Coded Slotted ALOHA over Rayleigh Block Fading Channels: BP Threshold and Converse Bound

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Outline

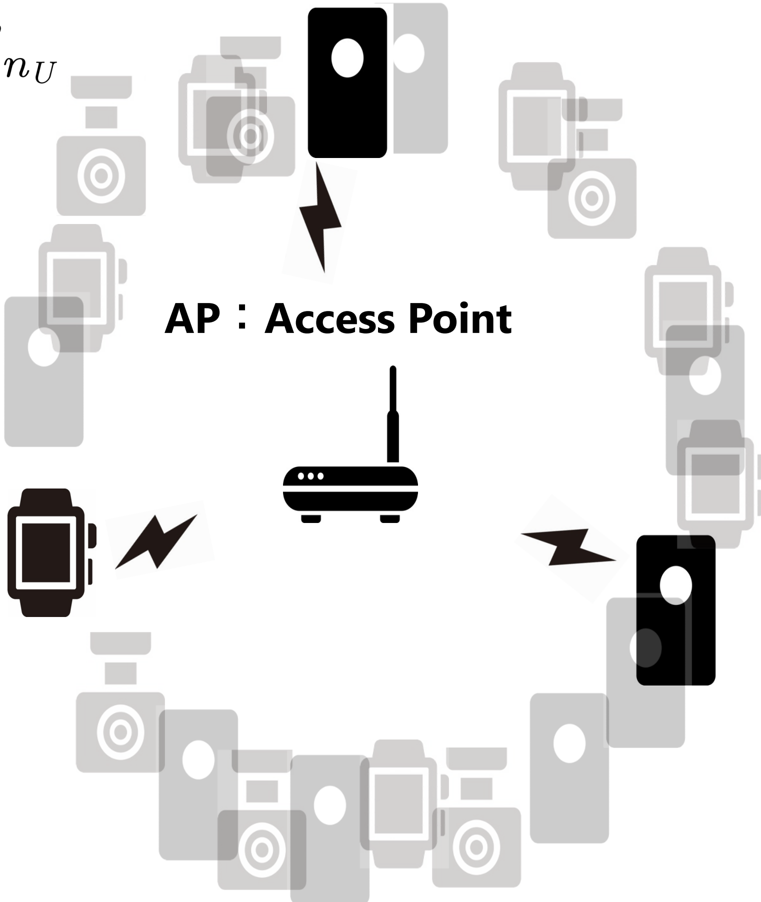
- **Problem Description**
  - ✓ **coded slotted ALOHA (C-SA) Systems**
    - **protocol: transmit & receive**
    - **performance metric**
- **Density Evolutions (Rayleigh Channels)**
  - ✓ **BP threshold**
- **Converse Bound**
- **Numerical Results**

Massive Sensor Networks

N_T **devices**

each device:

- **channel code** $\mathcal{C}_{\text{ch}}$
- **segment-oriented code set** $\mathcal{C}_{n_1}, \mathcal{C}_{n_2}, \ldots, \mathcal{C}_{n_U}$
- **activation probability** $\pi << 1$
- **a single message**



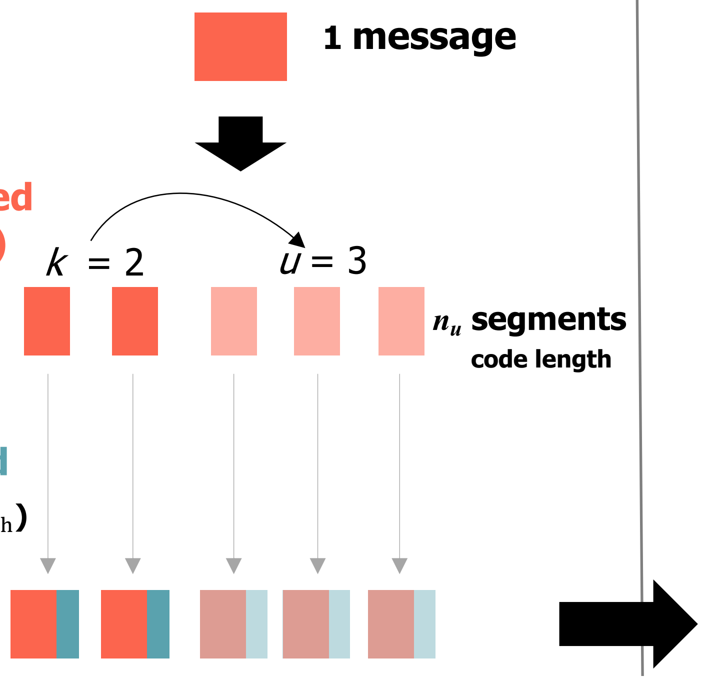
➡ **coded slotted ALOHA protocol**

Segment- & Channel-Oriented Codes

a device chooses $\mathcal{C}_{n_u}$ with probability Λ_{n_u}

➢ ENCODING

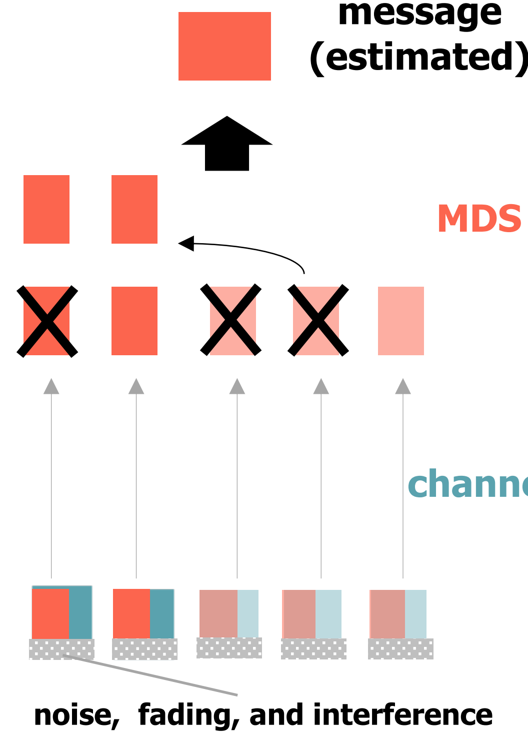
segment-oriented (MDS code $\mathcal{C}_{n_u}$)



channel-oriented
(code $\mathcal{C}_{\mathrm{ch}}$ with rate R_{ch})

