# CSC 252: Computer Organization Spring 2019: Lecture 3

Instructor: Yuhao Zhu

Department of Computer Science University of Rochester

#### **Action Items:**

- Trivia 1 is due tomorrow, midnight
- Main assignment due Feb. 1, midnight

- Programming Assignment 1 is out
  - Details: <a href="http://cs.rochester.edu/courses/252/spring2019/">http://cs.rochester.edu/courses/252/spring2019/</a>
     labs/assignment1.html
  - Due on Feb 1, 11:59 PM
  - Trivia due Friday, 1/25, 11:59 PM
  - You have 3 slip days (not for trivia)

20	21	22	23	24	25	26
				Today	Trivia	
27	28	29	30	31	Feb 1	2
					Due	

TA review sessions schedule is posted

#### **Review Session Schedule**

Two 1-hour review sessions are offered each week. Review sessions are not mandatory. During review sessions, TAs might review course materials from the past week, go over problem sets and past exams, provide an overview of programming assignments, etc. They will be interactive and you are encouraged to ask questions. It is up to the TAs to decide how to run it.

Tuesday 6 PM - 7 PM, in WH 2506 (Rotating across Olivia, Jessica, Max, Sam, Yawo)

Thursday 7:30 PM - 8:30 PM, in WH 2506 (Rotating across Amir, Minh, Yiyang, Yu)

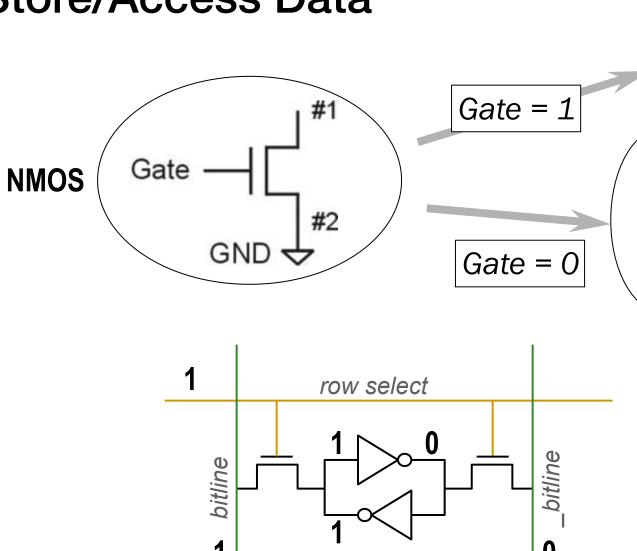
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  - http://www.cs.rochester.edu/courses/252/spring2019/

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  - If one doesn't know, ask another.
  - If all don't know, ask me.

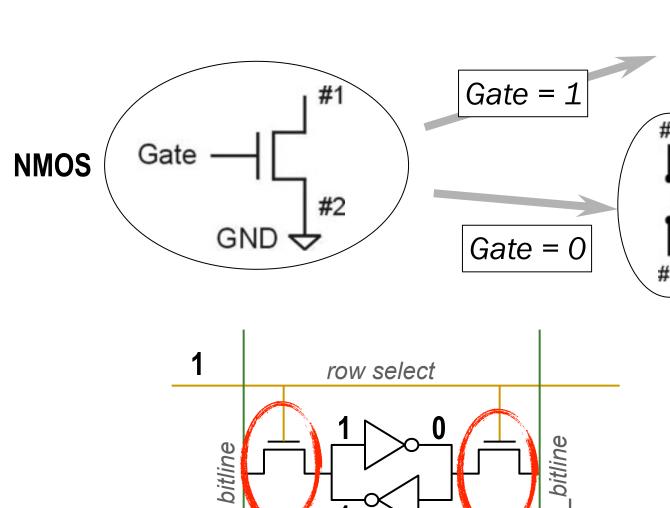
#### Previously in 252...

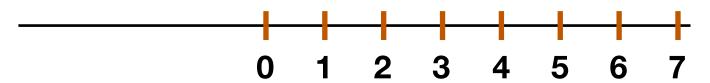
- Computers are built to understand bits: 0 and 1
  - 0: low (no) voltage; 1: high voltage
- Integer representations (Fixed-point really)

#### Store/Access Data

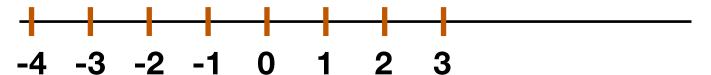


#### Store/Access Data

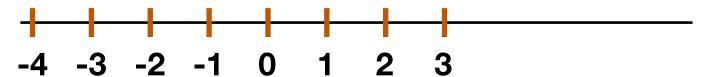




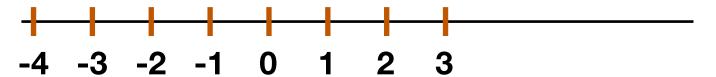
Unsigned	Binary
0	000
1	001
2	010
3	011
4	100
5 6	101
	110
7	111



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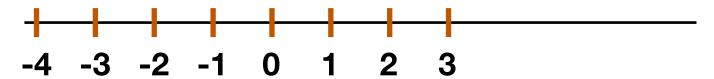


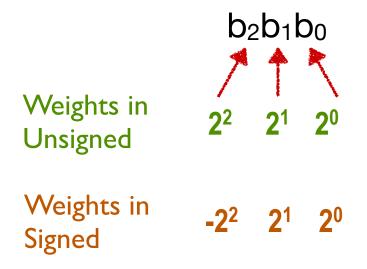
Signed	Unsigned	Binary
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1	1	001
2	2	010
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-4	4	100
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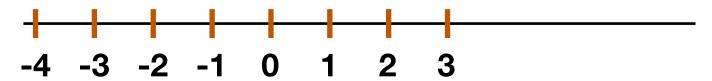


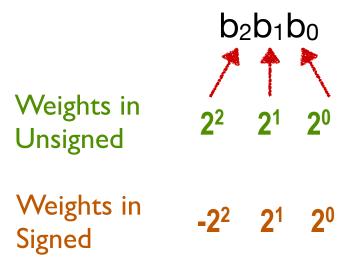
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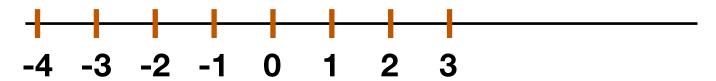
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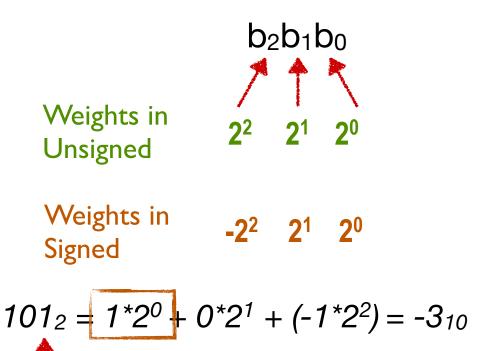




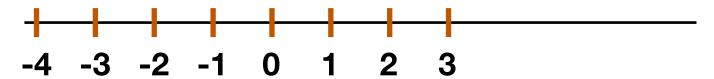
$$101_2 = 1^*2^0 + 0^*2^1 + (-1^*2^2) = -3_{10}$$

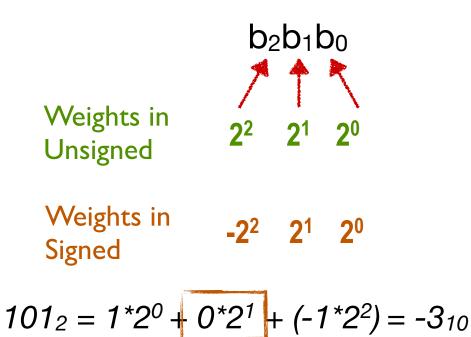
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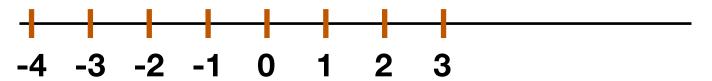
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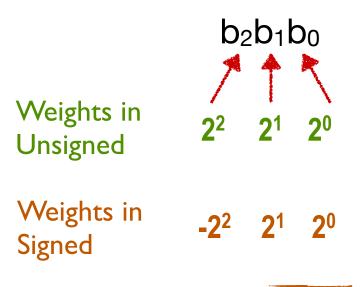




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- Only 1 zero
- There is (still) a bit that represents sign!
- Unsigned arithmetic still works

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	010	
+)	101	
	111	

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• 3 + 1 becomes -4 (called overflow. More on it later.)

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- Define a data type that captures all these attributes:
   unsigned char in C
  - Internally, an unsigned int variable is represented as a 8-bit, nonnegative, binary number

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  - The C language designers chose two's complement
  - This is where math meets computer science

C Data Type	32-bit	64-bit
(unsigned) char	1	1
(unsigned) short	2	2
(unsigned) int	4	4
(unsigned) long	4	8

	W			
	8	16	32	64
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

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## Data Types (in C)

			W	
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#### • C Language

- •#include <limits.h>
- Declares constants, e.g.,
  - •ULONG\_MAX
  - •LONG MAX
  - •LONG MIN
- Values platform specific

#### Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

## One Bit Sequence, Two Interpretations

 A sequence of bits can be interpreted as either a signed integer or an unsigned integer

Signed	Unsigned	Binary
0	0	000
1	1	001
2	2	010
3	3	011
-4 -3 -2	4	100
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-2	6	110
-1	7	111

## Signed vs. Unsigned Conversion in C

- What happens when we convert between signed and unsigned numbers?
- Casting (In C terminology)
  - Explicit casting between signed & unsigned

```
int tx, ty = -4;
unsigned ux = 7, uy;
tx = (int) ux; // U2T
uy = (unsigned) ty; // T2U
```

- Implicit casting
  - e.g., assignments, function calls

```
tx = ux;

uy = ty;
```

