CSC 252: Computer Organization Spring 2018: Lecture 2

Instructor: Yuhao Zhu

Department of Computer Science
University of Rochester

Action Items:
- **Programming Assignment 1 is out**
- **Trivia 1 is due on Friday, midnight**

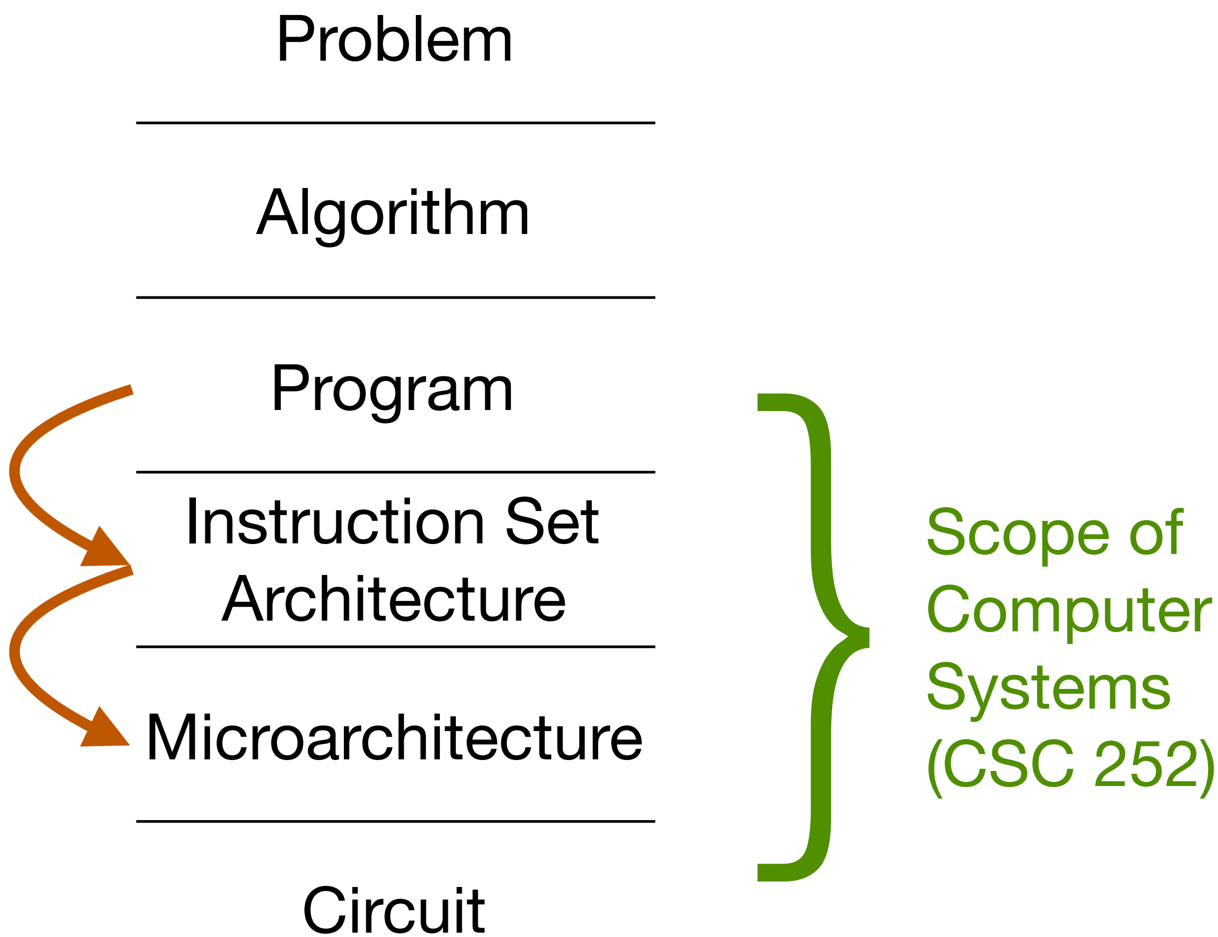
Slide Credits: Randal E. Bryant, David R. O'Hallaron

Announcement

- Programming Assignment 1 is out
  - Details: http://cs.rochester.edu/courses/252/spring2018/labs/assignment1.html
  - Due on Feb 2, 11:59 PM
  - Trivia due Friday, 1/26, 11:59 PM
  - You have 3 slip days (not for trivia)
- Ask the TAs if you have any questions regarding programming assignments

Previously in 252…

- How is a human-readable program translated to a representation that computers can understand?
- How does a modern computer execute that program?



Previously in 252…

C Program

```
void add() {
  int a = 1;
  int b = 2;
  int c = a + b;
}
```

Pre-processor
Compiler

→

Assembly program

```
movl    $1, -4(%rbp)
movl    $2, -8(%rbp)
movl    -4(%rbp), %eax
addl    -8(%rbp), %eax
```

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Assembler
Linker

Executable Binary

00011001 …
01101010 …
11010101 …
01110001 …

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Assembly program

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movl    $1, -4(%rbp)
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- Is ISA referring to assembly or binary?
  - They are the same thing; different representations.
- Instruction = Operator + Operand(s)

Previously in 252…

Assembly program *Executable Binary*

```
movl  $1, -4(%rbp)        00011001  …
movl  $2, -8(%rbp)        01101010  …
movl  -4(%rbp), %eax      11010101  …
addl  -8(%rbp), %eax      01110001  …
```

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Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

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- By encoding/interpreting sets of bits in various ways
  - Computers determine what to do (instructions)

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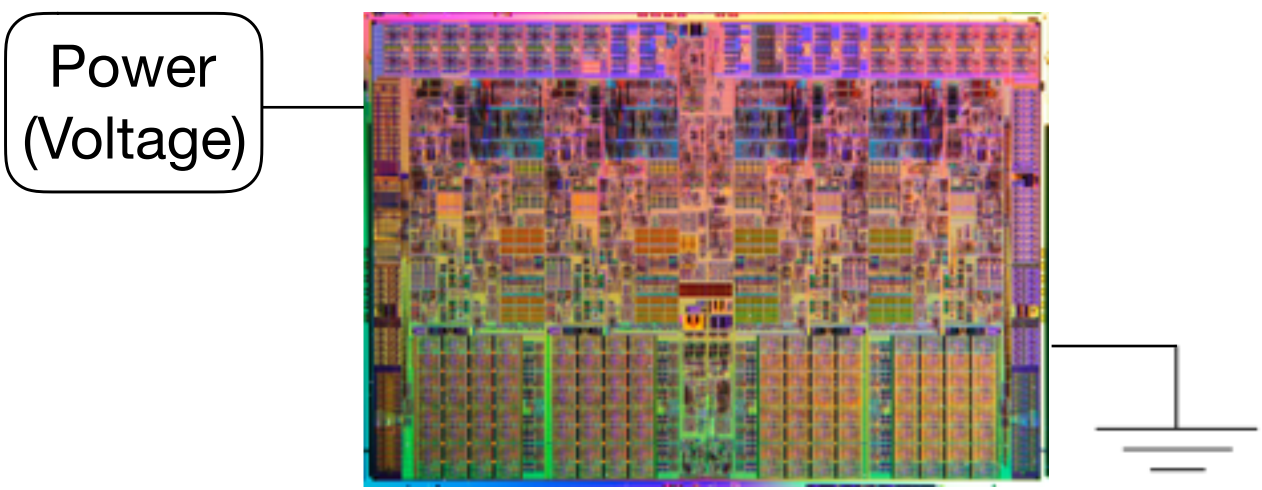