Rayleigh channel

➢ DECODING



MDS decoding

channel decoding

decoding of $(n_u{=}k{+}u,\ k)$-MDS code $\mathcal{C}_{n_u}$:

successful if at most k segments are erased

MDS code: maximum distance separable code

Channel decoding: threshold decoding

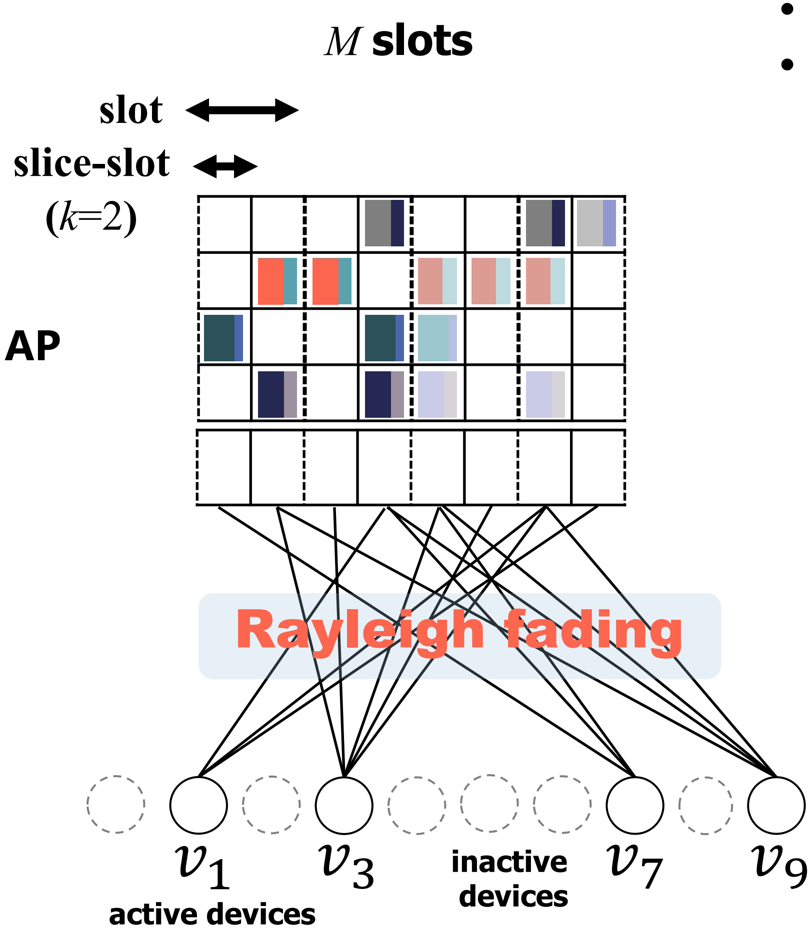
$$\text{SINR} = \frac{\Gamma_i}{1+\sum_{u\in\mathcal{R}\setminus i}\Gamma_u} > \eta_0 \qquad (\eta_0 = 2^{R_0}-1)$$

SNR threshold for decoding

no fading

Coded Slotted ALOHA (C-SA) [Paolini11]

➢ C-SA access protocol



each active device

- sends a single message
- choose code $\mathcal{C}_{n_u}$ with probability Λ_{n_u}
- sends its n_u coded segment over n_u slice-slots randomly

segment-oriented code set

$$\mathcal{C}_{n_1}, \mathcal{C}_{n_2}, \ldots, \mathcal{C}_{n_U}$$

average code length $\overline{n} = \sum_{u=1}^{U} \Lambda_{n_u} n_u$

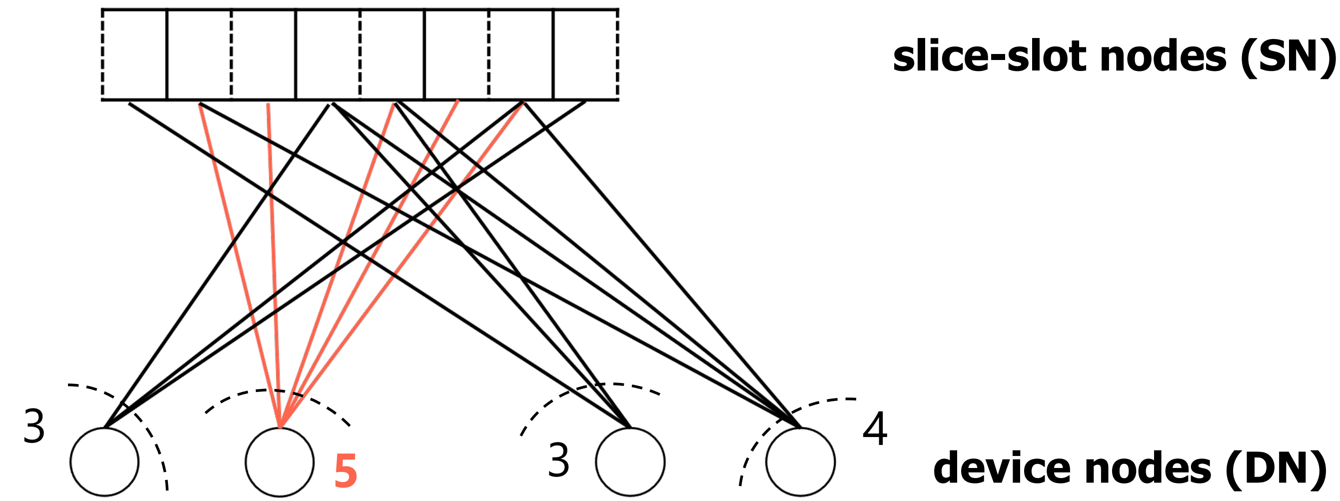
average code rate (energy efficiency) $\zeta = k / \overline{n}$

degree distribution $\{\Lambda_n\}_{n_1}^{n_U}$

to be optimized

[1] E. Paolini, G. Liva, and M. Chiani, “High throughput random access via codes on graphs: Coded slotted ALOHA,” in 2011 IEEE International Conference on Communications (ICC), 2011, pp. 1–6.

