

Introduction to Machine Learning, Spring 2022

Homework 2

(Due Friday, Apr. 8 at 11:59pm (CST))

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1 Problem1

Given a set of data $\{(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)\}$, where $y_i \in \{0, 1\}$. We want to conduct a binary classification, and the decision boundary is $\beta_0 + x^T \beta = 0$. When $\beta_0 + x^T \beta > 0$, the sample will be classified as 1, and 0 otherwise.

- (a) Define a function which enables to map the range of an arbitrary linear function to the range of a probability [2 points]
- (b) Derive the posterior probability of $P(y_i = 1|x_i)$ and $P(y_i = 0|x_i)$ [3 points]
- (c) Write the log-likelihood for N observations, which means:

$$l(\theta) = \log P(Y|X) = \sum_{i=1}^N \log(P(y_i|x_i))$$

(Using the expression of $P(y_i|x_i)$ in (b) and eliminate redundant items) [5 points]

Solution:

- (a)
- (b)
- (c)

Table 1: probability distribution for X

X	0	1	2	3
P	θ^2	$2\theta(1 - \theta)$	θ^2	$1 - 2\theta$

2 Problem2

- (a) Given a random variable X and its probability distribution is shown in Table 1. Now, we sample 8 times and get the results $\{3, 1, 3, 0, 3, 1, 2, 3\}$. Please derive the MLE estimate for $\theta(0 < \theta < \frac{1}{2})$. [4 points]
- (b) Now we discuss Bayesian inference in coin flipping. Let's denote the number of heads and the total number of trials by N_1 and N , respectively. Please derive the MAP estimate based on the following prior:

$$p(\theta) = \begin{cases} 0.5 & \text{if } \theta = 0.5 \\ 0.5 & \text{if } \theta = 0.3 \\ 0 & \text{otherwise,} \end{cases}$$

which believes the coin is fair, or is slightly biased towards tails. [4 points]

- (c) Suppose the true parameter is $\theta = 0.31$. Please compare the prior in (b) with the Beta prior distribution (You can review this part in Lecture 07). Which prior leads to a better estimate when N is small? Which prior leads to a better estimate when N is large? [2 points]

3 Problem3

According to the following Fig. 3, answer the following questions:

- (a) use the D-separation to discuss whether the following statements are true or not:
 - (1) Given x_4 , $\{x_1, x_2\}$ and $\{x_6, x_7\}$ are conditionally independent. [1(reason)+1(conclusion) points]
 - (2) Given x_6 , x_3 and x_2 are conditionally independent. [1(reason)+1(conclusion) points]
- (b) if all the nodes are observed and boolean variables, please complete the process of learning the parameter $\theta_{x_6|i,j}$ by using **MLE**, where $\theta_{x_6|i,j} = p(x_6 = 1 \mid x_3 = i, x_4 = j), i, j \in \{0, 1\}$. [6 points]

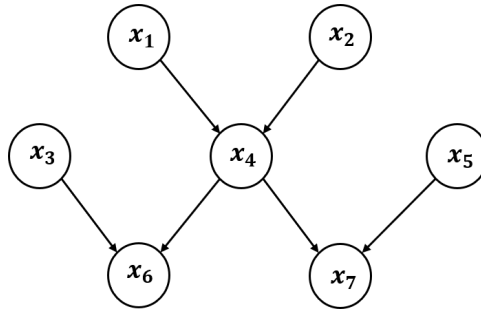


Figure 1: The Bayesian network for questions 3.