Chapter 8: Constant Cross-section Fin

ME 436 Aerothermal Fluids Laboratory
Yuhao Dong
Report 3, Group 5

11/01/2018
Mechanical Engineering Department
The City College of New York, USA

Abstract

In this lab, the fin is analyzed. When the heat is added to the fin, the heat passes through the entire fin, and the heat is spread to the surrounding through the fin. Basically, a fin can help the system (for example, a computer) to move the more heat away, and move it away faster. As the length of the fin gets farther away from the heat source, the temperature of the heat reduces at those locations. For this lab, we used the voltage of 39.92 V (almost 40 V) for the heat source. Also we used 8 pairs of thermocouples to measure the temperature distribution along the fin. The locations of the thermocouples are 65 mm, 110 mm, 180 mm, 280 mm, 394 mm, 504 mm, and 605 mm. The eighth thermocouple is measuring the ambient temperature. We set the time to be 30 minutes and used the computer to collect the data from the thermocouples. Once we have the time and the temperature, we could find the heat transfer coefficient. We could also calculate the theoretical heat transfer coefficient. The experimental heat transfer coefficient is $27.4388 \pm 15.8657\ \frac{W}{m^2C}$, and the theoretical heat transfer coefficient is $37.35\ \frac{W}{m^2C}$. We could also find the steady temperature for each location.

Introduction

The fins are the extended surfaces in a system. The fins can assist heat in moving away from the system by increasing the heat transfer rate. There are many different shapes of fins. Each of them could be using different equations to analysis. The shapes include annular, straight and spines. Usually the fins are very thing and very long. Moreover, the materials used for the fins are usually with high conductivity so the Biot number is very small. This could simplify the analysis because we could just assume that the fin temperature varies only along the fin, and it will be independent of the transverse direction.

The governing equation for these type of fins (constant cross-section fins) for steady state heat transfer is:

$$\frac{d^2T}{dx^2} - \frac{hC}{kA}(T - T_\infty) = 0 \quad (1)$$

T is the temperature and T_∞ is the ambient temperature. H means the heat transfer coefficient, k means the thermal conductivity, and C is the fin circumference. Based on the equation above, we could find the temperature distribution T = T(x) along the fin. And we could use that to calculate the total heat loss from the fin:

$$q_{loss} = \bar{h}\int_0^L (T - T_\infty)Cdx \quad (2)$$

We could determine the theoretical temperature by using equation below:

$$T(x,t) = T_{amb} \dots + \frac{q_o''}{k}\left\{\left[\left(\frac{e^{m(x-2L)} + e^{-mx}}{m(1-e^{-2mL})}\right) \dots - 2e^{m^2at}\left(\frac{1}{2m^2L} \dots + L\sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi}{L}x\right) e^{\frac{-n^2\pi^2}{L^2}\alpha t}}{m^2L^2 + n^2\pi^2}\right)\right]\right\} \quad (3)$$

In this equation, q_o'' is the power of the heater per unit area $q_o'' = \frac{V^2}{(A_c\mathrm{R})}$. Resistance is 178 ohms. K is the thermal conductivity of copper, which is equal to $401\frac{\mathrm{W}}{mk}$. m = $\sqrt{\frac{hc}{kA_c}}$ where C is the circumference of the fin, Ac is the cross-sectional area of the fin. L is the length, which is 0.605 m, and α is the thermal diffusivity, which is equal to $\alpha = \frac{\mathrm{k}}{\rho C_\rho}$ where $\rho = 8960 \frac{\mathrm{kg}}{\mathrm{m}^3}$ and $C_\rho = 386\frac{J}{kg\ K}$.

Experimental Setup and Procedure

For this lab, the experiment setups are quite simple. We have a circular constant cross-section fin which we will analysis. We also have 7 thermocouples attached on the fin. The location for thermocouple 1 is at x = 65 mm. The location for thermocouple 2 is at x = 110 mm. The location for thermocouple 3 is at x = 180 mm. For thermocouple 4 is at x = 280 mm. For thermocouple 5 is at x = 394 mm. For thermocouple 6 is at x = 504 mm, and for thermocouple 7 is at x = 605 mm. Thermocouple 8 measures the ambient temperature. We have an old computer for data analysis. There is a power source to supply the heat. If the experiment failed, we have a small fan to cool the fin. Moreover, we have a multimeter to measure the voltage we have. The voltage we have for our experiment is 39.92 V.

