CSC 252: Computer Organization Spring 2023: Lecture 2

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Department of Computer Science
University of Rochester

Announcement

- Make sure you can access CSUG machines!!!
- Programming assignment 1 will be posted this week.
  - I will send an announcement when it's out.
  - It is in C language. Seek help from TAs.
  - TAs are best positioned to answer your questions about programming assignments!!!
- Programming assignments do NOT repeat the lecture materials. They ask you to synthesize what you have learned from the lectures and work out something new.

Previously in 252…

Problem

Algorithm

Program

Instruction Set Architecture (ISA)

Microarchitecture

Circuit

Previously in 252…

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ISA is the contract between software and hardware.

Previously in 252…

Problem

Algorithm

	Renting
Service provider	Landlord
Service receiver	YOU
Contract	Lease
Contract's language	Natural language (e.g., English)

Circuit

:t
: and

Previously in 252…

Problem

Algorithm

	Renting	Computing
Service provider	Landlord	Hardware
Service receiver	YOU	Software
Contract	Lease	ISA
Contract's language	Natural language (e.g., English)	Assembly programming language

ct
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Circuit

Previously in 252…

- How is a human-readable program translated to a representation that computers can understand?

Problem

Algorithm

Program

Instruction Set Architecture (ISA)

Microarchitecture

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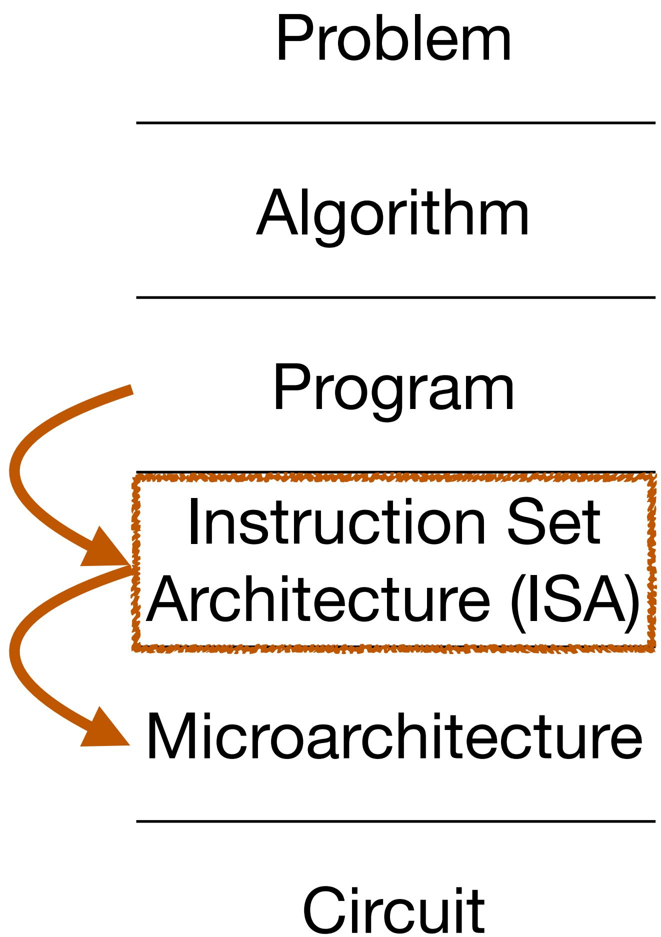
Previously in 252…

- How is a human-readable program translated to a representation that computers can understand?
- How does a modern computer execute that program?



ISA is the contract between software and hardware.

Previously in 252…

C Program

```
void add() {
  int a = 1;
  int b = 2;
  int c = a + b;
}
```

→

Assembly program

```
movl    $1, -4(%rbp)
movl    $2, -8(%rbp)
movl    -4(%rbp), %eax
addl    -8(%rbp), %eax
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Previously in 252…

Assembly program

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```

→

Executable Binary