Device Degree Distribution: Example

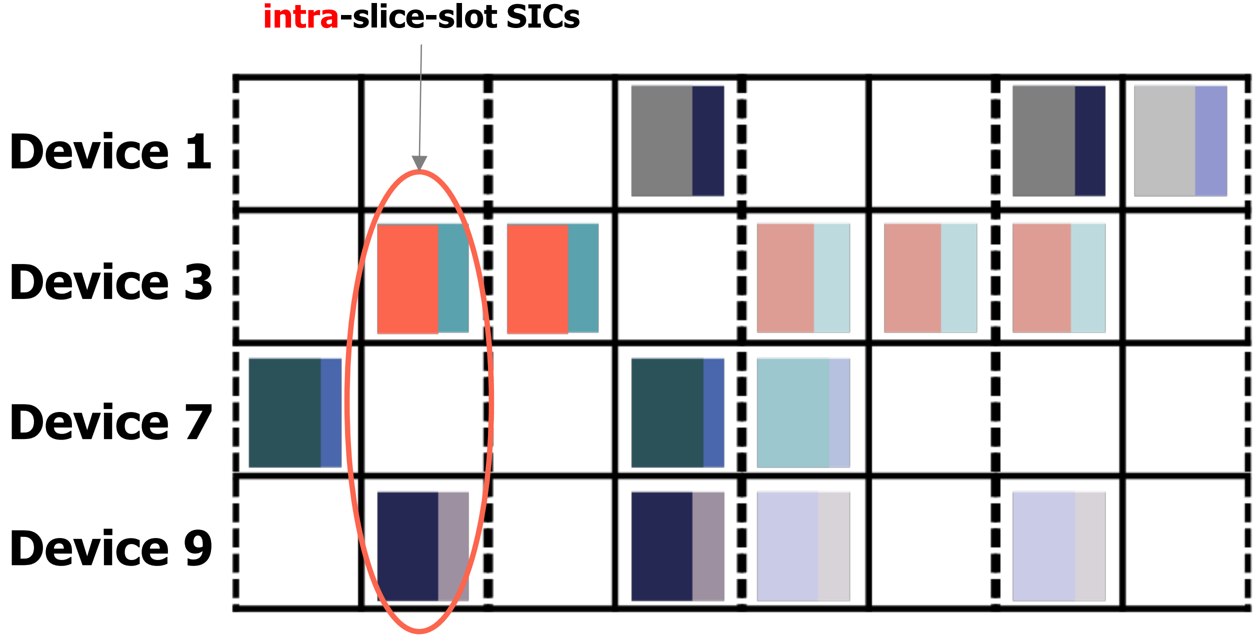


$$\Lambda(x) = \frac{2}{4}x^3 + \frac{1}{4}x^4 + \frac{1}{4}x^5$$

distribution in polynomial $\Lambda(x) = \sum_{n=n_1}^{n_U} \Lambda_n x^n$

Decoding over Fading: Example

$k = 2$



iterative decoding processing between

intra-slice-slot SICs

inter-slice-slot SICs

SIC: successive interference cancellation

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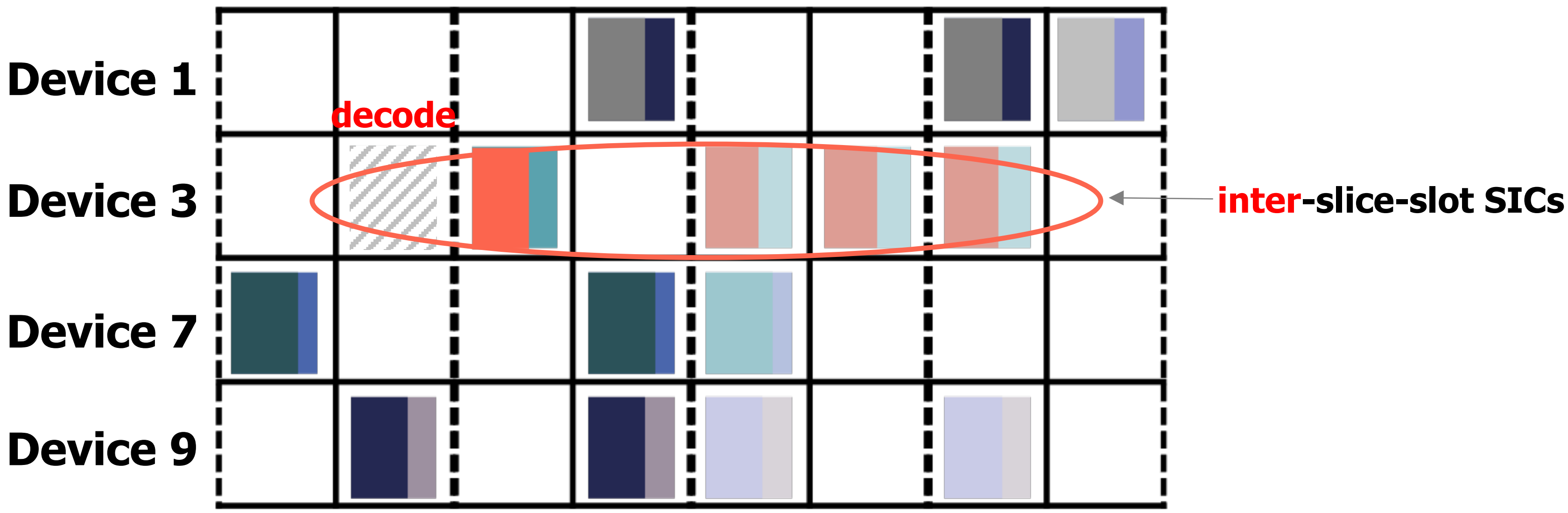

iterative decoding processing between **intra-slice-slot SICs** / **inter-slice-slot SICs**

