Chapter 9: Pelton Wheel

ME 436 Aerothermal Fluids Laboratory
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Abstract

This lab is about the Pelton wheel. By changing the flow rates and the rotational speeds, one can investigate the performance of the machine. There are two independent parameters which can be changed during the experiment: The load applied to the friction brake, and the position of the nozzle regulating spear. The load affects the rotational speed, and by changing the position of the nozzle regulating spear, the flow rate is changed. If the load is increased, the rotations will decrease. By increasing the pressure, the flow rate will decrease. The pressures we choose are 0.5 bar, 0.6 bar, 0.8 bar, 1 bar, and 1.2 bars. The load we have are: 350g, 450g, 550g, 650g, 750g, 850g, 1350g, and 1750g. We also recorded the corresponding spring mass and rotations for each case. By seeing the figure 6, we can make the conclusion that the maximum efficiency is not at the highest wheel speed. While increasing the wheel speed, the efficiency will increase with it. However, after it reaches the maximum efficiency, continuing increase the speed will decrease the efficiency. In this experiment, the largest uncertainty is about $\pm$10% of the efficiency. Overall, this experiment was successful.

Introduction

The modern Pelton wheel is a tangential partial turbine with double discharge bucket. The jet stream produces a force on the bucket, which also creates a torque on the shaft. All the available head is converted to kinetic energy. This system is very simple and cheap, and also it is very efficient. Therefore, it is very commonly used impulse turbine in the world. The Pelton wheel in our lab is Cussons p6240 Pelton wheel. Even though it is just a model, it has all characteristics of full size wheels.

In this experiment, torque can be calculated by using equation below:

$$T = (W - S)r \quad (1)$$

Where T is the torque, W is the brake load, S is the spring force, and r is the brake wheel radius.

The power input P_i is given by

$$P_i = PQ \qquad (2)$$

Where P is the pressure, and Q is the volume flow rate.

The power output can also be calculated:

$$P_o = TN \qquad (3)$$

Where T is torque, and N is the wheel speed. By having power output and power input, we could find the efficiency by using equation below:

$$\eta = \frac{P_o}{P_i} \qquad (4)$$

Experimental Setup and Procedure

In this experiment, a Cussons p6240 Pelton Wheel is used. It consists of a Degener 4717 model Pelton Wheel mounted on a base plate and fitted with a friction dynamometer. It has 16 elliptical ridged buckets at a mean diameter of 100 mm. The friction dynamometer consists of a 60 mm diameter brake wheel fitted with a fabric brake band. Also we have a tachometer which is needed to measure the rotation speed (rpm). Moreover, since the stopwatch is broken, we used our cellphone to measure the time for the flowrate.

Figure 1: Overview of the setup. The setups include a wheel on top of the table, a reader of the pressure, a container for water, and a tube connecting to the wheel.

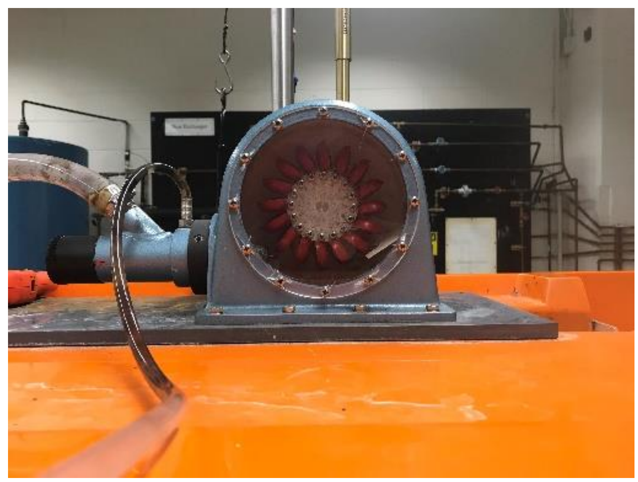

Figure 2: The Pelton wheel.

Figure 3: The weight carrier attached on the shaft.

Since there are two independent variables, which are the pressure and the load, we choose to set for one pressure value and test different loads, and then switch to another pressure value and repeat the same steps.