```
00011001   …
01101010   …
11010101   …
01110001   …
```

Previously in 252…

Assembly program		*Executable Binary*
movl $1, -4(%rbp)	→	00011001 …
movl $2, -8(%rbp)		01101010 …
movl -4(%rbp), %eax		11010101 …
addl -8(%rbp), %eax		01110001 …

- What's the difference between an assembly program and an executable binary?
  - They refer to the same thing — a list of instructions that the software asks the hardware to perform
  - They are just different representations
- Instruction = Operator + Operand(s)

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Assembly program		*Executable Binary*
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Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

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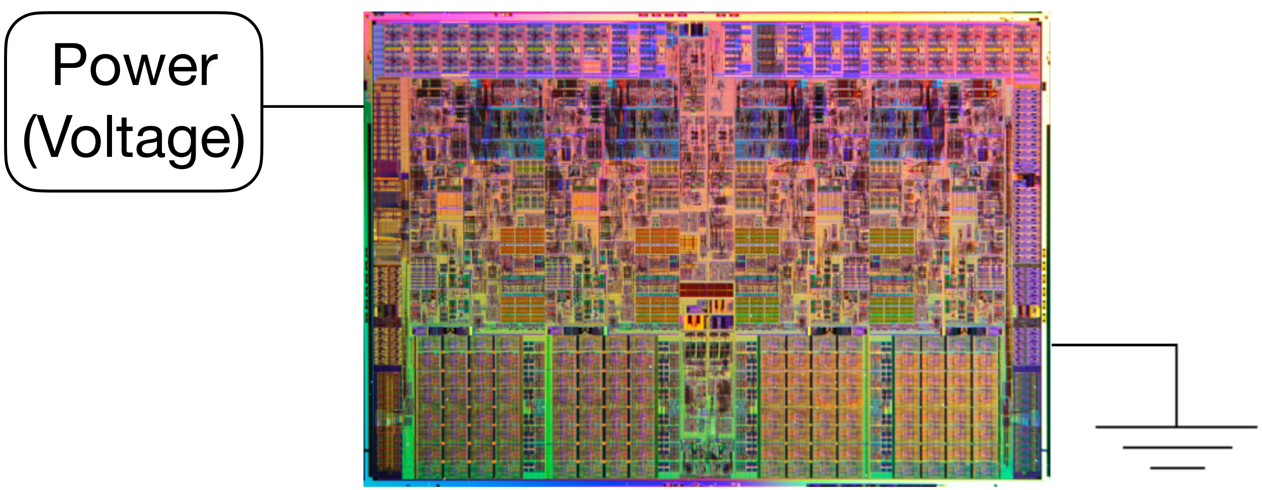

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    - Use high voltage to represent 1
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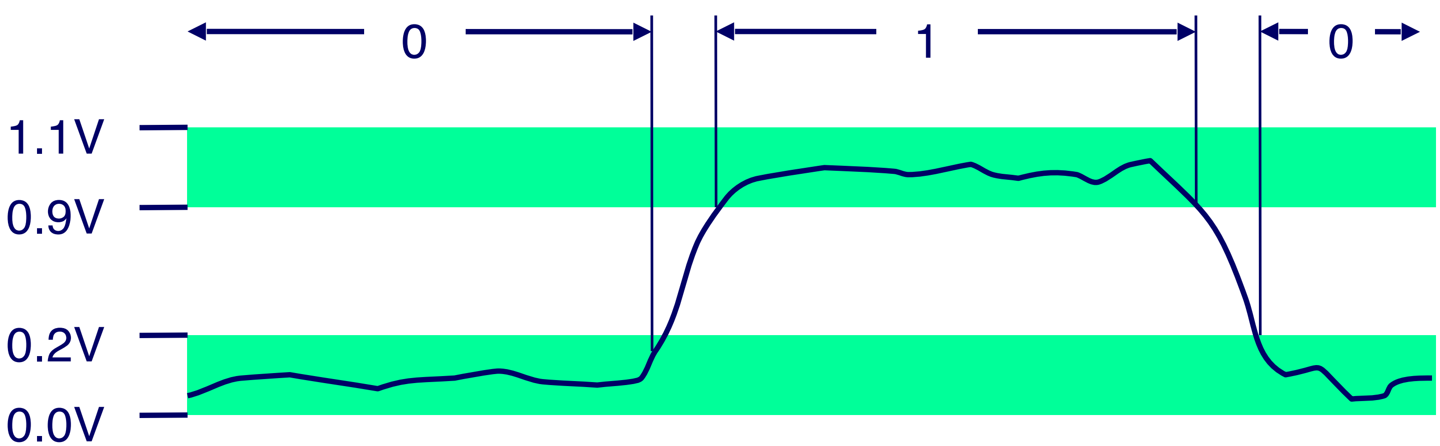

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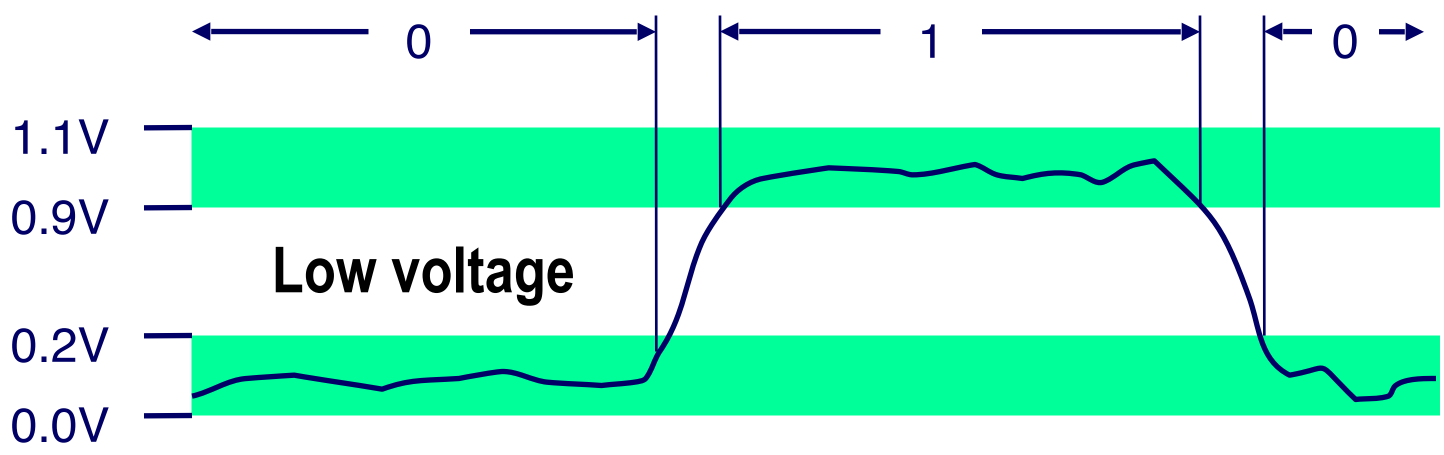

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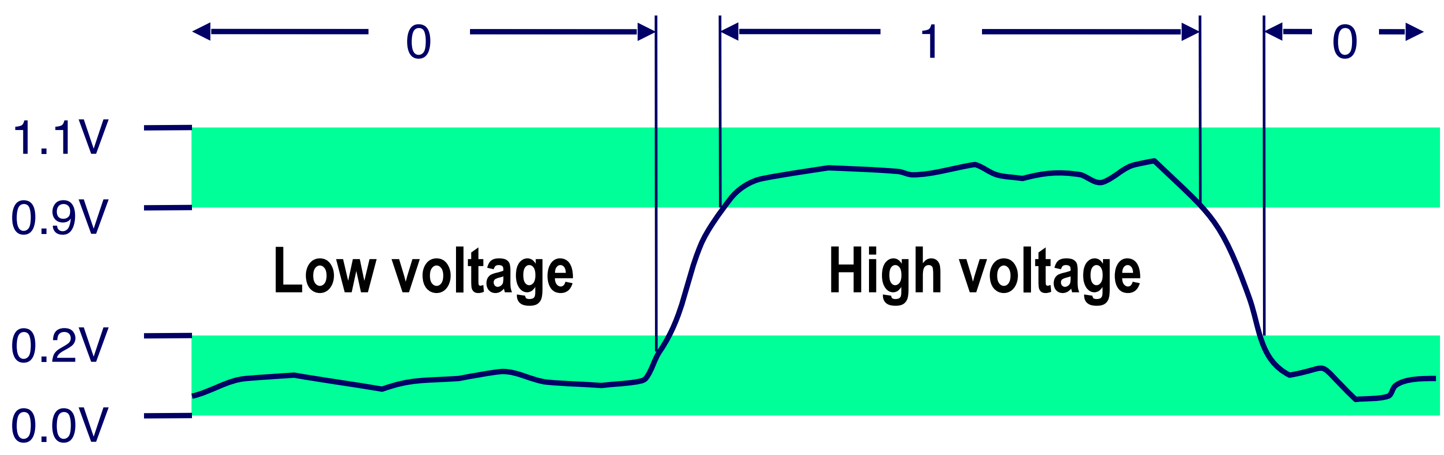

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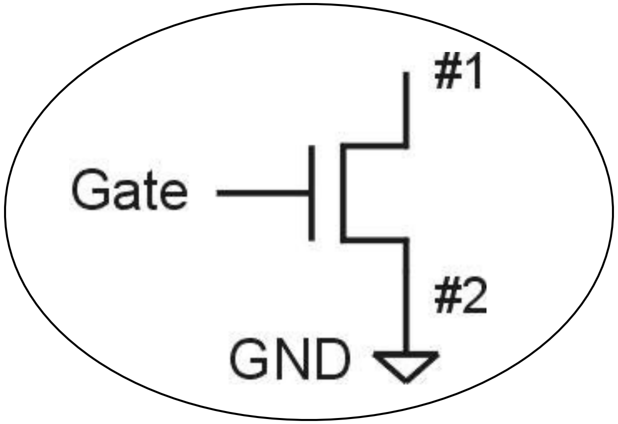

Terminal #2 must be connected to GND (0V).

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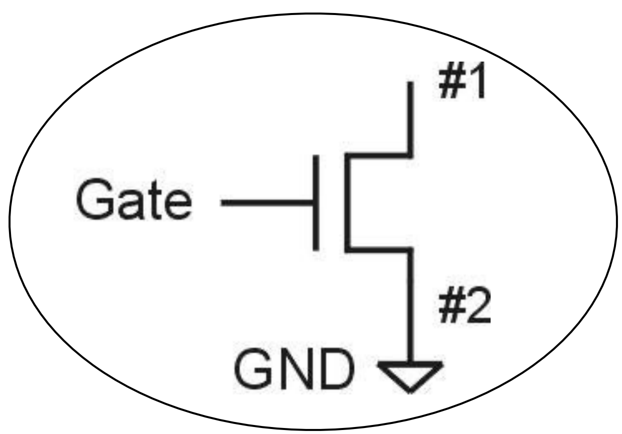
n-type (NMOS)

- when Gate has high voltage, short circuit between #1 and #2 (switch closed)



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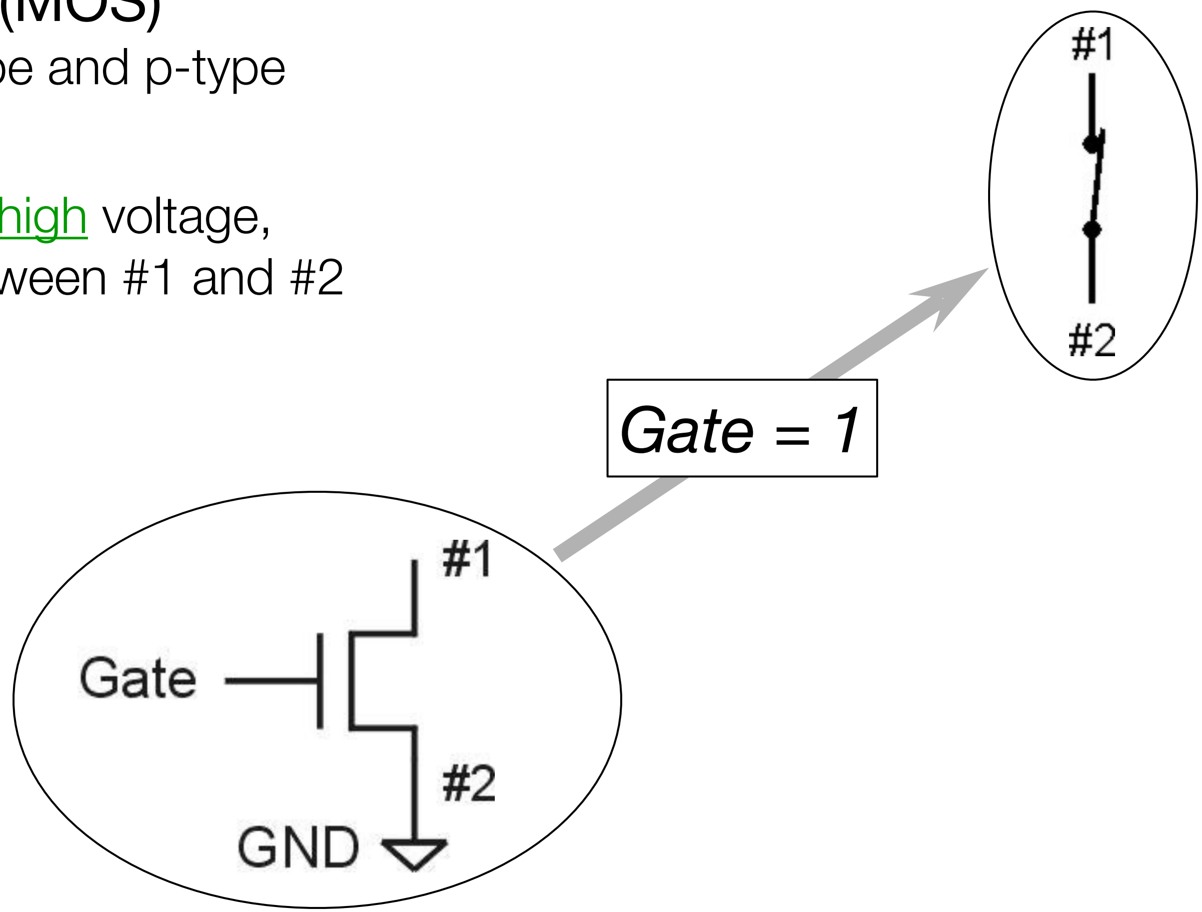