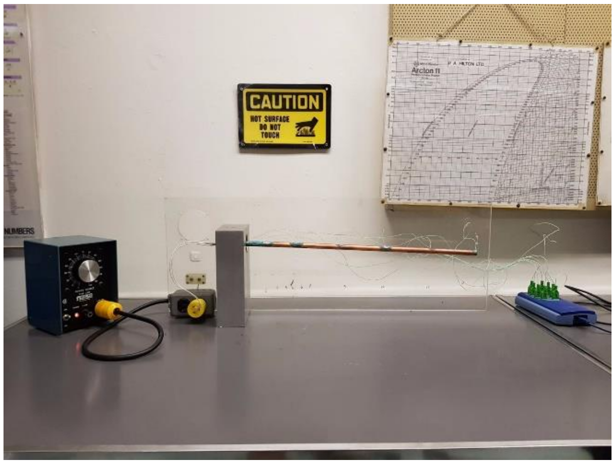

Figure 1: Overview of the set up. Object from left to right are: Power source, fin with thermocouple, and connection to the computer.

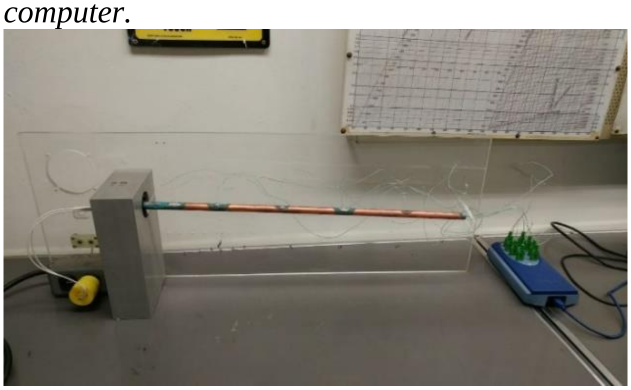

Figure 2: Fin with thermocouples attached.

Figure 3: Power supply with voltage about 40V.

First, we need to set the VariAC position to zero. We also need to check that all thermocouples are secured and in the right place. Then we start the Data Acquisition software. We first test the software by running it for 1 minutes without adding heat to the fin. Then we stopped it and step the sampling frequency to 1 Hz and set the sampling duration to be 30 minutes. The exact voltage we have is 39.92 V. While we start to collect data, we plug in the power to heat the fin at the same time. If the experiment failed during this process, we have to use the fan to cool the fin to the room temperature and start it over again. Once all data are collected by the software, we save the file.

Results

Table 1 showed both experimental and theoretical heat transfer coefficients. The experimental heat transfer coefficient we have is $27.4388 \pm 15.8657\ \frac{W}{m^2C}$, and the theoretical heat transfer coefficient is $37.35\frac{W}{m^2C}$. They should match with each other. However, by comparing them, we can see that they are not alike. There could be some error occurred during the experiment, which I will discuss more in detail in the conclusion section. Also it could be the uncertainty, which is very large ($\pm 15.8657\ \frac{W}{m^2C}$). Figure 4 shows the steady theoretical and experimental temperature vs length of the fin. As we can see, when the position is getting farther away from the heat source, the temperature is getting lower. The experimental temperatures are all greater than the theoretical temperatures. Figure 5 - 11 are the figures showing the temperature vs time for different locations. All graphs showed that the experimental temperature did not reach steady state after 30 minutes. All of the temperatures are continuously increasing, and the theoretical results showed that all the temperatures should reach the steady state within 30 minutes, so there must be some errors happened during the experiment. The uncertainty for this experiment is very large because the thermocouple is not very accurate. The uncertainty is $\pm 15.8657 \ \frac{W}{m^2 C}$.