First, we switched on the Hydraulics Bench pump and fully opened the bench regulating valve. Then we needed to remove all masses that attached on the shaft and the weight carrier. We measured the flow rate of the system by measuring the time for 10 L water to pass. Also we measured the rotational speed of the Pelton wheel by using the tachometer. After this, we attached the weight carrier on the shaft and recorded the speed, the net load in the brake band, the volume flow rate, and inlet pressure. After we recorded all information, we started to add more weight on the carrier and repeated the same steps. Moreover, we repeated the same steps for different pressure values.

Results

Figure 4 shows the torque vs wheel speed. The unit for torque is Nm, and the unit for wheel speed is rad/s. In this graph, we can see that increasing wheel speed will decrease the torque. This applies to all pressures we used. However, when the wheel speed is equal to zero, the torques are not very accurate. It is because that the maximum spring mass we have is 1 kg. Even if the spring mass is greater than that, it will still show 1 kg as the spring mass. Figure 5 shows the power output vs wheel speed for different pressures. As we can see, when the wheel speed increases, the power will increase with it at first. When the power output reaches the maximum, increasing wheel speed will decrease the power output and eventually the power output will be zero. In our case, Pressure of 1 bar will give us the maximum power output, and pressure of 0.5 bar gives us the minimum power output. Figure 6 shows the efficiency vs the wheel speed. We can see that the maximum efficiency is not at the highest wheel speed. While increasing the wheel speed, the efficiency will increase with it. However, after it reaches the maximum efficiency, continuing increase the speed will decrease the efficiency. Figure 7 shows the uncertainty for the efficiencies. The largest uncertainty is about $\pm$10% of the efficiency, which is realistic since the devices we have are not very accurate.

Conclusion

Overall, this experiment ran successfully. This experiment shows the relations between torque, power, and efficiency vs wheel speed. However, there were still some errors we made. The rpm we measured might not be very accurate because the number for tachometer is always changing. Also the spring mass was not accurate due to the exceed of the maximum spring mass it has. We could improve our experiment next time by fixing all the problems we made this time.

List of References

[1] Goushcha, O. *Aero-Thermal Fluids Laboratory ME 43600*. The City College of New York, 2018

Appendix A

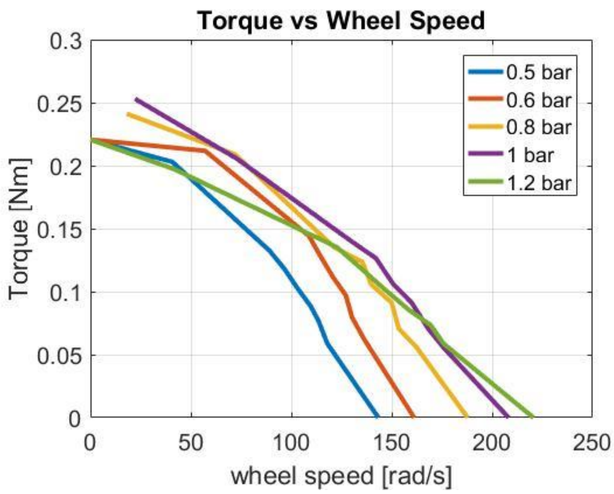

Figure 4: Torque (Nm) vs wheel speed (rad/s)

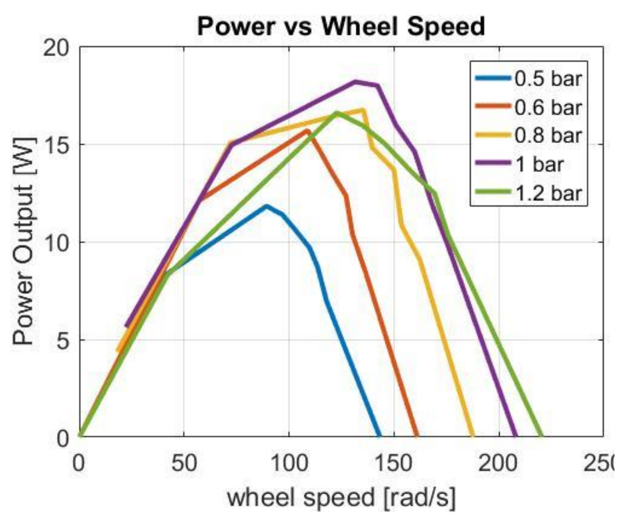

Figure 5: Power output (W) vs wheel speed (rad/s)

