

EE 301 Lab 5 – Sinusoids and Sinusoidal Correlation

In this lab we will gain experience with sinusoids and sinusoidal signal correlation in MATLAB.

1 What you will learn

This lab will be focused on complex numbers, sinusoids and complex exponentials in MATLAB. We will also gain an understanding of sinusoidal correlation with equivalent and different frequencies. An algorithm (FAPE) will be constructed to determine the amplitude, phase and frequency of a sinusoid.

2 Background Information and Notes

2.1 Sinusoids

Sinusoids are important signals due to the fact that most every signal in nature can be described in sinusoidal form or as a sum of sinusoids. An interesting property of sinusoids is that any linear time-invariant system whose input is a sinusoid will have an output that is a sinusoid of the same frequency. However, the amplitude and phase of the output may be different.

In many practical applications one needs to be able to identify the amplitude, phase, and/or frequency of a sinusoid. In communication systems, we can convey information by *modulating*, (or in other words perturbing) a sinusoidal signal (carrier). In order for the receiver to recover the transmitted information, one needs to know the amplitude, phase, and frequency of the carrier. Usually the carrier frequency is known in advance, however the phase and the amplitude are unknown.

In this lab, we will use correlation to extract information from a sinusoid. This process will involve the use of a complex exponential signal.

2.2 Complex numbers

Let us recall that a complex number $z = x + jy$ is defined by its *real part*, x , and its *imaginary part*, y , where $j = \sqrt{-1}$. This definition of the complex number is in rectangular form. It can also be written in *polar form* or *exponential form*, $z = re^{j\theta}$, where $r = |z|$ is the *magnitude* of the complex number and $\theta = \text{angle}(z)$ is the *angle*. We can convert between the two forms using the formulas

$$x = r \cos(\theta) \quad (3.1)$$

$$y = r \sin(\theta) \quad (3.2)$$

and

$$r = (x^2 + y^2)^{1/2} \quad (3.3)$$

$$\theta = \tan^{-1}(y/x) \quad (3.4)$$

One can also obtain the complex conjugate of a complex number, z^* , which is,

$$z^* = x - jy \quad (3.5)$$

$$z^* = re^{-j\theta} \quad (3.6)$$

One useful property of conjugation is that $zz^* = |z|^2$.

We can also use Euler's formula to express a relationship between the polar and rectangular forms of a complex number. Euler's formula is

$$e^{j\theta} = \cos(\theta) + j \sin(\theta) \quad (3.7)$$

And Euler's inverse formulas are:

$$\cos(\theta) = (e^{j\theta} + e^{-j\theta})/2 \quad (3.8)$$

$$\sin(\theta) = (e^{j\theta} - e^{-j\theta})/(2j) \quad (3.9)$$

2.3 Sinusoidal and complex exponential signals in continuous time

A continuous-time sinusoid $s(t)$, is given by the formula

$$s(t) = A \cos(\omega_0 t + \phi), \quad (3.10)$$

where $A > 0$ is the sinusoid's *amplitude*, ω_0 is the sinusoid's *frequency* given in *radian frequency* (radians per second), and ϕ is the sinusoid's *phase*. We can also represent the sinusoid as,

$$s(t) = A \cos(2\pi f_0 t + \phi), \quad (3.11)$$

where f_0 is the sinusoid's frequency given in Hertz (Hz, or cycles per second), where $\omega_0 = 2\pi f_0$.

The sinusoidal expression above is similar to another special signal known as the *complex exponential signal*. The form of a continuous-time complex exponential signal, $c(t)$, is

$$c(t) = Ae^{j(\omega_0 t + \phi)} \quad (3.12)$$

The relationship between the cosine of (3.10) and the complex exponential signal is as follows.

$$s(t) = A \cos(\omega_0 t + \phi) \quad (3.13)$$

$$s(t) = (A/2)[e^{j(\omega_0 t + \phi)} + e^{-j(\omega_0 t + \phi)}] \quad (3.14)$$

$$s(t) = (1/2)[c(t) + c^*(t)] \quad (3.15)$$

$$s(t) = \text{Re}\{Ae^{j(\omega_0 t + \phi)}\} \quad (3.16)$$

Notice that the cosine is merely the real part of our complex exponential signal.