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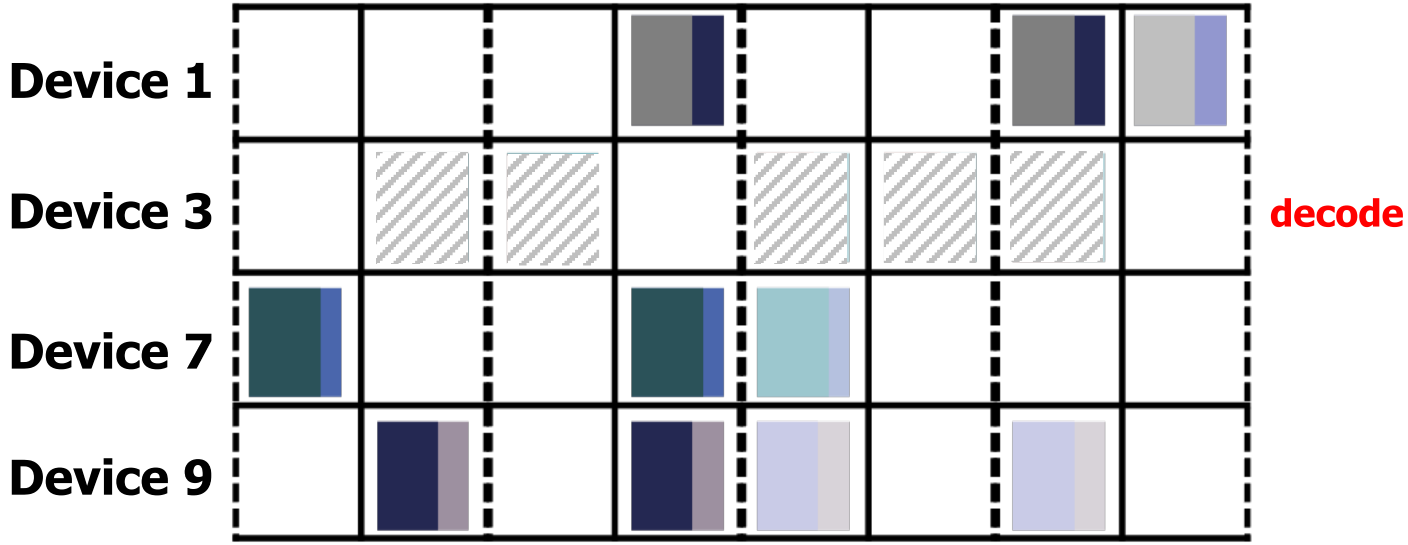

iterative decoding processing between **intra**-slice-slot SICs / **inter**-slice-slot SICs

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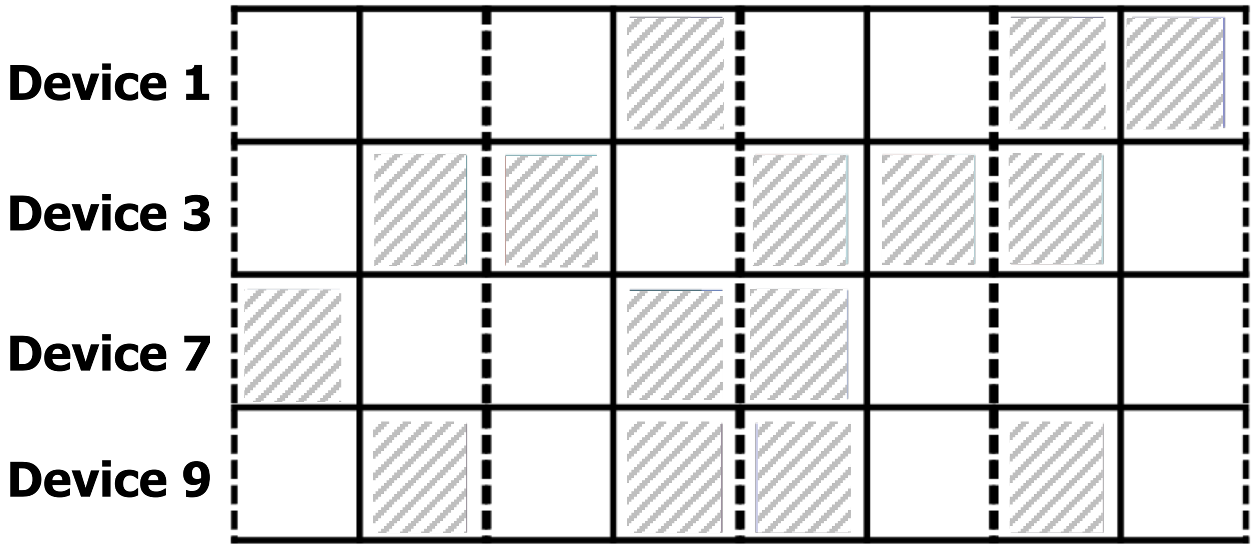


iterative decoding processing between **intra-slice-slot SICs** / **inter-slice-slot SICs**

SIC: successive interference cancellation

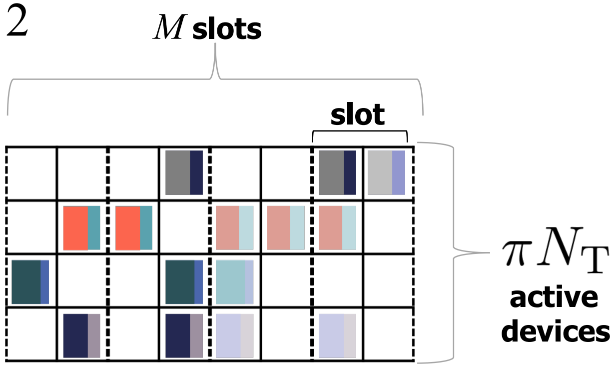
Performance Metric

- **average normalized offered traffic** G

N_{T}: **a total number of devices**
M: **number of slots**

$$\alpha = N_{\mathrm{T}}/M$$

ex. $k = 2$



1 packet/ 1 device
number of active devices

$$G = \frac{\pi N_{\mathrm{T}}}{M} = \pi\alpha \quad \text{[packets/ slot]}$$

activation prob.