Conclusion

The fin is used to assist moving heat away from the system. By measuring the temperature distribution and knowing the material properties, we can calculate the heat transfer coefficient both experimentally and analytically. However, the results for experimental and theoretical are different in our experiment. It could be that the thermocouples are not sensitive since the thermocouples are glued on the fin. It also could be that the copper fin we used are not very good. It looked dirty, so there might be some dust or other stuff stuck on the fin with might affect the measuring of the heat transfer coefficient. The temperature should reach the steady state after measuring for 30 minutes, but the experimental result shows the opposite. The temperatures continuously increased. It might because of the same error as stated above. Overall, this experiment could be more accurate if the experiment setup is renewed and the ambient temperature is controlled (other experiments and the heat source might increase the overall temperature of the room).

List of References

[1] Goushcha, O. *Aero-Thermal Fluids Laboratory ME 43600*. The City College of New York, 2018

Appendix A

Table 1: Experimental and theoretical heat transfer coefficients

$\bar{h}_{experimental}$	$27.4388 \pm 15.8657\ \frac{W}{m^2C}$
$\bar{h}_{theoretical}$	$37.35\frac{W}{m^2C}$

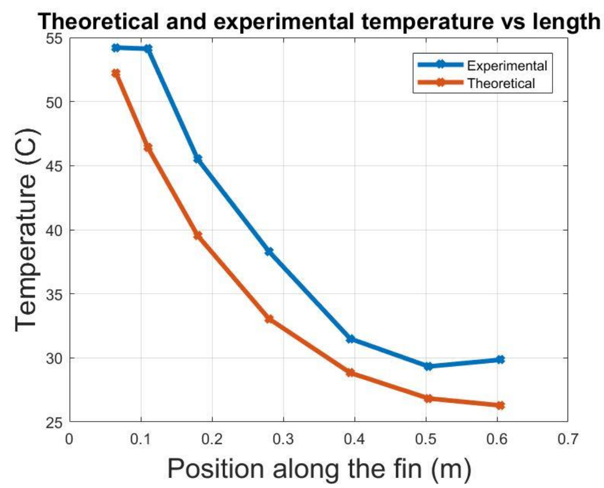

Figure 4: steady theoretical and experimental temperature distribution in the fin