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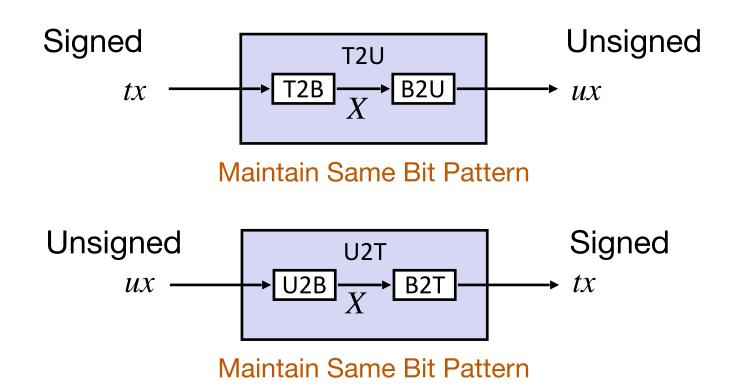
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## Mapping Between Signed & Unsigned

 Mappings between unsigned and two's complement numbers: Keep bit representations and reinterpret

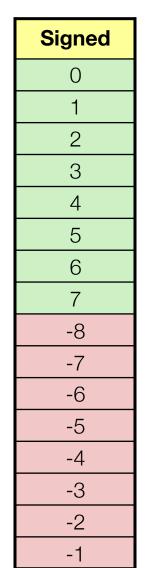


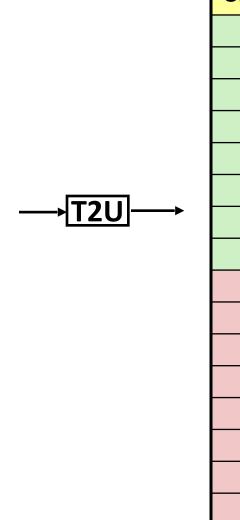
Bits
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

Signed
0
1
2
3
4
5
6
7
-8
-7
-6
-5
-4
-3
-2
-1

Unsigned
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

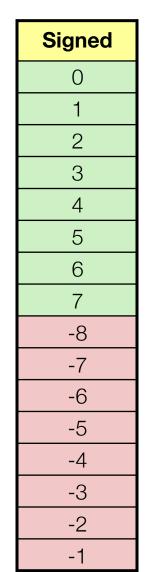
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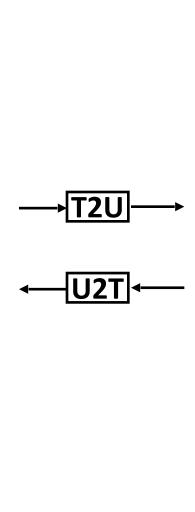




Unsigned
0
1
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9
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Unsigned
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

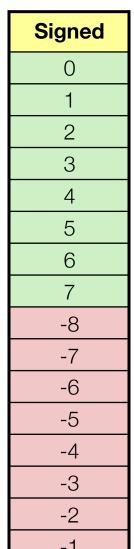
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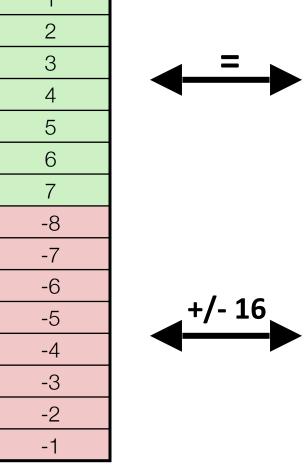
Signed
0
1
2
3
4
5
6
7
-8
-7
-6
-5
-4
-3
-2
-1

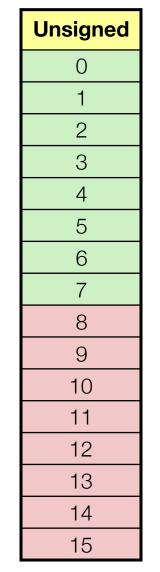


Unsigned
0
1
2
3
4
5
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7
8
9
10
11
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13
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15

Bits
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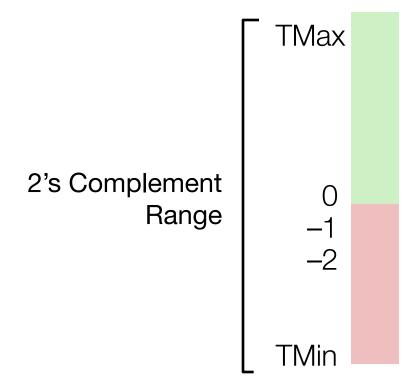


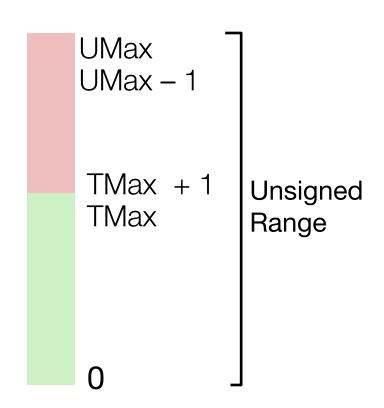


Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2		2
0011	3	_ = _	3
0100	4		4
0101	5		5
0110	6		6
0111	7		7
1000	-8		8
1001	-7		9
1010	-6		10
1011	-5	+/- 16	11
1100	-4		12
1101	-3		13
1110	-2		14
1111	-1		15

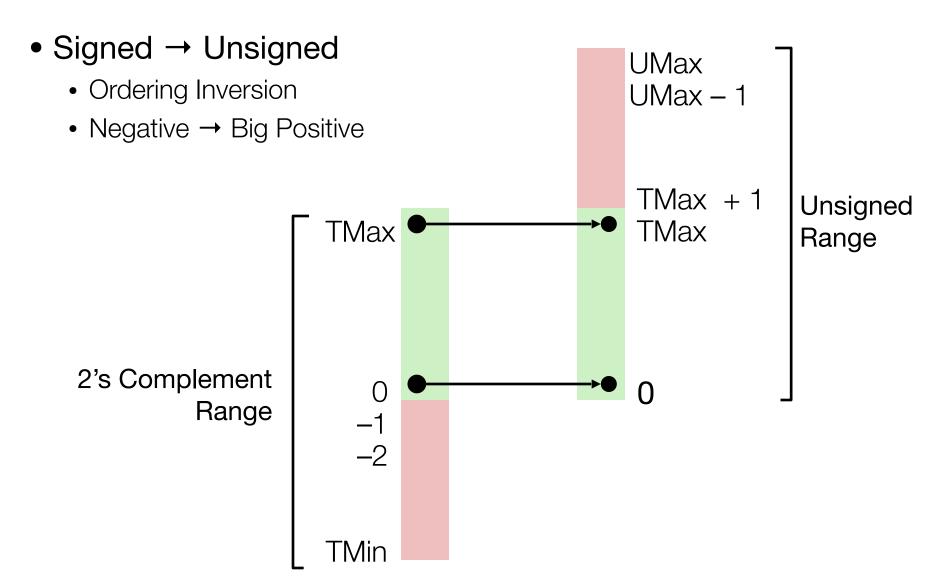
#### **Conversion Visualized**

- Signed → Unsigned
  - Ordering Inversion
  - Negative → Big Positive

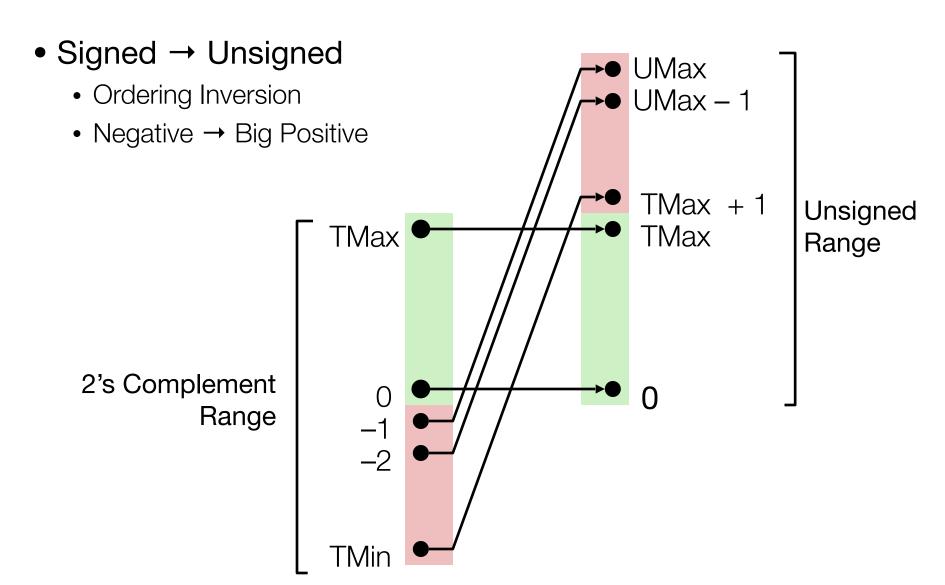




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#### The Problem

```
short int x = 15213;
int     ix = (int) x;
short int y = -15213;
int     iy = (int) y;
```

C Data Type	64-bit
char	1
short	2
int	4
long	8

#### The Problem

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- Converting from smaller to larger integer data type
- Should always be able to preserve the value, but how?

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- Converting from smaller to larger integer data type
- Should always be able to preserve the value, but how?