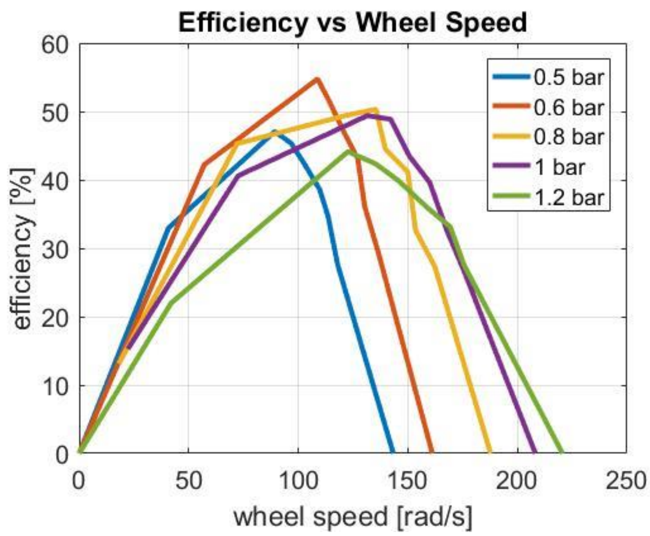

Figure 6: Efficiency (%) vs wheel speed (rad/s)

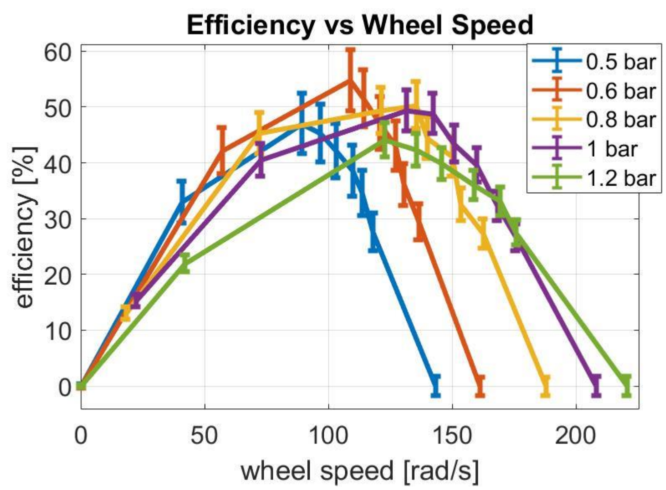

Figure 7: Uncertainty for the efficiencies.

Appendix B

Calculations

Convert rpm to rad/s:

$$1\,rpm = \frac{2\,\pi}{60}\frac{rad}{s}$$

Liter/s to m^3/s:

$$1\,\frac{L}{s} = 0.001\frac{m^3}{s}$$

Bar to pascal:

$$1\,bar = 100000\,pascal$$

Calculate torque:

$$T = (W - S)r$$

$$T = (0.03)(0.35 - 0.15)(9.806)$$

$$T = 0.058836\ Nm$$

Calculate power input:

$$P_i = PQ$$

$$P_i = (50000)(0.000502)$$

$$P_i = 25.1\ W$$

Calculate power output:

$$P_o = TN$$

$$P_o = (0.058836)(117.914)$$

$$P_o = 6.9376\ W$$

Calculate efficiency:

$$\eta = \frac{P_o}{P_i}$$

$$\eta = \frac{6.9376}{25.1}$$

$$\eta = 27.64\ \%$$

Uncertainty

The efficiency equation is shown below with two variables P_o and P_i.

$$\eta = \frac{P_o}{P_i}$$

Each of them will have variables as well. The equations for calculating P_o and P_i are shown below:

$$P_i = PQ$$

$$P_o = TN$$

For torque, equation is below, which is the equation we start first for uncertainty.

$$T = (W - S)r$$

For mass of the load, the uncertainty is 0. The uncertainty for spring mass is 0.01 kg. Since S is in Newton, we need to multiply the spring mass by 9.81. Therefore, uncertainty for S is

$$(0.01)(9.81) = 0.0981\ N$$

$$u_T = \sqrt{\left(\frac{\partial T}{\partial W} u_W\right)^2 + \left(\frac{\partial T}{\partial S} u_S\right)^2}$$

$$\frac{\partial T}{\partial S} = -r$$

Where r = 0.03 m.

