Yuhao Dong

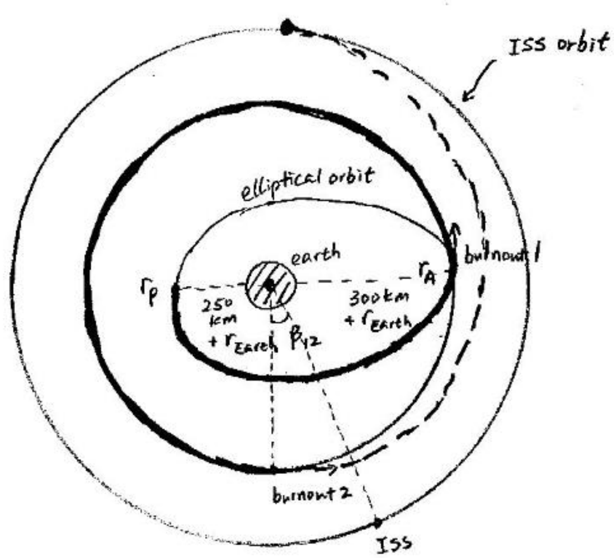
Orbital Mechanism

Design Project

12/12/18

As the problem stated, the spacecraft will be launched at the Kennedy Space Center in Florida. The launch date is June 15, 2019 and the launch location is 28.6266 ^{o}N, 80.6205 ^{o}W. There is no plane change required, and the launch site should lie in the plane of the orbit of the ISS. When T +580 sec, 1650 km downrange of the launch site, spacecraft is into the perigee of a 250 x 300 km elliptical orbit.

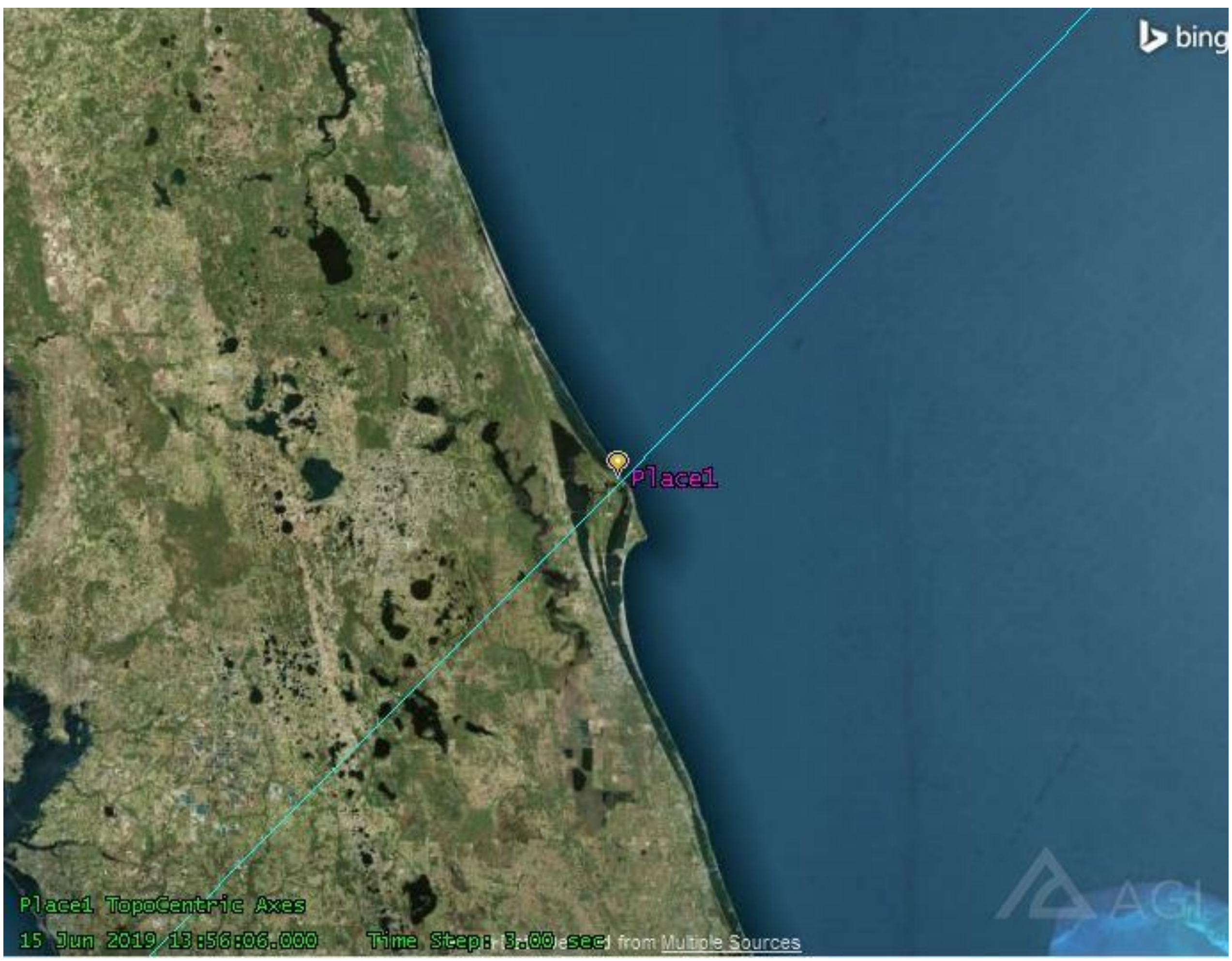
Plan of the project: The design I have for this project will have two burns (modified the design and explained the reason in later section). The spacecraft will continue flying (without adding or changing speeds) when it is into the perigee of the orbit. Once it reaches the apogee of the orbit, the first burn starts, which will make the spacecraft fly into a 300 km circular orbit (from elliptical to circular). Once the β_{12} between the ISS and the spacecraft matches with the desired angle, the 2nd burn starts. The spacecraft will go through 1/2 of the elliptical orbit to reach the ISS.

