Chapter 6: Viscous Flow in a Pipe

ME 436 Aerothermal Fluids Laboratory
Yuhao Dong
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Mechanical Engineering Department
The City College of New York, USA

Abstract

In this experiment, the pipe flow is been analyzed. The pressures in different locations of the pipe are measured to show the pressure drop in the pipe as the air flows. The pressures in different cross-sectional areas (Venturi tube) are also measured. The axial pressure profile is measured as well, and the velocity profile could be calculated by using the pressure profile. The pressures are collected by using Venturi tube and Pitot tube and used an inclined manometer to measure the data. This project helped us to understand the principle of flow rate, the relationship between the mean velocity and the flow rate in Venturi tube, the relationship between flow rate in Venturi tube and the flow rate we measured by pitot tube. Moreover, we compared the experimental data with the theoretical data. Flow rate in Venturi tube is 0.0904 m^3/s, and the flow rate we measured by pitot tube is 0.13 m^3/s. The mean flow rate for the Venturi tube is $0.959\frac{m}{s}$. The maximum velocity for the theoretical is 7.267 $\frac{m}{s}$. Overall, the experimental data matches with the theoretical ones.

Introduction

Viscous flow in a pipe shows the internal flow which the flow is completely bounded with the pipe surface. As a result, the velocity should be infinitely small (approaching to zero and eventually becomes zero at the boundary point) at the boundary and be largest at the center of the pipe. In this experiment, a round pipe is used. In principle, when the gas, in this experiment, air, is passing through the pipe, the pressure is dropping along the tube. The reason is that the energy is required for the air to overcome any frictional or viscous forces exerted by the pipe on the air. These energy losses contain two parts: head loss and minor loss. By measuring the pressure in different location of the tube, we could find the pressure diagram to analyze the pressure drop. Moreover, if the cross-sectional area is changed, the pressure will change with it as well. In the experiment, we need to measure the pressures along the pipe, the pressure for the Venturi and we can find the mean flow rate as well. The mean flow rate could also be calculated by using the equation below:

$$U_{Mean} = \sqrt{\frac{2}{\left(\frac{d_a}{d_b}\right)^4 - 1}\left(\frac{\Delta P_{Venturi}}{\rho}\right)} \qquad (1)\ [1]$$

Where d_a is the diameter of the pipe, d_b is the diameter of the Venturi tube.

The velocity profile is also very important. We could use the velocity profile to analyze the shear stress in the flow. Moreover, velocity profiles for laminar flow and turbulent flow are different. The type of flow could be determined by using equation below:

$$Re = \frac{VD}{v} \qquad (2)$$

Where Re is the Reynolds number, V is the average velocity, D is the diameter, and v is the kinematic viscosity of the fluid. Theoretically, the laminar flow (Re < 2000) velocity distribution could be calculated by using the equation below:

$$u_{theoretical}(r) = u_{max}\left(1 - \frac{r^2}{R^2}\right) \qquad (3)\ [1]$$

Where u(r) is the axial velocity, R is the inner diameter, r is the distance from the center to the point of the velocity we are calculating. u_{max} is the maximum velocity. We also can find the maximum velocity by using equation below:

$$V = \frac{1}{2}u_{max} \qquad (4)$$

Where V is the average velocity, which could be found by using equation below:

$$V = \frac{Q}{A} \qquad (5)$$

Where Q is the flow discharge and A is the pipe cross-sectional area.

We can get the velocity profile by the pressures we collected.

$$u = \sqrt{\frac{2\Delta P_{Pitot}}{\rho}} \qquad (6)[1]$$

And the flow rate is equal to:

$$Q_{pitot} = 2\pi \int_{r=0}^{r=wall} u\, r\, dr \qquad (7)[1]$$

In this experiment, we measure the pressure profile in increment of 0.5 inch, so for the theoretical calculation part, the same increment is applied. Moreover, in order to obtain the experimental data, we used the pitot tube to measure the pressure for an increment of 0.5 inch to obtain the full data.

Experimental Setup and Procedure

In this experiment, a long pipe, a blower, a Plexiglas tube, a venturi meter, and a pitot tube are used. The pipe has an inner diameter of 145 mm. This pipe serves as the main channel that the air is passing through. The blower is attached at the end of the pipe. The blower sucks the air into the tube. It has the same principle as a pump pushes the air into the channel. A straw is attached at the entrance section of the pipe to minimize the turbulent flow. Pitot tube is attached on the pipe for students to measure the pressure and the flow rate by moving it upward and downward, which provides different values of the pressure. The inclined manometer is connected to the pitot tube to measure the pressure values. A Venturi tube is attached on the pipe as well. The cross-sectional radii of the venturi meter are from 90 mm to 145 mm. The Venturi tube changes the cross-sectional area of the channel, which could cause the pressure change, and flow rate change.