$$u_T = \sqrt{(-0.03 * 0.0981)^2}$$

$$u_T = 0.002943\ Nm$$

For P_i, it is affected by pressure P and volume flow rate Q. The uncertainty for pressure is 0.05 bar. We need to convert to pascal, which is equal to 5000 Pa. For volume flow rate, there are two factors, which are the time and the volume.

$$Q = \frac{V}{t}$$

$$u_Q = \sqrt{\left(\frac{\partial Q}{\partial V} u_V\right)^2 + \left(\frac{\partial Q}{\partial t} u_t\right)^2}$$

$$\frac{\partial Q}{\partial V} = \frac{1}{t}$$

$$\frac{\partial Q}{\partial t} = -\frac{V}{t^2}$$

$$u_V = 0.5\ L = 0.0005\ m^3$$

$$u_t = 0.005\ sec$$

$$u_Q = \sqrt{\left(0.0005 \frac{1}{t}\right)^2 + \left(-0.005 \frac{V}{t^2}\right)^2}$$

Then we can find the uncertainty for P_i:

$$u_{Pi} = \sqrt{\left(\frac{\partial P_i}{\partial P} u_P\right)^2 + \left(\frac{\partial P_i}{\partial Q} u_Q\right)^2}$$

$$\frac{\partial P_i}{\partial P} = Q$$

$$\frac{\partial P_i}{\partial Q} = P$$

The next step is to find the uncertainty for P_o:

$$P_o = TN$$

For the uncertainty of wheel speed (N), we first need to find the uncertainty for the tachometer.

$$s_x = \sqrt{\frac{1}{N-1}\sum_{n=1}^{N}(x_n - \bar{x})}$$

Where N = 15

From lab manual page 12, we can find the t_{95} where v = N – 1 = 14

$$t_{95} = 2.145$$

For each single point, the uncertainty is 0.5 rpm, which is equal to 0.0524 radian per second.

$$s_{\bar{x}} = \frac{s_x}{\sqrt{N}} = 0.1875$$

$$u_x = \sqrt{\left(u_{single\ point}\right)^2 + (t_{95}s_{\bar{x}})^2}$$

$$u_N = 0.4056\, rps$$

$$u_{Po} = \sqrt{\left(\frac{\partial P_o}{\partial T} u_T\right)^2 + \left(\frac{\partial P_o}{\partial N} u_N\right)^2}$$

$$\frac{\partial P_o}{\partial T} = N$$

$$\frac{\partial P_o}{\partial N} = T$$

Once we got the uncertainty for both P_o and P_i, we can find the uncertainty for the efficiency.

$$\eta = \frac{P_o}{P_i}$$

$$u\eta = \sqrt{\left(\frac{\partial \eta}{\partial P_o} u_{po}\right)^2 + \left(\frac{\partial \eta}{\partial P_i} \delta_{pi}\right)^2}$$

$$\frac{\partial \eta}{\partial P_o} = \frac{1}{P_i}$$

$$\frac{\partial \eta}{\partial P_i} = -\frac{P_o}{p_i^2}$$

Sample efficiency for pressure = 0.5 bar, mass = 550 g, and spring mass is 0.25 kg is 0.045 (4.5%).