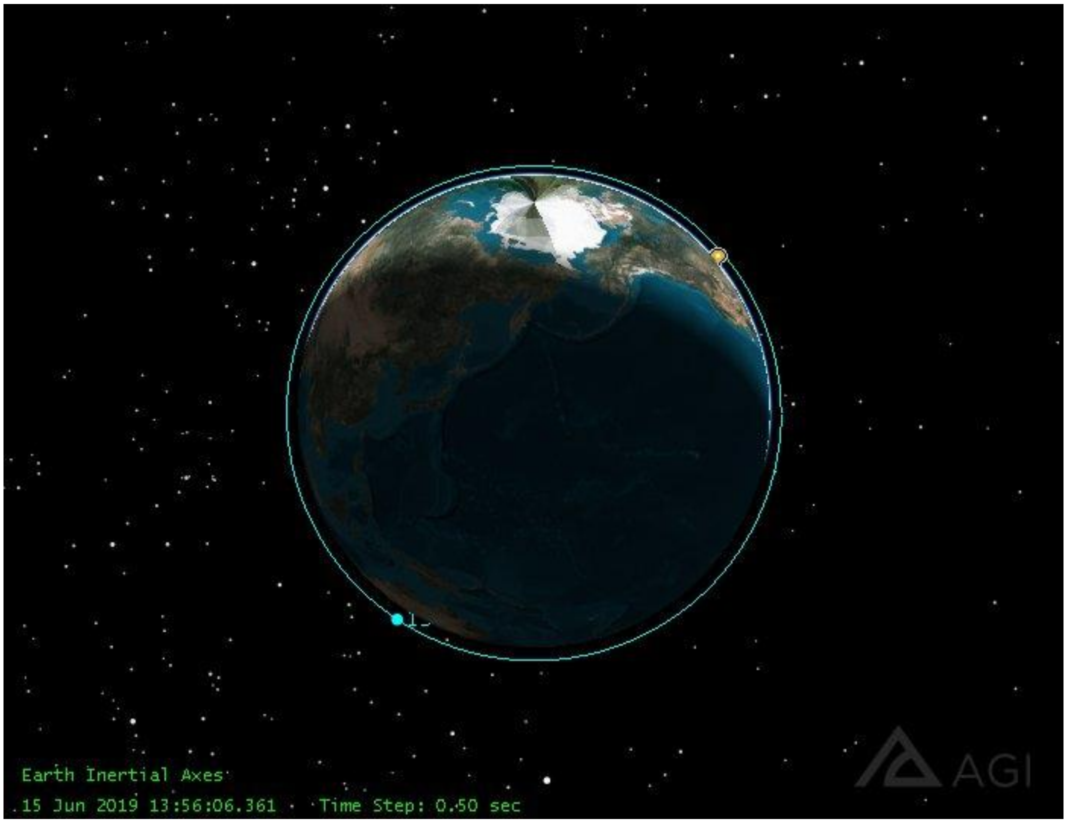


The advantage for this design is that since the two circular orbits, intermediate orbit (300 km) and the orbit for ISS (408 km) are very close, the velocity change will be very little. Also when they are docking, the velocity difference will also be very small. However, the disadvantage for this design is that since the velocity for these two orbits are very close to each other, the waiting time for they to have the specific β_{12} angle will be very long.

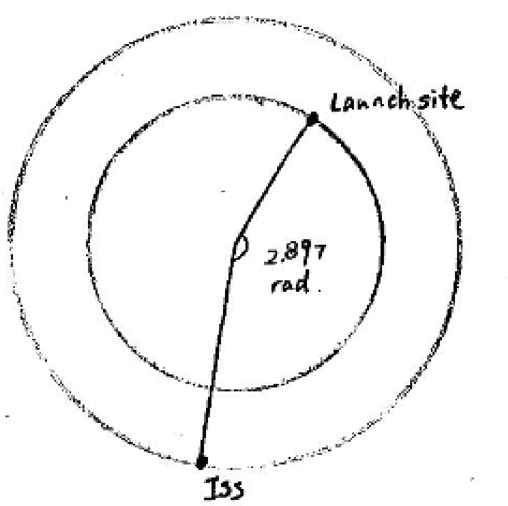
Assumption: The radius of Earth I used for this design is the average radius of the Earth (6368 km). Also I assumed the orbit for the ISS is a circular orbit with altitude of 408 km. Moreover, I assumed the two orbits (intermediate orbit for spacecraft and ISS orbit) will stay in the same plane.

1. Start time





By using the STK simulation software, we can find the time for the launch site lies on the orbit of the ISS and the angle between the ISS and the launch site at this time. The time is June 15, 2019 13:56:06 UTC time, which is same as 8:56:06 EDT time, and + 0 sec for the Mission Elapsed Time. The initial angle for them is 166^{o}, which is equal to 2.897 radian.