One can also express the complex exponential signal $c(t)$ using Euler's formula, as the sum of a real cosine wave and an imaginary sine wave:

$$c(t) = A \cos(\omega_0 t + \phi) + jA \sin(\omega_0 t + \phi) \quad (3.17)$$

2.4 Finding the amplitude and phase of a sinusoid with known frequency

Let us now use *correlation* to determine the amplitude and phase of a sinusoid with a known frequency. We will begin with a continuous-time sinusoid (the *target sinusoid*)

$$s(t) = A \cos(\omega_0 t + \phi) \quad (3.18)$$

with a known frequency ω_0 , with unknown amplitude A and phase ϕ . We will then calculate the in-place correlation between this target sinusoid with a *reference sinusoid*, $u(t)$, with the same frequency and with a known amplitude and phase.

Using a trig identity where $\cos(x)\cos(y) = (1/2)[\cos(x-y) + \cos(x+y)]$, then,

$$C(s, u) = \int_{t1}^{t2} A \cos(\omega_0 t + \phi) \cos(\omega_0 t) dt \quad (3.19)$$

$$C(s, u) = \frac{A}{2} \int_{t1}^{t2} \cos(\phi) + \cos(2\omega_0 t + \phi) dt \quad (3.20)$$

$$C(s, u) = \frac{A}{2} \left[\cos(\phi)t + \frac{1}{2\omega_0} \sin(2\omega_0 t + \phi) \right]_{t1}^{t2} \quad (3.21)$$

Since the frequency, ω_0 , is known, the limits of integration can be set to include an integer number of fundamental periods of the sinusoids. Then the second term evaluates to zero and we are left with,

$$C(s, u) = \frac{A}{2} \cos(\phi)(t2 - t1) \quad (3.22)$$

Now if we know the phase ϕ , then we can calculate the amplitude A of $s(t)$ from $C(s, u)$. And similarly, if the amplitude A is known, we can narrow the phase ϕ down to one of two values. However if both amplitude and phase are unknown, we cannot use this equation to our benefit.

However, if the interval over which we correlate is not a multiple of the fundamental period of $u(t)$, then the second term in equation (3.21) may not evaluate to zero. But most often ω_0 is much greater than one, and the second term will be so small that it can be neglected, and we are again left with equation (3.22).

To resolve the problem when both amplitude and phase are unknown, let us use a complex exponential,

$$c(t) = e^{j\omega_0 t} \quad (3.23)$$

as the reference signal.

$$C(s, c) = \int_{t1}^{t2} s(t) c^*(t) dt \quad (3.24)$$

$$C(s, c) = \int_{t1}^{t2} A \cos(\omega_0 t + \phi) e^{-j\omega_0 t} dt \quad (3.25)$$

$$C(s, c) = \int_{t1}^{t2} \frac{A}{2} \left[e^{j(\omega_0 t + \phi)} + e^{-j(\omega_0 t + \phi)} \right] e^{-j\omega_0 t} dt \quad (3.26)$$

$$C(s, c) = \frac{A}{2} \int_{t1}^{t2} \left[e^{j\phi} + e^{-j(2\omega_0 t + \phi)} \right] dt \quad (3.27)$$

$$C(s, c) = \frac{A}{2} \left[e^{j\phi} t + \frac{-1}{2j\omega_0} e^{-j(2\omega_0 t + \phi)} \right]_{t1}^{t2} \quad (3.28)$$

If we correlate over an integer number of periods of the target sinusoid, then the second term evaluates to zero and we are left with

$$C(s, c) = \frac{A}{2} e^{j\phi} (t2 - t1) \quad (3.29)$$

This correlation results in a complex number whose magnitude is directly proportional to the amplitude of the original sinusoid and whose angle is equal to its phase! We can manipulate equation (3.29) to obtain,

$$A = \frac{2}{t_2 - t_1} |C(s, c)| \quad (3.30)$$

$$\phi = \text{angle}(C(s, c)) \quad (3.31)$$

Notice that the form of equation (3.29) is just the phasor representation of our sinusoid.

Again if the interval over which we correlate is not a multiple of the fundamental period of $c(t)$, then the second term in equation (3.28) may not evaluate to zero. But the same argument that typically ω_0 is much greater than 1 applies, and we can neglect the second term.