Figure 1: The long pipe

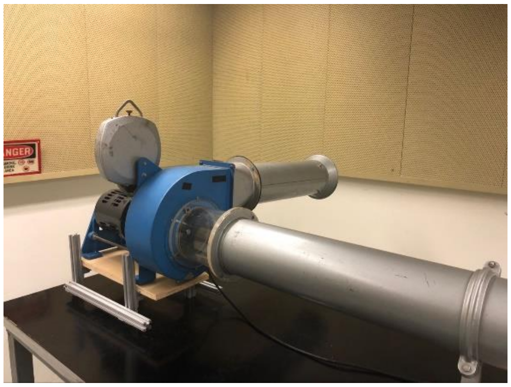

Figure 2: The blower

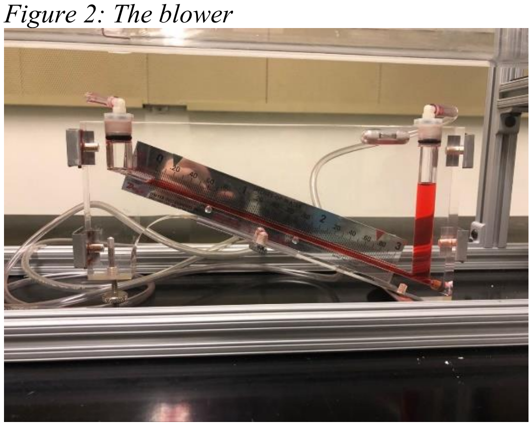

Figure 3: Inclined manometer

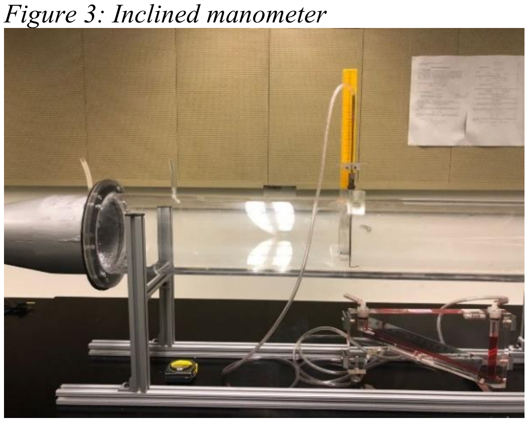

Figure 4: Pitot tube

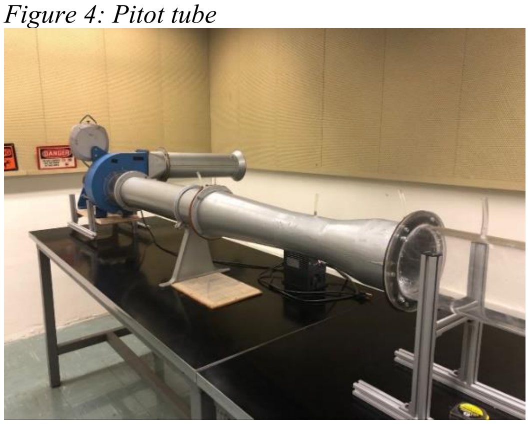

Figure 5: The Venturi tube

Before we started the experiment, we measured the locations of the pressure taps along the pipe from the entrance of the pipe. The locations are listed later in the report. After we measured the locations, Professor switched on the blower, and the air started to suck into the channel. Then we used the inclined manometer to measure the pressures along the pipe. After that, we also measured the pressure profile by using the pitot tube. We set the center of the pipe as 0 and used an increment of 0.5 inch of distance to obtain the pressure values.