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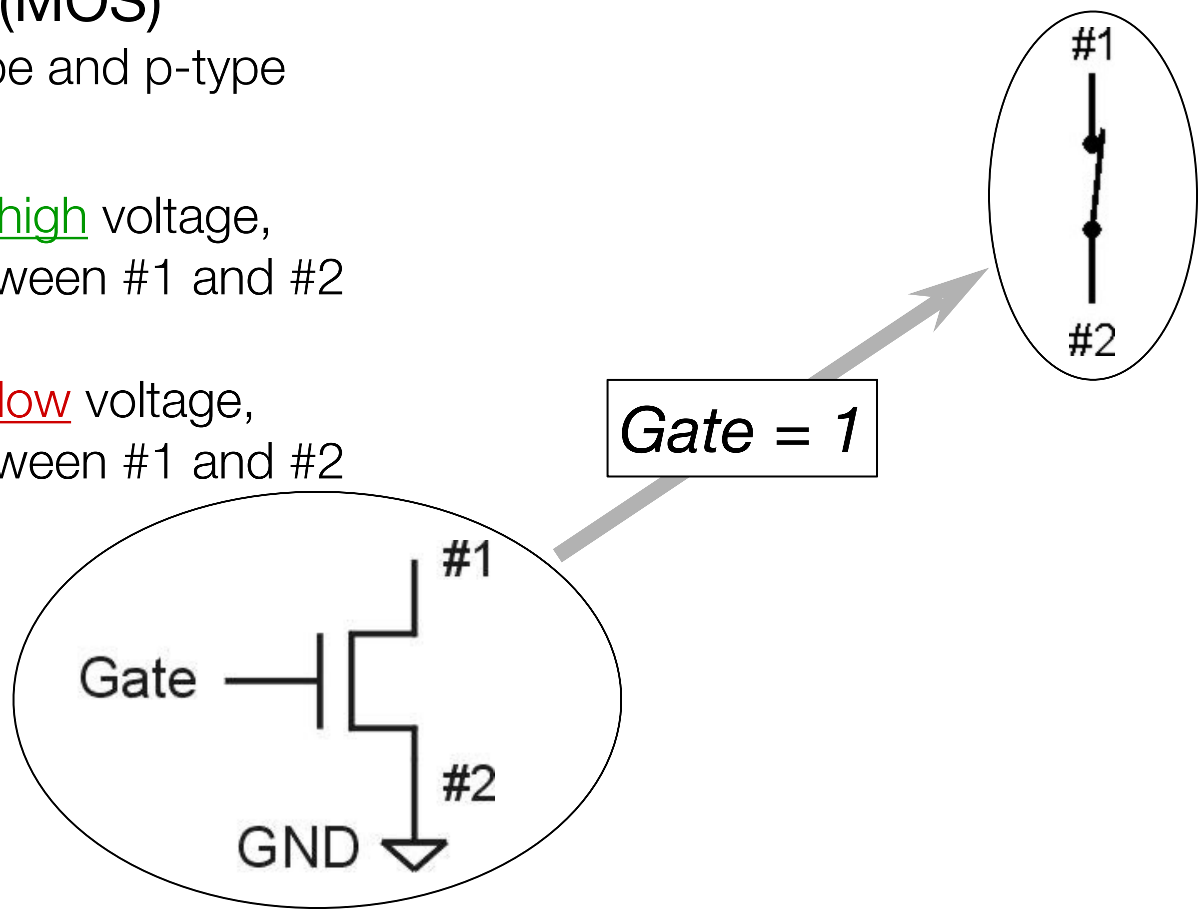

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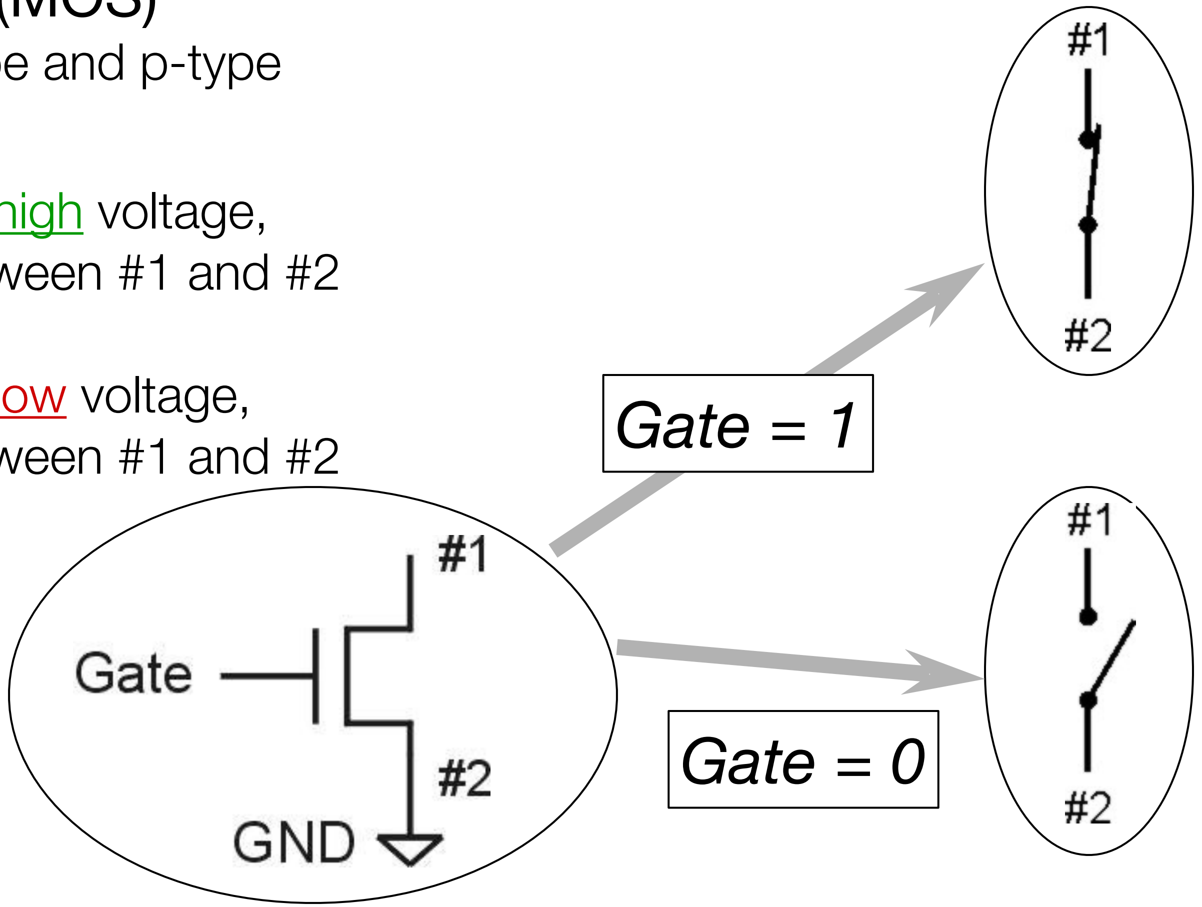

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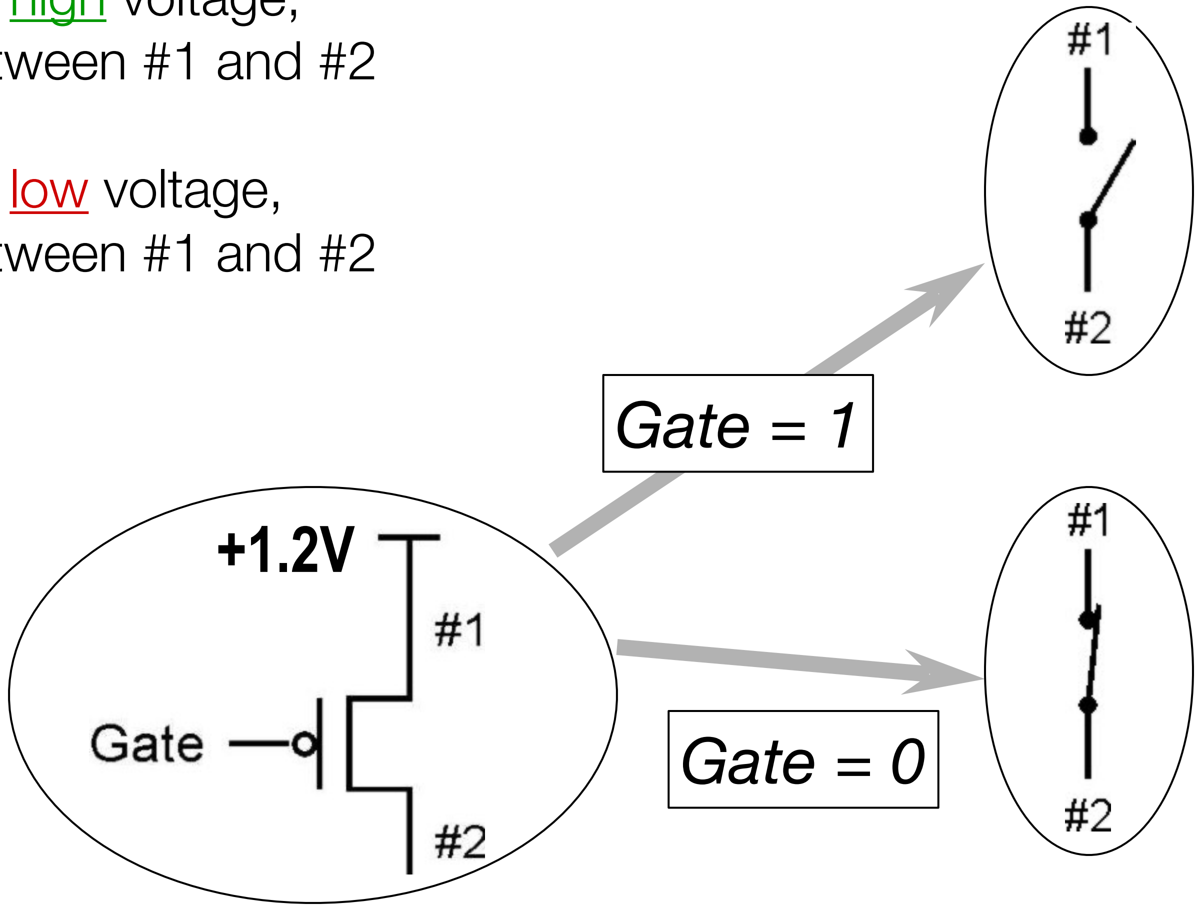
Why Bits?

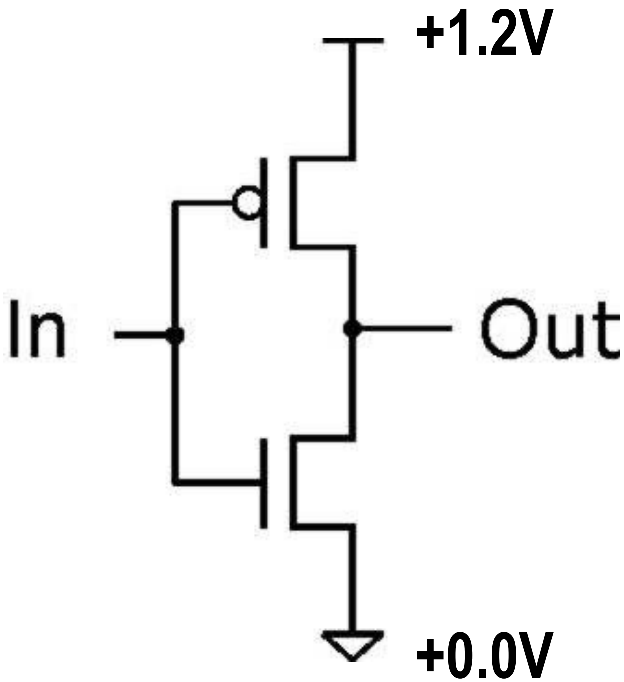
p-type is *complementary* to n-type (PMOS)

- when Gate has high voltage, open circuit between #1 and #2 (switch open)
- when Gate has low voltage, short circuit between #1 and #2 (switch closed)



Terminal #1 must be connected to +1.2V

Inverter



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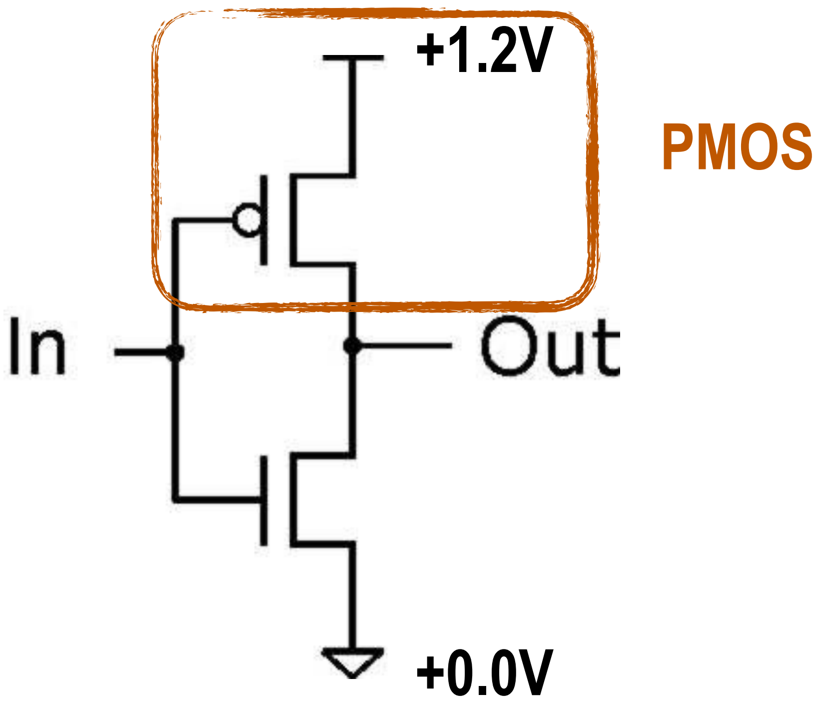

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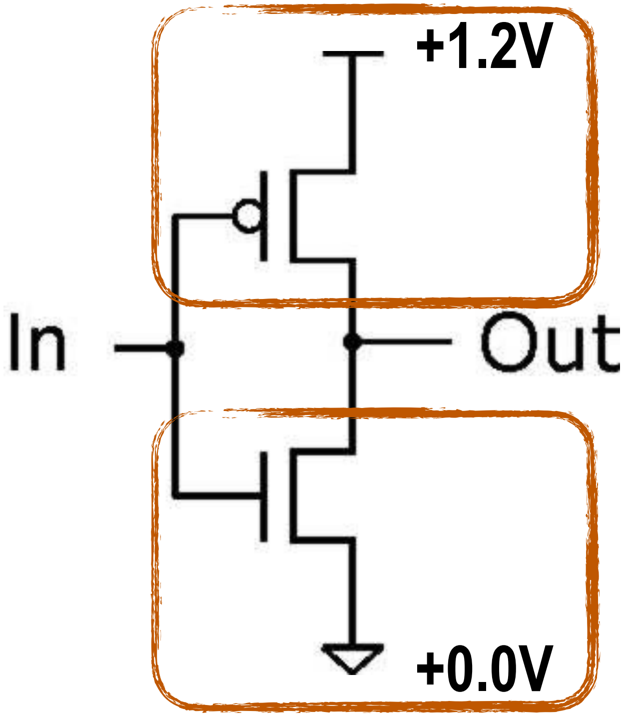
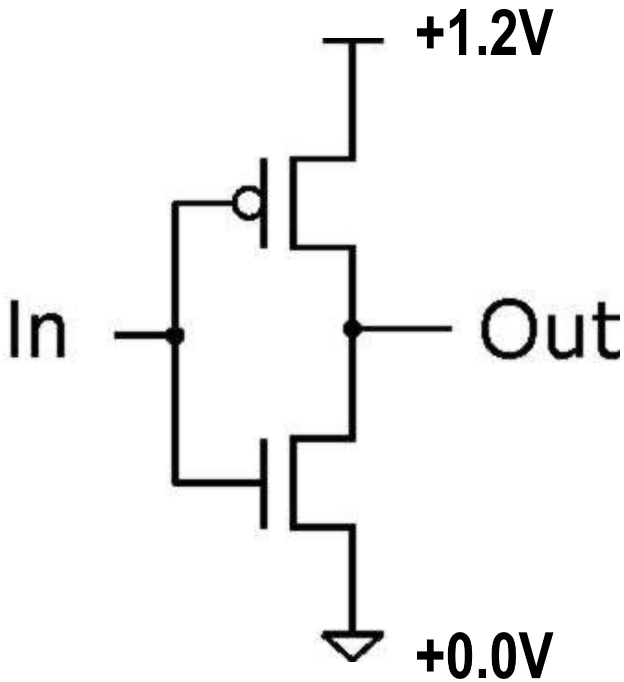
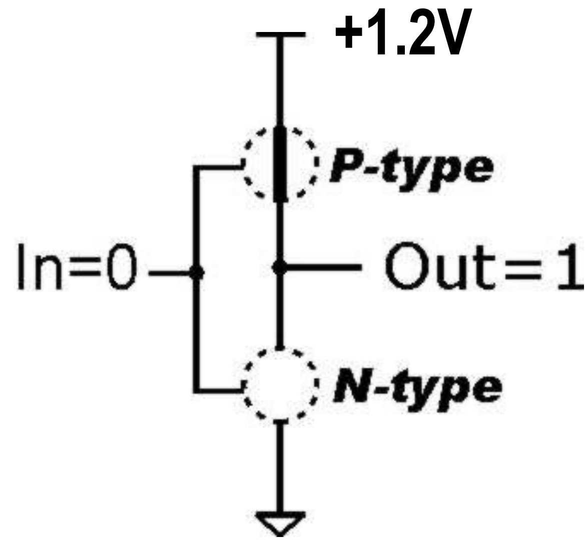


PMOS

NMOS

Inverter





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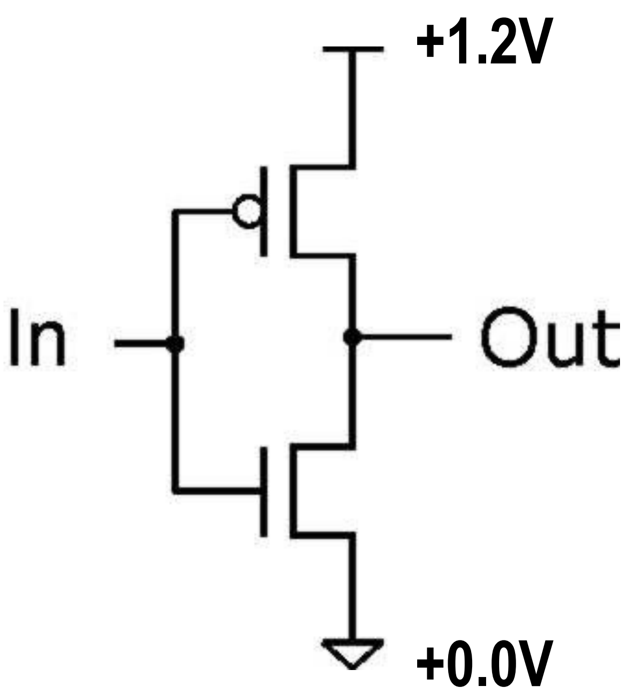
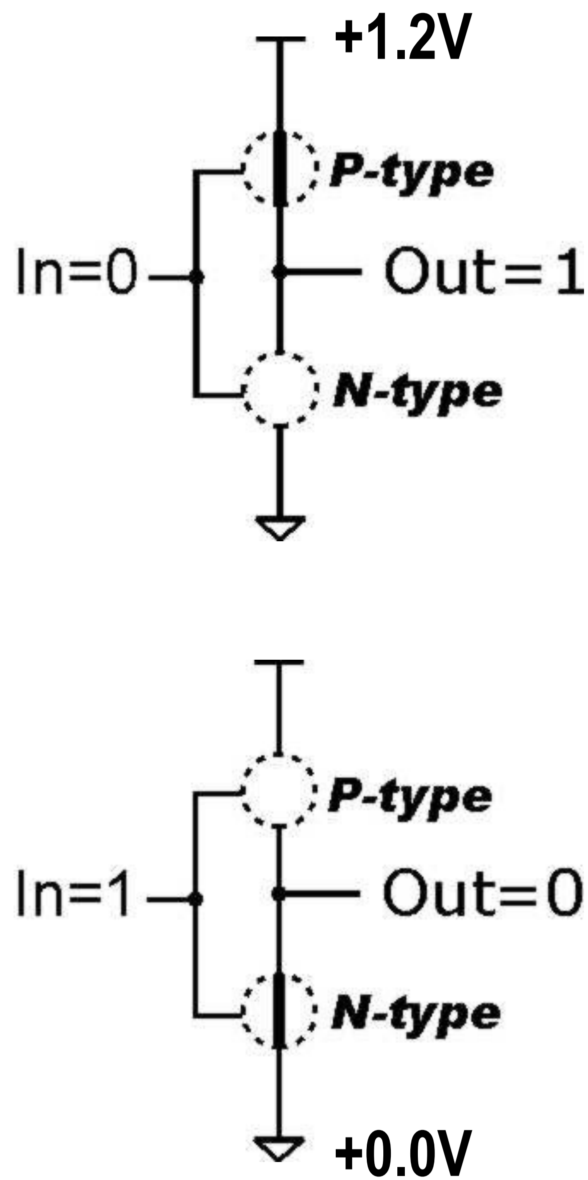
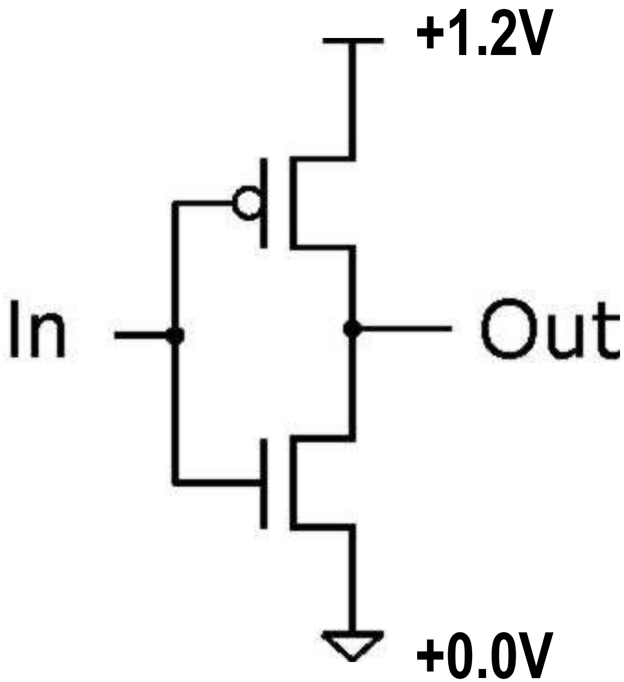
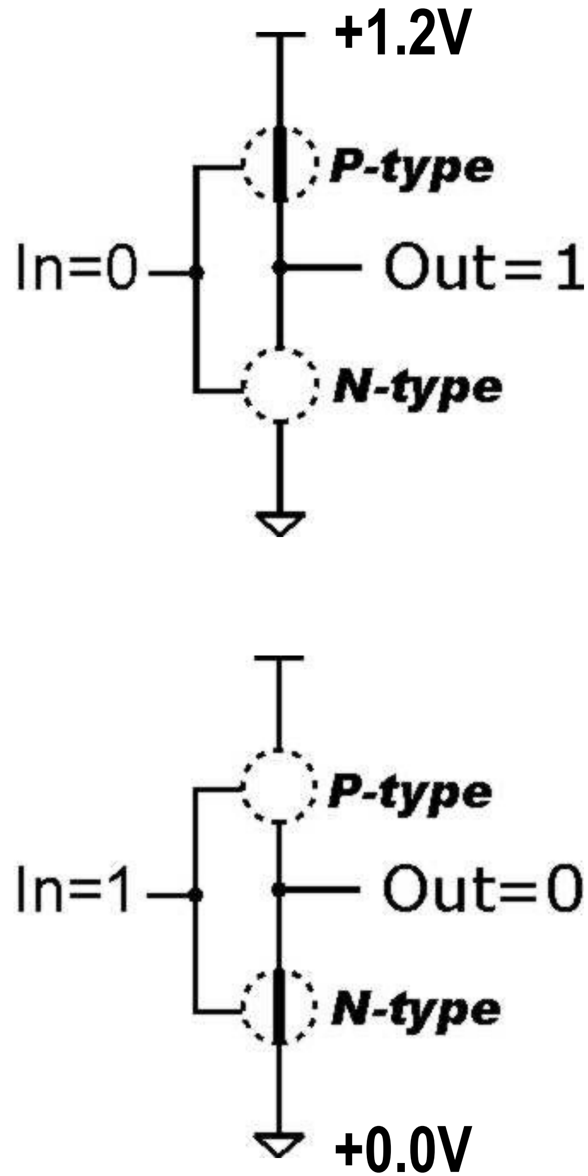

+1.2V
In
Out
+0.0V


+1.2V
P-type
In=0
Out=1
N-type
P-type
In=1
Out=0
N-type
+0.0V

Inverter



In	Out
0	1
1	0



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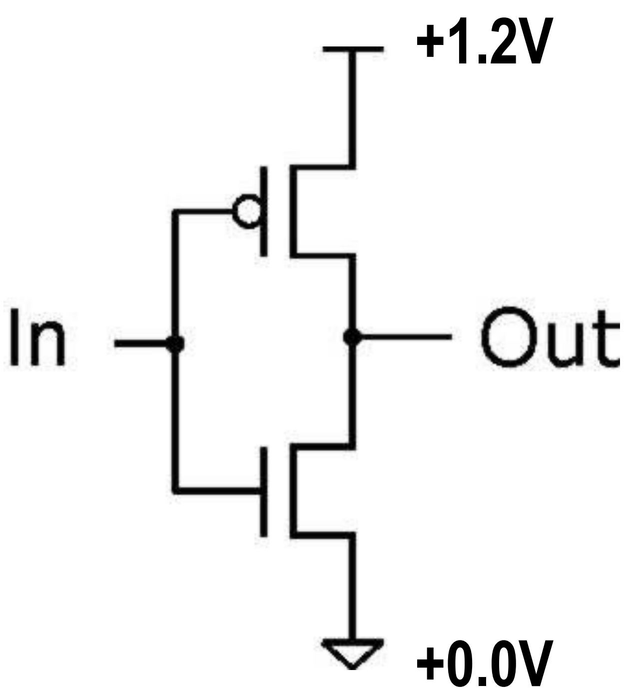

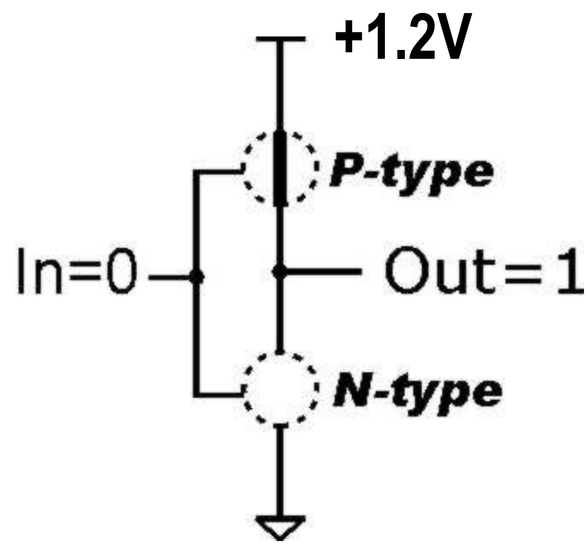

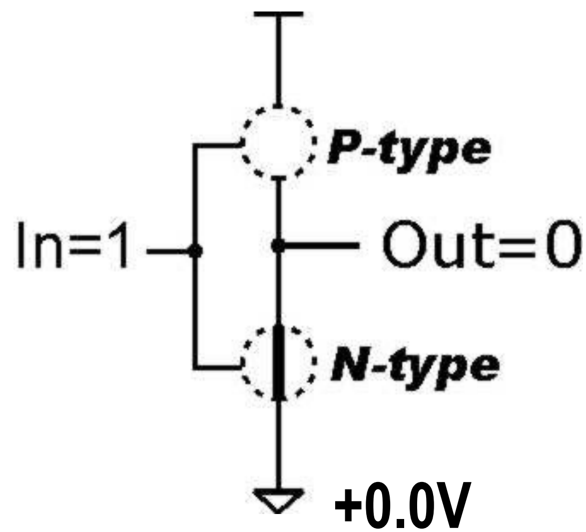

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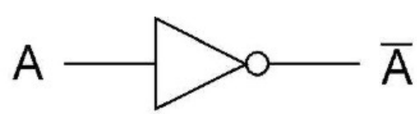
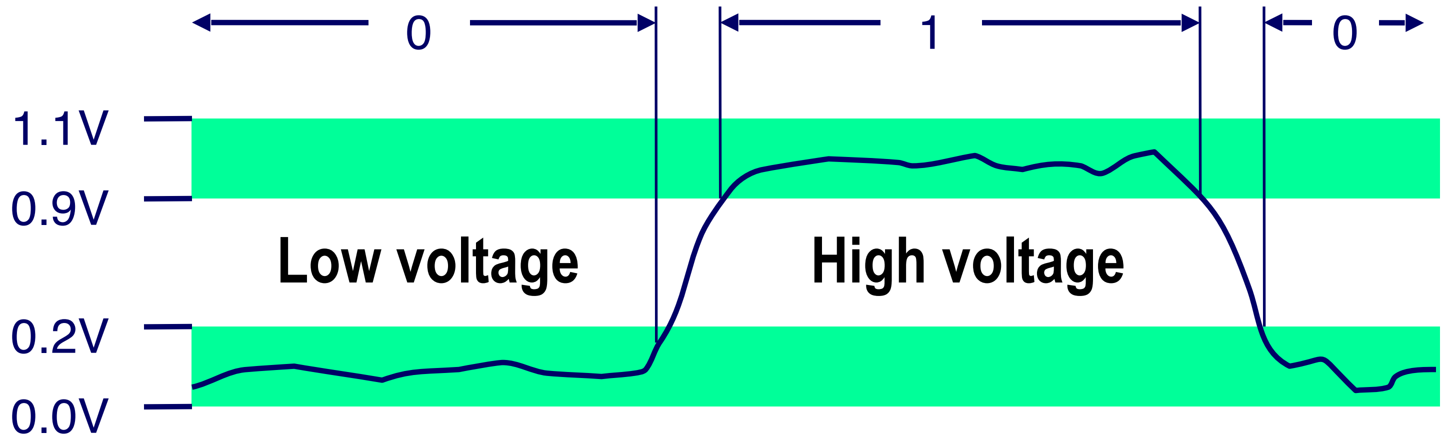
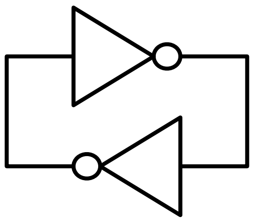


Store/Access Data





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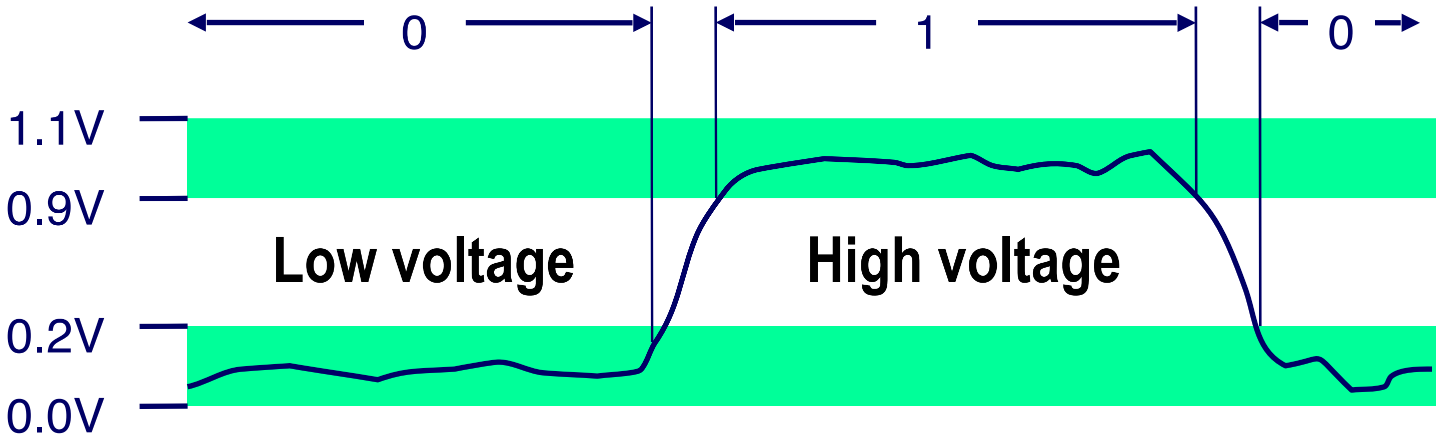