(PLR < target value)

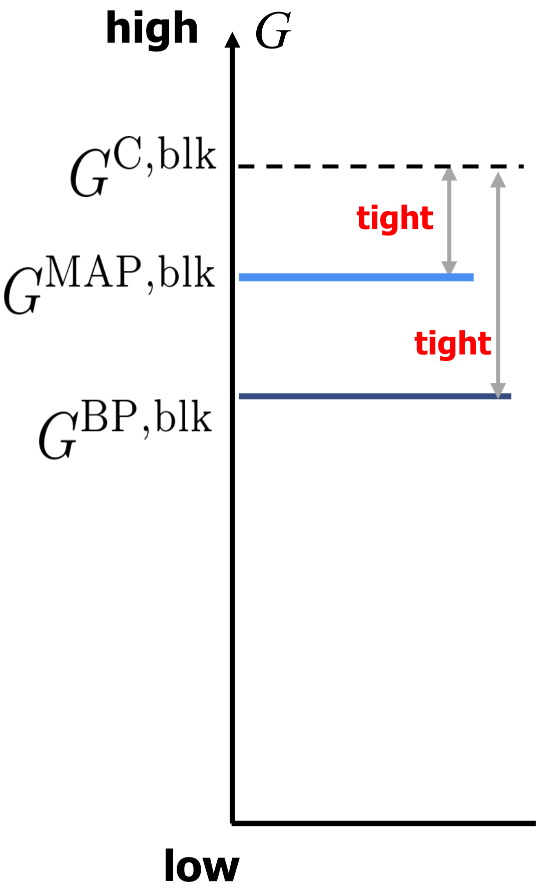
$\Lambda(x)$ **to be optimized**

maximum

PLR: **packet loss rate**

Threshold of traffic G

asymptotic ($M \to \infty$) performance



converse bound : impossible performance

MAP threshold : highest performance

high computation complexity

BP threshold : practical performance

by iterative decoding

MAP: maximum a posteriori **BP: belief propagation**

E. Paolini, C. Stefanovic, G. Liva, and P. Popovski, "Coded random access: Applying codes on graphs to design random access protocols," IEEE Commun. Mag., vol. 53, no. 6, pp. 144–150, 2015.

Previous Works

asymptotic ($M \to \infty$) performance

- **BP thresholds and converse bound were derived over**
  - ✓ ***collision* channel model**
  - ✓ ***erasure* channel model**
  - ✓ ***multi-packect reception* (MPR) model**
  - ✓ ***on-off fading* model**

- **BP thresholds and converse bound are unknown over**
  - ✓ **fading channel**

BP threshold of irregular repetition slotted ALOHA over Rayleigh is known.

a repetition code case of generic linear block code in C-SA [Clazzer17]

F. Clazzer, E. Paolini, I. Mambelli, and Cˇ . Stefanovic´, "Irregular repetition slotted ALOHA over the Rayleigh block fading channel with capture," in Proc. of IEEE ICC, 2017, pp. 1–6.

Our Objective

asymptotic ($M \to \infty$) performance

- **to derive BP threshold and converse bound over Rayleigh block channels**
  - ✓ **We formulate density evolution (DE) equations**
    - **We obtain BP threshold with iteration between the DE equations**
    - **We obtain converse bound from the sum of two EXIT curves**

- **We will show that**
  - ✓ **MDS coding provide more flexible values of energy efficiency and higher BP thresholds than repetition coding**
  - ✓ **Converse bound is tight**

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  - ✓ coded slotted ALOHA (C-SA) Systems
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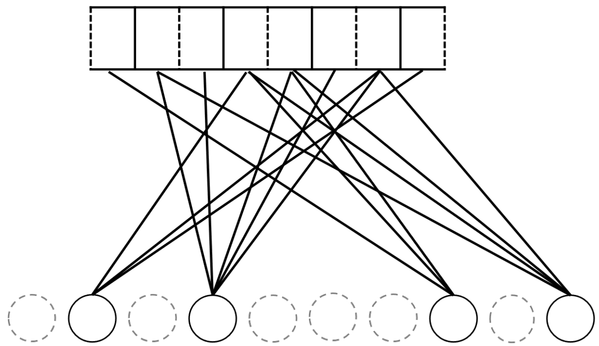
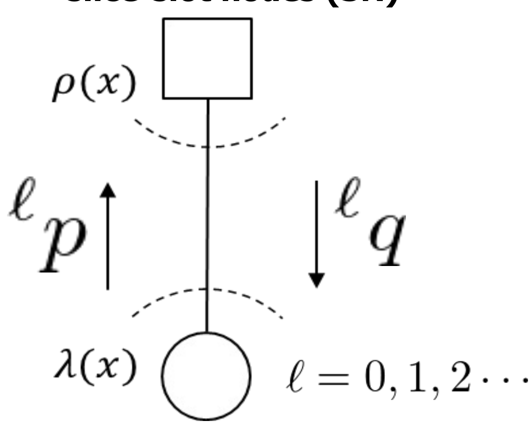
Bipartite Graph and Protograph

- **bipartite graph**



$M \to \infty$ **with constant** $\alpha = N_{\mathrm{T}}/M$

- **protograph**



degree distributions

(node-perspective)

$$\rho(x) = e^{-\frac{\alpha}{R}(1-x)}$$

Poission distribution in polynomial

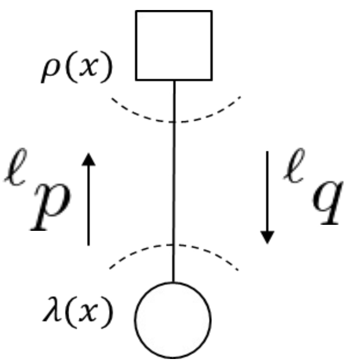
$$\lambda(x) = \sum_{u=n_1}^{n_c} \lambda_u x^{u-1}$$

Density Evolution (DE): Iterative Equations

- **average prob. that a DN-to-SN message is erased**

$$ {}^{(0)}p = \pi, \ {}^{(0)}q = 1 $$

$$ {}^{\ell}p = \pi \sum_{n=n_1}^{n_U} \lambda_n f_{\mathrm{b}}^{(n)}({}^{(\ell-1)}q) \triangleq \pi \tilde{\lambda}({}^{(\ell-1)}q) $$