Results

Table 1 shows the data we collected, and also the data which is converted to meter and Pa. Figure 6 is the graph for pressure vs distance along the tube. We can see that in the beginning, before the air enters the straw, the pressure is very high, but after it enters the straw, the pressure dropped a lot, and then decreased very slightly except when it enters the Venturi tube. The straw lined up the airs and minimized the turbulent flow. Table 2 is the pressure profile we collected during the lab. Figure 7 is the pressure vs radius graph for the pressure profile. It is not round smoothly because we only measured a limit number of points. Also it is not very symmetrical. The reason might be the gravity effect, or the errors during the lab. Table 3 compared the flow rate for Venturi tube and pitot tube. They are very similar. Flow rate in Venturi tube is 0.0904 m^3/s, and the flow rate we measured by pitot tube is 0.0959 m^3/s. And they should be similar because the flow rate should remain constant at all time. Figure 8 compared the theoretical results with experimental results. The x-axis is the velocity, and the y-axis is the radius. As we can see, at the boundary, theoretical velocity goes to zero, and our experimental results aren't. There are many factors that can cause this result. One big issue is that our sensor cannot really touch the boundary. It has thickness, and if it does not touch the boundary, the velocity wouldn't be zero. Another reason is the pitot tube is not located in the middle of the pipe (from the top view). It shifted a little bit, and that will affect the result. The uncertainty for the flow rate is $u_{flow\ rate} = 1.46 \times 10^{-3} \frac{m^3}{s}$, the smallest uncertainty for the velocity profile is 0.2795 m/s, and the largest uncertainty for the velocity profile is 0.5039m/s.

Conclusion

Based on our results, we can see that the flow rate will remain constant even if the cross-sectional area is changed. The pressure is dropped significantly if the cross-sectional area is reduced. The velocity profile is like a parabolic curve. Overall, this experiment was very successful. However, we could still make some improvements. We can wait a little bit more time for the manometer to remain stable, then read the number. Also we could measure the increment of the radius more precise.

List of References

[1] Goushcha, O. *Aero-Thermal Fluids Laboratory ME 43600*. The City College of New York, 2018

Appendix A

Table 1: Pressure along the tube

X (in)	X(m)	P (in H_2O)	P (Pa)
12.75	0.3238	-0.28	-34.8476
28.3	0.7188	-1.8	-223.956
52.5	1.3335	-1.84	-228.933
79.85	2.0282	-1.85	-230.177
103.7	2.6340	-1.88	-233.910
127.2	3.2309	-1.90	-236.398
151.5	3.8481	-1.92	-238.886
175.8	4.4653	-1.93	-240.131
211.5	5.3721	-1.94	-241.375
217.7	5.5296	-1.96	-232.863
227.6	5.7810	-2.86	-355.841
249.5	6.3373	-2.11	-262.526

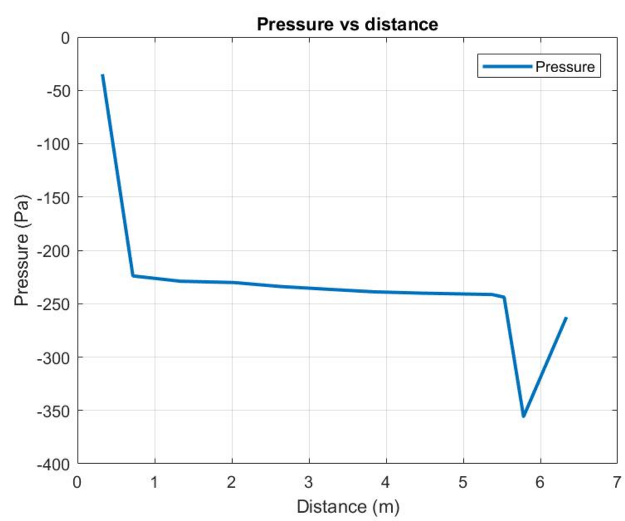

Figure 6: Pressure vs. distance

Table 2: Pressure Profile

R (in)	R (m)	ΔP (in H_2O)	ΔP (Pa)
2.55	0.0648	0.09	11.198
2.05	0.0521	0.16	19.907
1.55	0.0394	0.20	24.884
1.05	0.0267	0.22	27.372
0.55	0.0140	0.24	29.861
0.05	0.0013	0.26	32.349
-0.45	-0.0114	0.26	32.349
-0.95	-0.0241	0.26	32.349
-1.45	-0.0368	0.24	29.861
-1.95	-0.0495	0.18	22.396
-2.45	-0.0622	0.12	14.930
-2.55	-0.0648	0.08	9.954