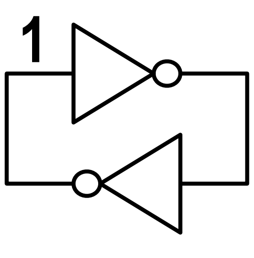

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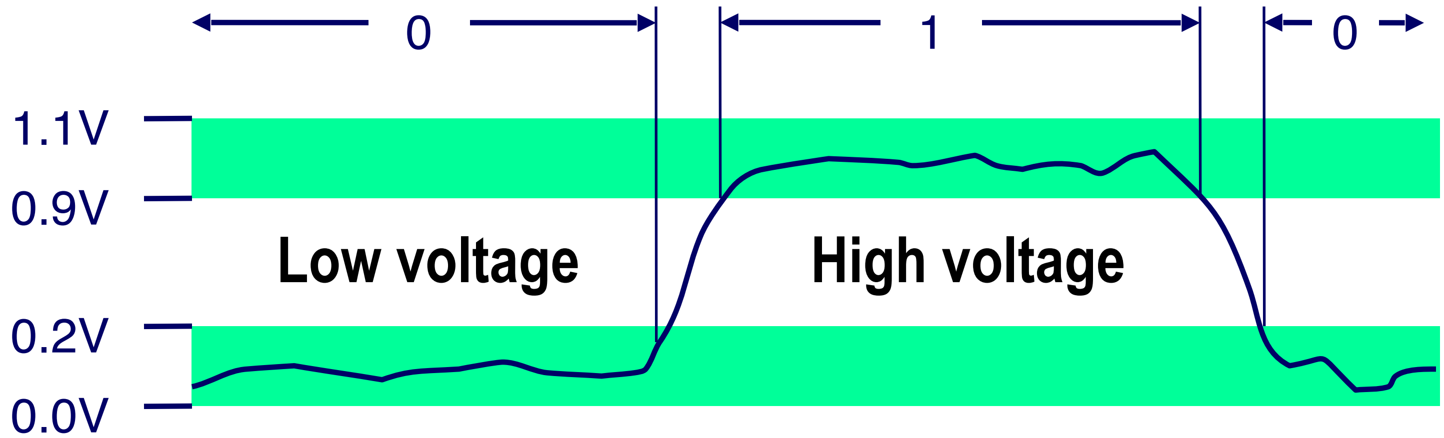

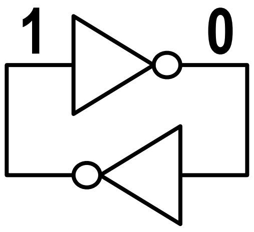

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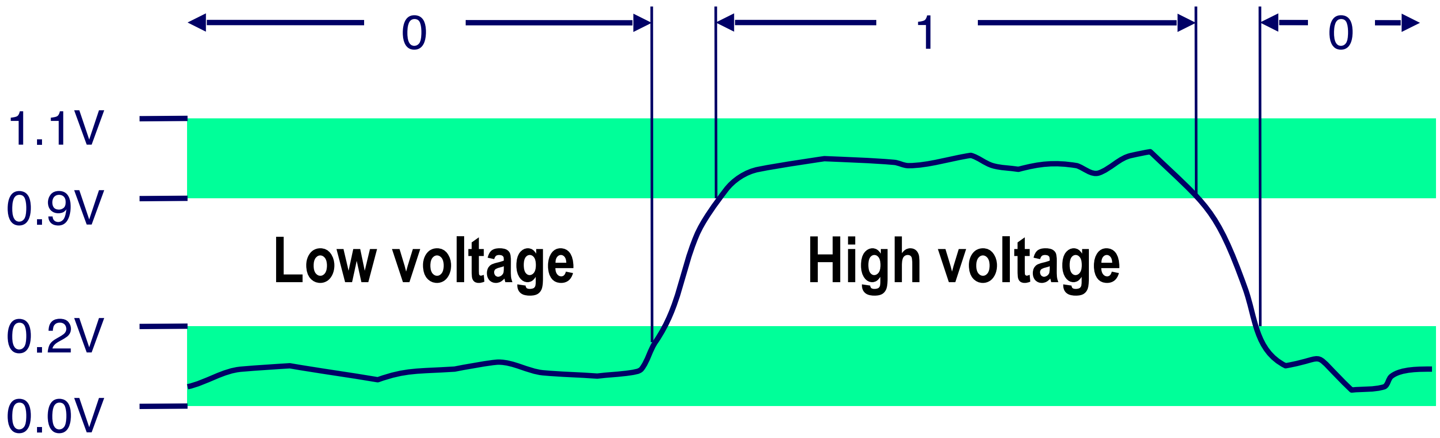

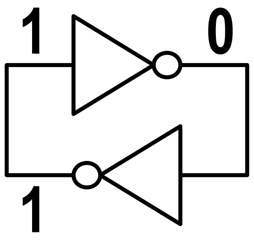

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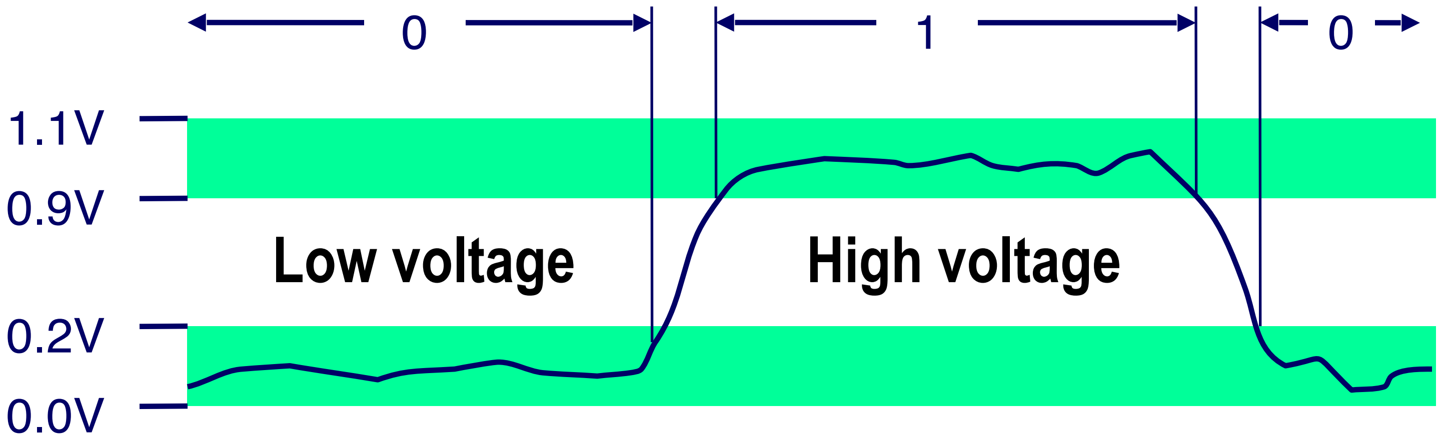
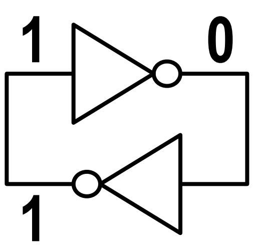


- Two cross coupled inverters store a single bit
  - Feedback path persists the value in the “cell”



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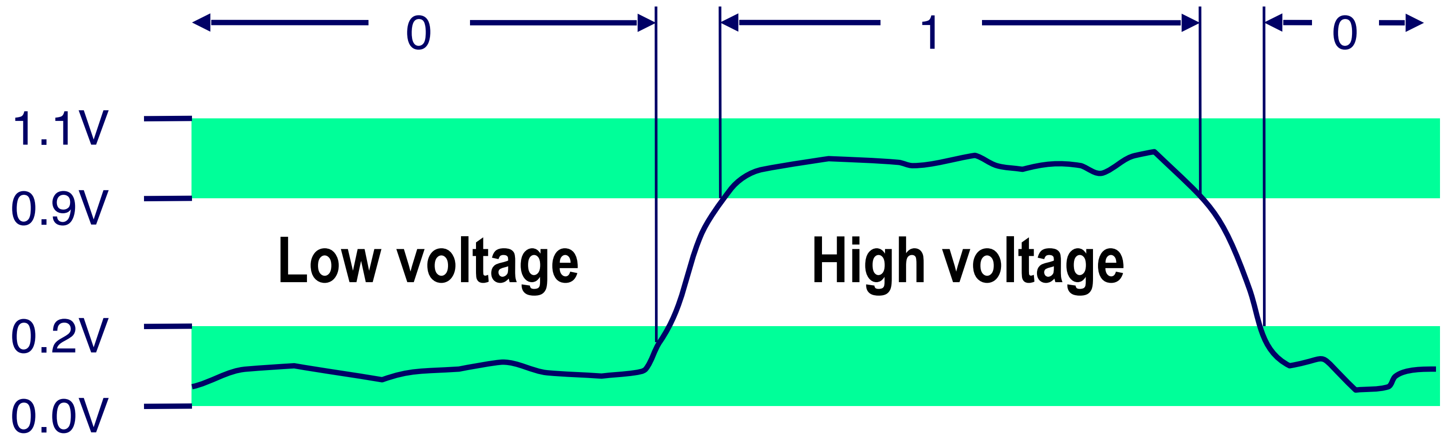

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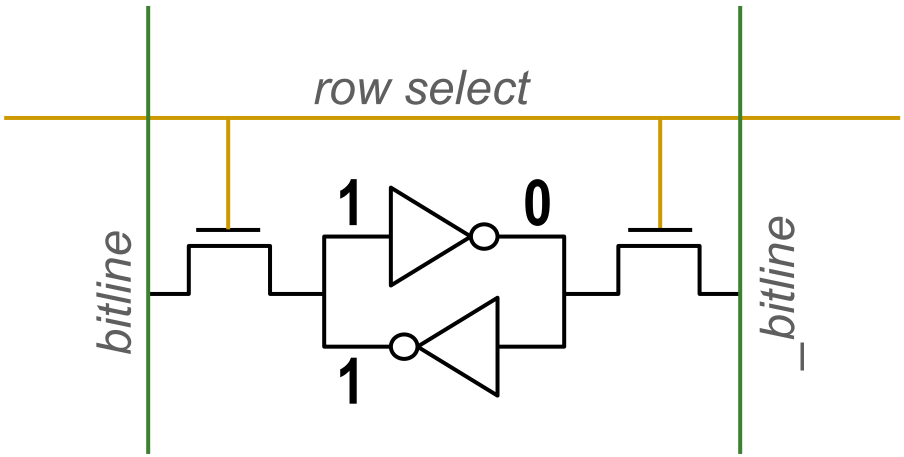

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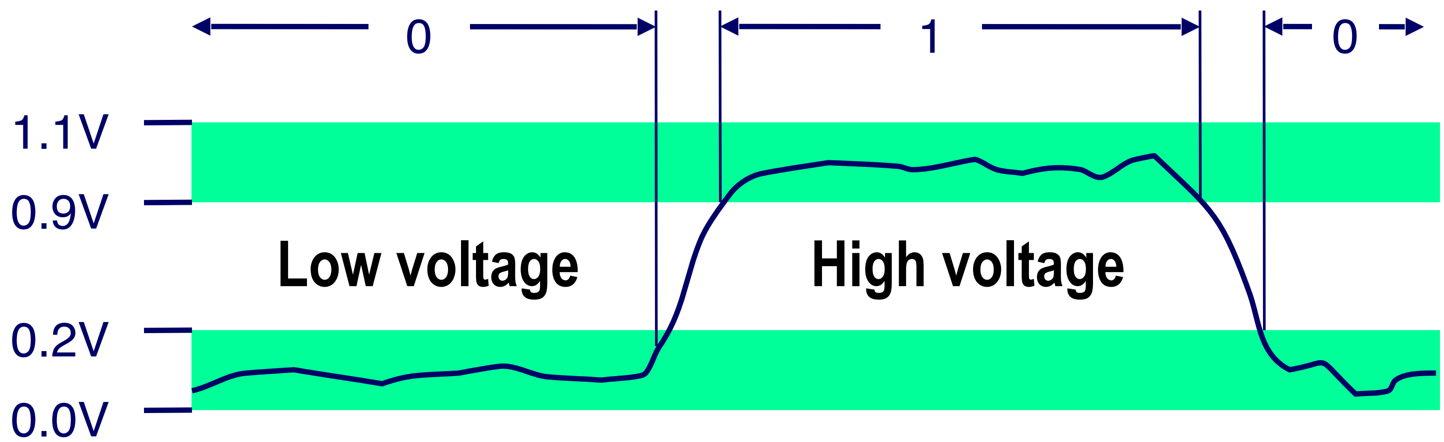

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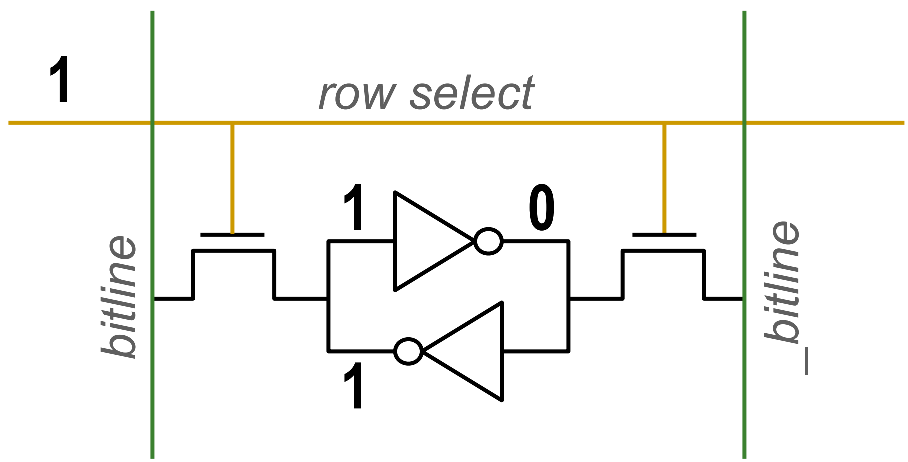

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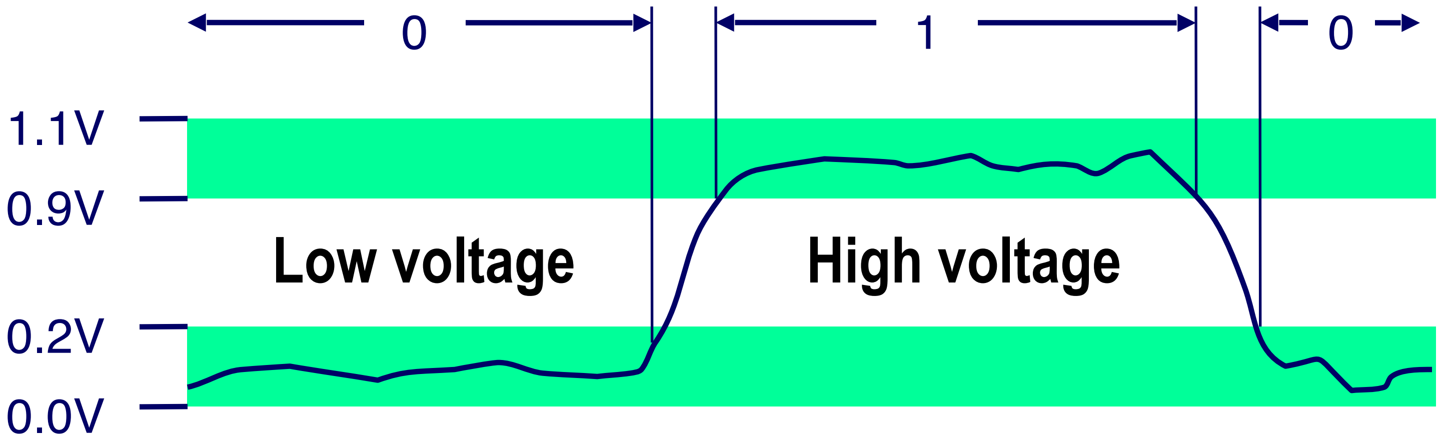

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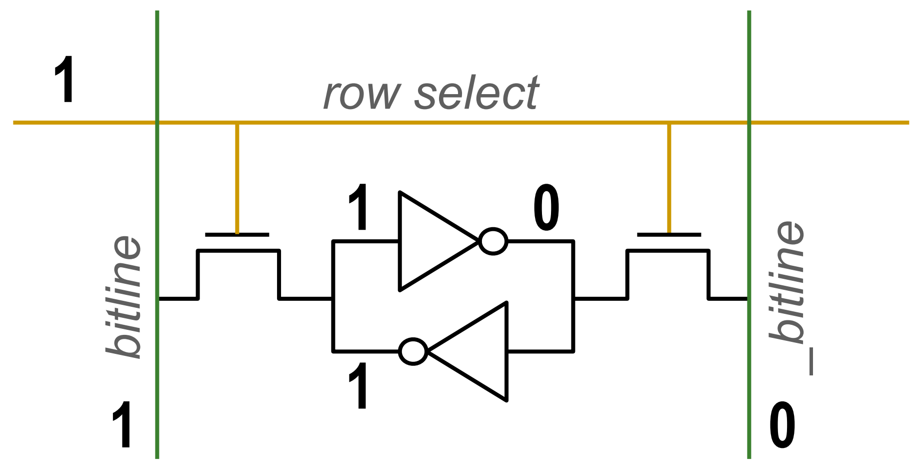

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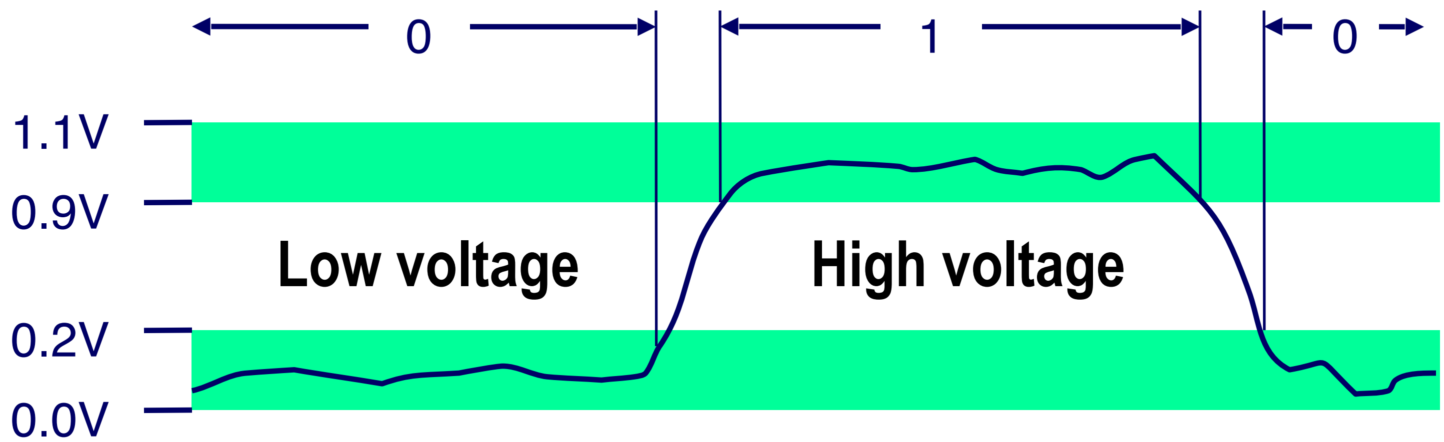

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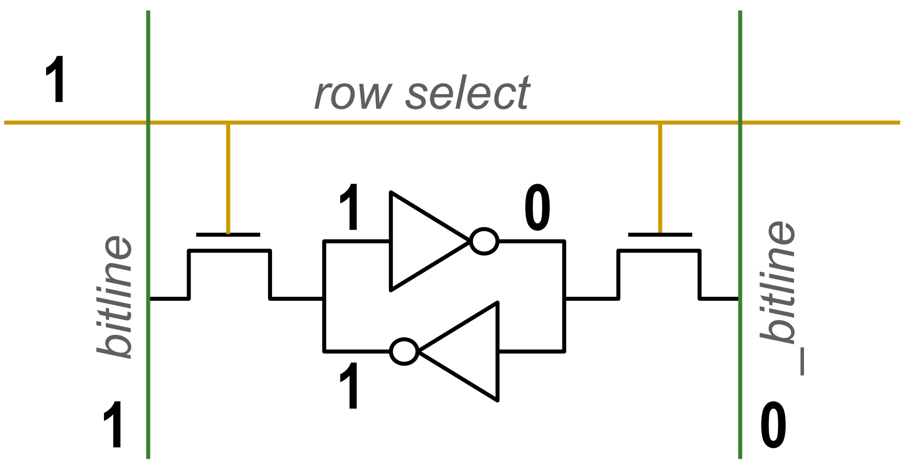
  - Feedback path persists the value in the "cell"
  - 4 transistors for storage



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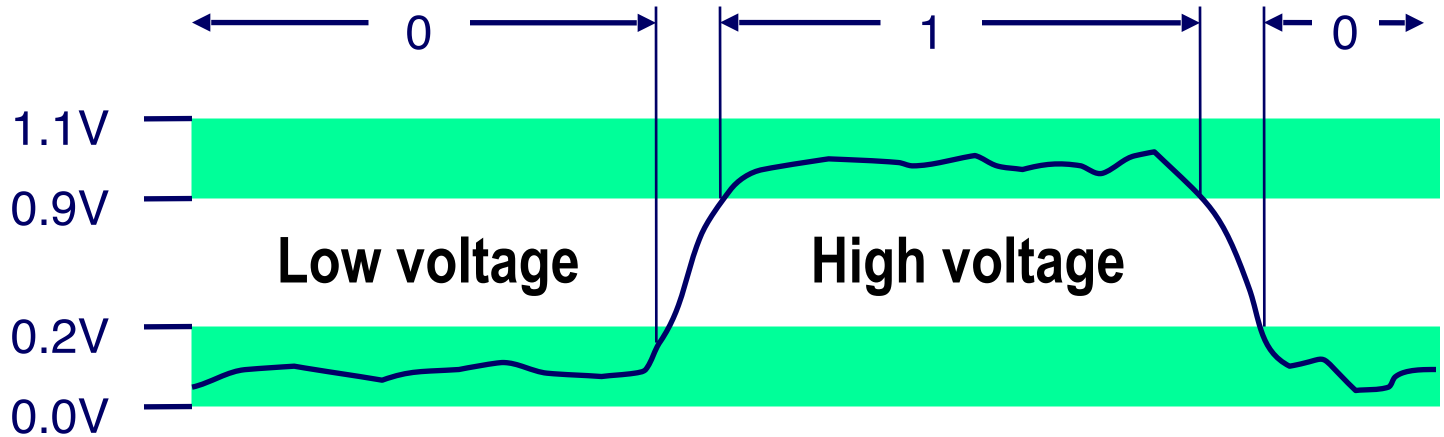

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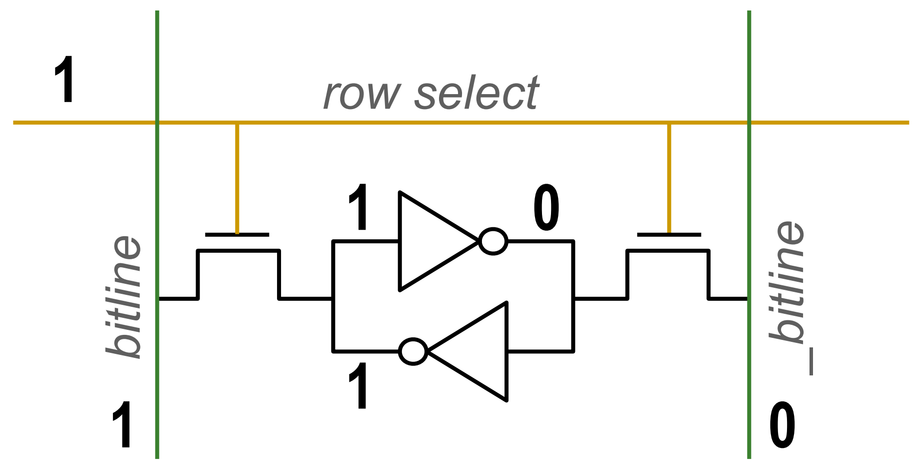