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
У	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

## Signed Extension

- Task:
  - Given w-bit signed integer x
  - Convert it to (w+k)-bit integer with same value

### Signed Extension

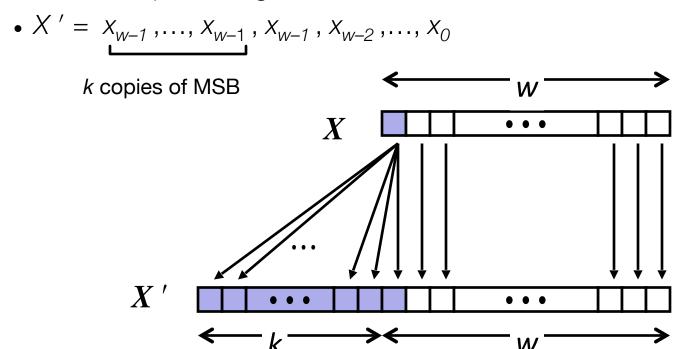
- Task:
  - Given w-bit signed integer x
  - Convert it to (w+k)-bit integer with same value
- Rule:
  - Make k copies of sign bit:

• 
$$X' = X_{w-1}, ..., X_{w-1}, X_{w-1}, X_{w-2}, ..., X_0$$

k copies of MSB

## Signed Extension

- Task:
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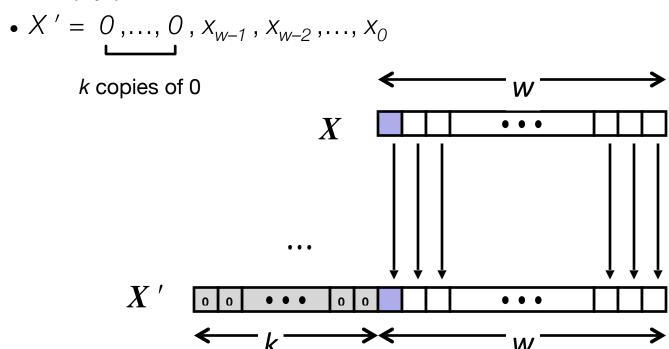
#### **Another Problem**

```
unsigned short x = 47981;
unsigned int ux = x;
```

	Decimal	Hex	Binary
x	47981	BB 6D	10111011 01101101
ux	47981	00 00 BB 6D	00000000 00000000 10111011 01101101

## **Unsigned (Zero) Extension**

- Task:
  - Given w-bit unsigned integer x
  - Convert it to (w+k)-bit integer with same value
- Rule:
  - Simply pad zeros:



#### **Yet Another Problem**

```
int x = 53191;
short sx = (short) x;
```

	Decimal	Hex	Binary
x	53191	00 00 CF C7	00000000 00000000 11001111 11000111
sx	-12345	CF C7	11001111 11000111

#### **Yet Another Problem**

```
int x = 53191;
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```

	Decimal	Hex	Binary
x	53191	00 00 CF C7	00000000 00000000 11001111 11000111
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# Questions?

#### Yet Another Problem

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int x = 53191;
short sx = (short) x;
```

	Decimal	Hex	Binary
x	53191	00 00 CF C7	00000000 00000000 11001111 11000111
sx	-12345	CF C7	11001111 11000111

- Truncating (e.g., int to short)
  - Can't always preserve the numerical value
  - C's implementation: leading bits are truncated, results reinterpreted

# Questions?

#### **Announcement**

- Check the course website before asking
  - http://www.cs.rochester.edu/courses/252/spring2019/
- Direct ALL questions regarding assignments to the TAs
  - They have done them. They have debugged them. They know them inside out.
  - If one doesn't know, ask another.
  - If all don't know, ask me.

#### Today: Representing Information in Binary

- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
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1	001
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<ul><li>3</li><li>4</li><li>5</li><li>6</li></ul>	100
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• Similar to Decimal Addition

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- Similar to Decimal Addition
- Suppose we have a new data type that is
   3-bit wide (c.f., short has 16 bits)

Normal
Case

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- Similar to Decimal Addition
- Suppose we have a new data type that is
   3-bit wide (c.f., short has 16 bits)
- Might overflow: result can't be represented within the size of the data type

<b>Normal</b>
Case

# Overflow Case

Unsigned	Binary
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2	010
3	011
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5	101
6	110
7	111

- Similar to Decimal Addition
- Suppose we have a new data type that is
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Normal
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Overflow Case



True Sum

- Similar to Decimal Addition
- Suppose we have a new data type that is
   3-bit wide (c.f., short has 16 bits)
- Might overflow: result can't be represented within the size of the data type

<b>Normal</b>
Case

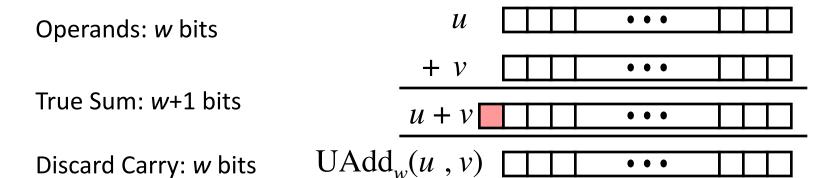
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7	111

3



True Sum
Sum with same bits

## **Unsigned Addition in C**



Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3 -2	101
-2	110
-1	111

 Has Identical Bit-Level behavior as unsigned addition (a big advantage over sign-magnitude)

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

 Has Identical Bit-Level behavior as unsigned addition (a big advantage over sign-magnitude)

Normal Case

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3 -2	101
-2	110
-1	111

- Has Identical Bit-Level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

Normal
Case

Overflow Case

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

- Has Identical Bit-Level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

Normal
Case

# Overflow Case

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

- Has Identical Bit-Level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

1	⁄lin	

**Signed** 

-3

**Binary** 

000

001

010

011

100

101

110

111

 /lin	******

Normal
Case

Negative Overflow

- Has Identical Bit-Level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

		Min	******
1 0	2		

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

#### Normal Case

Negative Overflow

- Has Identical Bit-Level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

Min
-----

**Signed** 

-3

-1

Normal
Case

**Binary** 

000

001

010

011

100

101

110

111

Negative Overflow

- Has Identical Bit-Level behavior as unsigned addition (a big advantage over sign-magnitude)
- Overflow can also occur

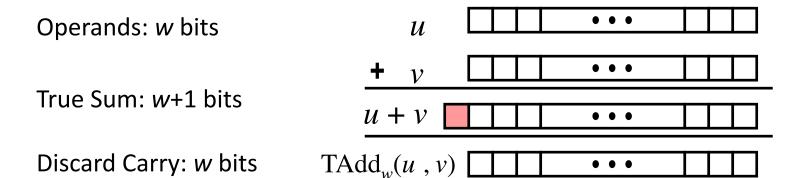
Max	
Min	

Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

# Overflow Case

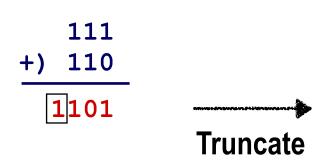
Negative Overflow

**Positive Overflow** 

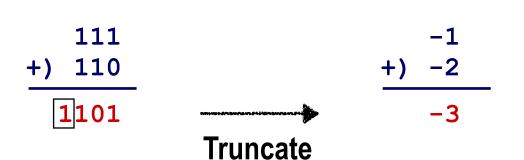


Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-4 -3 -2	101
-2	110
-1	111

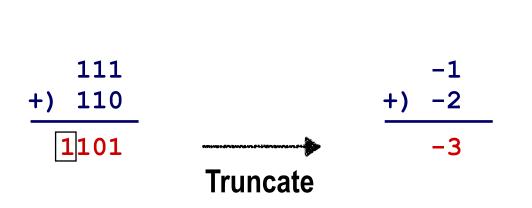
Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-4 -3 -2	101
-2	110
-1	111



Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3 -2	101
-2	110
-1	111

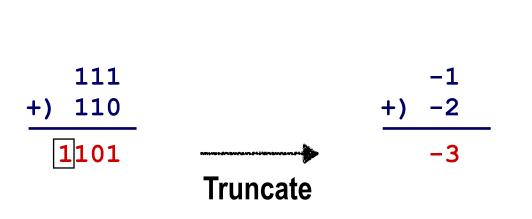


Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111



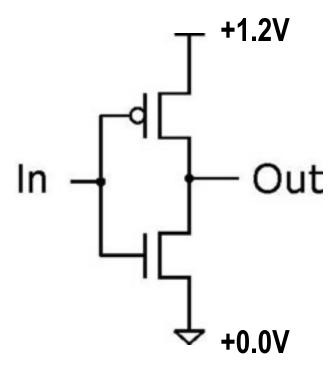
Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

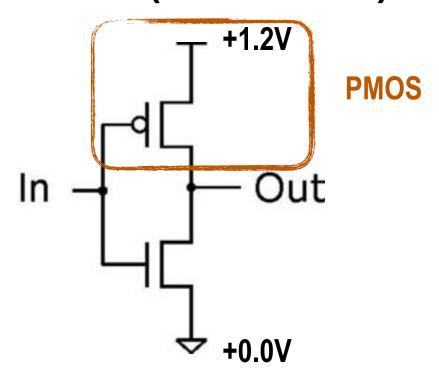
This is not an overflow by definition

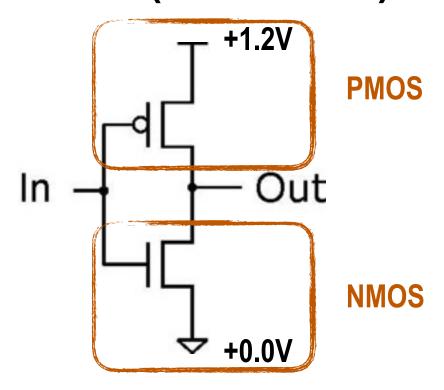


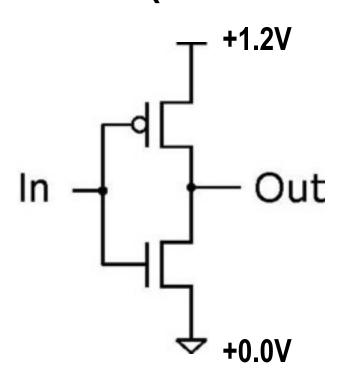
Signed	Binary
0	000
1	001
2	010
3	011
-4	100
-3	101
-2	110
-1	111

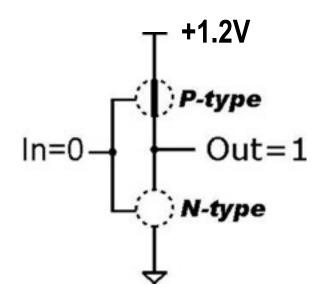
- This is not an overflow by definition
- Because the actual result can be represented by the bit width of the datatype (3 bits here)