The Amplitude and Phase Calculator

This lab will involve implementing a system that estimates the amplitude and phase of a sinusoid with a known frequency. Since we will do this calculation using a computer, we must work with discrete signals which are then sampled version of the signals $s(t)$ and $c(t)$. Denoting the sampling period, T_s the discrete-time signals are,

$$s[n] = s(nT_s) = A \cos(\omega_0 T_s n + \phi) \quad (3.32)$$

$$c[n] = c(nT_s) = e^{j\omega_0 T_s n} \quad (3.33)$$

As will be shown below, when T_s is small, the correlation of $s(t)$ and $c(t)$ is approximately the correlation between $s[n]$ and $c[n]$ with a factor T_s . Let $\{n_1, \dots, n_2\}$ denote the discrete-time interval corresponding to the continuous-time interval $[t_1, t_2]$, and let $N = n_2 - n_1 + 1$ denote the number of samples taken in the interval $[t_1, t_2]$, so that $t_2 - t_1 \approx NT_s$. Then,

$$C(s, c) = \int_{t_1}^{t_2} s(t) c^*(t) dt \quad (3.34)$$

$$C(s, c) = \sum_{n=n_1}^{n_2} \int_{nT_s}^{(n+1)T_s} s(t) c^*(t) dt \quad (3.35)$$

$$C(s, c) \approx \sum_{n=n_1}^{n_2} \int_{nT_s}^{(n+1)T_s} s(nT_s) c^*(nT_s) dt \quad (3.36)$$

$$C(s, c) = \sum_{n=n_1}^{n_2} s(nT_s) c^*(nT_s) T_s \quad (3.37)$$

$$C(s, c) = \sum_{n=n_1}^{n_2} s[n] c^*[n] T_s \quad (3.38)$$

$$C(s, c) = C_d(s, c) T_s \quad (3.39)$$

The above derivation assumes T_s is small, and $C_d(s, c)$ is the correlation between the

discrete-time signals $s[n]$ and $c[n]$. Thus,

$$C(s, c) \approx C_d(s, c) T_s \quad (3.40)$$

This equation will be used to estimate the amplitude and phase of a continuous-time sinusoid.

In the laboratory assignment, we will be implementing an “amplitude and phase calculator” (APC) as a MATLAB function. A diagram of this system is shown in Figure 3.1.

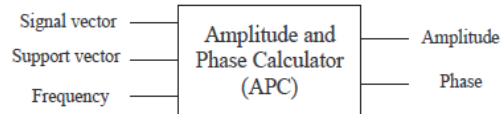


Figure 3.1: System diagram for the “amplitude and phase calculator”

The system takes three input parameters: a *signal vector* which contains the sinusoid itself, the *support vector* for the sinusoid, and the frequency of the reference sinusoid in radians per second. Note that in order to obtain the exact calculation of system’s output, the input sinusoid must be defined over an integer number of fundamental periods.

The output of APC is the sinusoid’s amplitude, and its phase in radians. The system calculates these outputs by first computing the in-place correlation given by equations (3.37) or (3.38), and then using this correlation value with equations (3.30) and (3.31) to compute the amplitude and phase. Note that in equation (3.30), one needs to replace $t_2 - t_1$ with $N = n_2 - n_1 + 1$ when implementing in discrete time.

2.5 Determining the frequency of a target sinusoid

Correlation can also be used to determine the frequency of a target sinusoid. Let the target sinusoid be defined by $s(t) = A \cos(\omega_s t + \phi)$, where ω_s , A , and ϕ are all unknown. We will then correlate $s(t)$ with a complex exponential signal, $c(t) = \exp(j\omega_c t)$, with frequency ω_c , where ω_s is not equal to ω_c :

$$C(s, c) = \int_{t_1}^{t_2} s(t) c^*(t) dt \quad (3.41)$$

$$C(s, c) = \int_{t_1}^{t_2} A \cos(\omega_s t + \phi) e^{-j(\omega_c t)} dt \quad (3.42)$$

$$C(s, c) = \int_{t_1}^{t_2} \frac{A}{2} \left[e^{j(\omega_s t + \phi)} + e^{-j(\omega_s t + \phi)} \right] e^{-j(\omega_c t)} dt \quad (3.43)$$

$$C(s, c) = \frac{A}{2} \int_{t_1}^{t_2} \left[e^{j[(\omega_s - \omega_c)t + \phi]} + e^{-j[(\omega_s + \omega_c)t + \phi]} \right] dt \quad (3.44)$$

$$C(s, c) = \frac{A}{2} \left[\frac{1}{\omega_s - \omega_c} e^{j[(\omega_s - \omega_c)t + \phi]} + \frac{1}{\omega_s + \omega_c} e^{-j[(\omega_s + \omega_c)t + \phi]} \right]_{t_1}^{t_2} \quad (3.45)$$

We will make a simplifying assumption and assume that $(\omega_s + \omega_c)$ is sufficiently large. Then the second term can be neglected and we have

$$C(s, c) \approx \frac{A}{2(\omega_s - \omega_c)} \left[e^{j[(\omega_s - \omega_c)t_2 + \phi]} - e^{j[(\omega_s - \omega_c)t_1 + \phi]} \right] \quad (3.46)$$

The above equation depends on the frequency difference $(\omega_s - \omega_c)$ between the target sinusoid and our reference signal. It can be shown that, the value of this correlation converges to the value of equation (3.29) as $(\omega_s - \omega_c)$ approaches zero.