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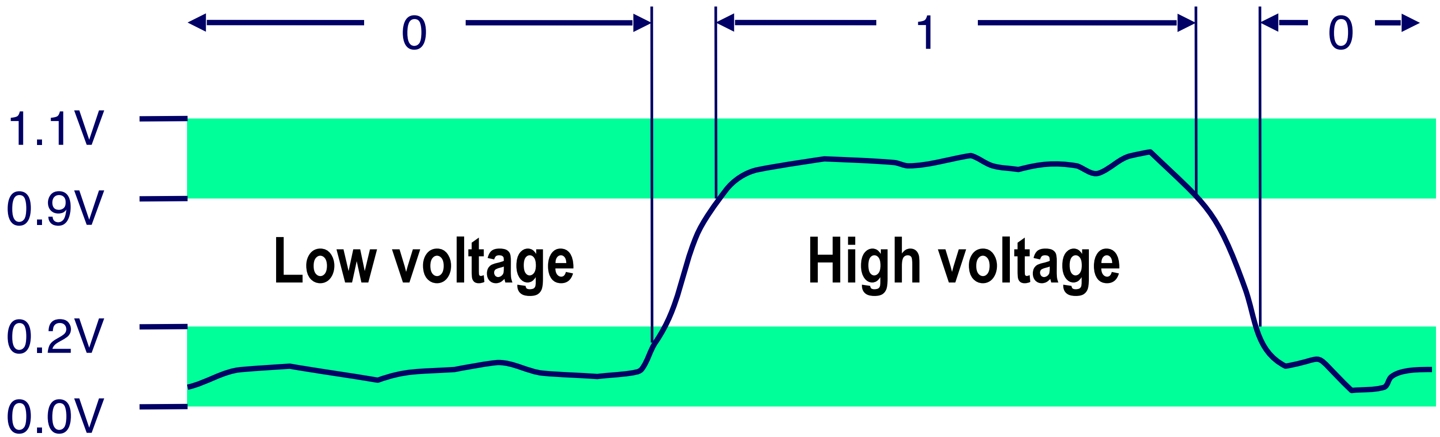

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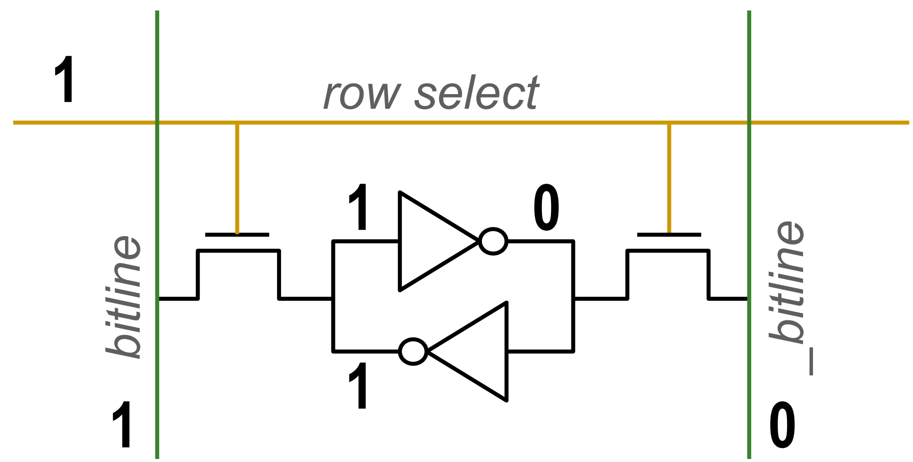
  - 4 transistors for storage
  - 2 transistors for access
  - A "6T" cell



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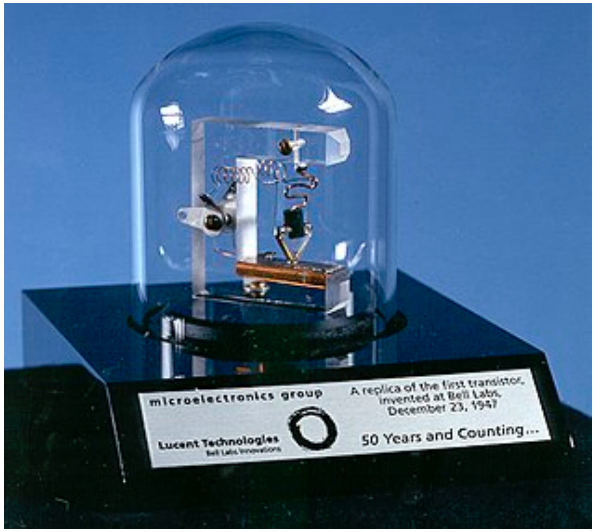
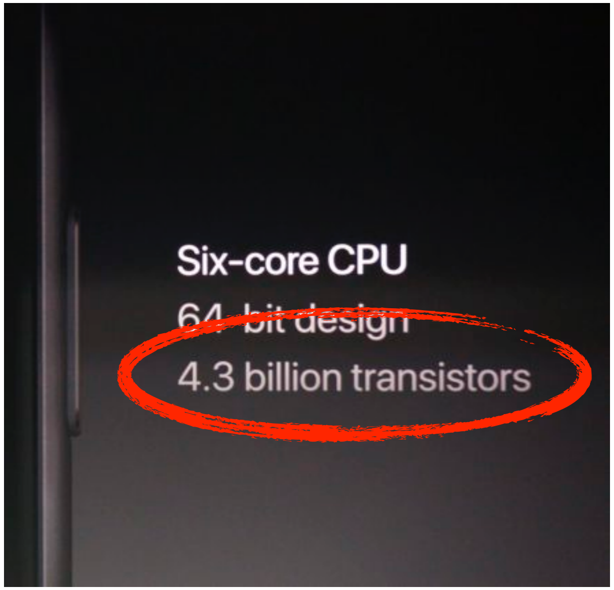
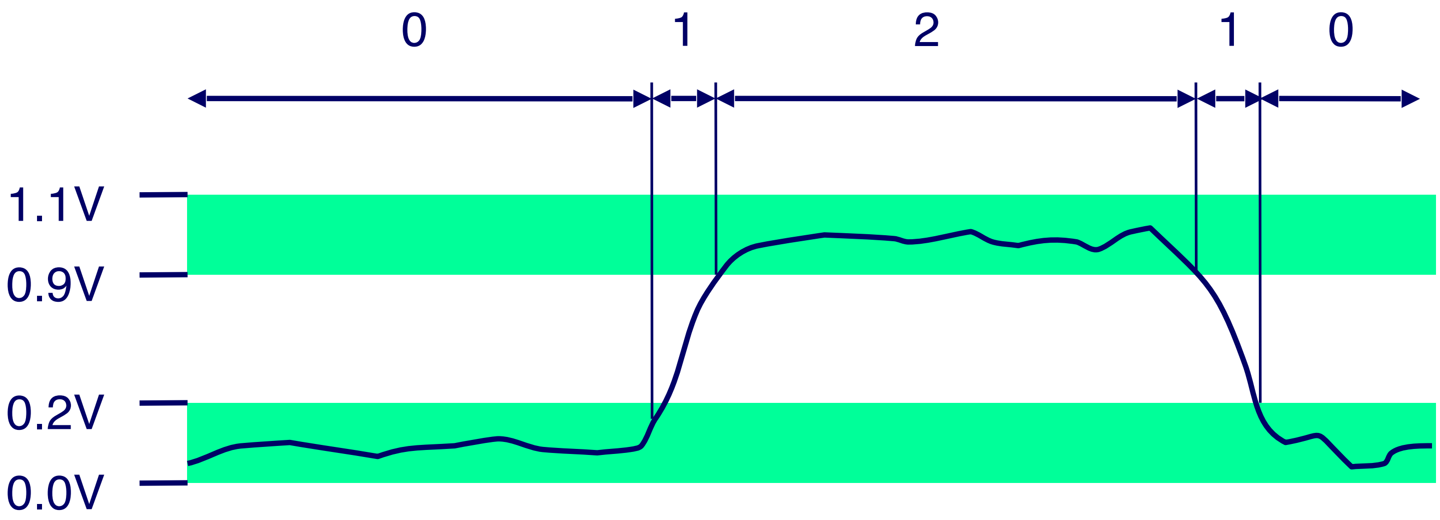




Why Limit Ourselves Only to Binary?

- Voltage is continuous. Why interpret it only as 0s and 1s?



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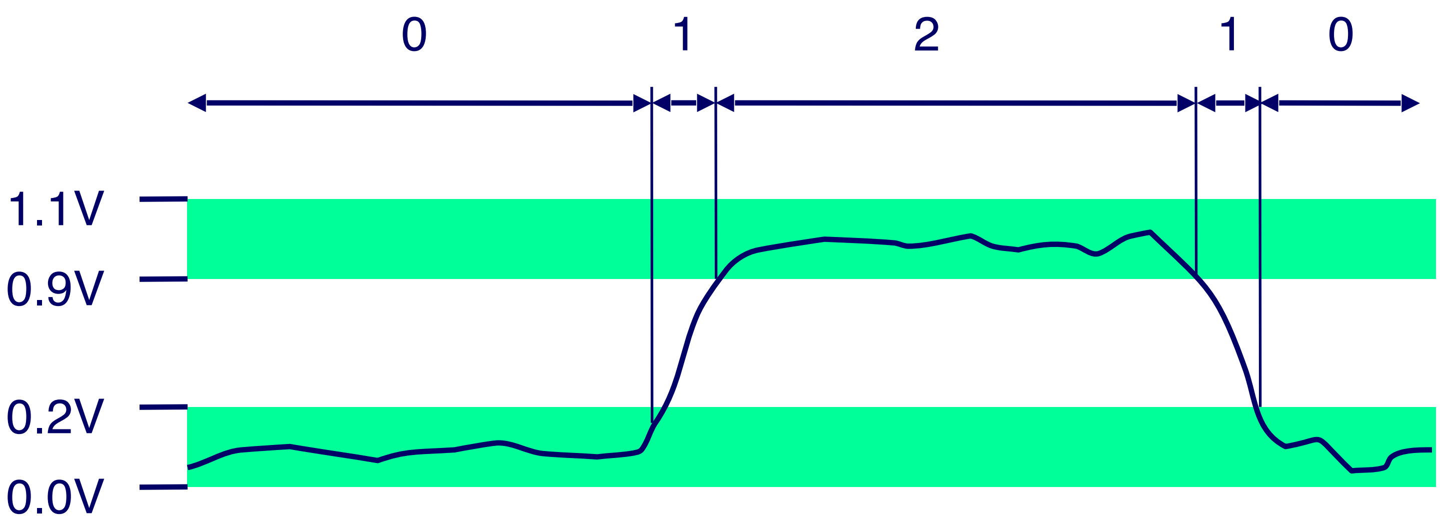
- Answer: Noise

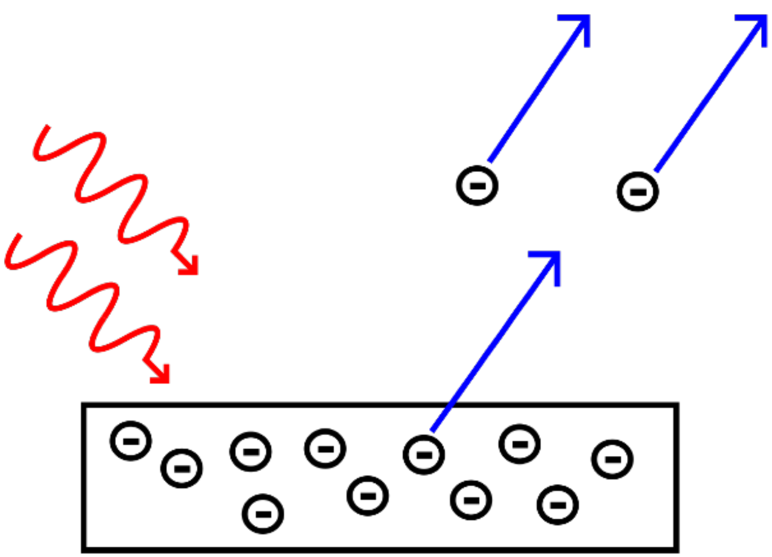


Can We Tolerate the Noise?

- But, there are applications that can tolerate noise

Can We Tolerate the Noise?

- But, there are applications that can tolerate noise
- Classic Example: Camera Sensor
  - Photoelectric Effect



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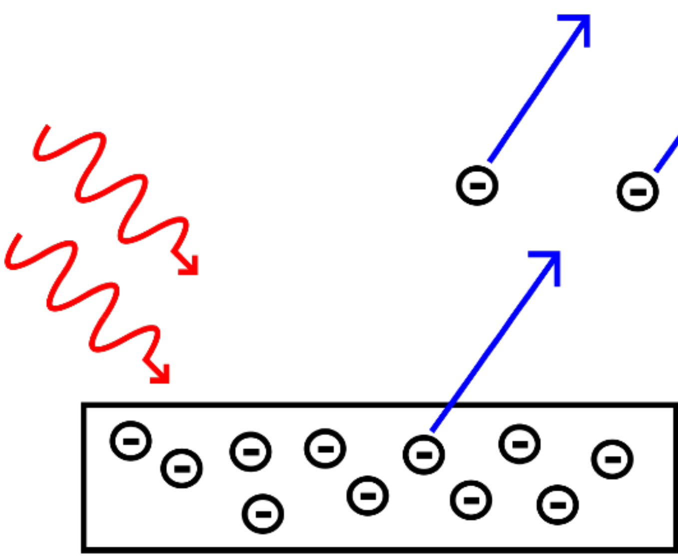

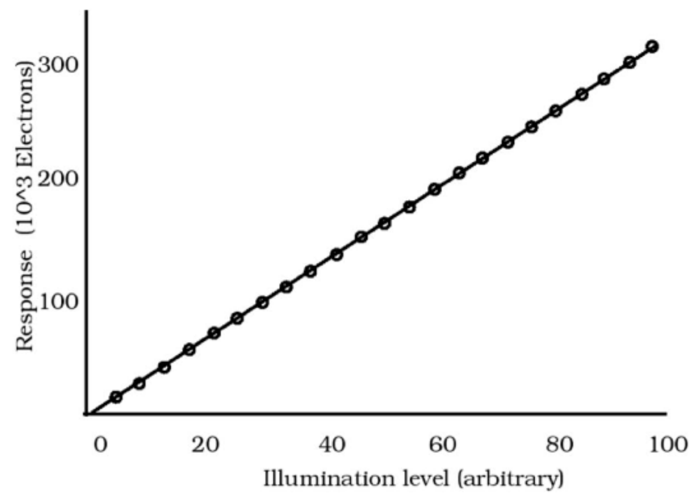

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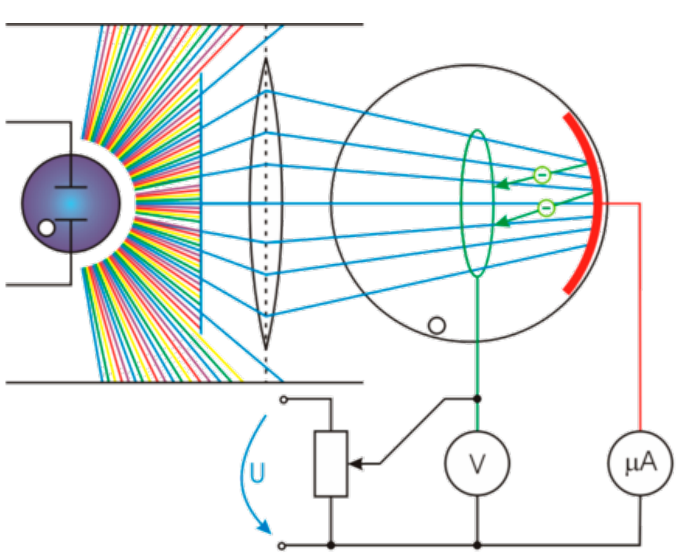

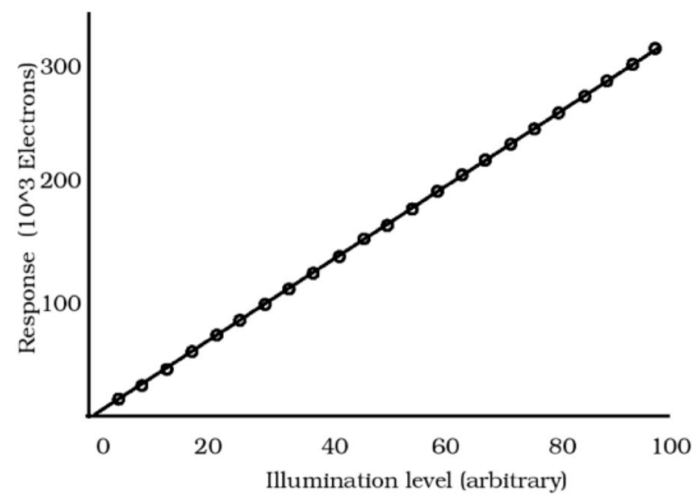

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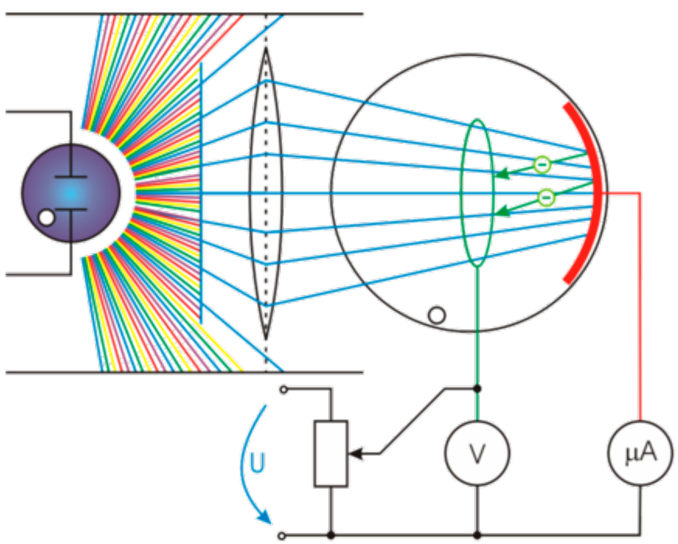

Binary Notation

Binary Notation

- Base 2 Number Representation (Binary)

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- C.f., Base 10 number representation (Decimal)

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Decimal	**Binary**
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

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```
  0110
+ 0101
------
  1011
```