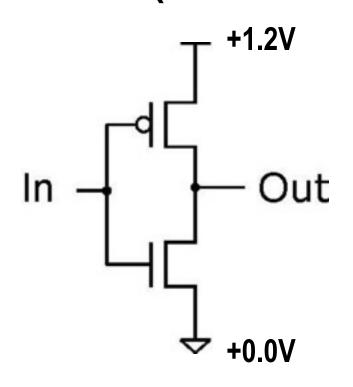


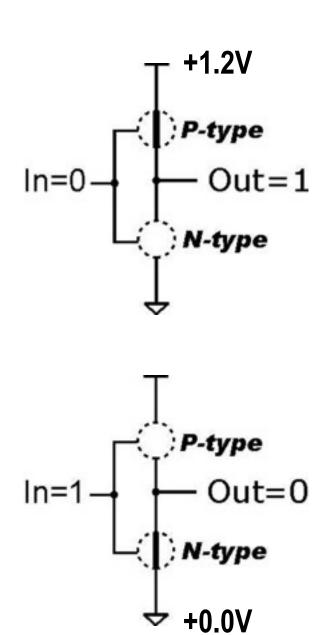


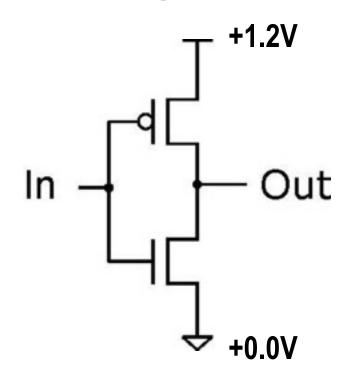


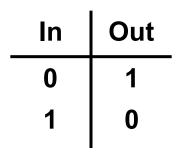


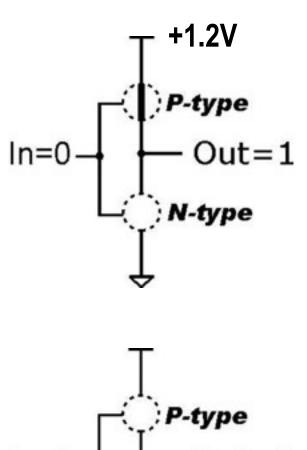


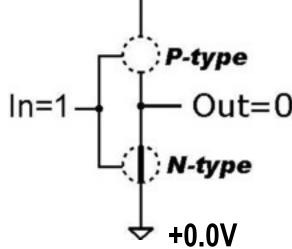


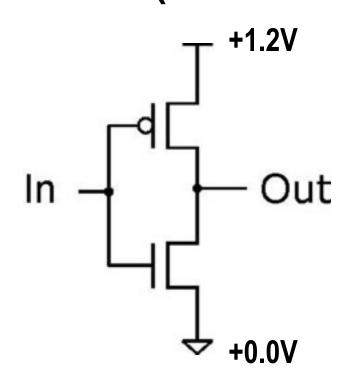


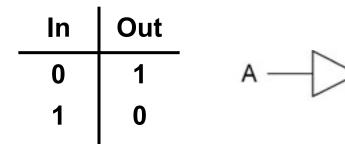


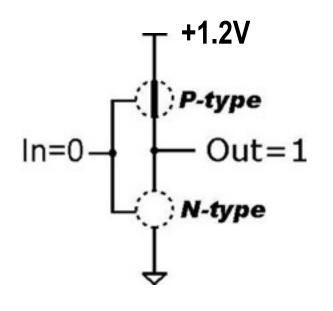


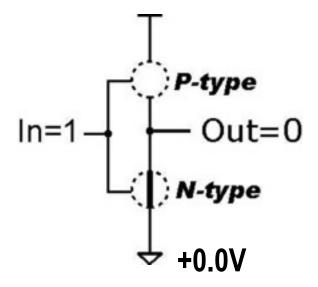




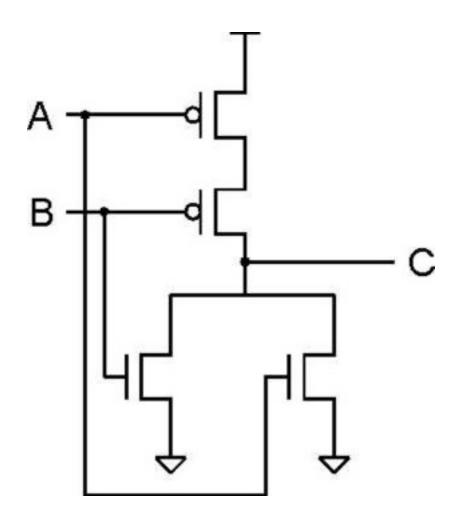


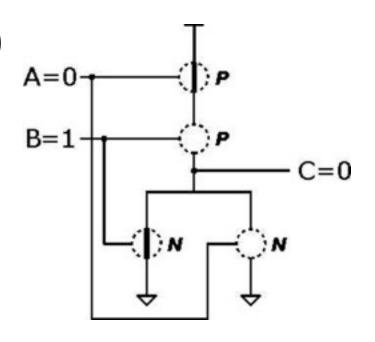






## NOR Gate (NOT + OR)

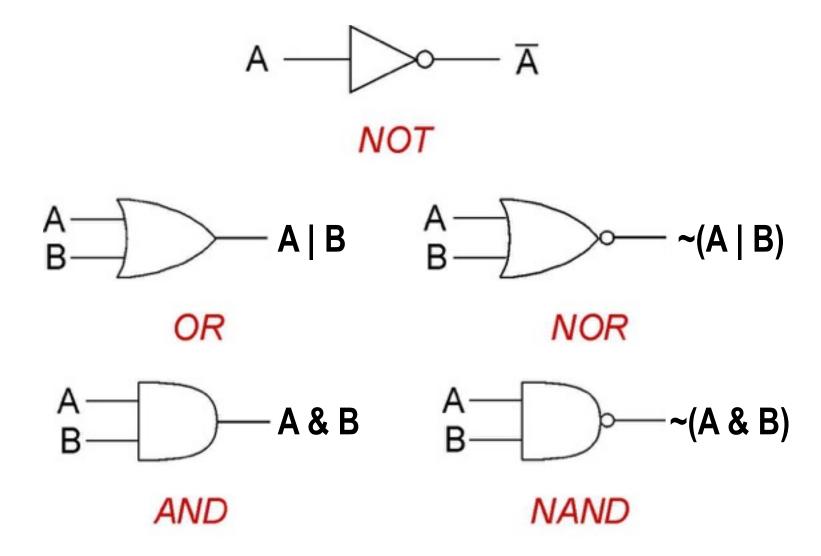




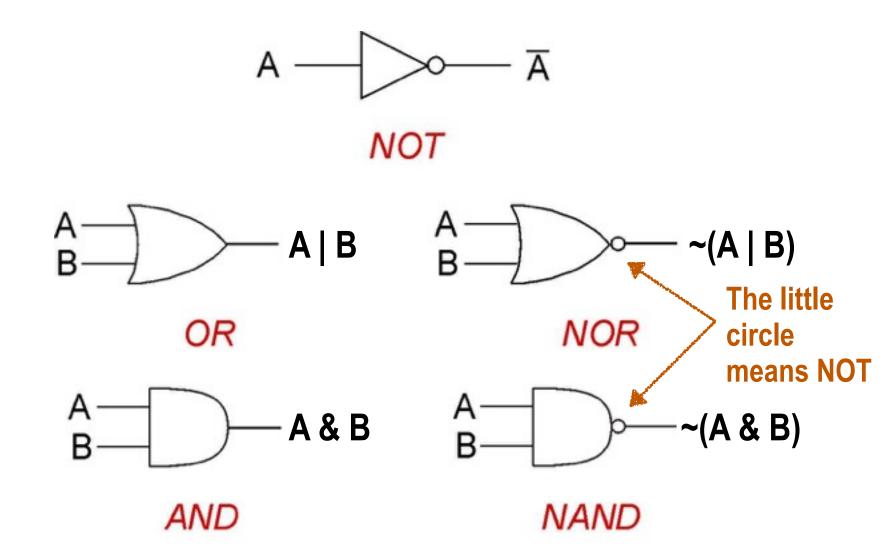
Α	В	С
0	0	1
0	1	0
1	0	0
1	1	0

Note: Serial structure on top, parallel on bottom.

#### **Basic Logic Gates**



#### **Basic Logic Gates**



В	$\mathbf{C}_{in}$	S	$\mathbf{C}_{\mathrm{ou}}$
			t
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1
0	0	1	0
0	1	0	1
1	0	0	1
1	1	1	1
	0 0 1 1 0	0 0 0 1 1 0 1 1 0 0 0 1	0 0 0 0 0 1 1 1 1 0 0 0 1 0 1 0



A	В	C <sub>in</sub>	S	$\mathbf{C}_{ou}$
				t
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = ( A \& B \& C_{in} )$$



	A	В	C <sub>in</sub>	S	C <sub>ou</sub>
					t
į=	0	0	0	0	0
	0	0	1	1	0
	0	1	0	1	0
	0	1	1	0	1
	1	0	0	1	0
	1	0	1	0	1
	1	1	0	0	1
	1	1	1	1	1

$$S = (\text{~A \& ~B \& C}_{in})$$
  
| (\times A & B & \times C\_{in})



A	В	C <sub>in</sub>	S	$\mathbf{C}_{\mathrm{ou}}$
				t
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = (\text{~A \& ~B \& C}_{in})$$
  
| (\tau A & B & \tau C\_{in})  
| (A & \tau B & \tau C\_{in})



A	В	C <sub>in</sub>	S	$\mathbf{C}_{ou}$
				t
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	. 1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = (\text{~A \& ~B \& C}_{in})$$

$$| (\text{~A \& B \& ~C}_{in})$$

$$| (\text{A \& ~B \& ~C}_{in})$$

$$| (\text{A & B & C}_{in})$$



A	В	C <sub>in</sub>	S	C <sub>ou</sub>
				t
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Add two bits and carry-in, produce one-bit sum and carry-out.

$$S = (\text{~A \& ~B \& C}_{in})$$

$$| (\text{~A \& B \& ~C}_{in})$$

$$| (\text{A \& ~B \& ~C}_{in})$$

$$| (\text{A & B & C}_{in})$$

 $C_{ou} = ( A \& B \& C_{in} )$ 



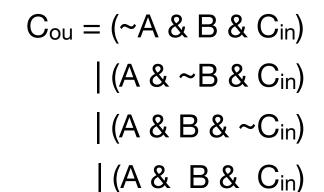
A	В	C <sub>in</sub>	S	C <sub>ou</sub>
				t
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

