_theta1,sin,t);
theta2graph = lsim(sys_theta2,sin,t);

figure
plot(t,theta1graph);
hold on
plot(t,theta2graph);
grid on
%% For P and Tf
P_num_sin = [((2/D2)*(1/n)*J2) ((2/D2)*(1/n)*bf) (0)];
Tf_num_sin = [(bf/n) (0)];

sys_p = tf(P_num_sin,den);
sys_Tf = tf(Tf_num_sin,den);

figure
lsim(sys_p,sin,t);
hold on
lsim(sys_Tf,sin,t);
```