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```
  0110
+ 0101
------
  1011
```

```
  6
+ 5
----
 11
```

Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
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Hexdecimal (Hex) Notation

- Base 16 Number Representation
  - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
  - Four bits per Hex digit
  - $11111110_2 = FE_{16}$
- Write $FA1D37B_{16}$ in C as
  - 0xFA1D37B
  - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Bit, Byte, Word

- Byte = 8 bits
  - Binary 00000000_2 to $11111111_{2;}$ Decimal: 0_{10} to 255_{10}; Hex: 00_{16} to FF_{16}
  - Least Significant Bit (LSb) vs. Most Significant Bit (MSb)

10110111

MSb **LSb**

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$$\boxed{1}011011\boxed{1}$$

MSb **LSb**

- Word = 4 Bytes (32-bit machine) / 8 Bytes (64-bit machine)
  - Least Significant Byte (LSB) vs. Most Significant Byte (MSB)

Today: Representing Information in Binary

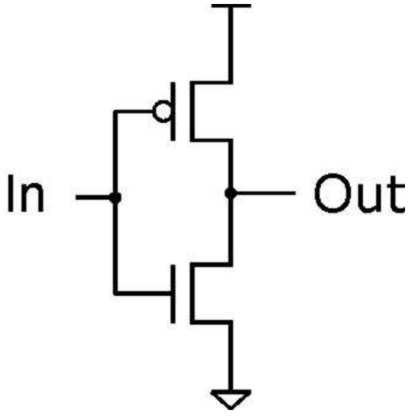
- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

Bit-level manipulations

Not

- **~A = 1 when A=0**

~	
0	1
1	0



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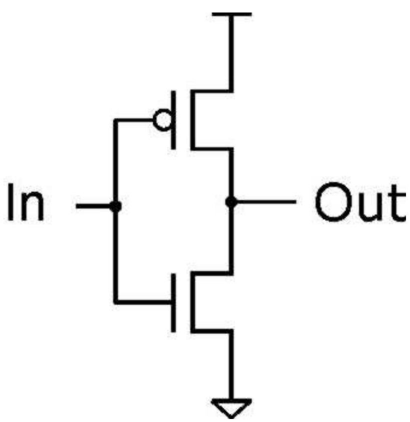

Or

- **A|B = 1 when either A=1 or B=1**

|	0	1
0	0	1
1	1	1

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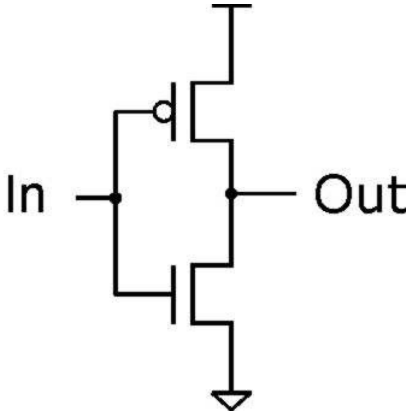

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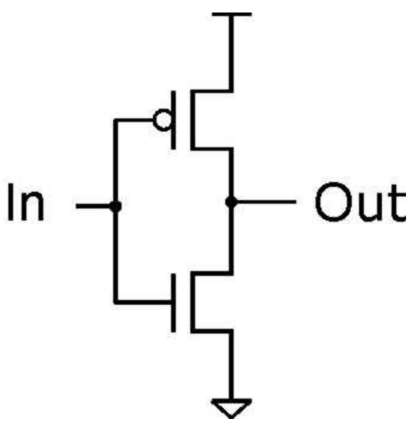

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Exclusive-Or (Xor)

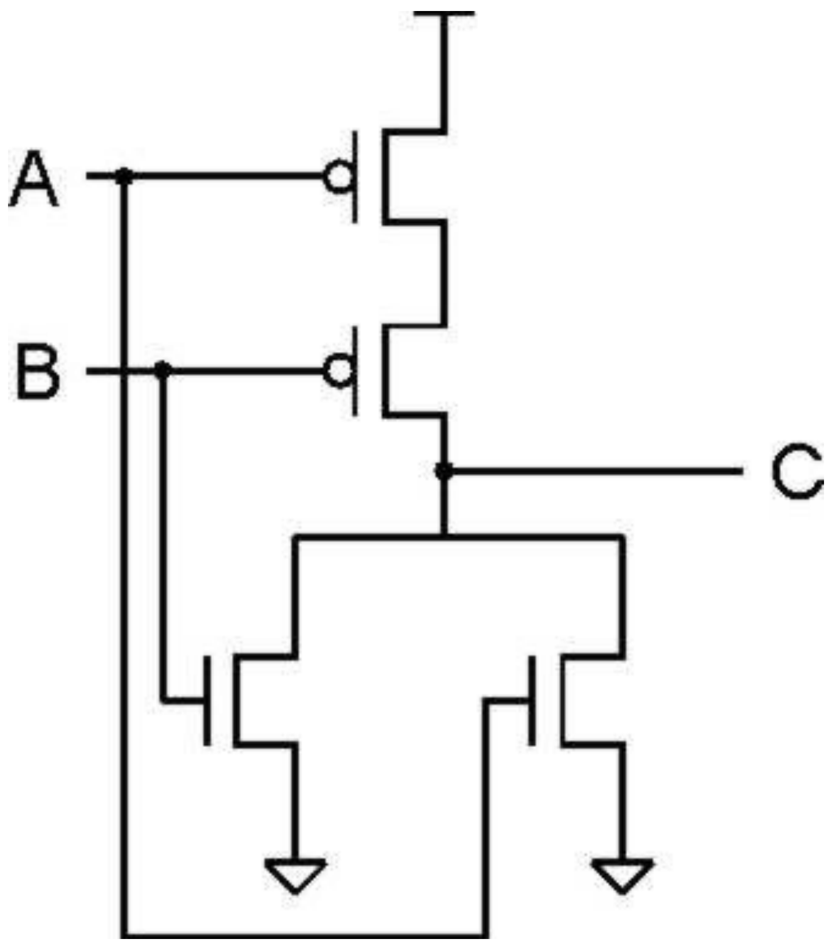
- A^B = 1 when either A=1 or B=1, but not both

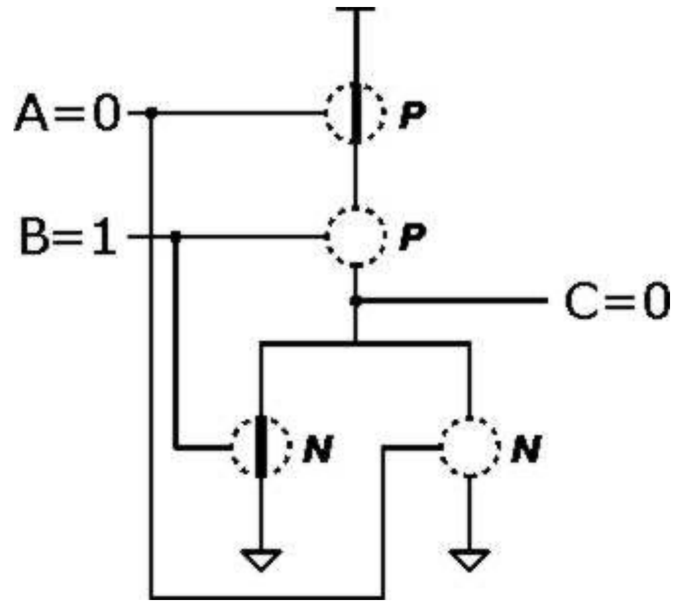
^	0	1
0	0	1
1	1	0

NOR (OR + NOT)

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

NOR (OR + NOT)





A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

Bit Vector Operations

- Operate on Bit Vectors
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
__________   __________   __________   __________
```