Matlab Code:

```
d=0.06;
g=9.806;
%rpm to radian per sec (rps)
rps = 2*pi/60;
%Liter to m^3
LtoM = 0.001;
%Bar to Pascal
BtoP = 100000;

mass = [0 350 450 550 650 750 850
1350 1750] / 1000; % Convert to kg

flow_rate = [10/19.92 10/20.95
10/24.07 10/27.18 10/31.91] *
LtoM; % in m^3/s
pressure = [0.5 0.6 0.8 1 1.2] *
BtoP; % in Pascal

% Spring mass (in kg)
spring1 = [0 0.15 0.19 0.25 0.3
0.35 0.4 0.66 1];
spring2 = [0 0.14 0.18 0.22 0.27
0.31 0.36 0.63 1.];
spring3 = [0 0.16 0.21 0.24 0.29
0.33 0.39 0.64 0.93];
spring4 = [0 0.16 0.21 0.24 0.29
0.32 0.38 0.65 0.89];
spring5 = [0 0.15 0.2 0.26 0.3 0.35
0.39 0.68 1];

% rotations (convert to radian per
sec)
```

```
rotation1 = [1369   1126    1085
1048    983.1    923.8   852.2
389.2   0] * rps;
rotation2 = [1539   1303    1244
1214    1152    1089    1039    545
0] * rps;
rotation3 = [1793   1550    1466
1432    1334    1293    1158
688.3   172.6] * rps;
rotation4 = [1988   1677    1606
1527    1440    1357    1255
692.5   212.1] * rps;
rotation5 = [2106   1678    1617
1515    1391    1292    1171
400.5   0] * rps;

%% Part 1 plot torque vs wheel
speed
T1 = (mass - spring1) * g * d/2;
T2 = (mass - spring2) * g * d/2;
T3 = (mass - spring3) * g * d/2;
T4 = (mass - spring4) * g * d/2;
T5 = (mass - spring5) * g * d/2;
disp('Torque for set1: ');
disp(T1);
disp('Torque for set2: ');
disp(T2);
disp('Torque for set3: ');
disp(T3);
disp('Torque for set4: ');
disp(T4);
disp('Torque for set5: ');
disp(T5);

figure
plot(rotation1,T1,rotation2,T2,rota
tion3,T3,rotation4,T4,rotation5,T5,
'LineWidth', 3)
xlabel ('wheel speed
[rad/s]','FontSize', 20)
ylabel('Torque [Nm]','FontSize',
20)
legend('0.5 bar','0.6 bar','0.8
bar','1 bar','1.2 bar');
title('Torque vs Wheel Speed');
set(gca,'FontSize',15);
grid on

%% Part 2 Calculate power input
Pi = pressure .* flow_rate;
disp('Power input: ');
disp (Pi);

%% Part 3 Plot Power output vs
wheel speed
Po1 = T1 .* rotation1;
```

```
Po2 = T2 .* rotation2;
Po3 = T3 .* rotation3;
Po4 = T4 .* rotation4;
Po5 = T5 .* rotation5;

disp('power output 1: ');
disp(Po1);
disp('power output 2: ');
disp(Po2);
disp('power output 3: ');
disp(Po3);
disp('power output 4: ');
disp(Po4);
disp('power output 5: ');
disp(Po5);

figure
plot(rotation1,Po1,rotation2,Po2,ro
tation3,Po3,rotation4,Po4,rotation5
,Po5,'LineWidth', 3)
xlabel ('wheel speed
[rad/s]','FontSize', 20)
ylabel('Power Output
[W]','FontSize', 20)
legend('0.5 bar','0.6 bar','0.8
bar','1 bar','1.2 bar')
title('Power vs Wheel Speed')
set(gca,'FontSize',15);
grid on

%% Part 4 Plot efficiency vs wheel
speed
eff1 = Po1 / Pi(1) * 100; %
percentage
eff2 = Po2 / Pi(2) * 100;
eff3 = Po3 / Pi(3) * 100;
eff4 = Po4 / Pi(4) * 100;
eff5 = Po5 / Pi(5) * 100;

figure
plot(rotation1,eff1,rotation2,eff2,
rotation3,eff3,rotation4,eff4,rotat
ion5,eff5,'LineWidth', 3)
xlabel ('wheel speed
[rad/s]','FontSize', 20)
ylabel('efficiency [%]','FontSize',
20)
legend('0.5 bar','0.6 bar','0.8
bar','1 bar','1.2 bar')
title('Efficiency vs Wheel Speed')
set(gca,'FontSize',15);
grid on

%% Uncertainty
uT = 0.002943;
```

```
time = [19.92 20.95 24.07 27.18
31.91];
uQ = sqrt((0.0005./time).^2+(-0.005
* 0.01 ./ time.^2).^2);
disp('uncertainty for Q: ');
disp(uQ);

uPi = sqrt((flow_rate * 5000).^2 +
(pressure .* uQ).^2);
disp('Uncertainty for Pi: ');
disp(uPi);

u_rpm = [897.2 903.4 898.7 896.9
901.8 888.9 890.1 891.7 893.7 894.5
898.6 898.8 909.3 911.9 908.1] *
rps;
s_x = std(u_rpm);
disp('s_x: ')
disp(s_x);

s_x_bar = s_x / sqrt(15);
disp('s_x_bar: ');
disp(s_x_bar);

u_N = sqrt((0.5*rps)^2 + (2.145 *
s_x_bar)^2);
disp('u_N: ');
disp(u_N);

u_Po1 = sqrt((rotation1 * uT).^2 +
(T1 * u_N).^2);
u_Po2 = sqrt((rotation2 * uT).^2 +
(T2 * u_N).^2);
u_Po3 = sqrt((rotation3 * uT).^2 +
(T3 * u_N).^2);
u_Po4 = sqrt((rotation4 * uT).^2 +
(T4 * u_N).^2);
u_Po5 = sqrt((rotation5 * uT).^2 +
(T5 * u_N).^2);

u_eff1 =
sqrt(((1/Pi(1,1))*u_Po1).^2 + ((-
Po1/Pi(1,1)^2).*uPi(1,1)).^2);
u_eff2 =
sqrt(((1/Pi(1,2))*u_Po2).^2 + ((-
Po2/Pi(1,2)^2).*uPi(1,2)).^2);
u_eff3 =
sqrt(((1/Pi(1,3))*u_Po3).^2 + ((-
Po3/Pi(1,3)^2).*uPi(1,3)).^2);
u_eff4 =
sqrt(((1/Pi(1,4))*u_Po4).^2 + ((-
Po4/Pi(1,4)^2).*uPi(1,4)).^2);
u_eff5 =
sqrt(((1/Pi(1,5))*u_Po5).^2 + ((-
Po5/Pi(1,5)^2).*uPi(1,5)).^2);

figure
errorbar (rotation1, eff1, 100 *
u_eff1,'linewidth',3)
hold on
errorbar (rotation2, eff2, 100 *
u_eff2,'linewidth',3)
hold on
errorbar (rotation3, eff3, 100 *
u_eff3,'linewidth',3)
hold on
errorbar (rotation4, eff4, 100 *
u_eff4,'linewidth',3)
hold on
errorbar (rotation5, eff5, 100 *
u_eff5,'linewidth',3)
xlabel ('wheel speed
[rad/s]','FontSize', 20)
ylabel('efficiency [%]','FontSize',
20)
legend('0.5 bar','0.6 bar','0.8
bar','1 bar','1.2 bar')
title('Efficiency vs Wheel Speed')
set(gca,'FontSize',15);
grid on
```

