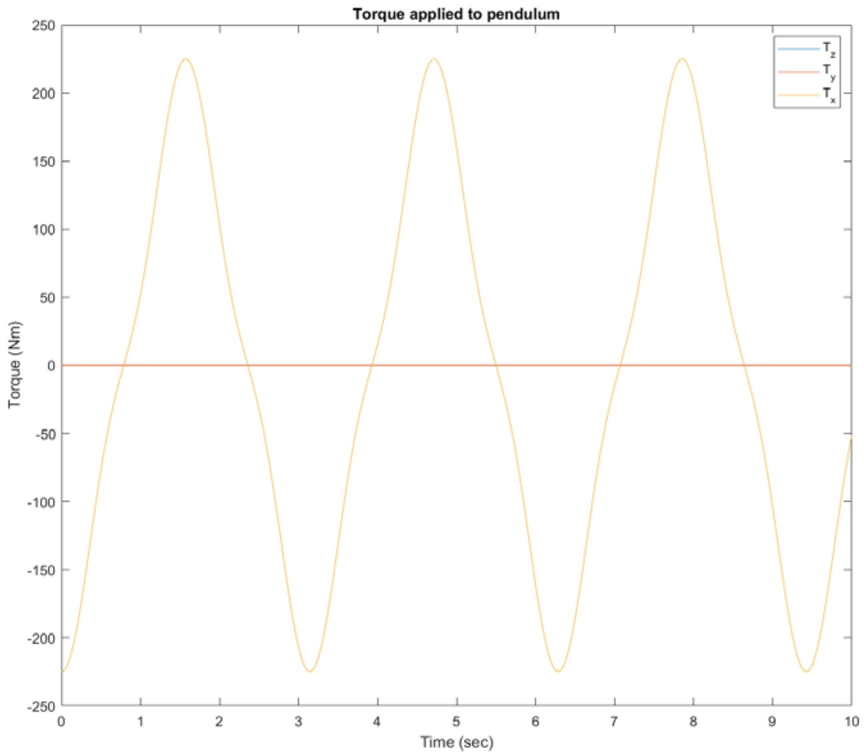
Assignment 7

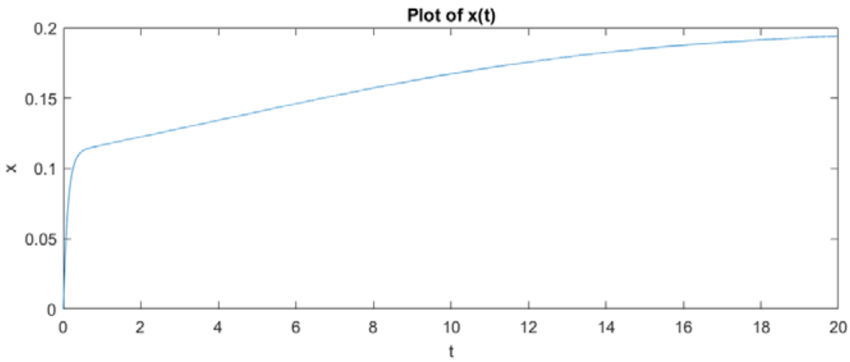
Problem 1.

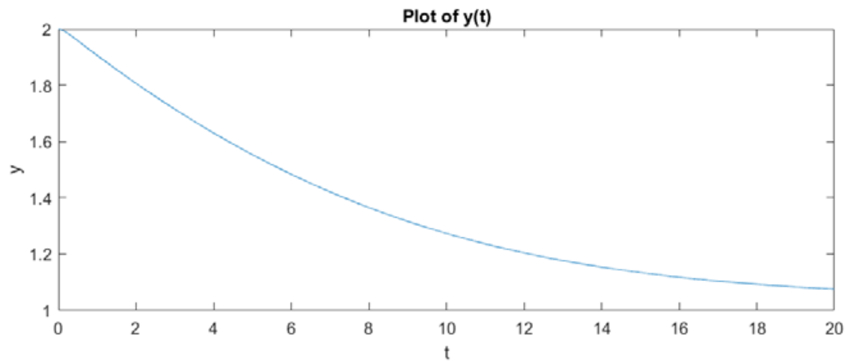
To perform the inverse dynamics analysis, please run the MATLAB file “simEngine3D_A7P1.m”. The plot that displays the value of the torque:



Problem 2.

Plots of x and y:





Problem 3.

(a)

Prob 3.

a) Proof:

① $y(1) = \frac{1}{1} + \frac{1}{1^2}\tan\left(\frac{1}{1}+\pi-1\right)$

$= 1 + \tan(\pi)$

$= 1 + 0$

$= 1$

So $y(t) = \frac{1}{t} + \frac{1}{t^2}\tan\left(\frac{1}{t}+\pi-1\right)$ satisfies the IC: $y(1)=1$.

② $\dot{y} = -\frac{1}{t^2} - \frac{2}{t^3}\tan\left(\frac{1}{t}+\pi-1\right) + \frac{1}{t^2}\cdot\left(-\frac{1}{t^2}\right)\cdot\left[1+\tan^2\left(\frac{1}{t}+\pi-1\right)\right]$

$= -\frac{1}{t^2} - \frac{1}{t^4} - \frac{2}{t^3}\tan\left(\frac{1}{t}+\pi-1\right) - \frac{1}{t^4}\tan^2\left(\frac{1}{t}+\pi-1\right)$

$-y^2 - \frac{1}{t^4}$

$= -\left[\frac{1}{t} + \frac{1}{t^2}\tan\left(\frac{1}{t}+\pi-1\right)\right]^2 - \frac{1}{t^4}$