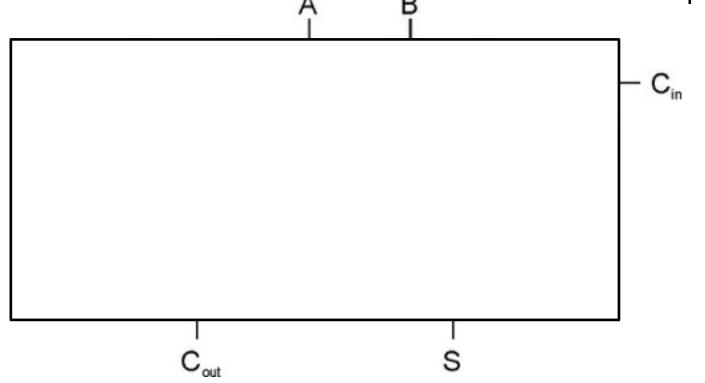
$$C_{ou} = (\text{~A \& B \& C}_{in})$$

$$| (A \& \text{~B \& C}_{in})$$

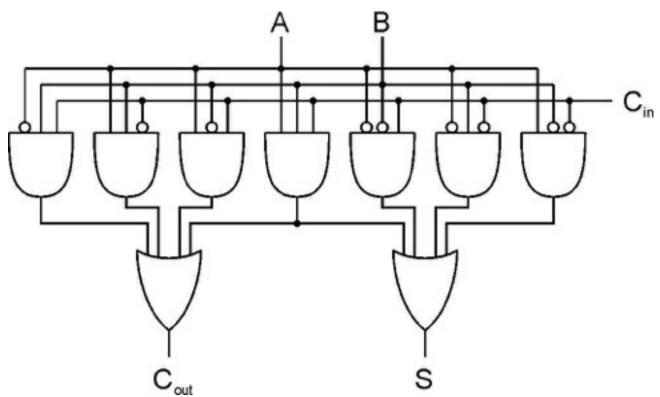
$$| (A \& B \& \text{~C}_{in})$$

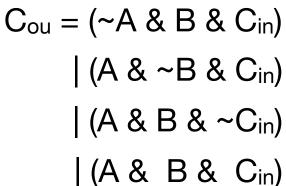
$$| (A \& B \& \text{~C}_{in})$$

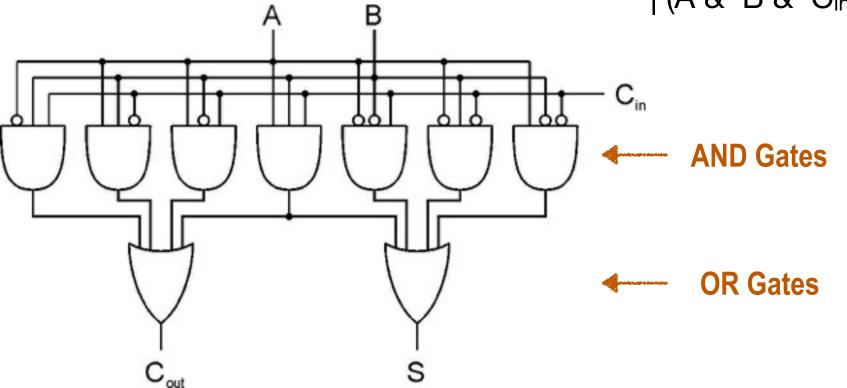


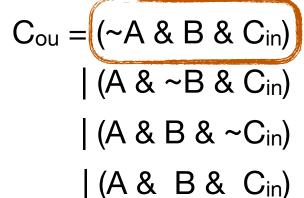


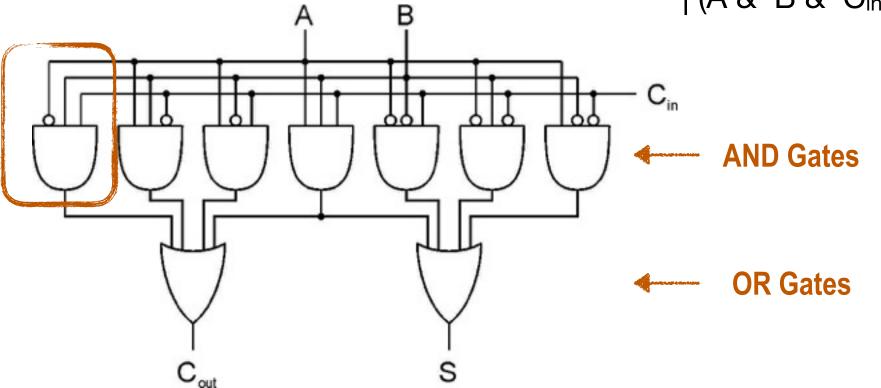
Add two bits and carry-in, produce one-bit sum and carry-out.

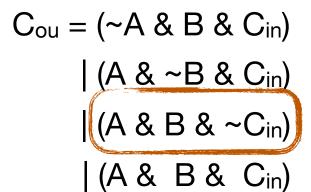


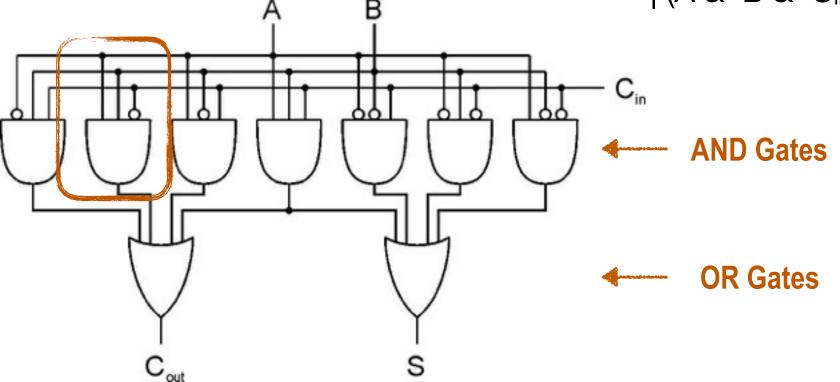




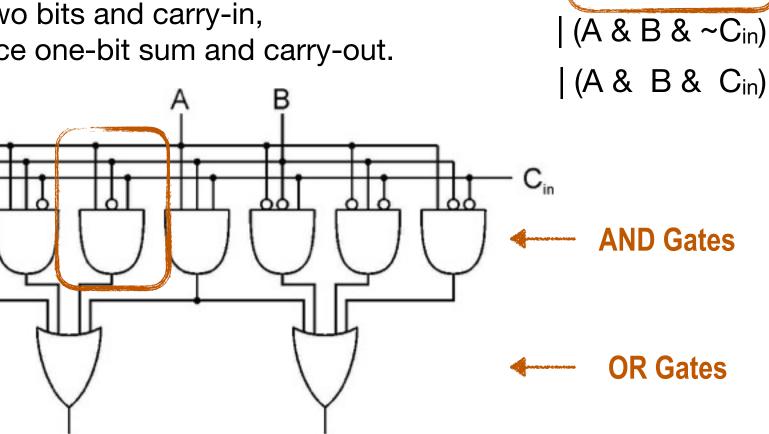






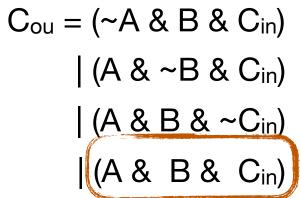


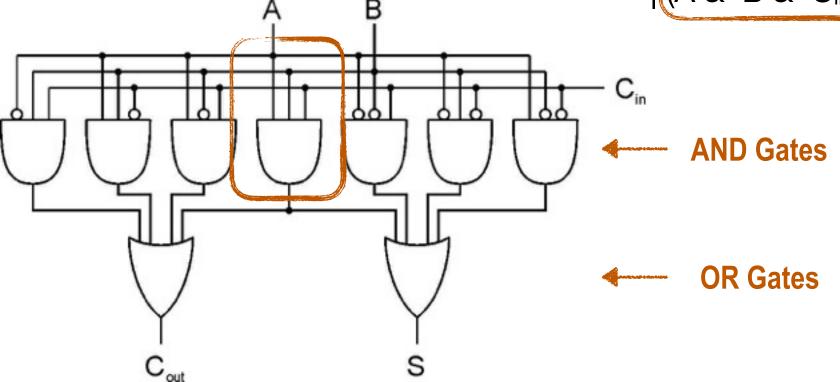
Add two bits and carry-in, produce one-bit sum and carry-out.