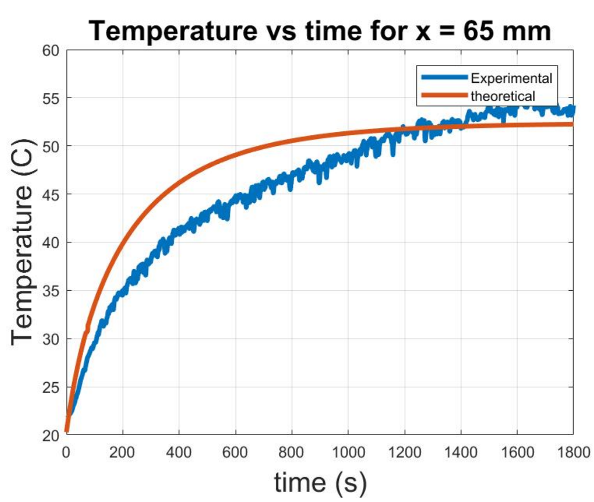

Figure 5: transient theoretical and experimental temperature for x = 65 mm

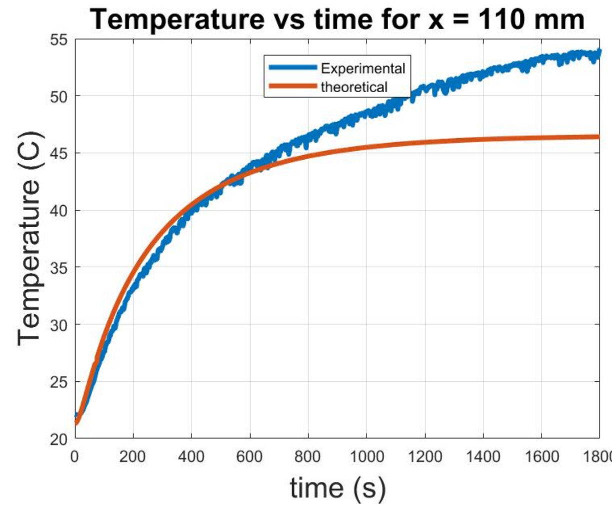

Figure 6: transient theoretical and experimental temperature for x = 110 mm

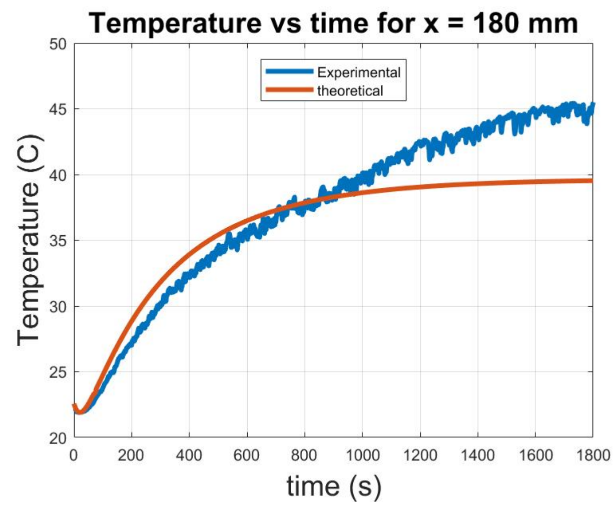

Figure 7: transient theoretical and experimental temperature for x = 180 mm

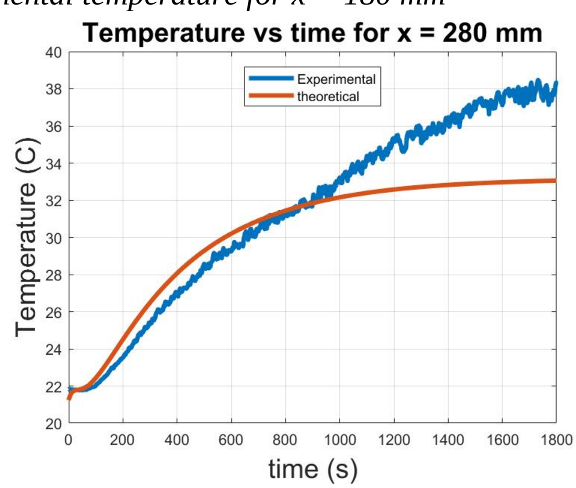

Figure 8: transient theoretical and experimental temperature for x = 280 mm

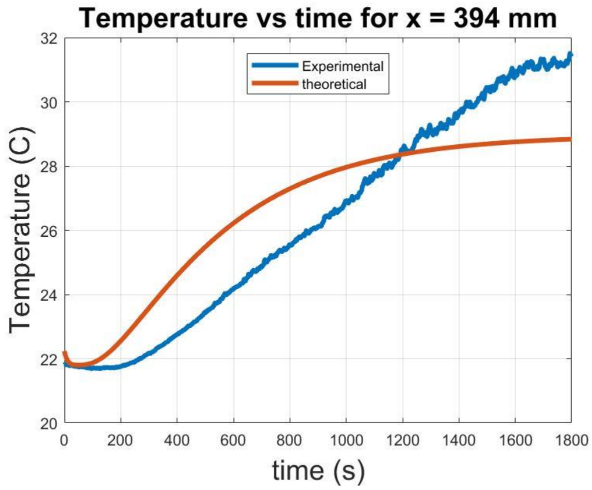

Figure 9: transient theoretical and experimental temperature for x = 394 mm

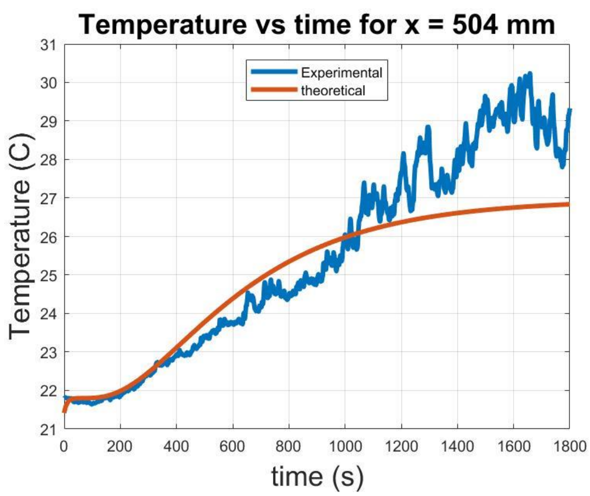

Figure 10: transient theoretical and experimental temperature for x = 504 mm

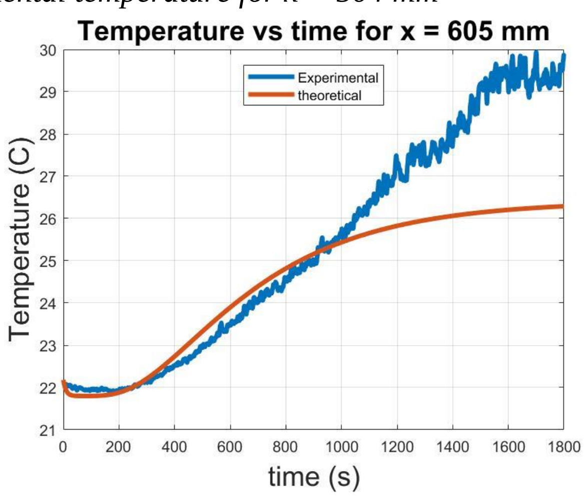

Figure 11: transient theoretical and experimental temperature for x = 605 mm

Appendix B

Calculations

$$T_\infty = 21.8 \ degree\ C$$

$$Voltage = 39.92\, V$$

$$R = 178\, \Omega$$

$$q = \frac{V^2}{R} = \frac{39.92^2}{178} = 8.95\, W$$

$$C = 2\, \pi\, r = 0.0377\, m$$

$$q = \bar{h}_{exp} \int_0^L (T_{ss}(x) - T_{amb}) C dx \quad (4)$$

$$\bar{h}_{exp} = \frac{q}{\int_0^L (T_{ss}(x) - T_{amb}) C dx} \quad (5)$$

Use trapezoid rule in matlab, we can find the result for the equation above.

$$\bar{h}_{exp} = 27.4388\ \frac{W}{m^2 C}$$

$$m = \sqrt{\frac{hC}{kA_c}} = \sqrt{\frac{27.4388\ (0.0377)}{401\ (1.131 * 10^{-4})}} \quad (6)$$

$$m = 4.776$$

$$q_o'' = \frac{V^2}{A_c R} = 7.9161 * 10^4\, \frac{W}{m^2} \quad (7)$$

$$\alpha = \frac{k}{\rho C_P} = \frac{401}{(\,8960)(386)} \quad (8)$$