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  --------      --------      --------      --------
  01000001
```

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   01101001       01101001       01101001
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   01000001       01111101
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```

Bit-Level Operations in C

- Operations &, |, ~, ^ Available in C
  - Apply to any “integral” data type
    - `long, int, short, char, unsigned`
  - View arguments as bit vectors
  - Arguments applied bit-wise
- Examples (Char data type)
  - `~0x41` → `0xBE`
    - `~01000001`$_2$ → `10111110`$_2$
  - `~0x00` → `0xFF`
    - `~00000000`$_2$ → `11111111`$_2$
  - `0x69 & 0x55` → `0x41`
    - `01101001`$_2$ `&` `01010101`$_2$ → `01000001`$_2$
  - `0x69 | 0x55` → `0x7D`
    - `01101001`$_2$ `|` `01010101`$_2$ → `01111101`$_2$

Aside: Logic Operations in C

- Contrast to Logical Operators
  - `&&, ||, !`
    - View 0 as “False”
    - Anything nonzero as “True”
    - Always return 0 or 1
    - Early termination (e.g., 0 && 1 && 1)
- Examples (char data type)
  - `!0x41` → `0x00`
  - `!0x00` → `0x01`
  - `!!0x41` → `0x01`

  - `0x69 && 0x55` → `0x01`
  - `0x69 || 0x55` → `0x01`
  - `p && *p` (avoids null pointer access)

Shift Operations

- Left Shift: `x << y`
  - Shift bit-vector **x** left **y** positions
      - Throw away extra bits on left
    - Fill with 0's on right
- Right Shift: `x >> y`
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- Undefined Behavior
  - Shift amount < 0 or ≥ total amount of bits

Argument x	01100010
<< 3	
Log. >> 2	
Arith. >> 2	

Argument x	10100010
<< 3	
Log. >> 2	
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    - Summary
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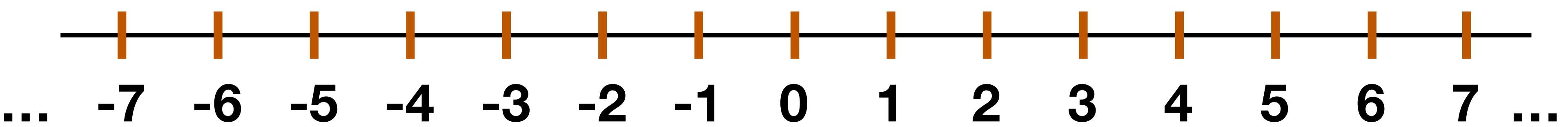
Representing Numbers in Binary

- Different types of number
  - Integer (Negative and Non-negative)
  - Fractions
  - Irrationals



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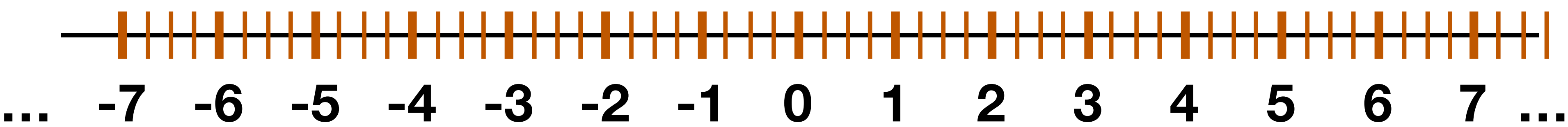

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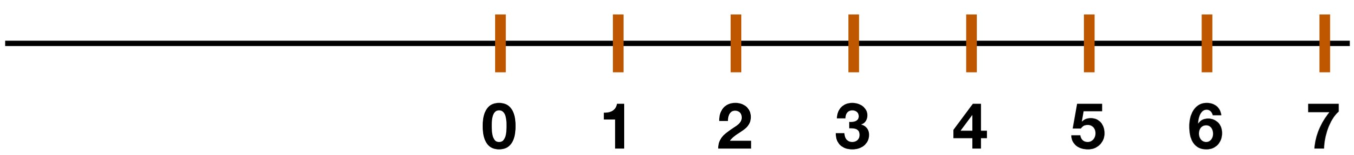

Encoding Negative Numbers

- So far we have been discussing non-negative numbers: so called unsigned. How about negative numbers?
- Solution 1: Sign-magnitude
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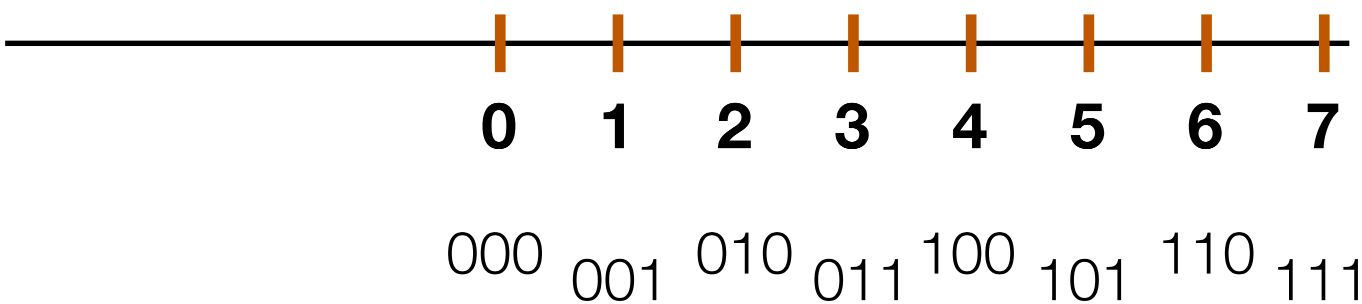

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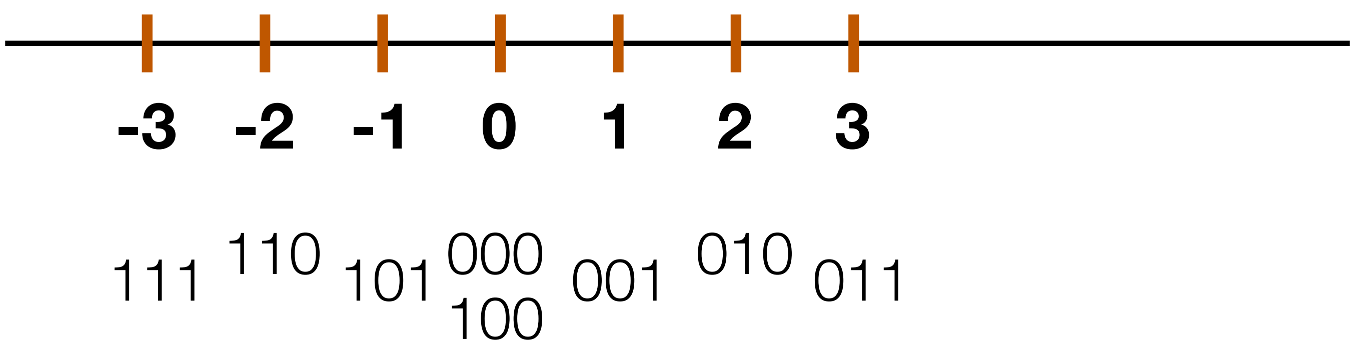

Sign-Magnitude Implications

- Bits have different semantics
  - Two zeros…
  - Normal arithmetic doesn’t work
  - Make hardware design harder

Signed Value	Binary
0	000
1	001
2	010
3	011
-0	100
-1	101
-2	110
-3	111

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------
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```

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-------         -------
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Encoding Negative Numbers

- Solution 2: Two’s Complement

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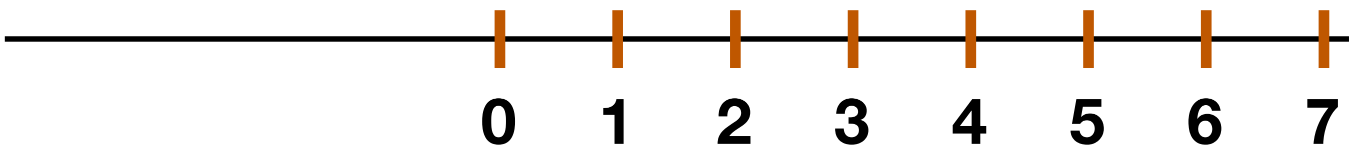

Unsigned	Binary
0	000
1	001
2	010
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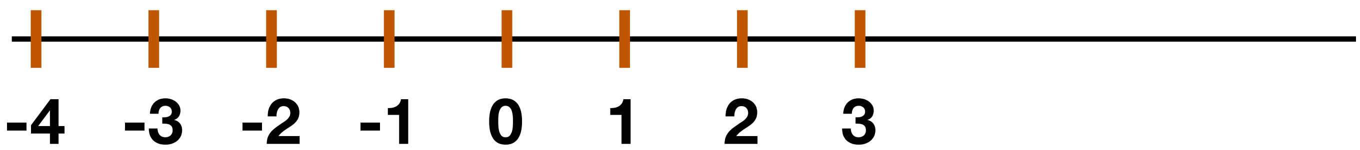
4	100
5	101
6	110
7	111

Encoding Negative Numbers

- Solution 2: Two's Complement



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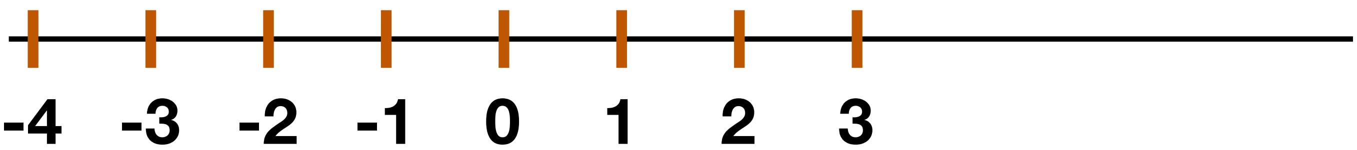

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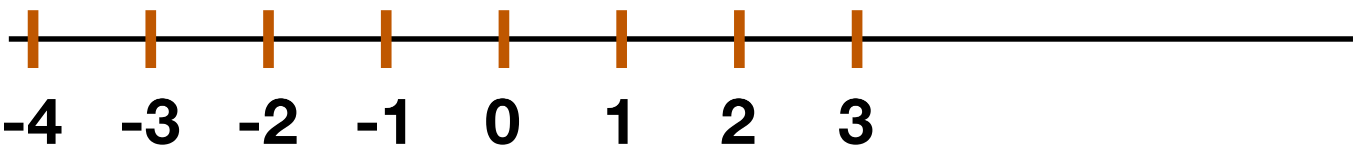

Signed Weight	Unsigned Weight	Bit Position
2^0	2^0	0
2^1	2^1	1
-2^2	2^2	2

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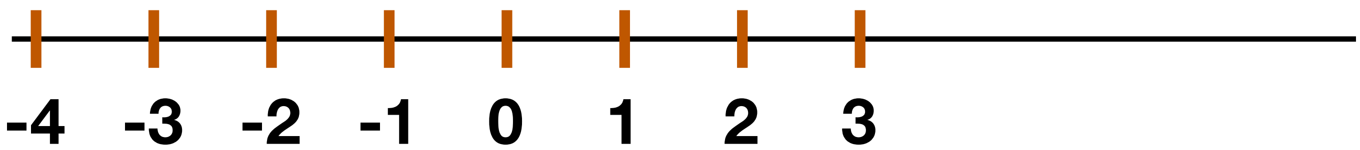

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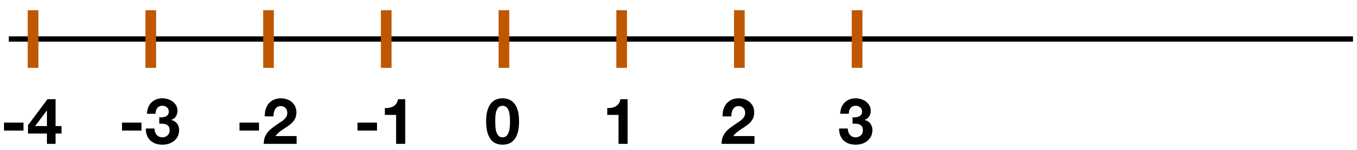

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$$101_2 = 1*2^0 + 0*2^1 - 1*2^2 = -3_{10}$$

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-4	4	100
-3	5	101
-2	6	110
-1	7	111

Two-Complement Encoding Example