Consider now the length-normalized correlation, $\tilde{C}(s, c)$, which is defined as

$$\tilde{C}(s, c) = \frac{C(s, c)}{t_2 - t_1} \quad (3.47)$$

From equation (3.29), one can see that when the reference and target signals have the same frequency, the length-normalized correlation does not depend on the length of the signal. However, when the signals have differing frequencies, from equations (3.46) and (3.47) one can see that the magnitude of the length-normalized correlation becomes smaller as we correlate over a longer period of time. In the limit as the correlation length goes to infinity, *the length-normalized correlation approaches zero unless the frequencies match exactly.*

Another interesting case occurs when we correlate over a *common period* of the target and reference signals, which occurs when the correlation interval includes an integer number of periods of *both* the target signal and reference signal. In this case, for signals of different frequencies, the correlation in equation (3.46) is zero. Note, however that the correlation is *not* zero when the frequencies match. This is the same requirement for equation (3.29) to be exact.

In order to determine the frequency of the target sinusoid we must use the “guess and check” method. We will need to check the correlation with complex exponentials of various frequencies. The complex exponential that yields the highest correlation, will be used to estimate of the frequency of the target signal. The next section, will describe the formal algorithm of this technique.

A frequency estimation algorithm

Let us suppose that we have a continuous-time target sinusoid $s(t)$ with length $[0, T]$ with an unknown amplitude, frequency, and phase. To estimate these parameters, we’ll calculate the length-normalized correlation between this signal and reference complex exponentials with various frequencies over the signal’s T second length. We will then look for the frequency that produces the largest correlation.

The frequencies of these complex exponentials are chosen to be multiples of $1/T$ so that the correlation is over an integer number of periods of each complex exponential. Thus, the frequencies will be $1/T$, $2/T$, etc. As in the previous section,

we will approximately compute the correlation from samples of $s(t)$ and each reference exponential, taken with some small sampling interval T_s . For convenience let's take N samples and choose $T_s = T/N$, for some large even integer N . We will learn in the future part of this course, that at the very least, two samples are needed from each period of the signal being sampled. Therefore, the highest frequency with which we will correlate is, approximately, $1/(2T_s)$. Thus, we will correlate $s(t)$ with complex exponentials at frequencies

$$\frac{1}{T}, \frac{2}{T}, \dots, \frac{N}{2T} = \frac{1}{2T_s} \quad (3.48)$$

Then, for $k = 1, 2, \dots, N/2$, the length normalized correlation of $s(t)$ with the complex exponential at frequency k/T is

$$X[k] \approx \frac{1}{T} \sum_{n=0}^{N-1} s(nT_s) \exp\left(-j2\pi \frac{k}{T} nT_s\right) T_s \quad (3.49)$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} s[n] \exp\left(-j2\pi \frac{k}{N} n\right) \quad (3.50)$$

where $T_s/T = 1/N$, and we have denoted the result as $X[k]$. Thus, the output of these correlations is the set of $N/2$ numbers $X[1], \dots, X[N/2]$. Note that $X[k]$ will generally be complex. To estimate the frequency of the target sinusoid, we must identify the value of k for which $|X[k]|$ is largest. With k_{\max} denoting this value, our estimated frequency, ω_{est} , is then,

$$\omega_{\text{est}} = 2\pi \frac{k_{\max}}{T} = 2\pi \frac{k_{\max}}{NT_s} \quad (3.51)$$

Once the frequency is known, we can then estimate the values of amplitude and phase. From equations (3.30) and (3.31), we have:

$$A_{\text{est}} = 2|X[k_{\max}]| \quad (3.52)$$

$$\phi_{\text{est}} = \text{angle}(X[k_{\max}]) \quad (3.53)$$

Note that this process requires some estimation and in the lab assignment we will see the effect of such an approximation.

A block diagram of the “frequency, amplitude, and phase estimator” (FAPE) system is given in Figure 3.2.