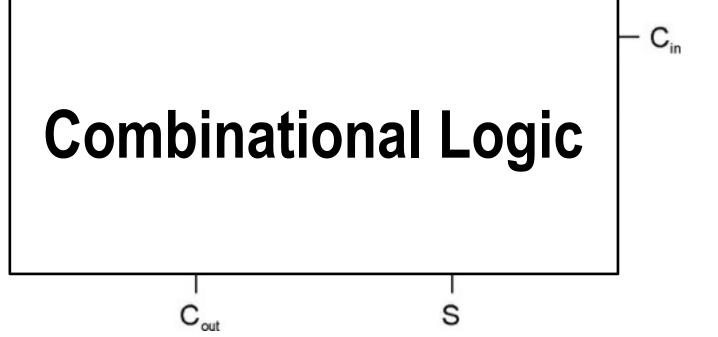
 $C_{ou} = ( A \& B \& C_{in} )$ 

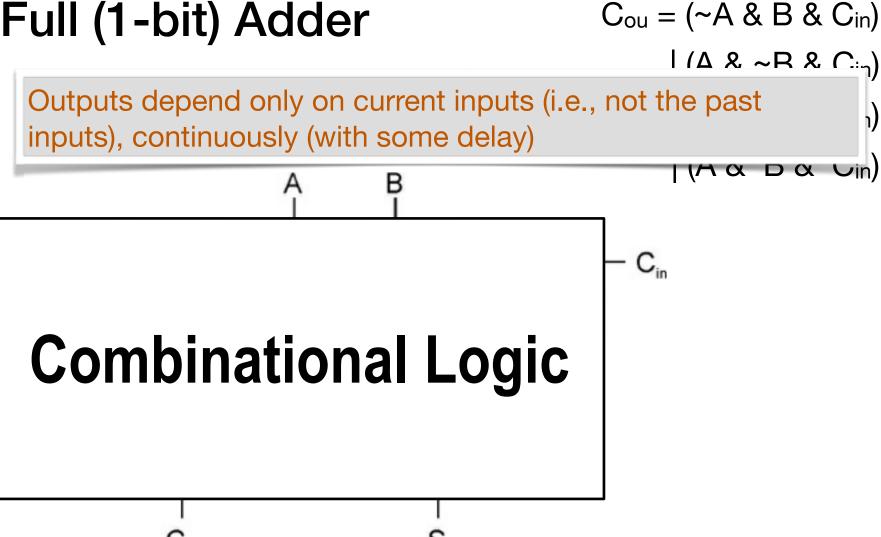
(A & ~B & C<sub>in</sub>)

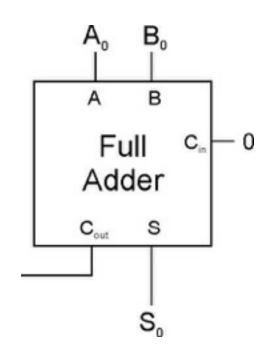


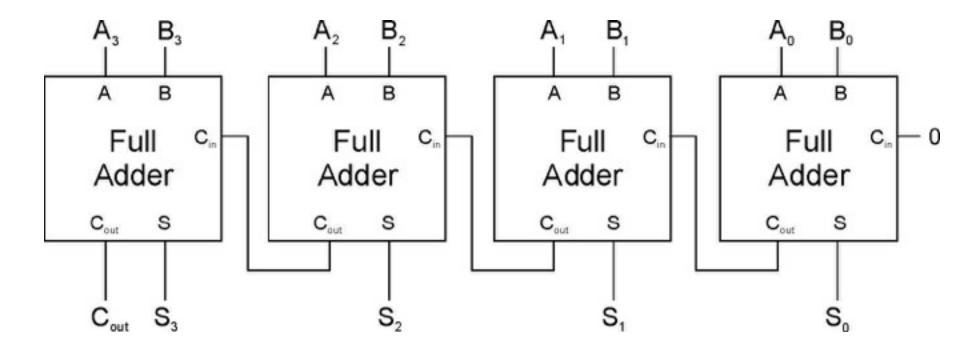


Add two bits and carry-in, produce one-bit sum and carry-out.

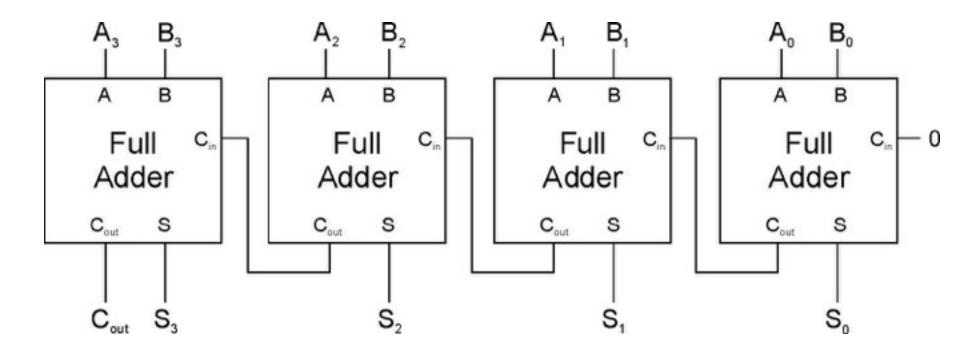




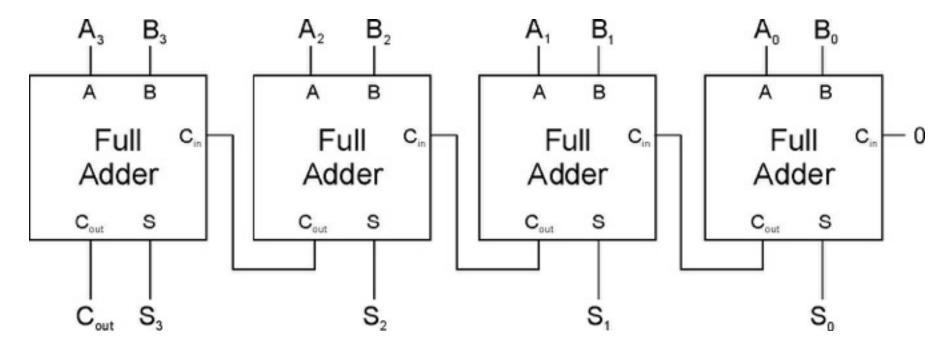




- Ripple-carry Adder
  - Simple, but performance linear to bit width



- Ripple-carry Adder
  - Simple, but performance linear to bit width
- Carry look-ahead adder (CLA)
  - Generate all carriers simultaneously

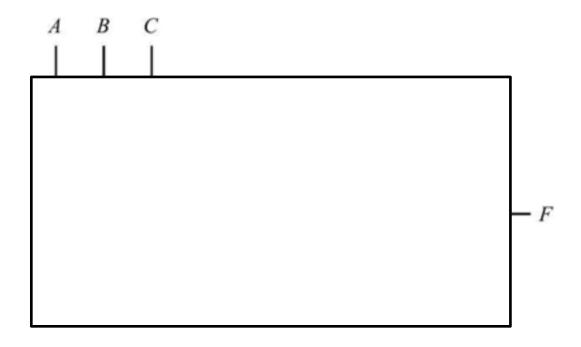


• Design digital components from basic logic gates

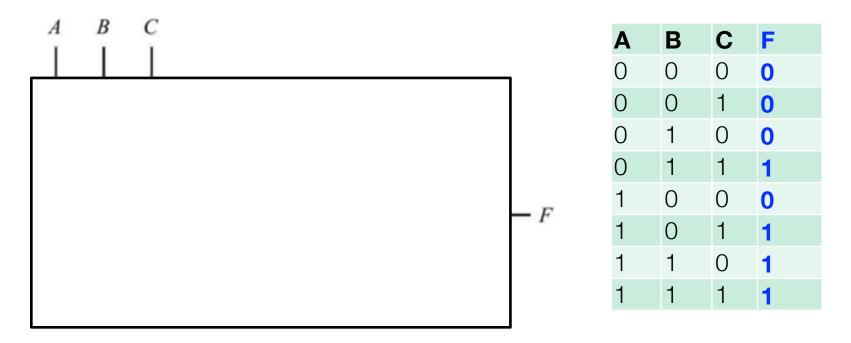
- Design digital components from basic logic gates
- Key idea: use the truth table!

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- Example: how to design a piece of circuit that does majority vote?

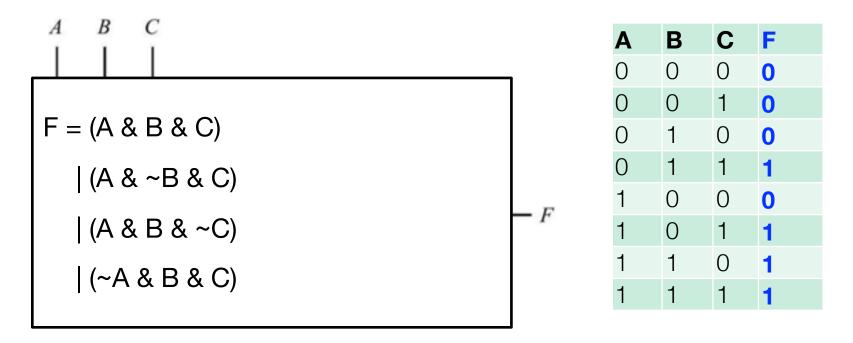
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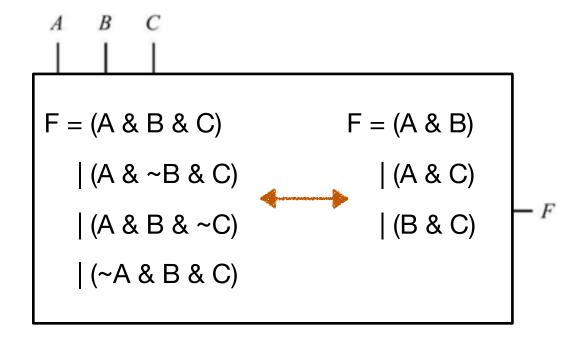
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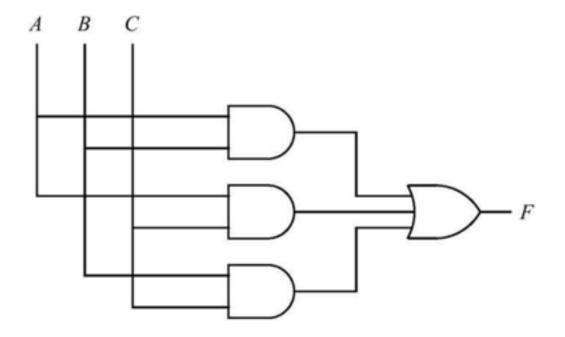


- Design digital components from basic logic gates
- Key idea: use the truth table!
- Example: how to design a piece of circuit that does majority vote?