```
x =      15213: 00111011 01101101
y =     -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Two-Complement Implications

- Only 1 zero
- Usual arithmetic still works
- There is a bit that represents the sign!
- Most widely used in today's machines

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+)  -3
-------
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```

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  - *UMin* = 0

    000…0
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Values for *W* = 16

	Decimal	Hex	Binary
`UMax`	**65535**	`FF FF`	`11111111 11111111`
`TMax`	**32767**	`7F FF`	`01111111 11111111`
`TMin`	**-32768**	`80 00`	`10000000 00000000`
`-1`	**-1**	`FF FF`	`11111111 11111111`
`0`	**0**	`00 00`	`00000000 00000000`

Numeric Ranges

- Unsigned Values
  - *UMin* = 0
    000…0
  - *UMax* = $2^w - 1$
    111…1

- Two's Complement Values
  - *TMin* = -2^{w-1}
    100…0
  - *TMax* = $2^{w-1} - 1$
    011…1
- Other Values
  - Minus 1
    111…1

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-1	**-1**	**FF FF**	**11111111 11111111**
0	**0**	**00 00**	**00000000 00000000**

Data Representations in C (in Bytes)

- By default variables are signed
- Unless explicitly declared as unsigned (e.g., `unsigned int`)
- Signed variables use two-complement encoding

C Data Type	32-bit	64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8

Data Representations in C (in Bytes)

	W			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

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- C Language
  - `#include <limits.h>`
  - Declares constants, e.g.,
    - `ULONG_MAX`
    - `LONG_MAX`
    - `LONG_MIN`
  - Values platform specific