$$\alpha = 1.1594 * 10^{-4}\, \frac{m^2}{s}$$

Theoretical temperature vales could be calculated by using equation below (Since the calculation is too complicated and involves many loops of calculation, the equation is solved by matlab).

$$T(x,t) \ldots$$
$$= T_{amb} \ldots$$
$$+ \frac{q_o''}{k}\left\{\left[\left(\frac{e^{m(x-2L)} + e^{-mx}}{m(1-e^{-2mL})}\right) \ldots\right.\right.$$
$$- 2e^{m^2 at}\left(\frac{1}{2m^2 L} \ldots\right.$$
$$\left.\left.\left. + L\sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi}{L}x\right) e^{\frac{-n^2\pi^2}{L^2}\alpha t}}{m^2L^2 + n^2\pi^2}\right)\right]\right\}$$

Based on the equation above, we can find the temperature distribution, and we could use the equation which used to calculate the experimental heat transfer coefficient to find the theoretical one.

$$q = \bar{h}_{theoretical} \int_0^L (T(x,t) - T_{amb}) C dx$$

$$\bar{h}_{theoretical} = \frac{q}{\int_0^L (T(x,t) - T_{amb}) C dx}$$

$$\bar{h}_{theoretical} = 37.35 \frac{W}{m^2 C}$$

Uncertainty

We need to find the uncertainty of the average experimental heat transfer coefficient (integration). The equation we have for the experimental heat transfer coefficient is

$$\bar{h}_{exp} = \frac{q}{\int_0^L (T_{ss}(x) - T_{amb}) C dx} \quad (10)$$

The uncertainty for the temperature is 0.005 degree C because the computer only stored 2 decimal places. The length is given in the manual, so we assume it does not have any uncertainty. Also the uncertainty for the thermocouple is about 1 ^{o}C.

