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EE301 LAB 2 Lab Report
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Questions 3-7

(3) $RC\frac{dV_{out}(t)}{dt} + V_{out}(t) = V_{in}(t).$

$i(t) = \frac{1}{R}[V_{in}(t) - V_c]$

$i(t) = \frac{dQ}{dt} = \frac{C\,dv_c(t)}{dt}$

$Ri(t) = V_{in} - V_{out}$

$RC = \frac{dv_c}{dt} + V_{out}(t) = V_{in}(t)$

(4) $V_{out} = V_0 e^{-t/RC} + \int_0^t \frac{1}{RC} e\, V_{in}(T)\,dT$

$V_{in}(t) = H(t) = u(t) = 1 \quad , t>0$

$Y_s = (t) = V_{out} = ?$

$Y_s(t) = V_0 e^{-t/RC} + \int_0^t \frac{1}{RC} e^{-(t-T)/RC}\,dT$

$Y_s(t) = \int_0^t \frac{1}{RC} e^{-(t-T/RC}\,dT$

$= \frac{1}{RC}\int_0^t e^{\frac{-t+T}{RC}} = \frac{1}{RC}\int_0^t e^{-t/RC} \cdot e^{T/RC}\,dT$

$= \frac{e^{-t/RC}}{RC}\int_0^t e^{T/RC}\,dT = \frac{e^{-t/RC}}{R} RC[e^{T/RC}]_0^t$

$= e^{-t/RC}[e^{t/RC} - e^0]$

$= e^0 - e^{-t/RC}$

$\boxed{Y_s(t) = 1 - e^{-t/RC}}$

(5) find impulse response $h(t)$

$$h(t) = \frac{d}{dt}\left[\frac{dy_s(t)}{dt}\right]$$

$$= \frac{d}{dt}\left[1 - e^{-t/RC}\right] = \frac{1}{RC} e^{-t/RC}$$

(6) $P_\Delta(t) = \frac{1}{\Delta}\left[u(t) - u(t-\Delta)\right]$, $\Delta > 0$

$$y(t) = x(t) * h(t)$$

$$= \int_{\infty}^{\infty} x(t)\, h(t-T)\, dt$$

$$= \int_{-\infty}^{\infty} x(t-T)\, h(T)\, dT$$

$$= \int_{-\infty}^{\infty} \frac{1}{\Delta}\left[u(t) - u(t-\Delta)\right] \cdot \frac{1}{RC} e^{-\frac{(t-T)}{RC}}\, dT$$

$$= \int_{0}^{\Delta} \frac{1}{\Delta} \cdot \frac{1}{RC} e^{\frac{(-t-T)}{RC}}\, dT$$

$$= \int_{0}^{\Delta} \frac{1}{\Delta RC} e^{-t/RC} \cdot e^{T/RC}\, dT$$

$$= \frac{e^{-t/RC}}{\Delta RC} \int_{0}^{\Delta} e^{\frac{T}{RC}}\, dt = \frac{e^{-t/RC}}{\Delta \cancel{RC}} \cancel{RC}\, e^{T/RC}\Big|_{0}^{\Delta}$$

$$= \frac{e^{-t/RC}}{\Delta}\left[e^{\Delta/RC} - e^{0}\right] = \boxed{\frac{e^{\frac{\Delta - t}{RC}} - e^{-t/RC}}{\Delta}}$$

$$\lim_{\Delta \to 0} \cancel{y(t) \Rightarrow RC \cdot e^{\frac{\Delta - t}{RC}} = \boxed{RC e^{-t/RC}}}$$

(7)

$$\lim_{\Delta \to 0} y(t) = \lim \frac{1}{\Delta} \left(e^{\frac{\Delta - t}{RC}} - e^{-t/RC} \right)$$

$$= \lim_{\Delta \to 0} \frac{1}{RC} e^{\frac{a-t}{RC}}$$

$$\boxed{\lim_{\Delta \to 0} = \frac{1}{RC} e^{-\frac{t}{RC}}}$$