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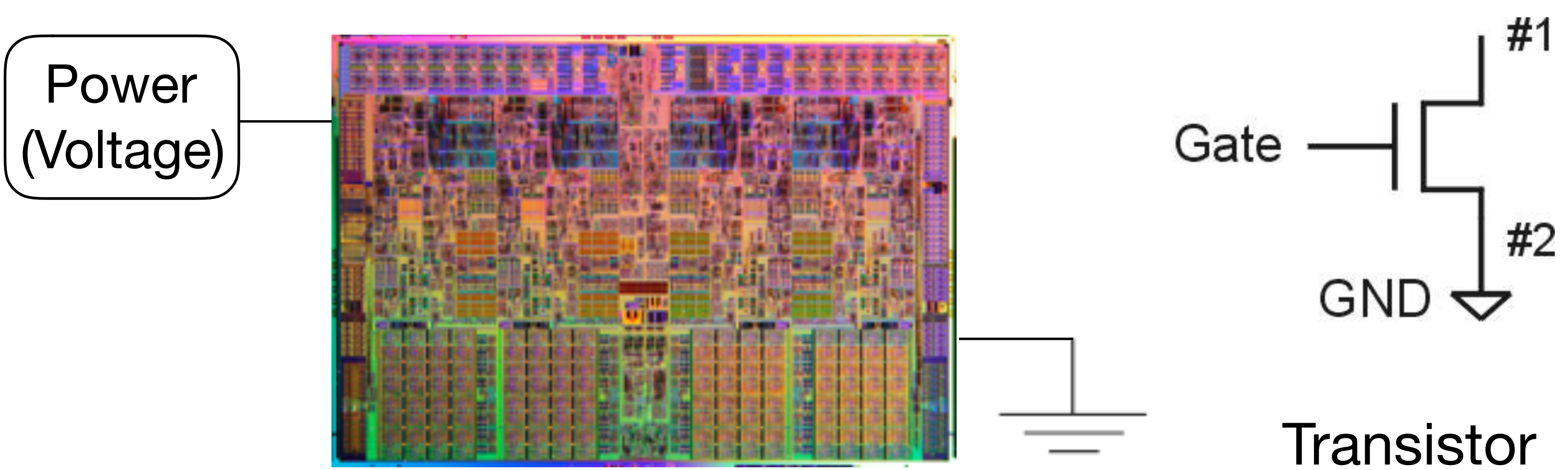

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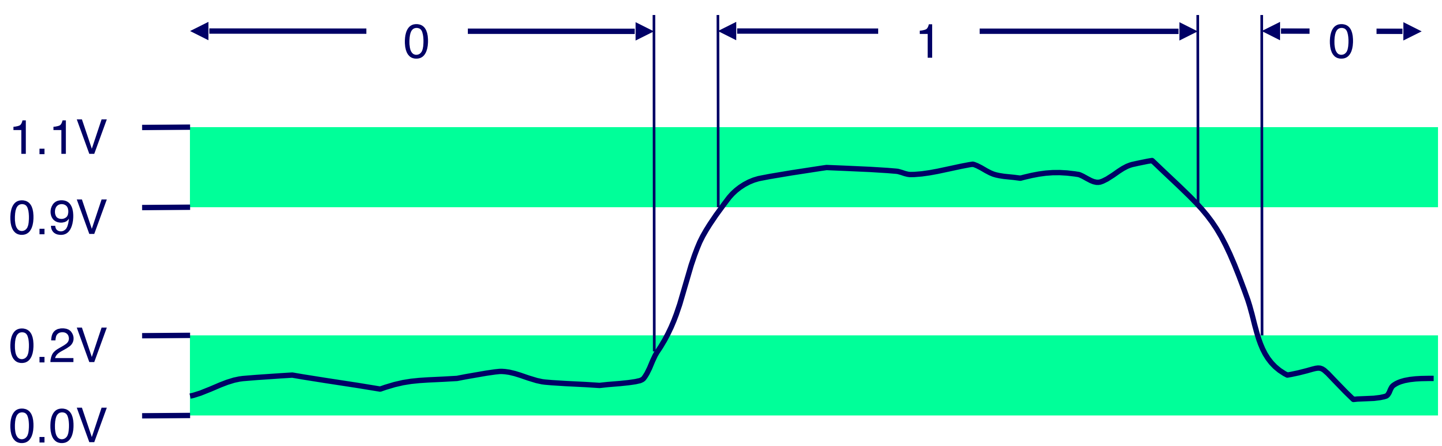
  - Transistor has two states: presence of a high voltage (“1”); presence of a low voltage (“0”)



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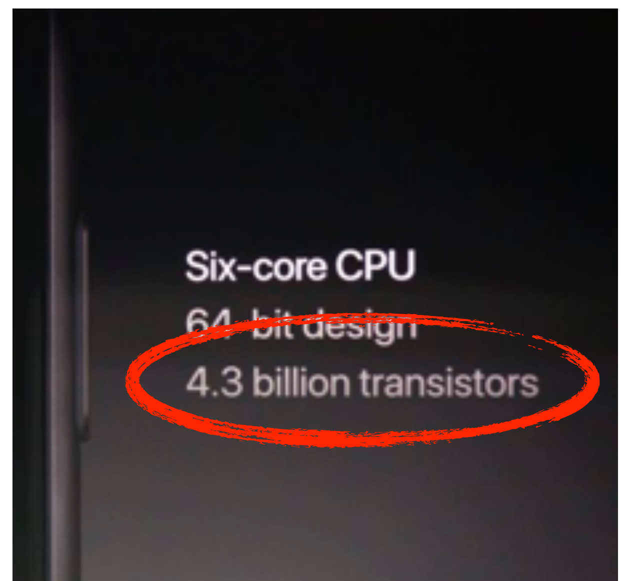

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- Classic Example: Camera Sensor
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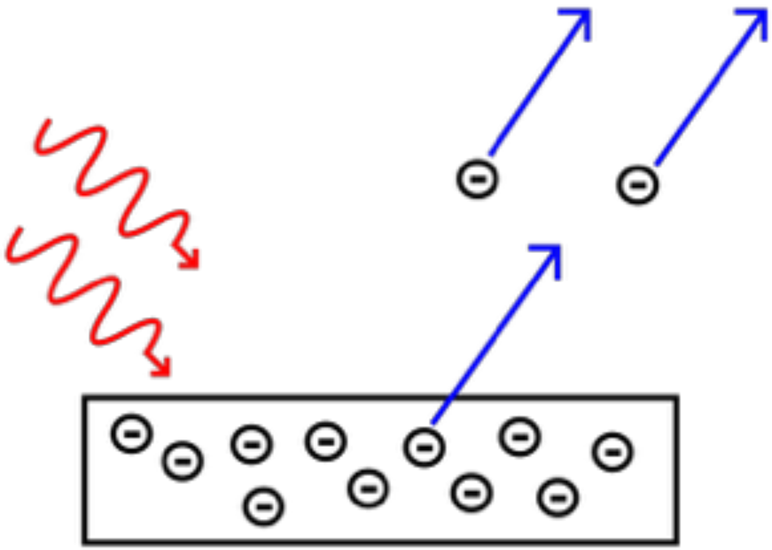