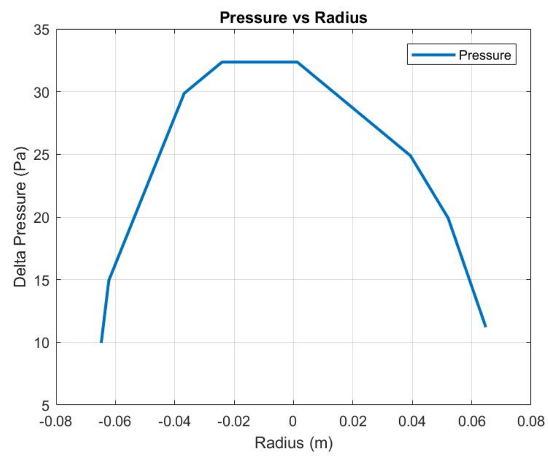

Figure 7: Delta pressure vs radius

Table 3: $Q_{venturi}$ vs Q_{pitot}

$Q_{venturi}$	0.0904 m^3/s
Q_{pitot}	0.0959 m^3/s

Table 4: Velocity Profile (experimental)

R (in)	R (m)	Velocity (m/s)
2.55	0.0648	4.2758
2.05	0.0521	5.7010
1.55	0.0394	6.3739
1.05	0.0267	6.6850
0.55	0.0140	6.9823
0.05	0.0013	7.2674
-0.45	-0.0114	7.2674
-0.95	-0.0241	7.2674
-1.45	-0.0368	6.9823
-1.95	-0.0495	6.0468
-2.45	-0.0622	4.9372
-2.55	-0.0648	4.0312

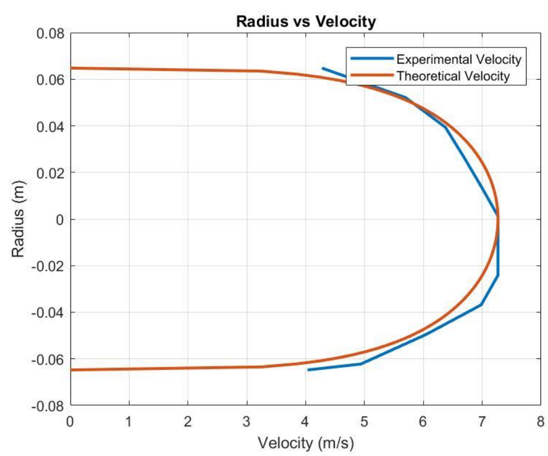

Figure 8: Radius vs Velocity graph

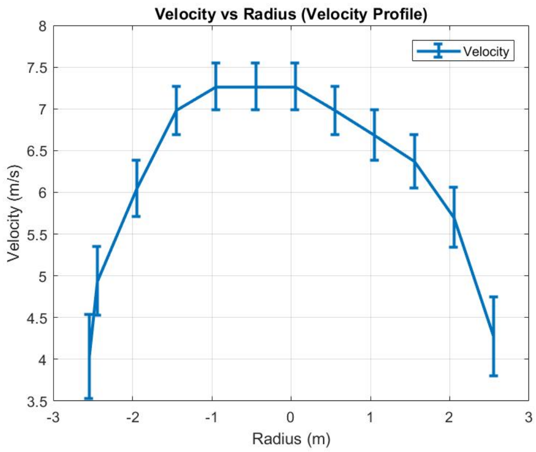

Figure 9: Velocity profile with error bars.

Table 5: Uncertainties for velocity profile

R (in)	R (m)	uncertainty
2.55	0.0648	0.4751
2.05	0.0521	0.3563
1.55	0.0394	0.3187
1.05	0.0267	0.3039
0.55	0.0140	0.2909
0.05	0.0013	0.2795
-0.45	-0.0114	0.2795
-0.95	-0.0241	0.2795
-1.45	-0.0368	0.2909
-1.95	-0.0495	0.3359
-2.45	-0.0622	0.4114
-2.55	-0.0648	0.5039

Appendix B

Calculations

In. H_2O convert to Pa:

$$Pressure = \rho g h = \rho g\ d\ sin\theta$$
$$P = 997 \times 9.81 \times 0.0254 \times sin\theta$$
$$P = 248.84 \times sin\theta$$

$$\Delta P_{venturi} = P_{venturi} - P_{pipe}$$
$$\Delta P_{venturi} = 355.8412 - 243.8632$$
$$\Delta P_{venturi} = 111.978\ Pa$$
$$U_{Mean} = \sqrt{\frac{2}{\left(\frac{d_a}{d_b}\right)^4 - 1}\left(\frac{\Delta P_{Venturi}}{\rho}\right)}$$