- **average prob. that a SN-to-DN message is erased**

$$ {}^{\ell}q = 1 - \chi(1 - {}^{\ell}p) \qquad \ell = 0, 1, 2 \cdots $$

➡ **belief propagation (BP) threshold**

$$ G^{\mathrm{BP,blk}} = \sup\{G = \alpha\pi : \underbrace{\Lambda({}^{(\infty)}q)}_{\text{average PLR}} < \mathrm{PLR}^*\} $$

Distribution $\Lambda(x)$ can be optimized by differential evolution algorithm

similar to LDPC decoding over erasure channel

α: **constant**

Density Evolution (cont.)

$$\bar{\epsilon}_m = 1 - \sum_{t=1}^{m} \frac{(m-1)!}{(m-t)!} \frac{e^{-\frac{1}{\bar{\Gamma}m}((1+\eta_0)^t - 1)}}{(1+\eta_0)^{t(m-(t+1)/2)}}$$

error probability of decoding m-collision within a slice-slot [Clazzer17]

$$\tilde{\lambda}({}^{(\ell-1)}q) = \sum_{u=n_1}^{n_c} \lambda_u \cdot f_{\mathrm{b}}^{(u)}({}^{(\ell-1)}q)$$

probability that among the other n_u-1 incoming message to the DN, at most k-1 segments are successfully recovered

$$f_{\mathrm{b}}^{(u)}(q) = \sum_{i=0}^{k-1} \binom{n_u - 1}{i} (1-q)^i q^{n_u - i - 1}$$

MDS code

κ-MaxDecoding: **the intra-slice-slot SIC is carried out only when the number of collisions is equal to or less than a given constant κ**

κ : a finite number

$$\chi(1 - {}^{\ell}p) \triangleq e^{-\frac{\alpha}{\zeta}\,{}^{\ell}p} \sum_{m=0}^{\infty} \frac{(\frac{\alpha}{\zeta}\,{}^{\ell}p)^m}{m!} (1 - \underline{\bar{\epsilon}_{m+1}})$$

due to fading

F. Clazzer, E. Paolini, I. Mambelli, and Cˇ . Stefanovic´,
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  - ✓ BP threshold
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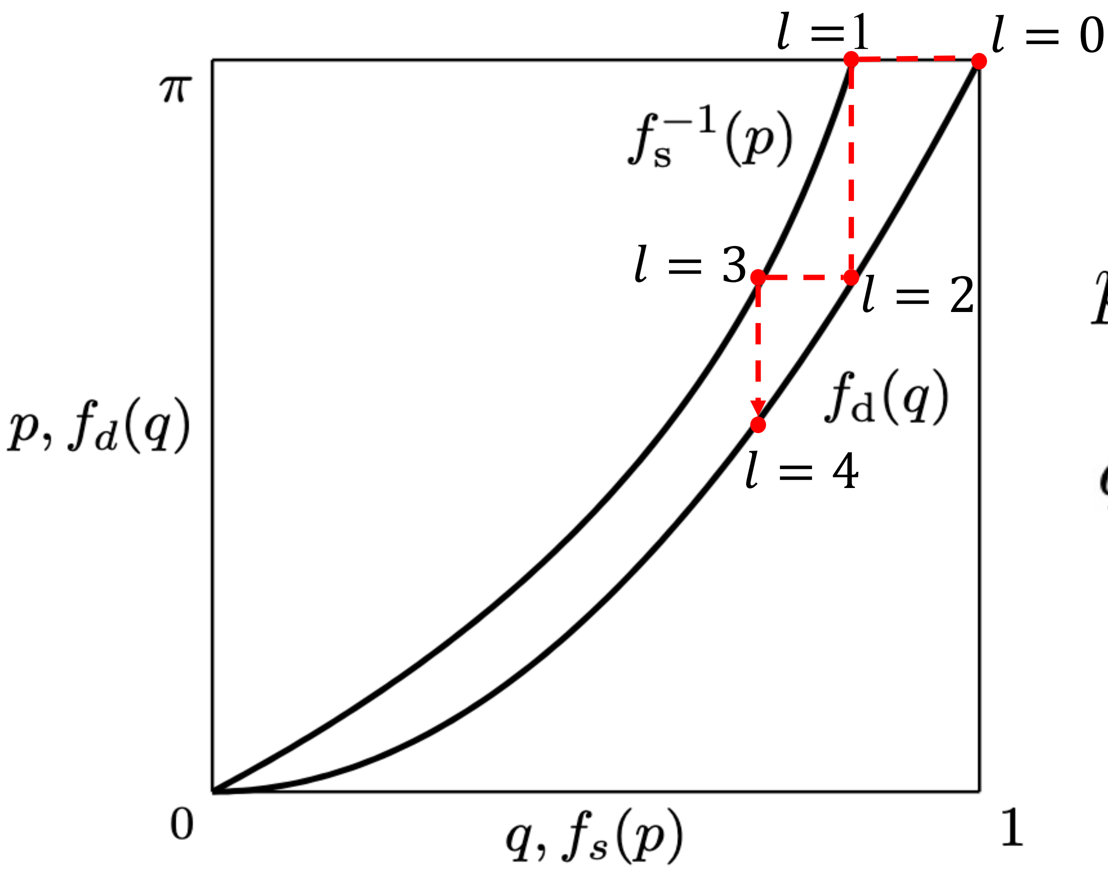
Extrinsic Information Transfer (EXIT) Chart