2. Reach the perigee

After 580 seconds, the spacecraft will enter the perigee of a 250 x 300 km elliptical orbit. The downrange is 1650 km of the launch site. The time when it enters the perigee of this orbit is June 15, 2019 9:05:46 EDT time, + 580 sec MET.

For the spacecraft:

$$580\, sec = 9\, min\, 40\ \ sec$$

$$radius\ of\ earth = 6368\ km$$

$$s = 1650\ km$$

$$s = r\ \theta$$

$$1650 = 6368\ \theta$$

$$\theta = 0.2911\, rad = 14.8458^{o}$$

The spacecraft moved 14.8458º from the launch site.

For the ISS:

$$r = 6368 + 408 = 6776\ km$$

$$V_c = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{3.986 \times 10^5}{6776}} = 7.66976\,\frac{km}{s}$$

$$r\ \omega = \mathrm{V}$$

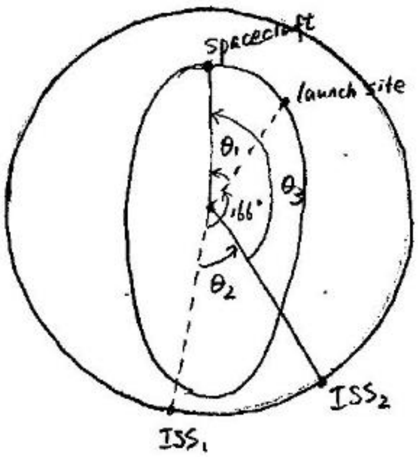
$$\omega = \frac{7.66976}{6776} = 0.001132\,\frac{rad}{s}$$

the time is 580 sec, so the angle it travelled is:

$$0.001132\ \times 580 = 0.65656\, rad = 37.62°$$

The angle between the spacecraft and the ISS is

$$166° - (37.62° - 14.8458°) = 143.2277°$$



$$\theta_1 = 14.8458°$$
$$\theta_2 = 37.62°$$
$$\theta_3 = 166° - (\theta_2 - \theta_1)$$
$$= 143.2277°$$