Note: density of air is 1.225 kg/m^3

d_a = pipe diameter = 145 mm = 0.145 m

d_b = Venturi diameter = 90 mm = 0.09 m

$$U_{Mean} = \sqrt{\frac{2}{\left(\frac{0.145}{0.090}\right)^4 - 1}\left(\frac{111.978}{1.225}\right)}$$
$$U_{Mean} = 5.645\frac{m}{s}$$
$$Q_{venturi} = A * U_{mean} = \frac{\pi \times 0.145^2}{4} * 5.645$$
$$Q_{venturi} = 0.0932\frac{m^3}{s}$$
$$Q_{venturi\ actually} = Q_{ideal} \times Cv$$

Where $c_v = 0.97$

$$Q_{venturi\ actually} = 0.0932 \times 0.97 = 0.0904$$
$$u = \sqrt{\frac{2\Delta P_{Pitot}}{\rho}} = \sqrt{\frac{2 * 32.349}{1.225}} = 7.267\frac{m}{s}$$

(maximum velocity. Rest of the velocity profiles are calculated in matlab.)

$$Q_{pitot} = 2\pi\int_{r=0}^{r=0.0648} u\ r\ dr = 0.0959\frac{m^3}{s}$$
$$u_{theoretical}(r) = u_{max}\left(1 - \frac{r^2}{R^2}\right)^{\frac{1}{n}}$$

For this part, n = 4 best matches our data.

$$u_{theoretical}(r) = 7.267 \times \left(1 - \frac{r^2}{0.0648^2}\right)^{\frac{1}{4}}$$

For maximum velocity:

$$u_{theoretical}(r) = 7.267 \times \left(1 - \frac{0^2}{0.0648^2}\right)^{\frac{1}{4}}$$
$$= 7.267 \frac{m}{s}$$

(Other velocities are listed in table 4)

Uncertainty

Uncertainty of the flow rate using Venturi tube, $Q_{venturi}$, and uncertainty of the velocity profile u(r).

Manometer hgt (δ_h): 0.01 in H_2O

Convert to Pa: 0.01*248.84 = 2.4884 Pa

For ΔP, there are two factors: $P_{venturi}, P_{pipe}$

Each of these factors has u_P = 2.4884 Pa

$$u_{Pventuri} = \sqrt{\left(\frac{\partial P_{Venturi}}{\partial P} u_P\right)^2 + \left(\frac{\partial P_{Venturi}}{\partial P} u_P\right)^2}$$

$$u_{Pventuri} = \sqrt{u_P{}^2 + u_P{}^2}$$

$$u_{Pventuri} = \sqrt{2.4884^2 + 2.4884^2}$$

$$u_{Pventuri} = 3.5191\, Pa$$

Uncertainty for the U_{mean} which is affected by pressure difference only, so there is only one factor:

$$u_{Umean} = \sqrt{\left(\frac{\partial U}{\partial P} u_{Pventuri}\right)^2} = \frac{\partial U}{\partial P} u_{Pventuri}$$

$$U_{Mean} = \sqrt{\frac{2}{\left(\frac{d_a}{d_b}\right)^4 - 1}\left(\frac{\Delta P_{Venturi}}{\rho}\right)}$$

$$U_{Mean} = \sqrt{\frac{2}{\left(\frac{d_a}{d_b}\right)^4 - 1}\left(\frac{1}{\rho}\right)} \times (\Delta P_{Venturi})^{1/2}$$

$$\frac{\partial U}{\partial P} = \sqrt{\frac{2}{\left(\frac{d_a}{d_b}\right)^4 - 1}\left(\frac{1}{\rho}\right)} \times \frac{1}{2} \times \frac{1}{\sqrt{\Delta P_{Venturi}}}$$

$$\frac{\partial U}{\partial P} = \sqrt{\frac{2}{\left(\frac{0.145}{0.090}\right)^4 - 1}\left(\frac{1}{1.225}\right)} \times \frac{1}{2} \times \frac{1}{\sqrt{111.978}}$$

$$\frac{\partial U}{\partial P} = 0.0252$$

$$u_{Umean} = \frac{\partial U}{\partial P} u_{Pventuri} = 0.0252 \times 3.5191$$

$$u_{Umean} = 0.0887 \frac{m}{s}$$

Uncertainty for the flow rate has only one factor only also, which is the mean velocity.