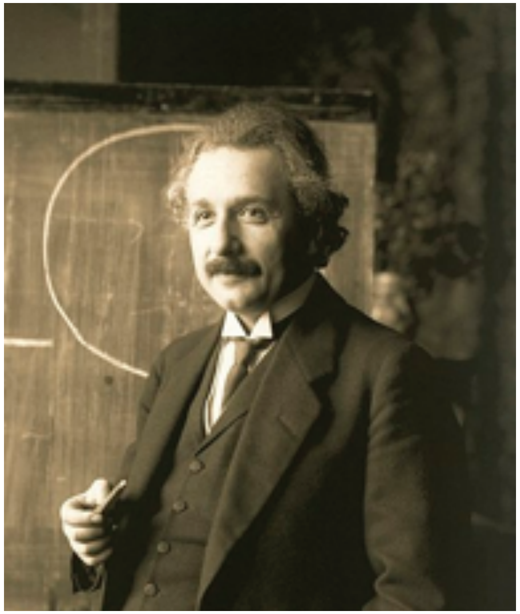

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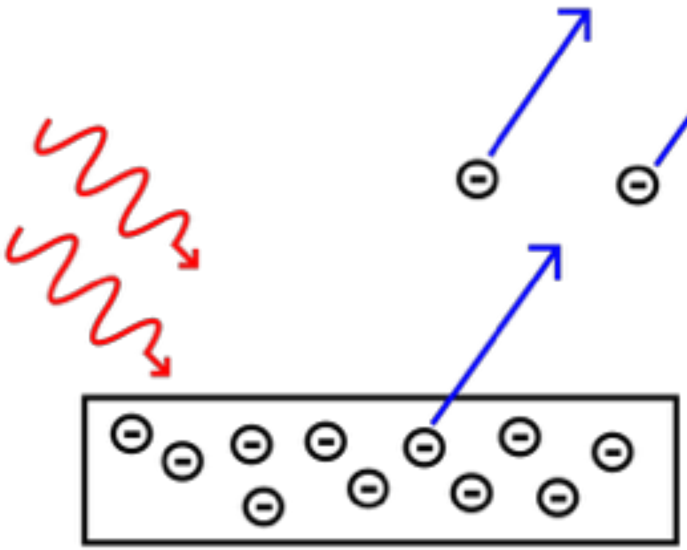

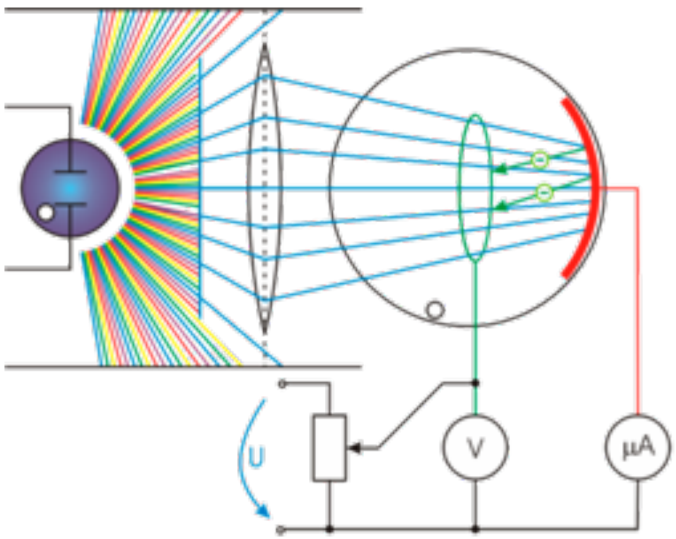

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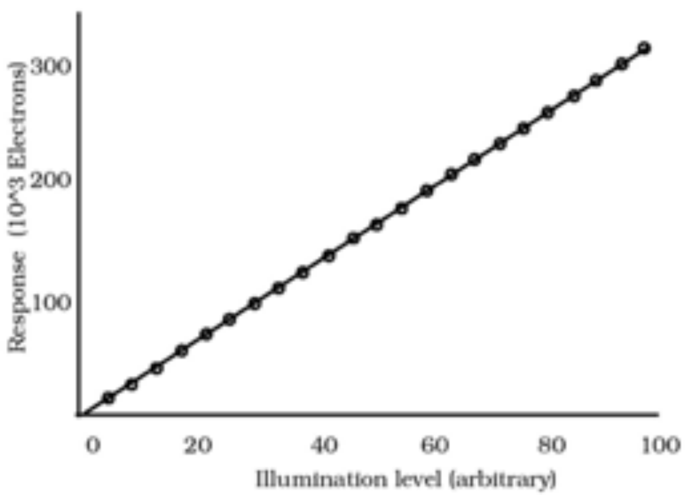

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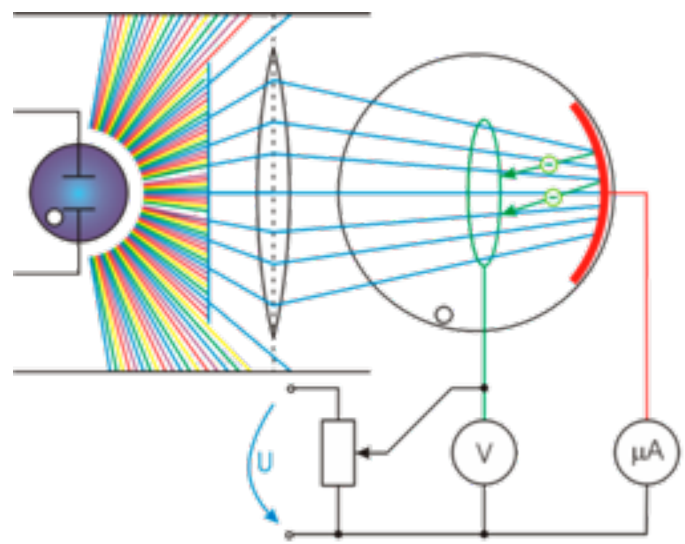

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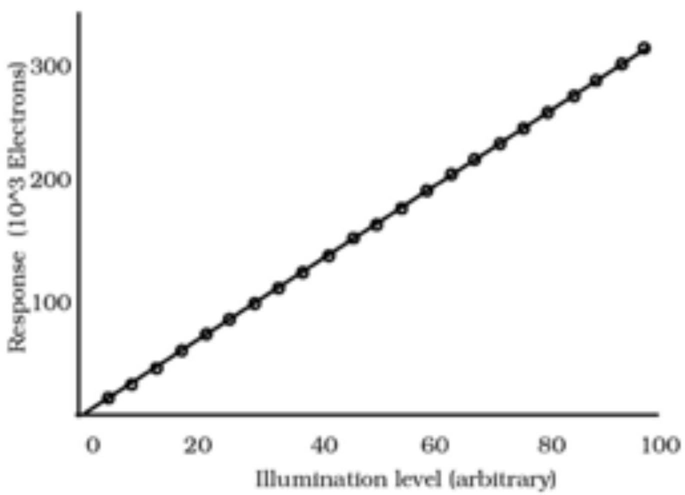