3. Wait until it reaches apogee

For the elliptical orbit,

$$r_p = 6368 + 250 = 6618\ km$$

$$r_A = 6368 + 300 = 6668\ km$$

$$T = 2\,\pi \sqrt{\frac{a^3}{\mu}}$$

$$2a = r_p + r_A$$

$$a = 6643\ km$$

$$T = 2\,\pi \sqrt{\frac{6643^3}{3.986 \times 10^5}} = 5388.4\ sec$$

$$\frac{1}{2}T = 2694\ sec = 44\ min\ 54\ sec$$

It will take 2694 seconds for the spacecraft to move from the perigee to apogee. The time for this stage will be June 15, 2019 9:50:40 EDT time, + 3274 sec MET.

For the ISS

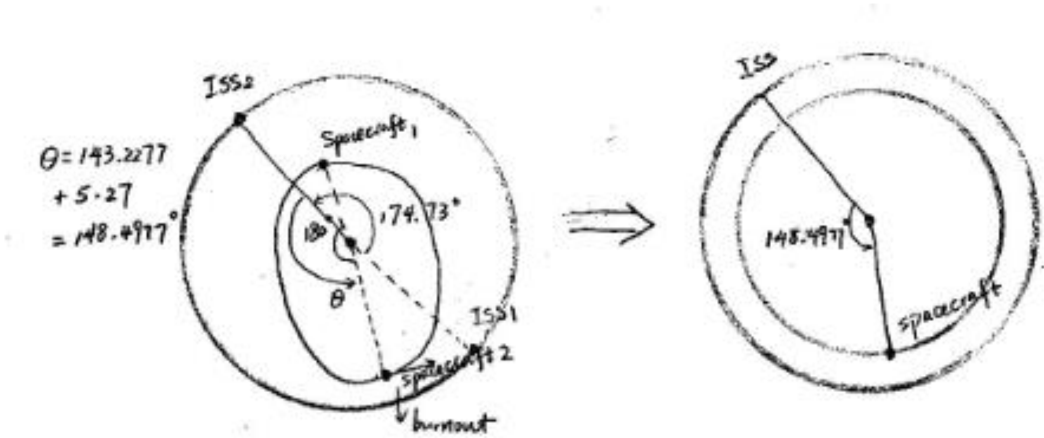
$$0.001132 \times 2694 = 3.0496\ rad = 174.73\ °$$

$$\Delta\theta = 180 - 174.73 = 5.27°$$

The angle between the spacecraft and the ISS will become

$$143.2277° + 5.27° = 148.4977°$$

At this point, the first burnout occurred (ΔV will be calculated in later section), and the orbit for the spacecraft is changing from the elliptical orbit to a 300 km radius circular orbit.



MODIFICATION OF THE DESIGN

The velocity difference when the spacecraft and the ISS when they are docking are relatively large if the spacecraft is having a 300 * 408 elliptical orbit while ISS is having a 408 km circular orbit. The velocities can be calculated by using equations below:

For ISS:

$$V_{iss} = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{3.986 \times 10^5}{6776}} = 7.66976 \frac{km}{s}$$

For Spacecraft:

$$V_s = V_{iss}\sqrt{1 - e}$$

Where

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{408 - 300}{408 + 300 + 6368 \times 2} = 0.0083$$

$$V_s = V_{iss}\sqrt{1 - 0.0083} = 7.63889\frac{km}{s}$$

The velocity difference is

$$7.66976 - 7.63889 = 0.03087\frac{km}{s} = 30.87\frac{m}{s}$$

30 m/s is very large, so the design needs to be modified.

I decide to make another burn for the spacecraft, which allows the spacecraft to fly to 407.5 km away from the surface of the earth (408 km is the distance for ISS). It is very close to the ISS orbit, so the velocity difference will be very close. I kept the 300 km orbit because the time for the next turn can be adjusted in this orbit. The 307.5 km orbit is too close to the 408 km ISS orbit so their velocities are very similar. If the docking time is in the night, they will wait a very long time until the next available time. Therefore, the time should be adjusted when the spacecraft is located in the 300 km orbit.