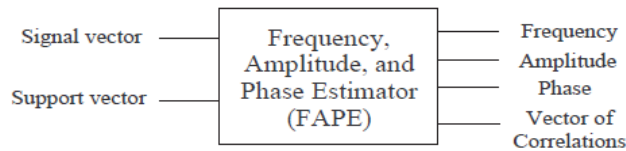


Figure 3.2: System diagram for the “frequency, amplitude and phase estimator”

2.6 Some useful Matlab commands

- **Constructing complex numbers:** MATLAB represents all complex numbers in rectangular form. To enter a complex number, simply type **5+6j** (for instance). Note that both **i** and **j** are used to represent (unless you have used one or the other as some other variable). To enter a complex number in polar form, type **2*exp(j*pi/3)** (for instance).
- **Extracting parts of complex numbers:** If **z** contains a complex number (or an array of complex numbers), you can find the real and imaginary parts using the commands **real(z)** and **imag(z)**, respectively. You can obtain the magnitude and angle of a complex number (or an array of complex numbers) using the commands **abs(z)** and **angle(z)**, respectively.
- **Complex conjugation:** To compute the complex conjugate of a value (or array) **z**, simply use the MATLAB command **conj(z)**.
- **Finding the index of the maximum value in a vector:** Sometimes we don't just want to find the maximum value in a vector; instead, we need to know where that maximum value is located. The **max** command will do this for us. If **v** is a vector and you use the command

```
>> [max_value, index] = max(v);
```

the variable **max_value** will contain largest value in the vector, and **index** contains position of **max_value** in **v**.

- MATLAB commands to help you visually determine the amplitude, frequency, and phase of a sinusoid: Sometimes you may need to determine the frequency, phase, and amplitude of a sinusoid from a MATLAB plot. In these cases, there are three commands that are quite useful. First, the command **grid on** provides includes a reference grid on the plot; this makes it easier to see where the sinusoid crosses zero (for instance). The **zoom** command is also useful, since you can drag a zoom box to zoom in on any part of the sinusoid. Finally, you can use **axis** in conjunction with **zoom** to find the period of the signal. To do so, simply zoom in on exactly one period of the signal and type **axis**. MATLAB will return the current axis limits as **[x_min, x_max, y_min, y_max]**.
- **Calling apc:** The function **apc**, which you will be writing in this laboratory, estimates amplitude and phase of a continuous-time target sinusoid from its samples. The input parameters are a (sampled) target sinusoid **s**, the sinusoid's support vector **t**, and the continuous-time frequency ω_0 in radians per second. We call **apc** like this:

```
>> [A, phi] = apc(s, t,  $\omega_0$ );
```


- **Calling fape:** The function **fape**, which you will be writing in this laboratory, implements the frequency, amplitude, and phase estimator system. This function accepts the samples of a target continuous-time sinusoid **s** and its support vector **t**, like this:

```
>> [frq, A, phi, X] = fape(s, t);
```

where **frq** is the estimated frequency in radians per second, **A** is the estimated amplitude, **phi** is the estimated phase, and **X** is the vector of correlations, $X[1]$, ..., $X[N/2]$ between **s** and each reference complex exponential.

3 Guided Exercises

1. Execute the following commands:

```
>> t = linspace(-0.5, 2, 1000);
>> plot(t, cos(linspace(-7.5, 27, 1000)), 'k:');
```

(a) (Extracting sinusoid parameters) Visually identify the amplitude, continuous time frequency, and phase of the continuous-time (sampled) sinusoid that you've just plotted.

- Include your estimated values in your report. Reduce your answers to decimal form.
- What is the phasor that corresponds to this sinusoid? Write it in both rectangular and polar form. (Again, keep your answers in decimal form. You should use MATLAB to perform these calculations.)

(b) (Checking your parameters) Verify your answers in the previous problem by generating a sinusoid using those parameters and plotting them on the above graph using **hold on**. Use **t** as your time axis/support vector. The new plot should be close to the original, but it does not need to be exactly correct.

- Include the resulting graph in your report. Remember to include a **legend**

2. (The Amplitude and Phase Calculator) In this problem we will complete and test a function which implements the “Amplitude and Phase Calculator”, as described in Section 2. Download the file **apc.m**. This is a “skeleton” M-file for the “amplitude and phase calculator”. Also, generate the following sinusoid (**s_test**) with its support vector (**t_test**):

```
>> t_test = 0:99;
>> s_test = 1.3*cos(t_test*pi/10 + 2.8);
```

(a) (Identify sinusoid parameters by hand) What are the amplitude, frequency in radians per second, and phase of **s_test**?