$$q = \frac{V^2}{R}$$

And the Voltage has uncertainty of 0.005 V. We need to find the uncertainty of q first.

$$u_q = \sqrt{\left(\frac{\partial q}{\partial V} u_V\right)^2} = \frac{\partial q}{\partial V} u_V \quad (11)$$

$$\frac{\partial q}{\partial V} = \frac{2V}{R} = \frac{2\,(39.92)}{178} = 0.4485$$

$$\frac{\partial q}{\partial V} u_V = 0.4485 * 0.005 = 0.00224\, V$$

Both circumference and length are fixed (given in the manual so we assume they don't have any uncertainty neither). Therefore, the only two parts that affect the uncertainty of the heat transfer coefficient are the q and the temperature. Since $T_{ss}(x)$ contained 7 measurements, the uncertainty for $T_{ss}(x)$ is:

$$u_{Tss} = \sqrt{7\ {u_T}^2} = \sqrt{(7)(1.005)^2}$$

$$u_{Tss} = 2.659\, degree\ C$$

Moreover, there are still 2 temperatures exist ($T_{ss}(x)$ and T_{amb}), the total uncertainty for temperatures is:

$$u_{Ttotal} = \sqrt{{u_{Tss}}^2 + {u_T}^2}$$

$$u_{Ttotal} = \sqrt{2.659^2 + 1.005^2}$$

$$u_{Ttotal} = 2.8426\, degree\ C$$

Now we know that uncertainty for q is 0.00224 V and uncertainty for temperature is

$2.8426\ degree\ C$, we could find the uncertainty for the experimental heat transfer coefficient.

First, we need to find the uncertainty for the integration $\int_0^L (T_{ss}(x) - T_{amb}) C dx$. Since there are 7 different locations, there will be 7 different uncertainties.

$$u_{integral} = \sqrt{\left(\frac{\partial integral}{\partial T} u_T\right)^2} \qquad (12)$$

$$u_{integral} = \frac{\partial integral}{\partial T} u_T$$

$$\frac{\partial integral}{\partial T} = L\ C$$

Note: L equal to 7 different lengths.

$$u_{integral} = L\ u_T\ C$$

Uncertainties are listed below:

Table 2: Uncertainties for the integration

0.0070	0.0118	0.0193	0.0300
0.0422	0.0540	0.0648	

$$u_h = \sqrt{\left(\frac{\partial h}{\partial q} u_q\right)^2 + \left(\frac{\partial h}{\partial integral} u_{integral}\right)^2}$$

$$\frac{\partial h}{\partial q} = \frac{1}{\int_0^L (T_{ss}(x) - T_{amb}) C dx} = 4.1723$$

$$\frac{\partial h}{\partial integral} = -\frac{q}{\left(\int_0^L (T(x,t) - T_{amb}) C dx\right)^2}$$

$$\frac{\partial h}{\partial integral} = -155.8486$$

$$u_h = \sqrt{(4.1723\ u_q)^2 + (-155.8486\ u_{integral})^2}$$

where $u_q = 0.00224$

The uncertainties for heat transfer coefficient are listed below:

Table 3: Uncertainties for the heat transfer coefficient (unit: $\frac{W}{m^2C}$)

1.0856	1.8372	3.0063	4.6765
6.5805	8.4176	10.1045	

The average heat transfer coefficient is:

$$u_h = \sqrt{(u_{h1})^2 + (u_{h2})^2 + (u_{h3})^2 + (u_{h4})^2 + \cdots (u_{h5})^2 + (u_{h6})^2 + (u_{h7})^2}$$