The velocity for the 407.5 * 408 km orbit at its apogee:

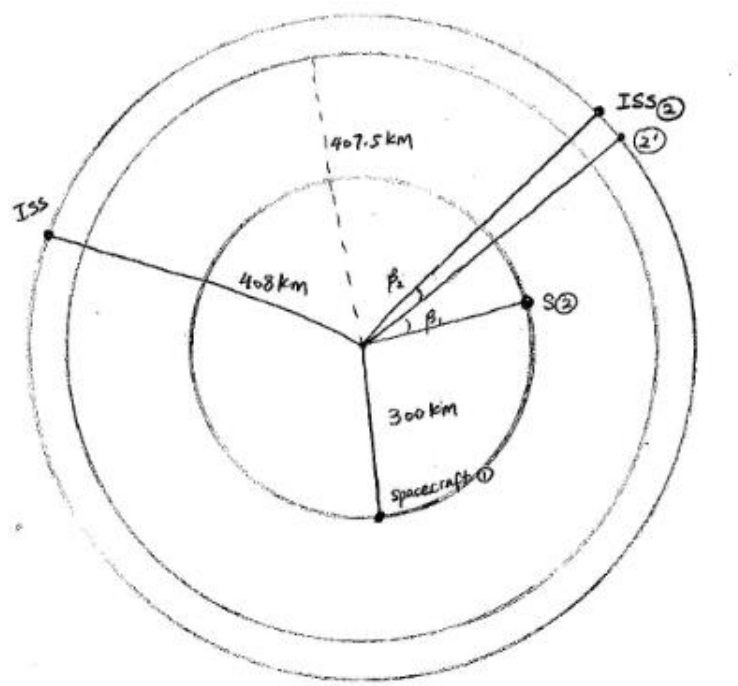
$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{408 - 407.5}{408 + 407.5 + 6368 \times 2} = 3.68963 \times 10^{-5}$$

$$V_s = V_{iss}\sqrt{1 - e} = 7.66962\frac{km}{s}$$

The velocity difference is

$$7.66976 - 7.66962 = 1.4\ \times 10^{-4}\frac{km}{s} = 0.14\frac{m}{s}$$

0.14/s is reasonable for the spacecraft to dock with ISS.



4. β1

$$R = \frac{r_2}{r_1} = \frac{6775.5}{6668} = 1.0161$$

From inner orbit to outer orbit

$$\beta = \pi \left[1 - \left(\frac{1+R}{2R}\right)^{\frac{3}{2}}\right] = 0.037309\, rad = 2.1377°$$

5. β2

$$R = \frac{r_2}{r_1} = \frac{6776}{6775.5} = 1.0000738$$

$$\beta = \pi \left[1 - \left(\frac{1+R}{2R}\right)^{\frac{3}{2}}\right] = 1.738616 \times 10^{-4} rad = 0.00996°$$

6. Total waiting time

$$n_{ISS} = \omega = 0.001132 \frac{rad}{s}$$

$$n_s = \sqrt{\frac{\mu}{r_s}} = \sqrt{\frac{3.986 \times 10^5}{6668^3}} = 0.0011595\,\frac{rad}{s}$$

$$2\pi + \alpha = n_s(\Delta t)$$

$$\frac{(148.4977 + 2.1377 + 0.00996)\pi}{180} + \alpha = n_{iss}(\Delta t)$$

Then solve for Δt:

$$2.62926 + \alpha = 0.001132\,(\Delta t)$$

$$2\pi + \alpha = 0.0011595(\Delta t)$$

$$\Delta t = 132868\,\mathrm{sec} = 36\,hr\;53\,\mathrm{min}\;28\,sec$$

The waiting time is 132868 seconds, so the time for this stage will be June 16, 2019 22:45:08 EDT time, + 136142 sec MET.