$$u_{flow\ rate} = \sqrt{\left(\frac{\partial Q}{\partial U} u_{Umean}\right)^2} = \frac{\partial Q}{\partial U} u_U$$

$$Q_{venturi} = A * U_{mean}$$

$$\frac{\partial Q}{\partial U} = A$$

$$u_{flow\ rate} = A \times u_U = \frac{\pi \times 0.145^2}{4} \times 0.0887$$

$$u_{flow\ rate} = 1.46 \times 10^{-3} \frac{m^3}{s}$$

Uncertainty of the velocity profile

$$u = \sqrt{\frac{2\Delta P_{Pitot}}{\rho}} = \sqrt{\frac{2}{\rho}} \times \Delta P_{Pitot}{}^{1/2}$$

The only factor for velocity profile is pressure: 0.01*248.84 = 2.4884 Pa

$$u_{velocity} = \sqrt{\left(\frac{\partial u}{\partial P} u_P\right)^2} = \frac{\partial u}{\partial P} u_P$$

$$\frac{\partial u}{\partial P} = \sqrt{\frac{2}{\rho}} \times \frac{1}{2} \times \Delta P_{Pitot}{}^{-1/2}$$

$$u_{velocity} = \sqrt{\frac{2}{\rho}} \times \frac{1}{2} \times \Delta P_{Pitot}{}^{-1/2} \times u_P$$

$$= \sqrt{\frac{2}{1.225}} \times \frac{1}{2} \times 32.349^{-1/2} \times 2.4884$$

$$u_{velocity} = 0.2795 \frac{m}{s}$$

(This is only one uncertainty. Rest of uncertainties are calculated by Matlab.)

Matlab Code:

```
clear all;
clc;

da=0.145;
db=0.09;
p=1.225; %density
deltap=355.8412-243.8632;
umean=sqrt(2/((da/db)^4-1)*(deltap/p));
disp(umean);

A=pi*(da^2)/4;
Q=A*umean;
cv=0.97;
Qrate=Q*cv;
disp(Qrate);

%%
tometer = 0.0254; % convert to meter
topa = 248.84*sin(pi*30/180); % convert to pascal
x =[12.75,28.3,52.5,79.85,103.7,127.2,151.5,175.8,211.5,217.7,227.6,249.5]; % unit in inches
p =[-0.28,-1.8,-1.84,-1.85,-1.88,-1.90,-1.92,-1.93,-1.94,-1.96,-2.86,-2.11];% unit in " H2O
X = x * tometer;
P = p * topa;

figure
disp('distance in meter: ');
disp(X);
disp('pressure in Pa: ');
disp(P);

plot(X,P,'linewidth',2);
xlabel('Distance (m)');
ylabel('Pressure (Pa)');
title('Pressure vs distance');
legend('Pressure');
grid on;

%%
% pressure profile

tometer = 0.0254; % convert to meter
topa = 248.84*sin(pi*30/180); % convert to pascal

r = [2.55, 2.05, 1.55, 1.05, 0.55, 0.05, -0.45, -0.95, -1.45, -1.95, -2.45, -2.55];
dp = [0.09, 0.16, 0.2, 0.22, 0.24, 0.26, 0.26, 0.26, 0.24, 0.18, 0.12, 0.08];

R = r * tometer;
DP = dp * topa;

figure
disp('radius in meter: ');
disp(R);
disp('delta pressure in Pa: ');
disp(DP);

plot(R,DP,'linewidth',2);
xlabel('Radius (m)');
ylabel('Delta Pressure (Pa)');
title('Pressure vs Radius');
legend('Pressure');
grid on;

%%
%Experimental u(r)
rho = 1.225; %kg/m^3
u = sqrt(2*DP/rho); %pitot tube velocities
disp('Velocity profile');
disp(u);

figure
error = sqrt(2/1.225)*(1/2)*2.4884*DP.^(-1/2);
errorbar (r,u,error,'linewidth',2);
grid on;
ylabel('Velocity (m/s)');
xlabel('Radius (m)');
title('Velocity vs Radius (Velocity Profile)');
legend('Velocity');

figure
plot(u,R,'linewidth',2);
xlabel('Velocity (m/s)');
ylabel('Radius (m)');
title('Radius vs Velocity');
grid on;

hold on;

%Analytical
umax=max(u);
r = linspace (-2.55*tometer,2.55*tometer);
utheoretical=umax*(1-(((r).^2)/(2.55*0.0254)^2)).^(1/4);
plot(utheoretical,r, 'linewidth',2);
legend('Experimental Velocity', 'Theoretical Velocity');

%%
%velocity uncertainty
uvelocity = sqrt(2/1.225)*(1/2)*DP.^(-1/2)*2.4884;
disp(uvelocity);
```