$$u_h = 15.8657 \frac{W}{m^2C}$$

Matlab Code:

```
finfile=load('fin.txt');
Tamb=finfile(end,end); % 21.8
Tss=finfile(end,2:8); % 54.21,
54.12, 45.5, 38.3, 31.5, 29.33,
29.85
Ttotal=finfile(:,2:8);
time=finfile(:,1);
position=[65,110,180,280,394,504,60
5]/1000; % convert to m
V = 39.92;
R = 178; % Resistance given in the
manual
D = 12 / 1000; % diameter of the
fin
r = D / 2; % radius of the fin
q= V^2 / R;
C = 2 * pi * r;
hexp=q/(trapz(position,(Tss -
Tamb)*C));
disp('Experimental h: ');
disp(hexp);

Ac = pi*r^2;
qo = V^2 / (Ac * R);
k = 401;
m=sqrt(hexp*C/(Ac*k));
L = 0.605;
p = 8960;
Cp = 386;
a = k / (p*Cp);

for t=0:1800
```

```
    for x=1:7
        for n=1:5
            sumall(n) =
((cos((n*pi*position(x))/L))*(exp(-
((n^2)*(pi^2)*a*t)/L^2)))...

/(((m^2)*(L^2))+((n^2)*(pi^2)));
        end
        T(x,t+1) =
Tamb+qo/k*((exp(m*(position(x)-
2*L))+exp(-m*position(x)))...
        /(m*(1-exp(-2*m*L)))-
(2*(exp(-
(m^2)*a*t)))*((1/(L*2*m^2))+L*sum(s
umall)));
    end
end

htheoretical=q./(trapz(position,(T(
[1:7],end) - 21.8))*C);
disp('theoretical h: ');
disp(htheoretical);

figure
plot(position,Tss,'linewidth',2,'Ma
rker','x');
hold on
plot(position,T([1:7],end),'linewid
th',2,'Marker','x')
title('Steady Theoretical and
experimental temperature vs
length');
xlabel('Position along the fin
(m)')
ylabel('Temperature (C)')
legend('Experimental','Theoretical'
)

T = T.';

figure
plot(time,Ttotal(:,1),'linewidth',2
)
hold on
plot(time,T(:,1),'linewidth',2.5)
xlabel('time (s)')
ylabel('Temperature (C)')
legend('Experimental',
'theoretical');
title('Temperature vs time for x =
65 mm');

figure
plot(time,Ttotal(:,2),'linewidth',2
)
hold on
plot(time,T(:,2),'linewidth',2.5)
xlabel('time (s)')
ylabel('Temperature (C)')
legend('Experimental',
'theoretical');
title('Temperature vs time for x =
110 mm');

figure
plot(time,Ttotal(:,3),'linewidth',2
)
hold on
plot(time,T(:,3),'linewidth',2.5)
xlabel('time (s)')
ylabel('Temperature (C)')
legend('Experimental',
'theoretical');
title('Temperature vs time for x =
180 mm');

figure
plot(time,Ttotal(:,4),'linewidth',2
)
hold on
plot(time,T(:,4),'linewidth',2.5)
xlabel('time (s)')
ylabel('Temperature (C)')
legend('Experimental',
'theoretical');
title('Temperature vs time for x =
280 mm');

figure
plot(time,Ttotal(:,5),'linewidth',2
)
hold on
plot(time,T(:,5),'linewidth',2.5)
xlabel('time (s)')
ylabel('Temperature (C)')
legend('Experimental',
'theoretical');
title('Temperature vs time for x =
394 mm');

figure
plot(time,Ttotal(:,6),'linewidth',2
)
hold on
plot(time,T(:,6),'linewidth',2.5)
xlabel('time (s)')
ylabel('Temperature (C)')
legend('Experimental',
'theoretical');
title('Temperature vs time for x =
504 mm');
```

```
figure
plot(time,Ttotal(:,7),'linewidth',2
)
hold on
plot(time,T(:,7),'linewidth',2.5)
xlabel('time (s)')
ylabel('Temperature (C)')
legend('Experimental',
'theoretical');
title('Temperature vs time for x =
605 mm');


Tuncertainty = position * 2.8426 *
0.0377;
disp(Tuncertainty);
dhdq=1./(trapz(position,(T([1:7],en
d) - 21.8))*C);
disp(dhdq);
dhdintegral=-
q./((trapz(position,(T([1:7],end) -
21.8))*C)^2);
disp(dhdintegral);
uh = sqrt((4.1723*0.00224)^2 + (-
155.8486 .* Tuncertainty).^2);
disp('uncertainties for h: ');
disp(uh);
averageuncertainty =
sqrt(1.0856^2+1.8372^2+3.0063^2+4.6
765^2+6.5805^2+8.4176^2+10.1045^2);
disp('average uncertainty: ')
disp(averageuncertainty);
```