7. Time for spacecraft to travel from 300 km orbit to 407.5 km orbit:

$$a = \frac{r_p + r_a}{2} = \frac{6775.5 + 6668}{2} = 6721.75\,km$$

$$T = 2\,\pi\sqrt{\frac{a^3}{\mu}}$$

$$\Delta t = \frac{1}{2}T = \pi\sqrt{\frac{a^3}{\mu}}$$

$$\Delta t = \pi\sqrt{\frac{6721.75^3}{3.986 \times 10^5}} = 2742.471\,sec = 45\,min\;42\,sec$$

The time for this stage is June 17, 2019 4:30:50 UTC time, which is equal to June 16, 2019 23:30:50 EDT time.

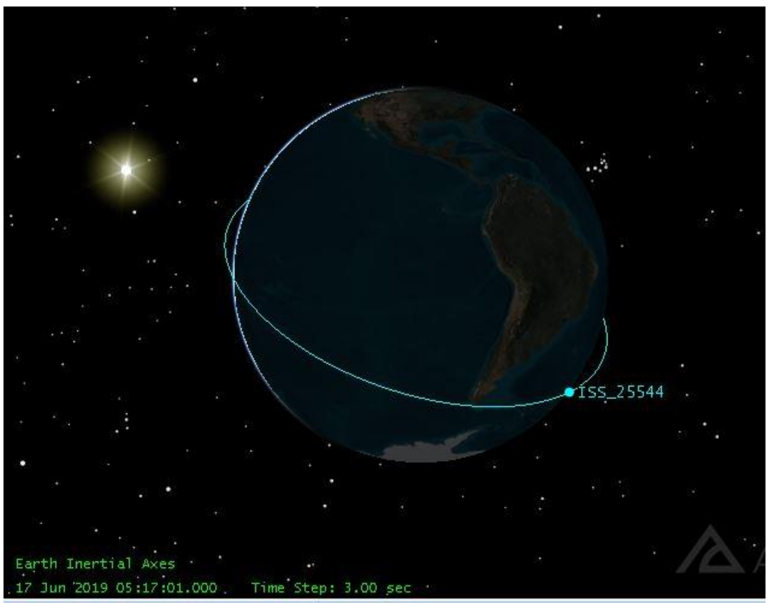
8. Time for spacecraft to travel from 407.5 km orbit to 408 km orbit

$$= \frac{r_p + r_a}{2} = \frac{6775.5 + 6776}{2} = 6775.75\ km$$

$$\Delta t = \frac{1}{2}T = \pi \sqrt{\frac{a^3}{\mu}} = 2775.347 sec = 46\ min\ 15\ sec$$

The time for this stage is June 17, 2019 5:17:05 UTC time

However, during this time, the ISS is behind of earth, so there is no sun light on the ISS. Therefore, it is not a proper time to rendezvous and docking with the ISS. We will need to wait for the next turn.



9. Waiting time until next turn

The inner spacecraft will have to run one additional orbit (angle of 2π)

$$2\pi = (n_s - n_{iss})(\Delta t)$$

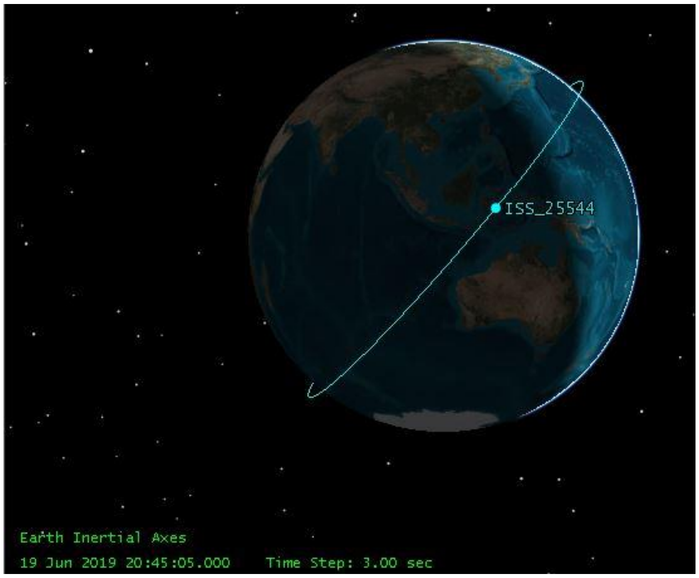
$$2\pi = (0.0011595 - 0.001132)(\Delta t)$$

$$\Delta t = 228479.47\ sec$$

10. Next try to reach ISS orbit.

$$228479.47 + 136142 + 2742.471 + 2775.347 = 370139\ sec$$

370139 sec from the starting time will be June 19, 2019 20:45:05 UTC time, which is also not a proper time, so we need to wait for one more turn



11. Next try to reach ISS orbit.

$$370139 + 228479 = 598618 \; sec$$

598618 sec from the starting time will be June 22, 2019 12:13:04 UTC time, which the ISS is in front of the earth. Therefore, the final time when the spacecraft reaches ISS is June 22, 2019 12:13:04 UTC time, which is equal to June 22, 2019 7:13:04 EDT time.