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Decimal	**Binary**
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

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```
   0110
 + 0101
 ------
   1011
```

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```
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+ 0101
------
  1011
```

```
   6
+  5
----
  11
```

Decimal	Binary
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Hexdecimal (Hex) Notation

- Base 16 Number Representation
  - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
  - Four bits per Hex digit
  - $11111110_2 = FE_{16}$
- Write $FA1D37B_{16}$ in C as
  - 0xFA1D37B
  - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Bit, Byte, Word

- Byte = 8 bits
  - Binary 00000000_2 to 11111111_2; Decimal: 0_{10} to 255_{10}; Hex: 00_{16} to FF_{16}
  - Least Significant Bit (LSb) vs. Most Significant Bit (MSb)

$$\boxed{1}011011\boxed{1}$$

MSb **LSb**

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- Word = 4 Bytes (32-bit machine) / 8 Bytes (64-bit machine)
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Questions?

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Boolean Algebra

- Developed by George Boole in 19th Century
  - Algebraic representation of logic
    - Encode "True" as 1 and "False" as 0

And

- **A&B = 1 when both A=1 and B=1**

&	0	1
0	0	0
1	0	1

Or

- **A|B = 1 when either A=1 or B=1**

|	0	1
0	0	1
1	1	1

Not

- **~A = 1 when A=0**

~	
0	1
1	0

Exclusive-Or (Xor)

- **A^B = 1 when either A=1 or B=1, but not both**

^	0	1
0	0	1
1	1	0

General Boolean Algebras

- Operate on Bit Vectors
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
```

- All of the Properties of Boolean Algebra Apply

General Boolean Algebras

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& 01010101   | 01010101   ^ 01010101   ~ 01010101
  --------     --------     --------     --------
  01000001
```

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  --------     --------     --------     --------
  01000001     01111101     00111100     10101010
```

- All of the Properties of Boolean Algebra Apply

Bit-Level Operations in C

- Operations &, |, ~, ^ Available in C
  - Apply to any "integral" data type
    - `long, int, short, char, unsigned`
  - View arguments as bit vectors
  - Arguments applied bit-wise
- Examples (Char data type)
  - `~0x41` → `0xBE`
    - $\sim 01000001_2 \rightarrow 10111110_2$
  - `~0x00` → `0xFF`
    - $\sim 00000000_2 \rightarrow 11111111_2$
  - `0x69 & 0x55` → `0x41`
    - 01101001_2 & $01010101_2 \rightarrow 01000001_2$
  - `0x69 | 0x55` → `0x7D`
    - $01101001_2 \mid 01010101_2 \rightarrow 01111101_2$

Contrast: Logic Operations in C

- Contrast to Logical Operators
  - `&&, ||, !`
    - View 0 as "False"
    - Anything nonzero as "True"
    - Always return 0 or 1
    - Early termination
- Examples (char data type)
  - `!0x41` → `0x00`
  - `!0x00` → `0x01`
  - `!!0x41` → `0x01`

  - `0x69 && 0x55` → `0x01`
  - `0x69 || 0x55` → `0x01`
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  - `p && *p` (avoids null pointer access)

Watch out for && vs. & (and || vs. |)… one of the more common oopsies in C programming

Shift Operations

- Left Shift: `x << y`
  - Shift bit-vector **x** left **y** positions
      - Throw away extra bits on left
    - Fill with 0's on right
- Right Shift: `x >> y`
  - Shift bit-vector **x** right **y** positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on left
- Undefined Behavior
  - Shift amount < 0 or ≥ word size

Argument x	`01100010`
<< 3	
Log. >> 2	
Arith. >> 2	

Argument x	`10100010`
<< 3	
Log. >> 2	
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  - Shift amount < 0 or ≥ word size

Argument x	01100010
<< 3	00010*000*
Log. >> 2	*00*011000
Arith. >> 2	*00*011000

Argument x	10100010
<< 3	00010*000*
Log. >> 2	101000
Arith. >> 2	

Shift Operations

- Left Shift: `x << y`
  - Shift bit-vector **x** left **y** positions
      - Throw away extra bits on left
    - Fill with 0's on right
- Right Shift: `x >> y`
  - Shift bit-vector **x** right **y** positions
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  - Logical shift
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<< 3	00010*000*
Log. >> 2	*00*011000
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Arith. >> 2	101000

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Argument x	01100010
<< 3	00010*000*
Log. >> 2	*00*011000
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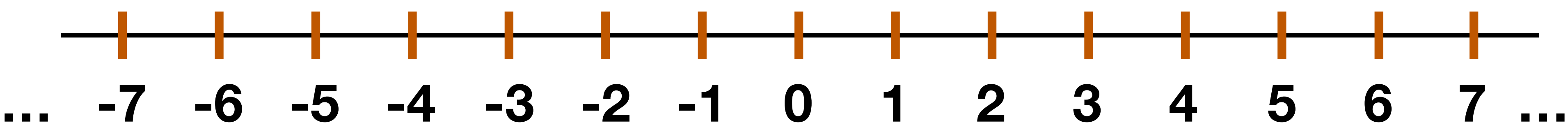
<< 3	00010*000*
Log. >> 2	*00*101000
Arith. >> 2	*11*101000

Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

Representing Numbers in Binary

- What numbers do we need to represent in bits?
  - Integer (Negative and Non-negative)
  - Fractions
  - Irrationals



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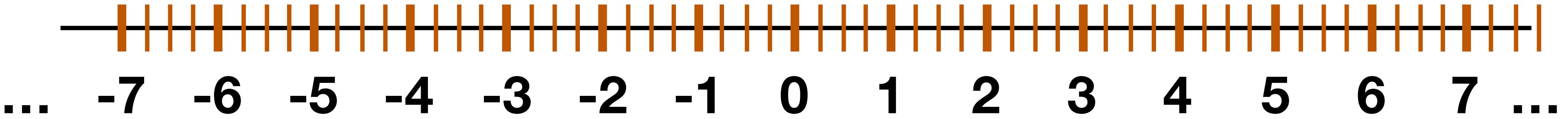

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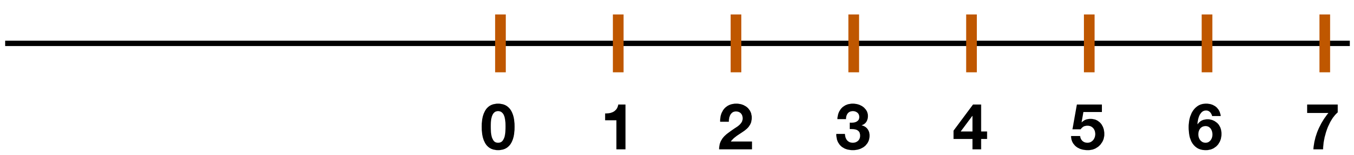

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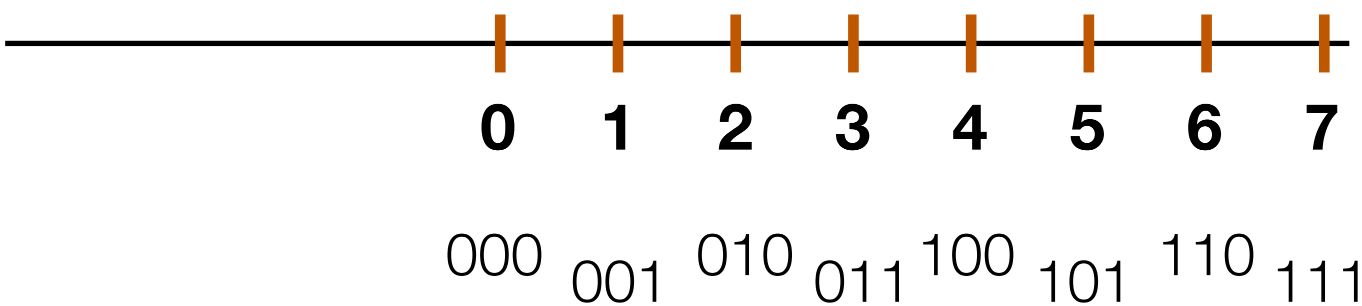

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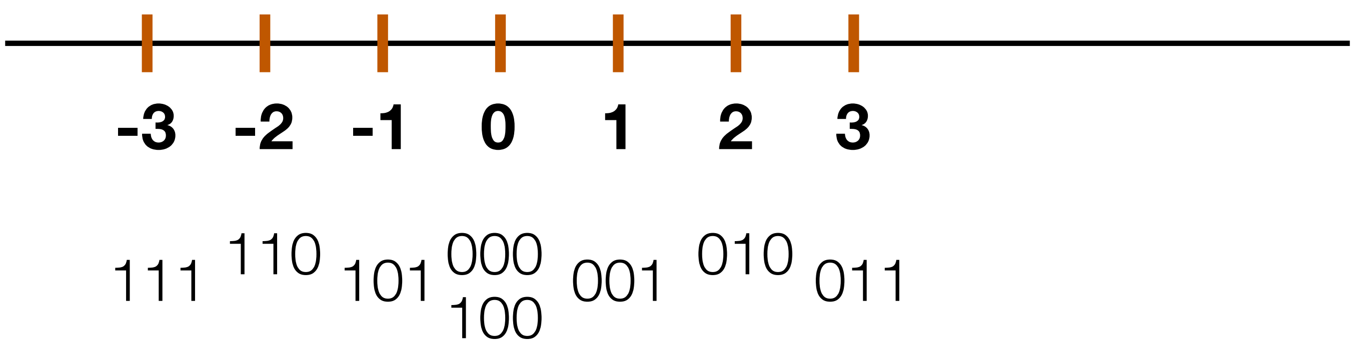

Sign-Magnitude Implications

- Bits have different semantics
  - Two zeros…
  - Normal arithmetic doesn’t work
  - Make hardware design harder

Signed Value	Binary
0	000
1	001
2	010
3	011
-0	100
-1	101
-2	110
-3	111

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```
    010
+)  101
-------
    111
```