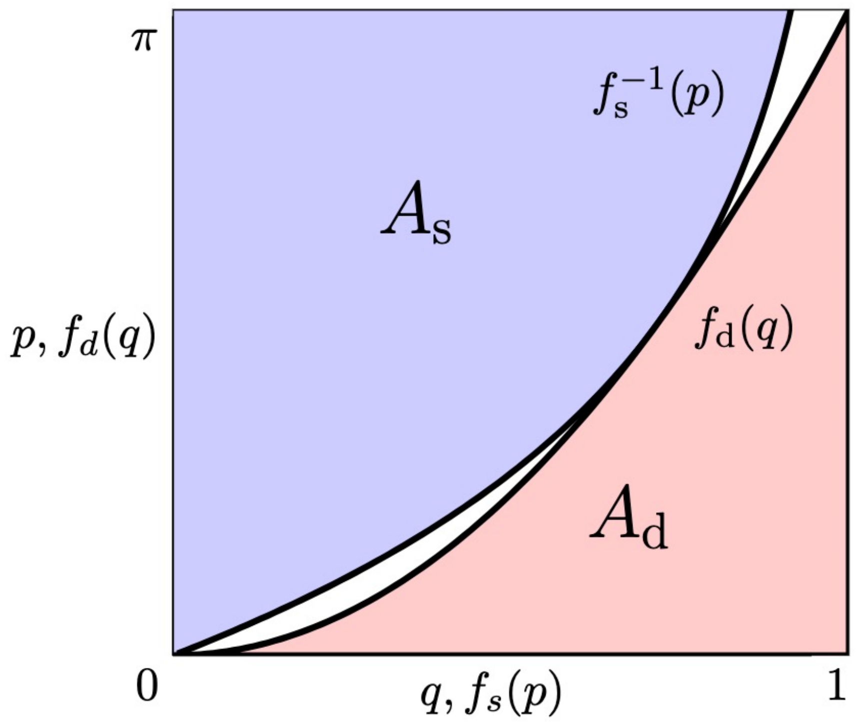
$$p = \pi\tilde{\lambda}(q) \triangleq f_{\mathrm{d}}(q)$$

$$q = 1 - \chi(1-p) \triangleq f_{\mathrm{s}}(p)$$

A necessary condition for successful decoding is to have an open tunnel between the two curves.

Sum of Two Areas < Area of Entire Domain

π : **activation prob.**



$$A_d + A_s < \pi$$

$$A_s = \int_0^{\pi} f_s(p)\mathrm{d}p$$

$$A_d = \int_0^{1} f_d(q)\mathrm{d}q$$

$$G^{C,blk} = \sum_{m=0}^{\infty}\left((1-\bar{\epsilon}_{m+1})(1-e^{-G^{C,blk}\frac{1}{\zeta}}\sum_{r=0}^{m}\frac{(G^{C,blk}\frac{1}{\zeta})^r}{r!})\right)$$

$G = \pi\alpha$

The solution of the equation give the converse bound $G^{C,blk}$ **.**

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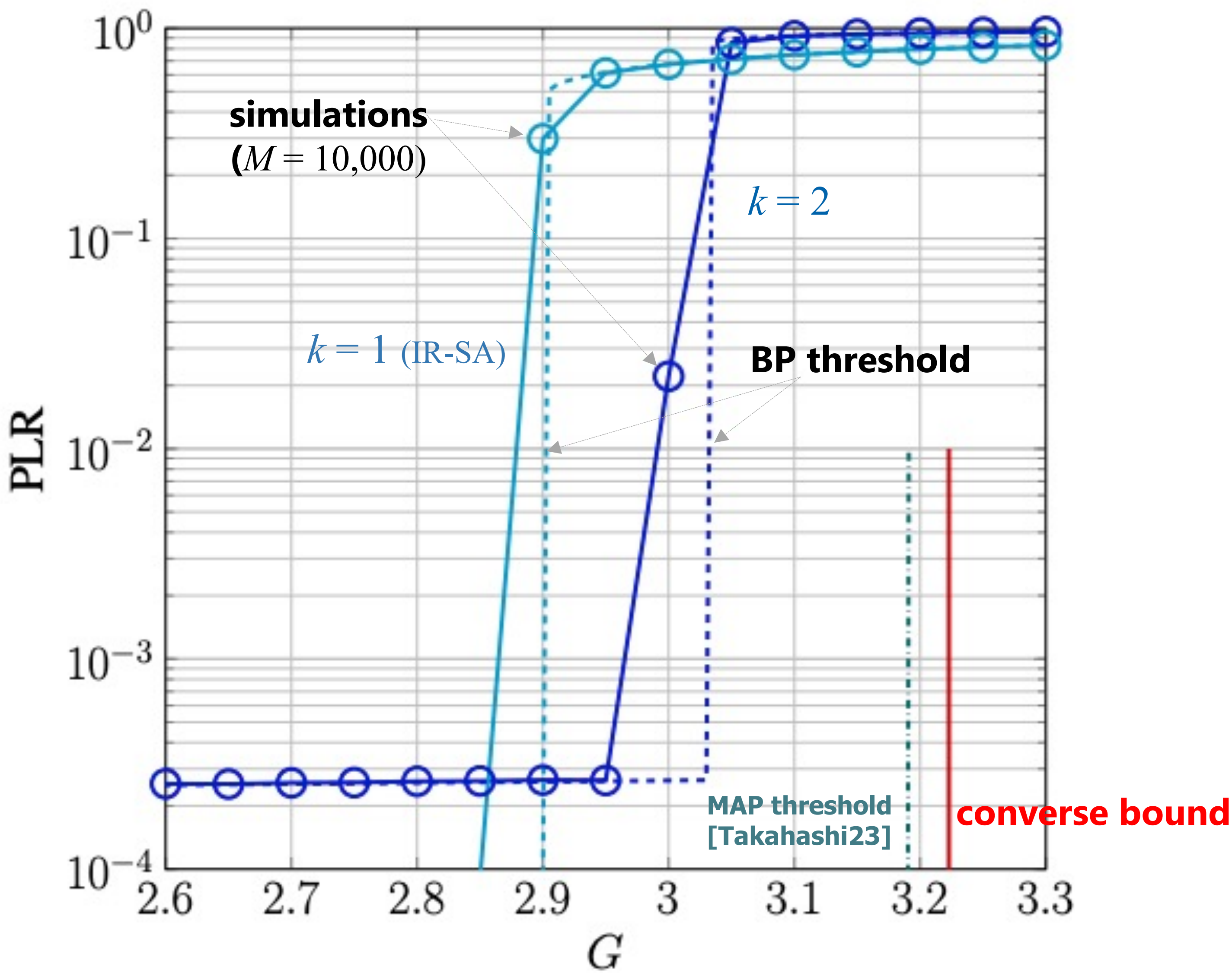
Specifications in Analysis and Simulations

Paramters	Values
ratio of number of devices to slots α	100
SNR threshed of decoding η_0	1
average received SNR per device $\bar{\Gamma}$	20 dB
number of fading samples	10000
target PLR*	$\leq 10^{-2}$