12. Burns

a. Burn 1: From ellipse to 300 km circular orbit

$$\Delta V_A = \sqrt{\frac{\mu}{r_2}}\left[1 - \sqrt{1-e}\right]$$

Where

$$e = \frac{r_a - r_p}{r_a + r_p} = \frac{6668 - 6618}{6668 + 6618} = 0.0037634$$

$$\Delta V_A = \sqrt{\frac{3.986 \times 10^5}{6668}}\left[1 - \sqrt{1 - 0.0037634}\right] = 0.014562\,\frac{km}{s}$$

b. Burn 2: From 300 km circular orbit to 407.5 km orbit

$$\Delta V_p = \sqrt{\frac{\mu}{r_1}}\left[\sqrt{1+e} - 1\right]$$

Where

$$e = \frac{r_2 - r_1}{r_2 + r_1} = \frac{6775.5 - 6668}{6775.5 + 6668} = 0.0079964$$

$$V_p = \sqrt{\frac{3.986 \times 10^5}{6668}}\left[\sqrt{1 + 0.0079964} - 1\right] = 0.030851\,\frac{km}{s}$$

MODIFICATION OF THE DESIGN

Once the spacecraft reaches the 407.5 km orbit, it will not have a burn to circularize this orbit. Instead, it will directly have a burn to make it transfer from the apogee of 300 * 407.5 km orbit to 407.5 * 408 km elliptical orbit to dock with ISS

$$V_{a-inner} = V_c\sqrt{1-e}$$

Where

$$V_c = \sqrt{\frac{\mu}{407.5 + 6368}} = 7.67\,\frac{km}{s}$$

$$e = \frac{r_a - r_p}{r_a + r_p} = 0.01519$$

$$V_{a-inner} = 7.61151\,\frac{km}{s}$$

$$V_{p-outer} = V_c\sqrt{1 + e}$$

Where

$$V_c = 7.67\,\frac{km}{s}$$

$$e = \frac{r_a - r_p}{r_a + r_p} = 3.68963 \times 10^{-5}$$

$$V_{p-outer} = 7.67014\,\frac{km}{s}$$

$$\Delta\mathrm{V} = 7.67014 - 7.61151 = \ 0.05863\,\frac{km}{s}$$

$$V_{tot} = 0.014562 + 0.030851 + 0.05863 = 0.104043\,\frac{km}{s}$$

Timeline of events

	EDT time	MET time (sec)	description	ΔV
Event 1	June 15, 2019 8:56:06	+ 0	The spacecraft is launched from the Kennedy Space Center	
Event 2	June 15, 2019 9:05:46	+ 580	The spacecraft enters the perigee of the 250 x 300 km elliptical orbit	
Event 3	June 15, 2019 9:50:40	+ 3274	The spacecraft enters the apogee of the 250 x 300 km elliptical orbit. As soon as it enters the apogee, the first burnout occurred, which will change the orbit to a 300 km circular orbit.	$0.014562\frac{km}{s}$
Event 4	June 22, 2019 5:42:06	+ 593100	The second burnout occurred, which transferred the spacecraft from 300 km circular orbit to 300 * 407.5 km elliptical orbit.	$0.030851\frac{km}{s}$
Event 5	June 22, 2019 6:27:48	+ 595842	The third burnout occurred. The spacecraft is moving from the apogee of the 300 * 407.5 km elliptical orbit to the ISS orbit.	$0.05863\frac{km}{s}$
Event 6	June 22, 2019 7:13:04	+598618	The spacecraft rendezvoused and docked with the ISS	
				$V_{tot} = 0.104043\frac{km}{s}$