(b) (Write the APC) Complete the function **apc**. You should use the signal **s_test** to test the operation of your function. You may also use the compiled function **apc_demo.p** to test your results.

- Include the code for **apc** in your lab document

(c) (Test APC on a sinusoid with unknown parameters) Download the file **lab5_data.mat**. This **.mat** file contains the support vector (**t_samp**) and signal vector (**s_samp**) for a sampled continuous-time sinusoid with a continuous-time frequency of $\omega_0 = 200\pi$ radians.

- From **t_samp**, determine the sampling period, T_s , of this signal.
- Use **apc** to determine the amplitude and phase of the sinusoid exactly.

(d) (APC in a non-ideal case) What happens if we use **apc** to correlate over a non-integral number of periods of our target sinusoid? We will investigate this question in this problem and the next. First, let's examine a single non-integral number of periods. Generate the following sinusoid:

```
>> apc_support = 0:80;
```

```
>> apc_test = cos(apc_support*pi/15);
```

This is a sinusoid with a frequency of $\hat{\omega}_0 = \pi/15$ radians per second, unit amplitude, and zero phase shift.

- Plot **apc_test** and include the plot in your report.
- What is the fundamental period of **apc_test**?
- Approximately how many periods are included in **apc_test**?
- Use **apc** to estimate the amplitude and phase of this sinusoid. What are the amplitude and phase errors for this signal?

(e) (APC in many non-ideal cases) Now we wish to examine a large number of different lengths of this sinusoid. You will do this by writing a **for loop** that repeats the previous part for many different values of the length of the incoming sinusoid. Specifically, write a **for loop** with loop counter **support_length** ranging over values **10:500**. In each iteration of the loop, you should

- i. Set **apc_support** to **0:(support_length)**,
- ii. Recalculate **apc_test** using the new **apc_support**,
- iii. Use **apc** to estimate the amplitude and phase of **apc_test**, and
- iv. Store these estimates in two separate vectors.

Put your code in an M-file script so that you can run it easily.

- Include your code in the MATLAB appendix.
- Use **subplot** to plot the amplitude and phase estimates as a function of support length in two subplots of the same figure. You should be able to see both local oscillation of the estimates and a global decrease in error with increased support length.
- At what support lengths are the amplitude estimates correct (i.e., equal to 1)?
- What minimum support length do we need to be sure that the phase error is less than 0.01 radians?

3. (The Frequency, Amplitude, and Phase Estimator) In this problem, we'll explore the frequency, amplitude and phase estimator, as described in Section 2. Download the file **fape.m**. This is a “skeleton” M-file for the “frequency, amplitude, and phase estimator” system.

(a) (Write the FAPE) Complete the **fape** function. You can use **t_test** and **s_test** from Problem 2 check your function's results.

- Include the completed code in your report
- What are the frequency (in radians per second), amplitude, and phase estimates returned by **fape** for **t_test** and **s_test**? Are these estimates correct?
- Use **stem** and **abs** to plot the magnitude of the vector of correlations returned by **fape** versus the associated frequencies.
- What do you notice about this plot? What can you deduce from this fact? (Hint: Consider what this plot tells you about the returned estimates.)

(b) (Running FAPE on in a non-ideal case) In this problem, we'll see what happens to FAPE when the target sinusoid does not include an integral number of periods. **lab5_data.mat** contains the variables **fape_test_t** (a support vector) and **fape_test_s** (its associated sinusoidal signal). Run **fape** on this signal.

- What are the frequency in radians per second, amplitude, and phase estimates that are returned?
- Use **stem** and **abs** to plot the magnitude of the returned vector of correlations.

- Plot **fape_test_s** and a new sinusoid that you generate from the parameter estimates returned by FAPE on the same figure (using **hold on**). Use **fape_test_t** as the support vector for the new sinusoid. Make sure you use different line types and include a legend.
- What can you say about the accuracy of estimates returned by FAPE?
- Compare the plot of the correlations generated in this problem and in Problem 3a. What do these different plots tell you?

4 Review

1. None.

5 Lab Report

1. The first page of your Lab report should be a cover sheet with your name, USC ID and Lab #. Please note that all reports should be typed.
2. Answer all the questions which were asked in the lab report. Kindly display the code lines you executed to arrive at your answer along with figures to support them. Please give written explanation or put comment lines where necessary. Please note that each figure should have proper labels for the x and y axis and should have a suitable title.
3. Answer the review questions.
4. Submit a printout of your completed M-file documenting all the lab exercises.