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-------
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      2
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-------
     -3
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    010          2
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Encoding Negative Numbers

- Solution 2: Two’s Complement

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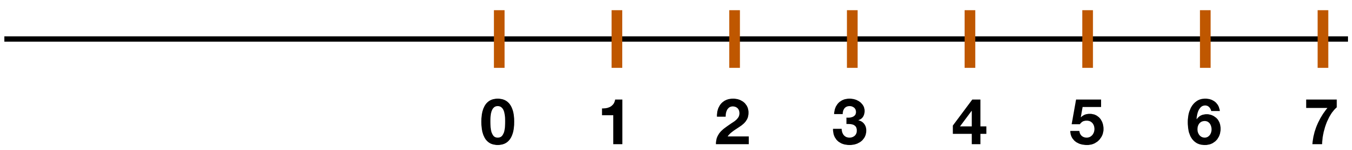

Unsigned	Binary
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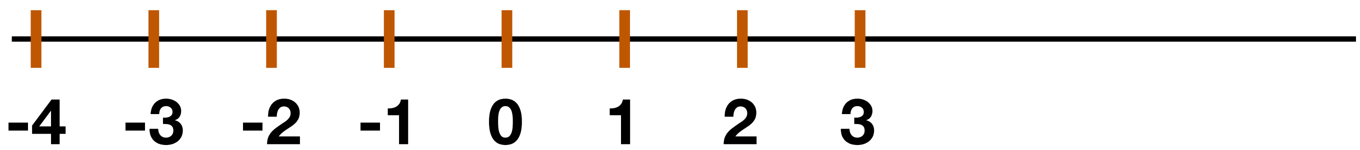
Encoding Negative Numbers

- Solution 2: Two's Complement



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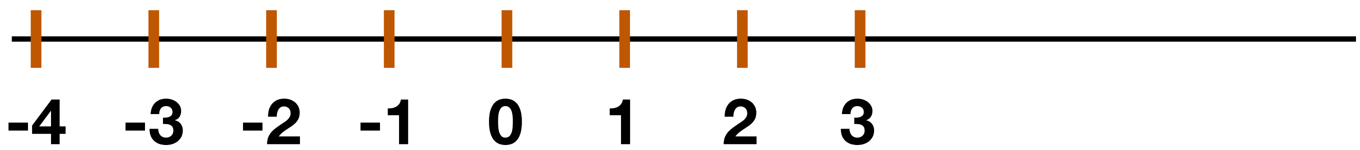

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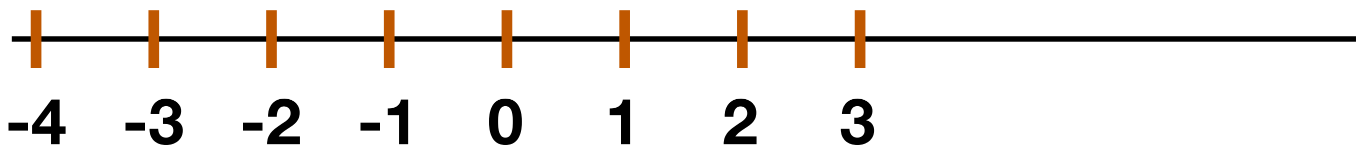

Signed Weight	Unsigned Weight	Bit Position
0	0	0
1	1	1
2	2	2
-4	4	3

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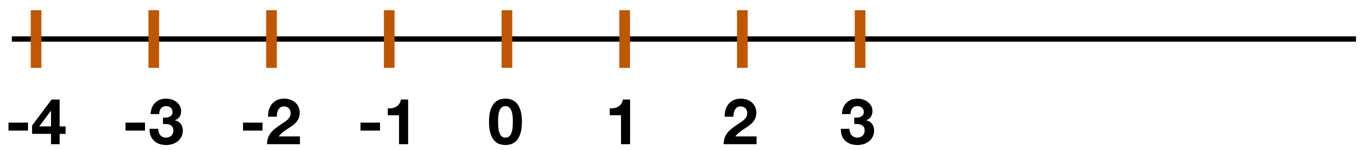

