Spatio-Temporal SIC-Based Slotted ALOHA with Multi-Antenna Reception over Rayleigh Fading Channels

Yuhei Takahashi*, Daiki Fukui*, Guanghui Song†, Tomotaka Kimura*, Zilong Liu‡, and Jun Cheng*

* Department of Intelligent Information Engineering and Sciences, Doshisha University, Kyoto, 610-0321, Japan
(e-mail: {cyjk1101, ctwk0108}@mail4.doshisha.ac.jp tomkimur@mail.doshisha.ac.jp; jcheng@ieee.org)

† The State Key Laboratory of Integrated Services Networks, Xidian University, Xi'an 710071, China
(e-mail:songguanghui@xidian.edu.cn)

‡ School of Computer Science and Electronic Engineering, University of Essex, Colchester, CO4 3SQ, UK
(e-mail:zilong.liu@essex.ac.uk)

***Abstract*—We analyze the spatio-temporal successive interference cancellation (SIC)-based slotted ALOHA with multi-antenna reception and real-time feedback over Rayleigh fading channels. For the two-device SA case, we derive exact closed-form state-transition probabilities of the Markov process modeling the decoding procedure. The analysis enables exact computation of the sum rate and supports joint optimization of transmission probability and coding rate. Numerical results confirm that incorporating multiple antennas into the spatio-temporal SIC framework yields significant performance improvements.**

***Index Terms*—slotted ALOHA, successive interference cancellation, Rayleigh fading, multi-antenna**

I. Introduction

Random access schemes, which evolved from slotted ALOHA (SA), are regarded as promising solutions to support massive machine-type communication (mMTC), as these schemes allow devices to send their data without pre-establishing connections and pre-requesting channel resources. In conventional SA, collided packets at an access point (AP) are discarded (i.e., collision channel model). To allow devices to detect collisions and retransmit, the AP provides real-time feedback (ACK) at the end of each slot.

The collision channel model is often considered pessimistic, since fading causes power variations among signals in collision slots, allowing for the *capture effect* to occur, when sufficiently strong signals may be decoded. The AP can successfully decode some packets if their signal-to-interference-plus-noise ratio (SINR) exceeds a decoding threshold [1]. An *intra-slot SIC* (successive interference cancellation) enhances decodability by iteratively canceling already decoded packets [2].

The aforementioned schemes [1], [2] assume a single-antenna AP. In contrast, several studies have explored SA protocols with multi-antenna or multi-receiver settings by leveraging *spatial diversity*. The throughput of capture effect was first analyzed for a two-antenna AP under Rayleigh fading [3], and later extended to include shadowing [4]. On the other hand, SA with multiple receivers has been studied in [5]. In [6], [7], the throughput analysis was simplified by adopting the on–off fading channel model together with the inclusion–exclusion principle.

In parallel with spatial diversity, *temporal diversity* has been investigated to enhance throughput by randomly transmitting packets or their replicas into a subset of slots within a frame, a scheme known as irregular repetition slotted ALOHA (IR-SA) [8]–[10]. For IRSA with multi-antenna reception, the asymptotic throughput was given using a density evolution based analysis [11]. IR-SA, however, lacks real-time feedback, causing redundant transmissions and delayed decoding, which hurt efficiency and delay-sensitive use.

To address these issues while still exploiting temporal diversity, conventional SA systems introduced in [1], [2] can be modified by storing the collided packets or the residual signal containing undecoded packets to a buffer for further inter-slot SIC processing [12], [13]. In Rayleigh fading channels, iterative decoding for two-device SA combining intra- and inter-slot SIC has also been analyzed using a Markov process [13]. To the best of our knowledge, the potential of further enhancing such buffered systems through spatial diversity has not yet been explored.

This paper considers a K_{T}-device SA system where an L-antenna AP employs spatio-temporal SIC with buffering and ACKs, extending [12] [13]; this system is termed the spatio-temporal SIC-based SA system with multi-antenna reception. For the two-device case, we derive closed-form probabilities, formulate the sum rate via a Markov model, and optimize it.

II. System model

A. Random Uplink Transmission and Re-transmission

As depicted in Fig. 1, we consider the SA system in which K_{T} single-antenna devices communicate with a common AP equipped with L antennas. Let $\mathcal{K}$ and $\mathcal{L}$ denote the sets of active devices and antenna indices, respectively, where $|\mathcal{K}| = K_a,\ |\mathcal{L}| = L$. The antennas in AP are spaced sufficiently far apart to ensure independent fading values.

The system employs a shared code $\mathcal{C}$ at code rate R. This encoder could be a combination of a channel encoder and a

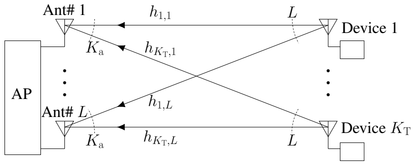


Fig. 1: A K_T-device SA system, where at slot t, K_a devices are active and send their packets to the AP equipped with L antenna elements. The AP process the signals received during the current slot and any residual signals from past slots to recover the transmitted packets.

high-order modulator. We assume that each device possesses an individual message stream. Each message is encoded into a codeword, known as a packet, using code $\mathcal{C}$. Following the SA protocol, the complete transmission period is segmented into time slots, whereby each slot's duration is designed to accommodate the transmission of a single packet. During time slot t, each device transmits or re-transmits its packet to AP with transmission probability p and at power P. Once a packet is transmitted successfully, the device refreshes its message stream in preparation to send a new packet. By assumption, the devices are synchronized in terms of symbols and slots.