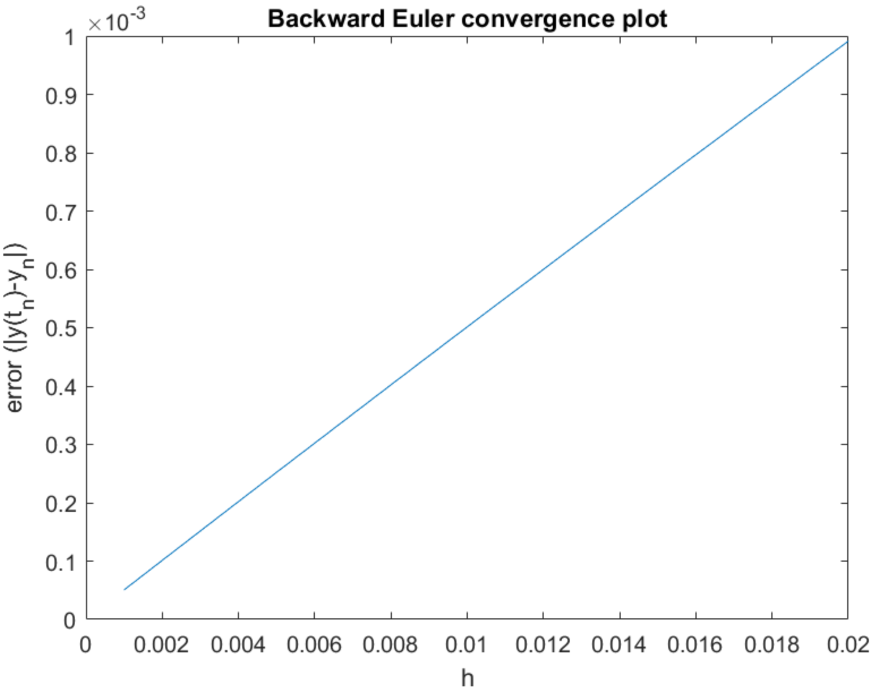
$= -\frac{1}{t^2} - \frac{2}{t^3}\tan\left(\frac{1}{t}+\pi-1\right) - \frac{1}{t^4}\tan^2\left(\frac{1}{t}+\pi-1\right) - \frac{1}{t^4}$

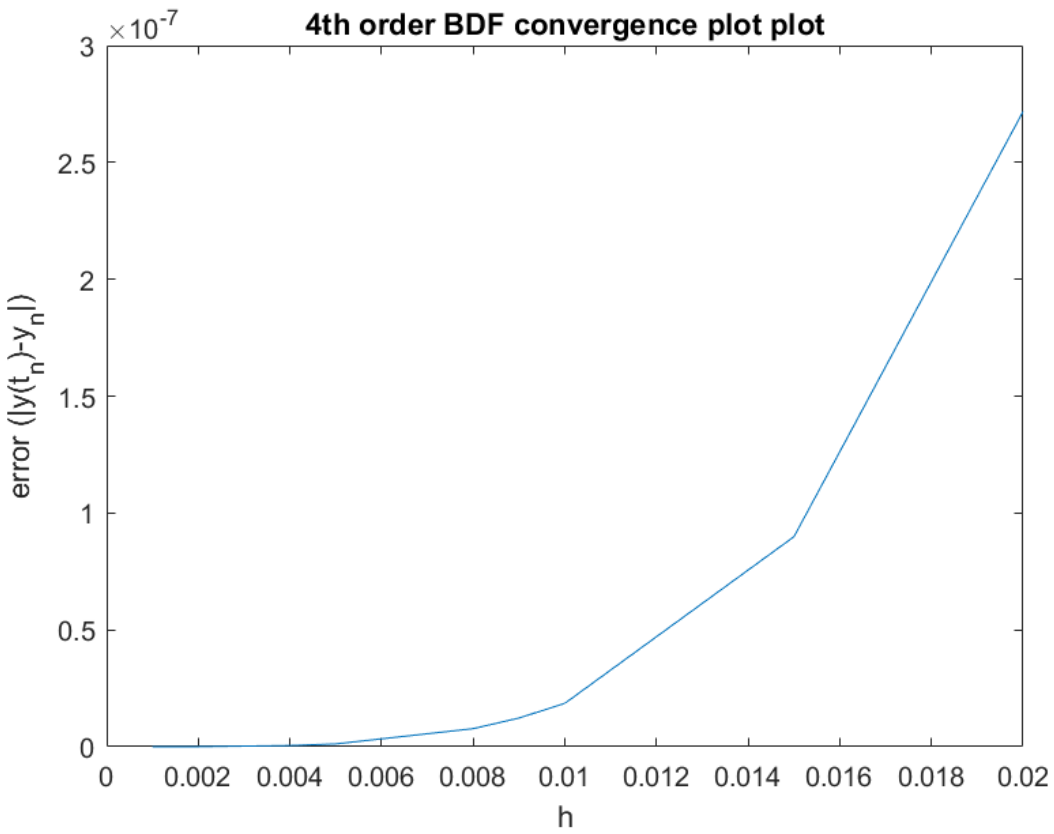
Therefore, LHS $= \dot{y} = -y^2 - \frac{1}{t^4} =$ RHS

$y(t) = \frac{1}{t} + \frac{1}{t^2}\tan\left(\frac{1}{t}+\pi-1\right)$ satisfies the scalar ODE.

$\square$

(b) and (c)





(d)

From the plots, we can see the convergence of Backward Euler has a slope of 1, and the convergence of 4^{th} order BDF is a quartic curve. To show this, I add a log-log plot where the slopes are 1 and 4 respectively. This result makes sense as Backward Euler is a first order method and 4^{th} order BDF is a 4^{th} order method.