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2	2	010
3	3	011
-4	4	100
-3	5	101
-2	6	110
-1	7	111

Two-Complement Encoding Example

```
x =      15213: 00111011 01101101
y =     -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Two-Complement Implications

- Only 1 zero
- Usual arithmetic still works
- There is a bit that represents sign!
- Most widely used in today's machines

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+)  101
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```

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+)   -3
-------
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```

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- Unsigned Values
  - *UMin* = 0

    000…0
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Values for *W* = 16

	Decimal	Hex	Binary
`UMax`	**65535**	`FF FF`	`11111111 11111111`
`TMax`	**32767**	`7F FF`	`01111111 11111111`
`TMin`	**-32768**	`80 00`	`10000000 00000000`
`-1`	**-1**	`FF FF`	`11111111 11111111`
`0`	**0**	`00 00`	`00000000 00000000`

Numeric Ranges

- Unsigned Values
  - *UMin* = 0
    000…0
  - *UMax* = $2^w - 1$
    111…1

- Two's Complement Values
  - *TMin* = -2^{w-1}
    100…0
  - *TMax* = $2^{w-1} - 1$
    011…1
- Other Values
  - Minus 1
    111…1

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`0`	**0**	`00 00`	`00000000 00000000`

Data Representations in C (in Bytes)

- By default variables are signed
- Unless explicitly declared as unsigned (e.g., `unsigned int`)
- Signed variables use two-complement encoding

C Data Type	32-bit	64-bit
`char`	1	1
`short`	2	2
`int`	4	4
`long`	4	8

Data Representations in C (in Bytes)

	W			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
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- C Language
  - `#include <limits.h>`
  - Declares constants, e.g.,
    - `ULONG_MAX`
    - `LONG_MAX`
    - `LONG_MIN`
  - Values platform specific

Can We Represent Fractions in Binary?

- What does 10.01_2 mean?
- C.f., Decimal
  - $12.45 = 1*10^1 + 2*10^0 + 4*10^{-1} + 5*10^{-2}$
- $10.01_2 = 1*2^1 + 0*2^0 + 0*2^{-1} + 1*2^{-2} = 2.25_{10}$

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Decimal	Binary
0	00.00
0.25	00.01
0.5	00.10
0.75	00.11
1	01.00
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1.5	01.10
1.75	01.11
2	10.00
2.25	10.01
2.5	10.10
2.75	10.11
3	11.00
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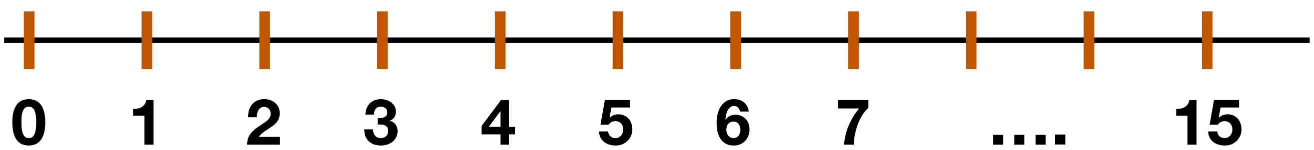

Decimal	Binary
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0.25	00.01
0.5	00.10
0.75	00.11
1	01.00
1.25	01.01
1.5	01.10
1.75	01.11
2	10.00
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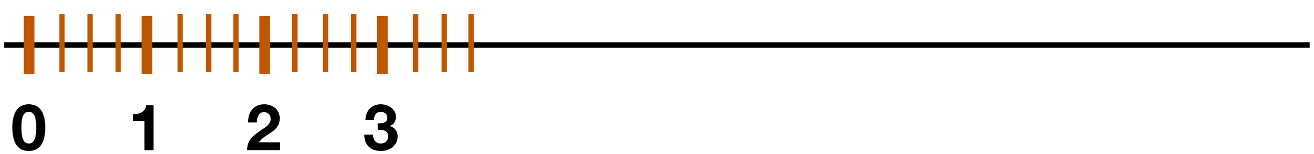

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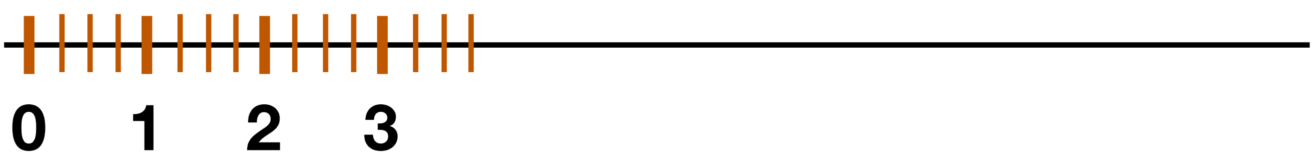


```
   01.10
+  01.01
--------
   10.11
```

```
   1.50
+  1.25
-------
   2.75
```

Decimal	Binary
0	00.00
0.25	00.01
0.5	00.10
0.75	00.11
1	01.00
1.25	01.01
1.5	01.10
1.75	01.11
2	10.00
2.25	10.01
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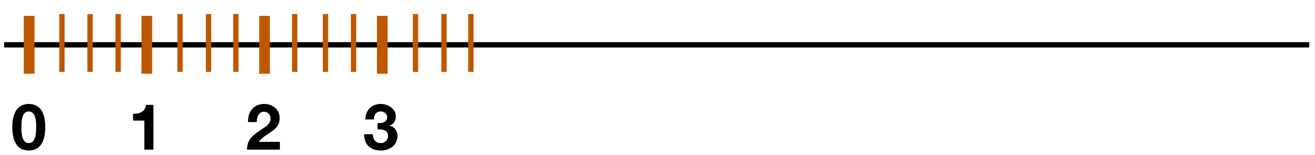

Integer Arithmetic Still Works!