Set 1	Pressure (Barr)	0.5		Flowrate (L/s)	0.502008032				
Mass(kg)	0	350	450	550	650	750	850	1350	1750
Spring Mass (kg)	0	0.15	0.19	0.25	0.3	0.35	0.4	0.66	**1**
Rotation (rpm)	1369	1126	1085	1048	983.1	923.8	852.2	389.2	0

Set 2	Pressure (Barr)	0.6		Flowrate (L/s)	0.477326969				
Mass(kg)	0	350	450	550	650	750	850	1350	1750
Spring Mass (kg)	0	0.14	0.18	0.22	0.27	0.31	0.36	0.63	1
Rotation (rpm)	1539	1303	1244	1214	1152	1089	1039	545	0

Set 3	Pressure (Barr)	0.8		Flowrate (L/s)	0.415454923				
Mass(kg)	0	350	450	550	650	750	850	1350	1750
Spring Mass (kg)	0	0.16	0.21	0.24	0.29	0.33	0.39	0.64	0.93
Rotation (rpm)	1793	1550	1466	1432	1334	1293	1158	688.3	172.6

Set 4	Pressure (Barr)	1		Flowrate (L/s)	0.367917586				
Mass(kg)	0	350	450	550	650	750	850	1350	1750
Spring Mass (kg)	0	0.16	0.21	0.24	0.29	0.32	0.38	0.65	0.89
Rotation (rpm)	1988	1677	1606	1527	1440	1357	1255	692.5	212.1

Set 5	Pressure (Barr)	1.2		Flowrate (L/s)	0.313381385				
Mass(kg)	0	350	450	550	650	750	850	1350	1750
Spring Mass (kg)	0	0.15	0.2	0.26	0.3	0.35	0.39	0.68	**1**
Rotation (rpm)	2106	1678	1617	1515	1391	1292	1171	400.5	0

uncertainties	
Time	0.005
Volume	0.5
Pressure	0.05
Spring Force	0.01
Tachometer	TBD

N	Tach Uncertainties
1	897.2
2	903.4
3	898.7
4	896.9
5	901.8
6	888.9
7	890.1
8	891.7
9	893.7
10	894.5
11	898.6
12	898.8
13	909.3
14	911.9
15	908.1