Α	В	С	F
<b>A</b> 0	0	0	0
0	0	1	0
0 0 0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

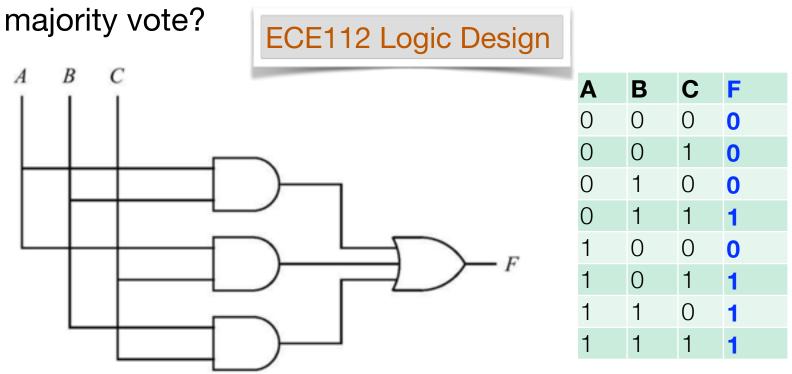
- Design digital components from basic logic gates
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- Example: how to design a piece of circuit that does majority vote?



Α	В	С	F
<b>A</b> 0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

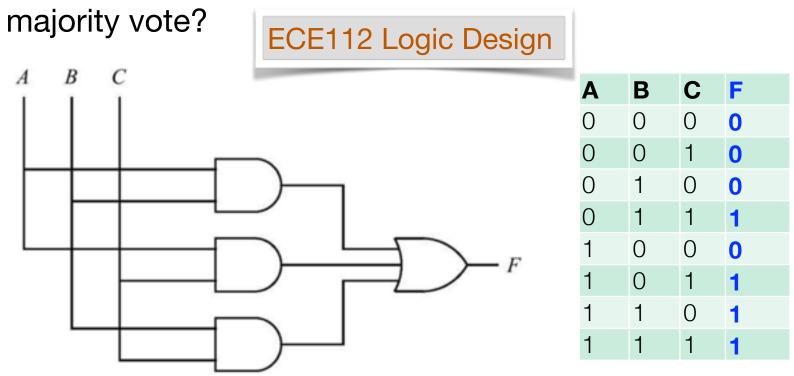
- Design digital components from basic logic gates
- Key idea: use the truth table!

Example: how to design a piece of circuit that does



# Questions?

- Design digital components from basic logic gates
- Key idea: use the truth table!
- Example: how to design a piece of circuit that does



• Goal: Computing Product of w-bit numbers x, y

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#### **Original Number (w bits)**

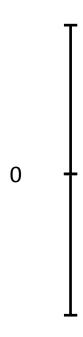
OMax 
$$2^{w-1}-1$$
 OMin  $-2^{w-1}$ 

Goal: Computing Product of w-bit numbers x, y

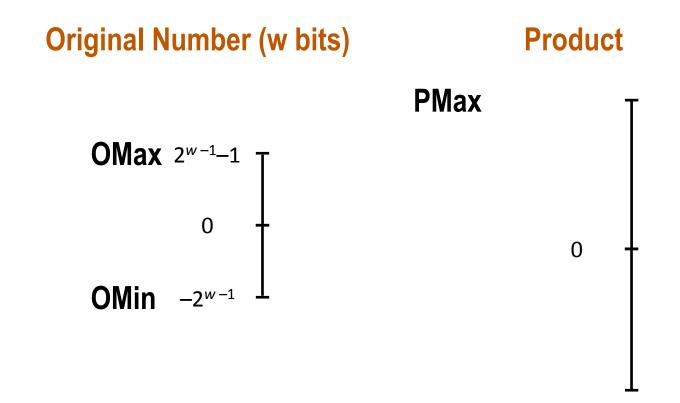
#### **Original Number (w bits)**

# OMax $2^{w-1}-1$ T OMin $-2^{w-1}$

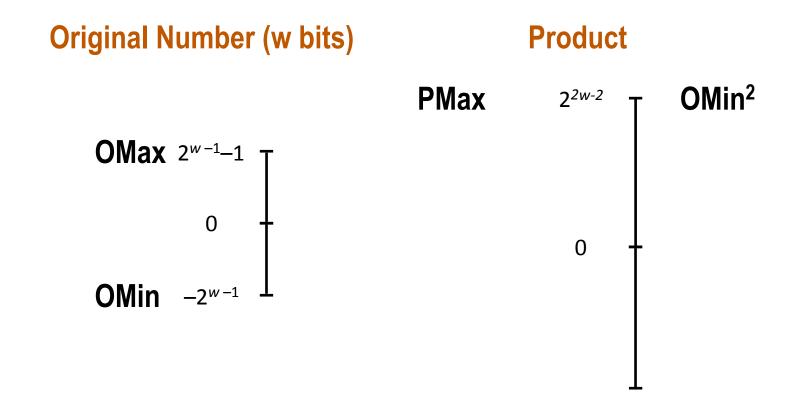
#### **Product**



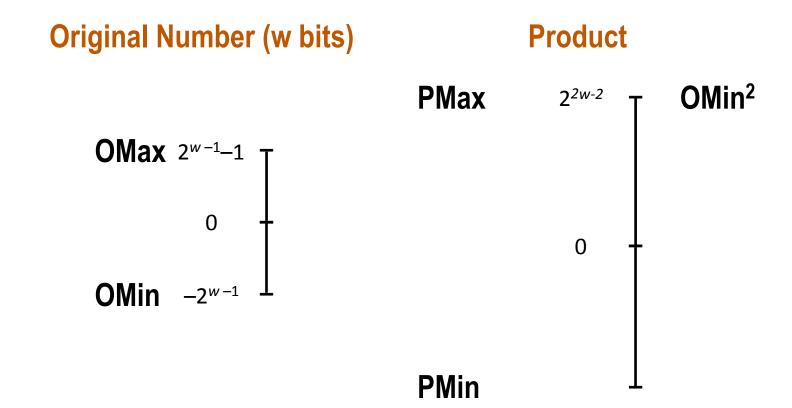
• Goal: Computing Product of w-bit numbers x, y



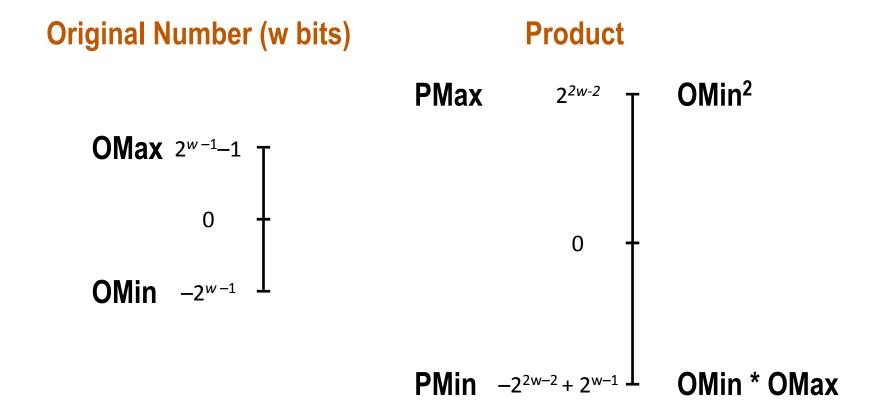
Goal: Computing Product of w-bit numbers x, y



Goal: Computing Product of w-bit numbers x, y



Goal: Computing Product of w-bit numbers x, y

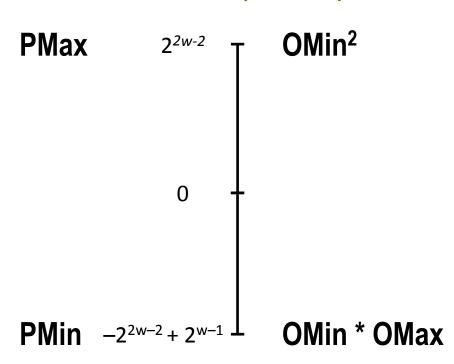


Goal: Computing Product of w-bit numbers x, y

# OMax $2^{w-1}-1 = 0$ OMin $-2^{w-1}$

**Original Number (w bits)** 

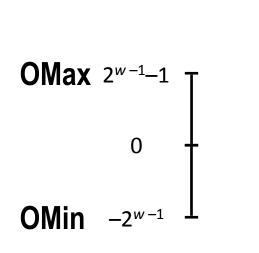
#### **Product (2w bits)**

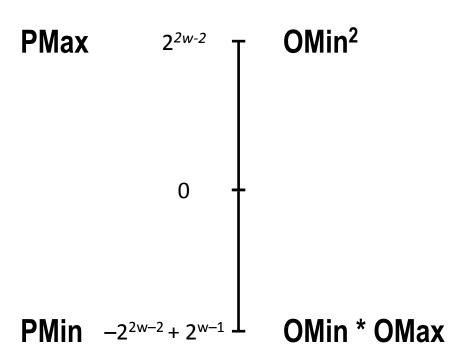


- Goal: Computing Product of w-bit numbers x, y
- Exact results can be bigger than w bits
  - Up to 2w bits (both signed and unsigned)

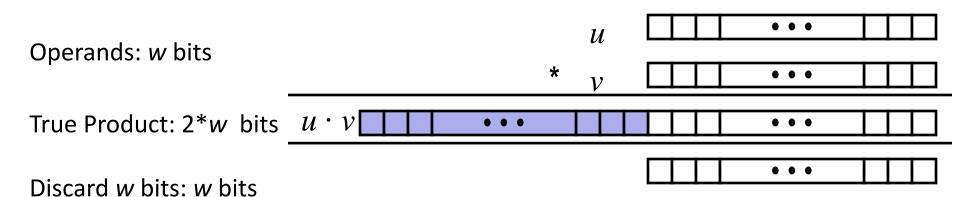
**Original Number (w bits)** 

**Product (2w bits)** 





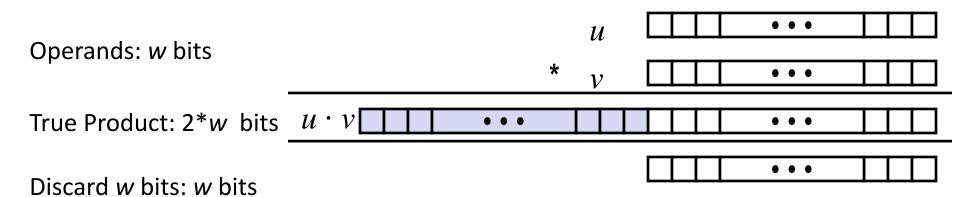
# Unsigned Multiplication in C



- Standard Multiplication Function
  - Ignores high order w bits
- Implements Modular Arithmetic

$$UMult_{w}(u, v) = u \cdot v \mod 2^{w}$$

# Signed Multiplication in C



- Standard Multiplication Function
  - Ignores high order w bits
  - Some of which are different for signed vs. unsigned multiplication
  - Lower bits are the same