```
  01.10
+ 01.01
-------
  10.11
```

```
  1.50
+ 1.25
------
  2.75
```

Decimal	Binary
0	00.00
0.25	00.01
0.5	00.10
0.75	00.11
1	01.00
1.25	01.01
1.5	01.10
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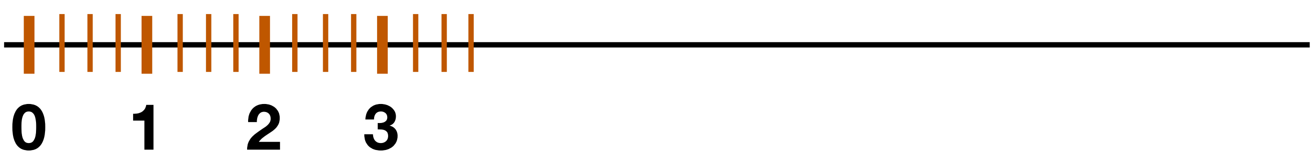
2.25	10.01
2.5	10.10
2.75	10.11
3	11.00
3.25	11.01
3.5	11.10
3.75	11.11

Fixed-Point Representation

- Fixed interval between two representable numbers as long as the binary point stays fixed
  - Each bit represents 0.25_{10}
- Fixed-point representation of numbers
  - Integer is one special case of fixed-point