Distribution $\Lambda(x)$ were optimized by differential evolution algorithm

BP Thresholds of C-SA

$(\kappa = 15)$



BP Thresholds & Converse Bound $(\zeta = 1/2)$

ζ	$G^{\mathrm{BP,blk}}$				$G^{\mathrm{C,blk}}$
	$k=1$	$k=2$	$k=3$	$k=4$	
1/2	2.8769	2.9231	2.9377	2.9405	3.1315

BP thresholds are close to converse bounds with increasing k.

energy efficiency $\zeta = k / \overline{n}$

(average code rate of MDS codes)

BP Thresholds of SC-C-SA & Converse Bounds

$(\kappa = 15)$

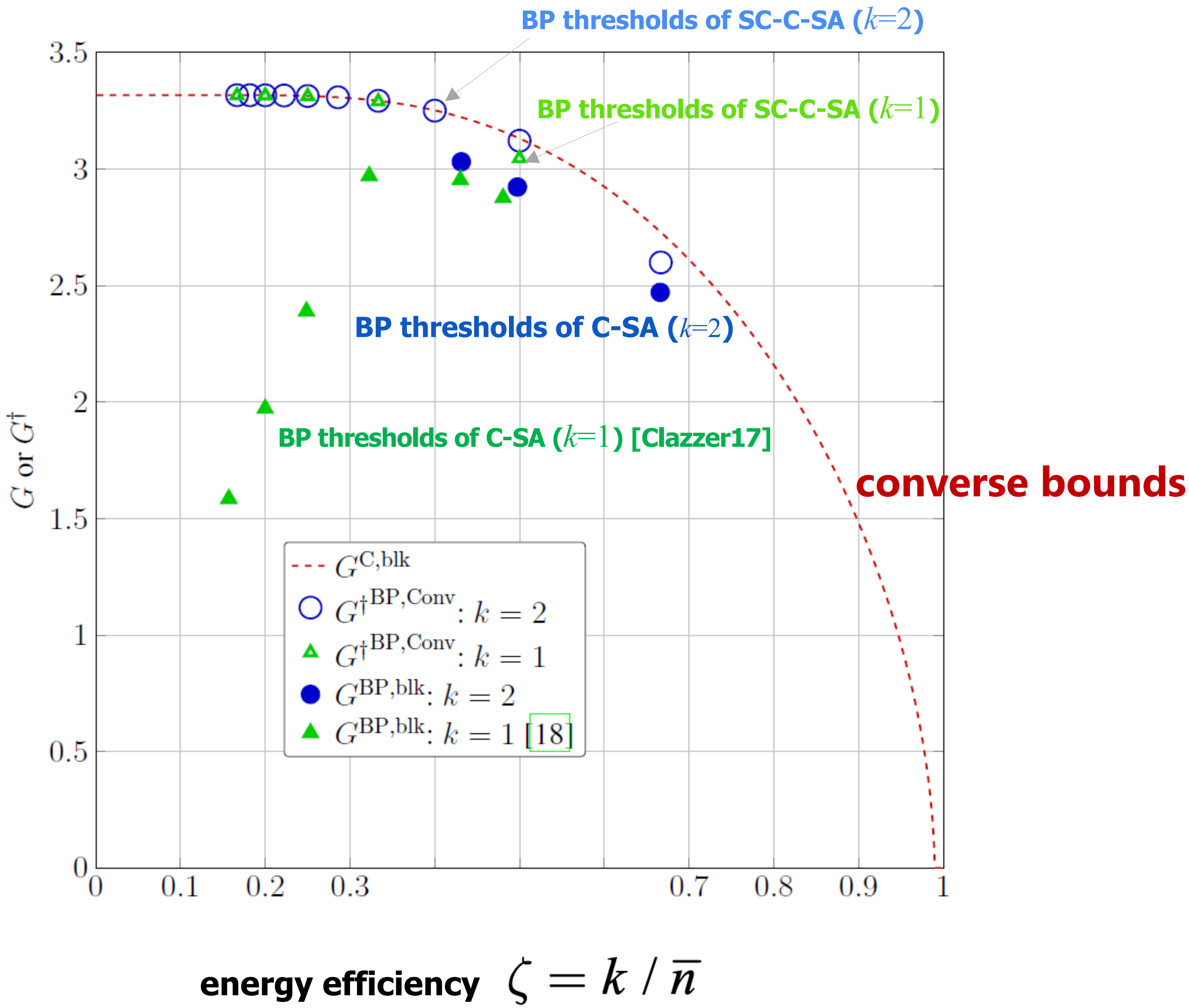
SC-C-SA：spatially-coupled slotted ALOHA

ζ	$G^{\dagger\,\mathrm{BP,Conv}}$			$G^{\mathrm{C,blk}}$
	$k=1$	$k=2$	$k=3$	
3/4	-	-	2.2668	2.4063
2/3	-	2.5992	-	2.7281
3/5	-	-	2.9098	2.9245
1/2	3.0463	3.1218	3.1295	3.1315
3/7	-	-	3.2251	3.2270
2/5	-	3.2511	-	3.2520
3/8	-	-	3.2706	3.2707
1/3	3.2906	3.2932	3.2933	3.2934
3/10	-	-	3.3048	3.3048
2/7	-	3.3082	-	3.3084
3/11	-	-	3.3108	3.3108
1/4	3.3129	3.3139	3.3139	3.3139
3/13	-	-	3.3155	3.3155
2/9	-	3.3160	-	3.3161
3/14	-	-	3.3164	3.3164
1/5	3.3159	3.3169	3.3169	3.3169
3/16	-	-	3.3172	3.3172
2/11	-	3.3172	-	3.3172
3/17	-	-	3.3173	3.3173
1/6	3.3163	3.3173	3.3174	3.3174

tight

BP thresholds of SC-C-SA are close to converse bounds

Converse Bounds & BP Thresholds (C-SA & SC-C-SA)



Summary

- **We derive BP threshold and converse bound for coded slotted ALOHA systems over Rayleigh channels**
  - ✓ **We formulate density evolution (DE) equations**
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- **Our numerical results show that**
  - **MDS coding provide more flexible values of energy efficiency and higher BP thresholds than repetition coding**
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Thank you for your attention.

Source code can be found here.

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