# Power-of-2 Multiply with Shift

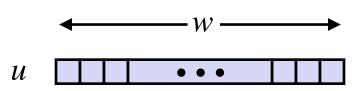
#### Operation

- u << k gives u \* 2<sup>k</sup>
- $001_2 << 2 = 100_2 (1 * 2^2 = 4)$
- Both signed and unsigned

## Power-of-2 Multiply with Shift

#### Operation

- u << k gives u \* 2<sup>k</sup>
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## Power-of-2 Multiply with Shift

#### Operation

- u << k gives u \* 2<sup>k</sup>
- $001_2 << 2 = 100_2 (1 * 2^2 = 4)$
- Both signed and unsigned

True Product: w+k bits  $u \cdot 2^k$  ••• 0 0 ••• 0 0

 $\mathcal{U}$ 

#### Power-of-2 Multiply with Shift

#### Operation

•  $u << k \text{ gives } u * 2^k$ •  $001_2 << 2 = 100_2 (1 * 2^2 = 4)$ • Both signed and unsigned

True Product: w+k bits  $u \cdot 2^k$ Discard k bits (if overflow)

#### Power-of-2 Multiply with Shift

#### Operation

•  $u << k \text{ gives } u * 2^k$ •  $001_2 << 2 = 100_2 (1 * 2^2 = 4)$ • Both signed and unsigned

True Product: w+k bits  $u \cdot 2^k$ 

Discard k bits (if overflow)

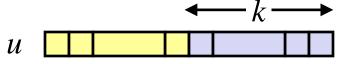
0 ••• 0 0

#### Most machines shift and add faster than multiply

- Compiler generates this code automatically
- u << 3 == u \* 8
- (u << 5) (u << 3) == u \* 24

- Implement power-of-2 divide with shift
  - u  $>> k \text{ gives } \lfloor u / 2^k \rfloor (\lfloor 2.34 \rfloor = 2)$
  - Uses logical shift

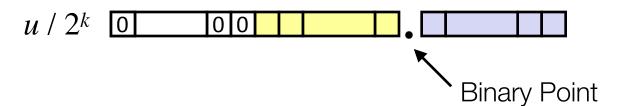
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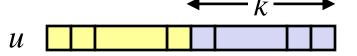


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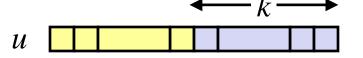
True Product: w+k bits

 $u/2^k$  0 00 .

Discard *k* bits after binary point

 $\lfloor u/2^k \rfloor$  Binary Point

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True Product: w+k bits

Discard *k* bits after binary point

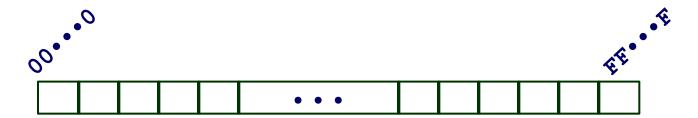
$$\lfloor u/2^k \rfloor$$
 O O Binary Point

- $234_{10} >> 2 = 2.34_{10}$ , truncated result is 2 (  $\lfloor 2.34 \rfloor = 2$ )
- $1101_2 >> 2 = 0011_2$  (true result:  $11.01_2$ .  $\lfloor 13 / 4 \rfloor = 3$ )

#### Today: Representing Information in Binary

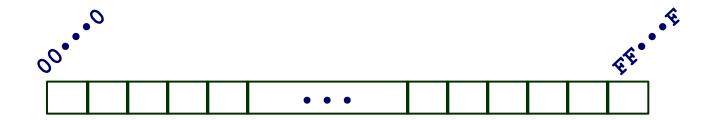
- Why Binary (bits)?
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - Summary
- Representations in memory, pointers, strings

#### **Byte-Oriented Memory Organization**



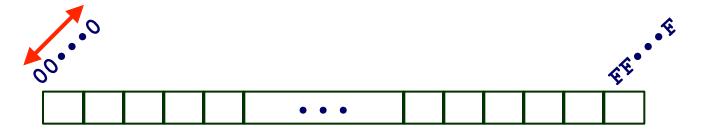
- Programs refer to data by address
  - Conceptually, envision it as a very large array of bytes: byte-addressable
    - In reality, it's not, but can think of it that way
  - An address is like an index into that array
    - and, a pointer variable stores an address

#### **Machine Words**



- Any given computer has a "Word Size"
  - Nominal size of a memory address
  - Until recently, most machines used 32 bits (4 bytes) as word size
    - Limits addresses to 4GB (2<sup>32</sup> bytes)
  - Increasingly, machines have 64-bit word size
    - Potentially, could have 18 EB (exabytes) of addressable memory
    - That's 18.4 X 10<sup>18</sup>

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#### Example Data Representations (in Bytes)

Word Size

4

8

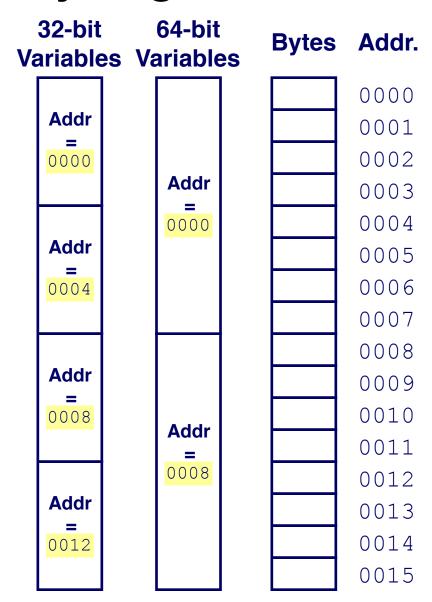
C Data Type	3 <b>2</b> -bit	64-bit
char	1	1
short	2	2
int	4	4
long	4	8
float	4	4
double	8	8
pointer	4	8

#### Example Data Representations (in Bytes)

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#### Word-Oriented Memory Organization

- Addresses Specify Byte Locations
  - Address of first byte in word
  - Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)

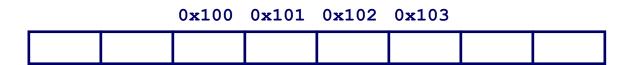


• How are the bytes within a multi-byte word ordered in memory?

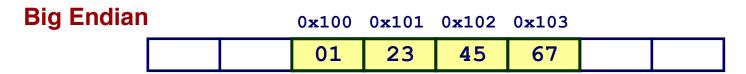
- How are the bytes within a multi-byte word ordered in memory?
- Example
  - Variable x has 4-byte value of 0x01234567
  - Address given by &x is 0x100

0x100	0x101	0x102	0x103	

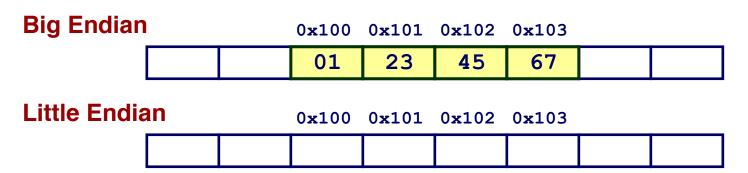
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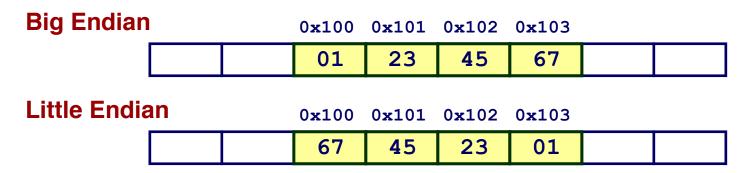
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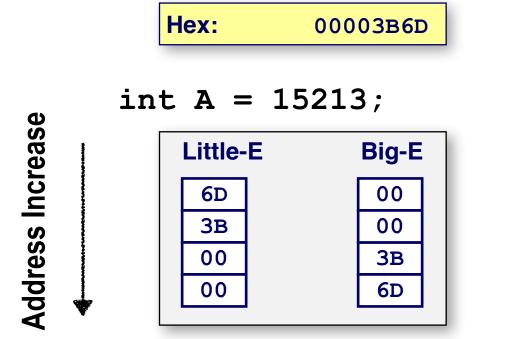
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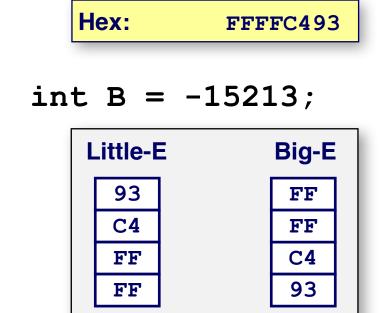


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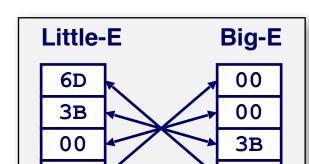
**Hex:** 00003B6D

Hex: FFFFC493

int A = 15213;

00

Address Increase



6D

int B = -15213;



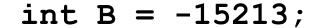
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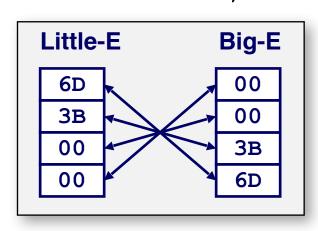
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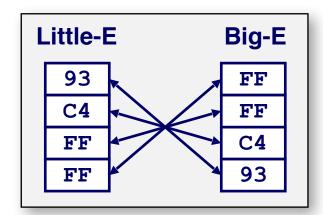
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Address Increase







#### **Announcement**

- Check the course website before asking
  - http://www.cs.rochester.edu/courses/252/spring2019/
- Direct ALL questions regarding assignments to the TAs
  - They have done them. They have debugged them. They know them inside out.
  - If one doesn't know, ask another.
  - If all don't know, ask me.