```
   01.10          1.50
+  01.01        + 1.25
--------        ------
   10.11          2.75
```

Decimal	Binary
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0.25	00.01
0.5	00.10
0.75	00.11
1	01.00
1.25	01.01
1.5	01.10
1.75	01.11
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2.25	10.01
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2.75	10.11
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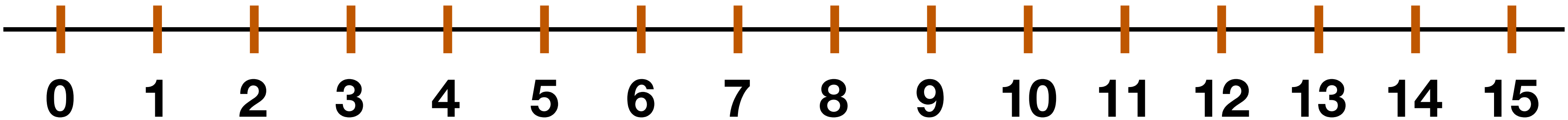
Aside: Quantization



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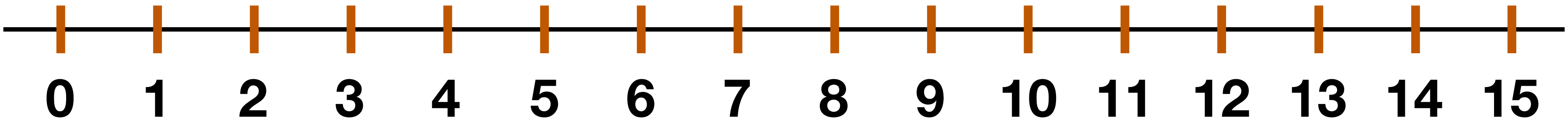

- Representing all integers precisely requires 4 bits

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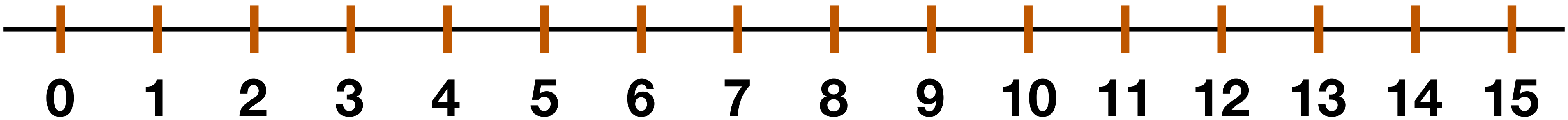

- Representing all integers precisely requires 4 bits
- What if we can tolerate some imprecisions
  - 1, 2, 3 are approximated by 0
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  - We would only need 2 bits

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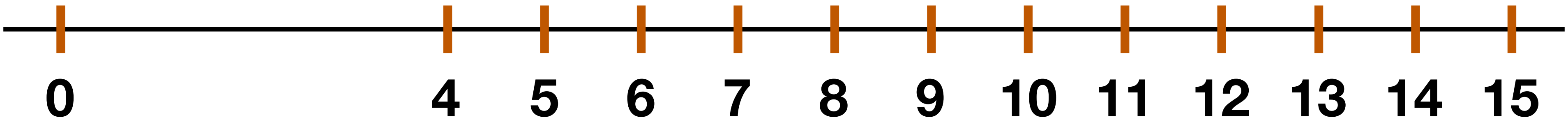

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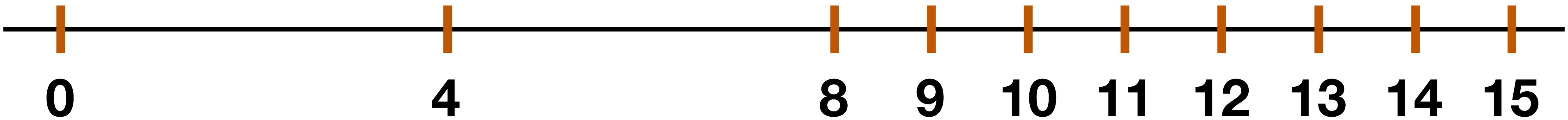

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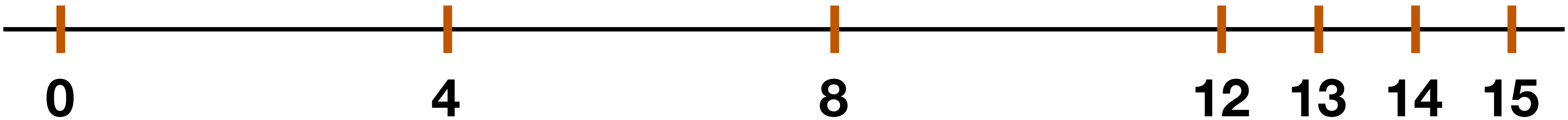

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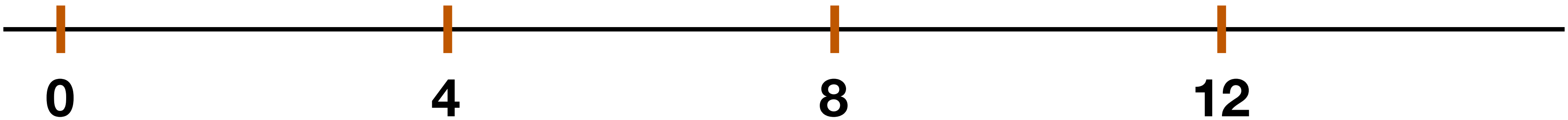
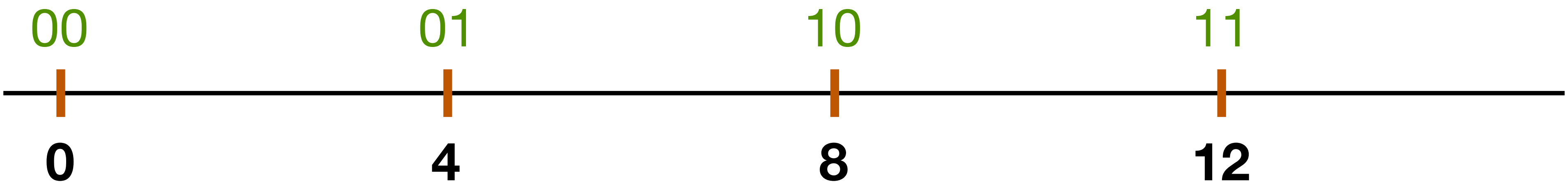
Aside: Quantization



- Representing all integers precisely requires 4 bits
- What if we can tolerate some imprecisions
  - 1, 2, 3 are approximated by 0
  - 5, 6, 7 are approximated by 4…
  - We would only need 2 bits

Aside: Quantization



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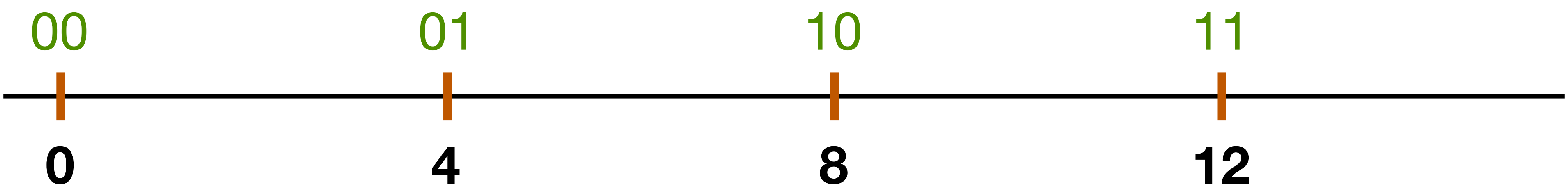

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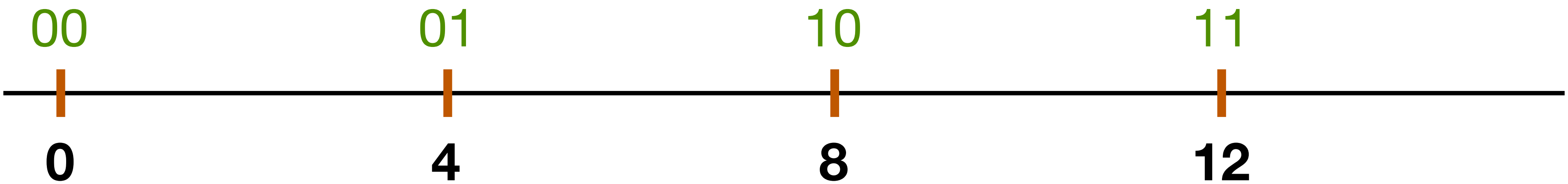
- That is, 1 bit represents 4_{10}
  - 10_2 becomes $4 * (1 * 2^1) = 8$
  - Every time we increment a bit, the value is incremented by 4
  - 1, 2, 3 are represented approximately by 10_2

Aside: Quantization



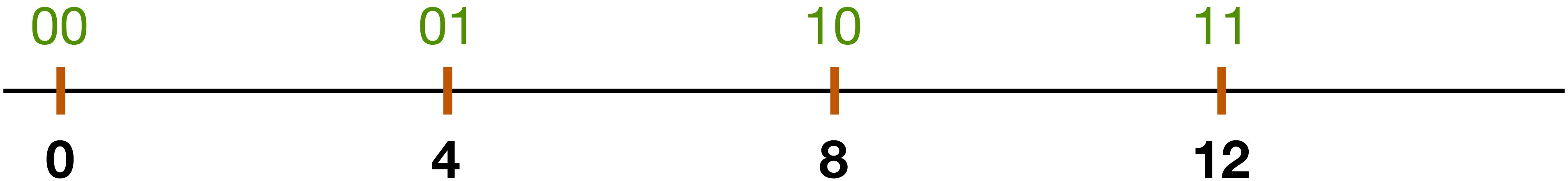
- Representing all integers precisely requires 4 bits
- What if
    - 1, 2
    - 5, 6
    - We

Note that this is different from "base 4"

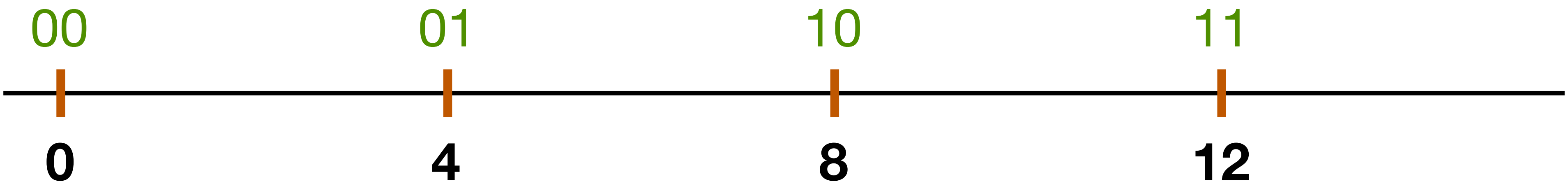
- $10_4 = 1 * 4^1 + 0 * 4^0 = 4$
- Every increment still only increments 1

- That is, 1 bit represents 4_{10}
    - 10_2 becomes $4 * (1 * 2^1) = 8$
    - Every time we increment a bit, the value is incremented by 4
    - 1, 2, 3 are represented approximately by 10_2

Aside: Quantization

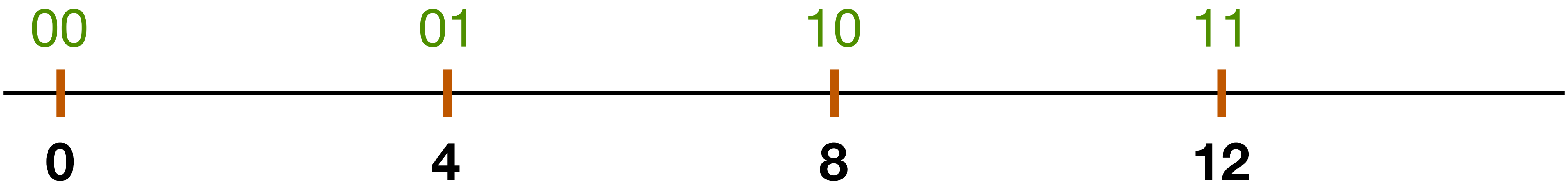


Aside: Quantization



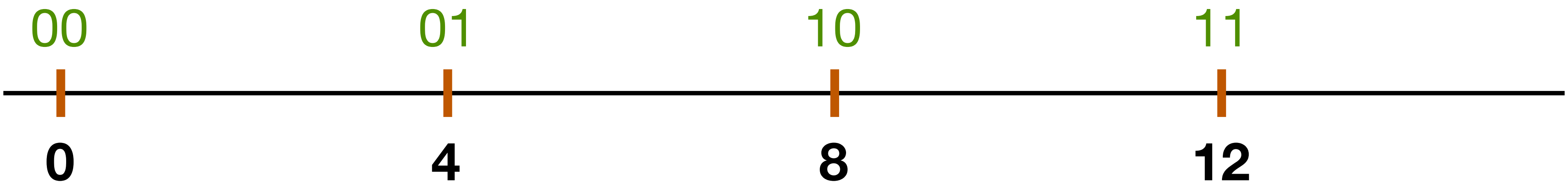
- Saves storage space and improves computation speed
  - 50% space saving
  - 4-bit arithmetic becomes 2-bit arithmetic

Aside: Quantization



- Saves storage space and improves computation speed
  - 50% space saving
  - 4-bit arithmetic becomes 2-bit arithmetic
- Many real-world applications can tolerate imprecisions
  - Image processing
  - Computer vision
  - Real-time graphics
  - Machine learning (Neural networks)

Aside: Quantization



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Aside: Quantization

Questions?

00	01	10	11
0	**4**	**8**	**12**

- Saves storage space and improves computation speed
  - 50% space saving

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