In the receiver, AP iteratively decodes from the L received signals at slot t and residual signals from past slots, feeding real-time confirmation messages, i.e., ACKs, back to the devices at the end of each slot.

B. Spatio-Temporal SIC Decoding

Consider a generic slot, e.g., slot t, at some point during decoding. For the sake of concise notation, we omit index t in this subsection. Assuming K_a ($\leq K_\mathrm{T}$) devices are active and transmit their packets to the slot, the resultant L superimposed signals, one per antenna, within the slot are $\boldsymbol{y}_i = \sum_{k\in\mathcal{K}} h_{k,i}\boldsymbol{x}_k + \boldsymbol{z}_i, \quad i = 1, 2, \ldots, L$. Here, device k is active and has accessed the slot. The vector $\boldsymbol{x}_k \in \mathcal{C}$ represents its codeword (or packet) of code $\mathcal{C}$. Vector $\boldsymbol{z}_i \sim \mathcal{CN}(\mathbf{0}, \sigma^2 I)$ is an additive, circularly symmetric complex Gaussian (CSCG) noise with mean $\mathbf{0}$ and diagonal covariance matrix $\sigma^2 I$ with noise power σ^2 and identity matrix I.

The channel coefficient $h_{k,i}$ expresses the fading of the channel from the active device k to Ant# i of AP during the slot, and it is kept constant for one slot and then modified for the next slot. We consider Rayleigh block fading channels, i.e., the channel coefficient $h_{k,i} \sim \mathcal{CN}(0, \sigma_\mathrm{h}^2)$, $|h_{k,i}|$ is Rayleigh distributed, and the received signal-to-noise ratio (SNR) γ_{ki} is exponentially distributed with $p_\gamma(x) = 1/\bar{\gamma} \cdot e^{-\frac{x}{\bar{\gamma}}}, \quad x > 0$, where $\bar{\gamma}$ is the average received SNR of the packet.

The following spatio-temporal SIC decoding process, influenced by transmission and spatial diversities, involves a complex iterative procedure integrating both intra- and inter-slot, as well as intra- and inter-antenna SICs (see Algorithm 1).

1) Intra-Slot SIC: In the AP, decoding within a slot involves an iterative process alternating between intra-antenna and inter-antenna SICs. It is assumed that the AP has full awareness of the channel state information (CSI) and the number of packet collisions.

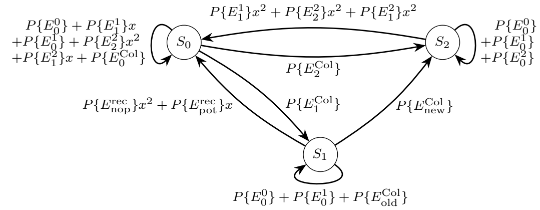


Fig. 2: State transition diagram of the Markov process for the two-device case [13].

a) Intra-Antenna SIC: At Ant# i, the SINR of the packet from device k, assuming $\gamma_{1i} > \gamma_{2i} > \cdots > \gamma_{Ki}$, can be calculated as

$$\eta_{k,i} \triangleq \frac{P|h_{k,i}|^2}{\sigma^2 + \sum_{u=k+1}^{K_\mathrm{a}} P|h_{u,i}|^2} = \frac{\gamma_{ki}}{1 + \sum_{u=k+1}^{K_\mathrm{a}} \gamma_{ui}}. \quad (1)$$

Assuming a threshold-based decoding, the packet $\boldsymbol{x}_k$ is successfully decoded if the SINR $\eta_{k,i}$ exceeds a certain threshold η_0, i.e., $\Pr\{\text{packet } \boldsymbol{x}_k \text{ decoded}\} = \begin{cases} 1, & \eta_{k,i} > \eta_0, \\ 0, & \eta_{k,i} \leq \eta_0, \end{cases}$ where the SNR threshold η_0 satisfies $R = \log_2(1+\eta_0)$ for code $\mathcal{C}$ of rate R. Upon successful decoding, the packet is removed from the received signal $\boldsymbol{y}_i$. Subsequent decoding proceeds with the interference-removed signal. This iterative process is termed *intra-antenna SIC*.

b) Inter-Antenna SIC: The previously described intra-antenna SIC process enables the successful decoding of packets. These decoded packets are then canceled from the signals of other antennas within the same slot. This procedure is referred to as *inter-antenna SIC*.

The above two processes are repeated across all antenna elements until no further decoding is possible for any packets on any antenna element. This iterative decoding procedure is referred to as *intra-slot SIC* due to its execution within a slot.

Due to random fading, the complete recovery of all K_a packets cannot be ensured within a slot. The slot t is *m-recovery* if the AP can successively recover m ($< K_\mathrm{a}$) packets from $\boldsymbol{y}_i$, $i = 1, 2, \ldots, L$, with the intra-slot SIC. After removing the recovered packets from $\boldsymbol{y}_i$, any residual signals, each per antenna, are stored into a residual buffer for further processing.

2) Inter-Slot SIC: Following an intra-slot SIC at slot t, if a recovered packet in the slot is identified as a re-transmission, the interference from its replicas will be canceled from the associated residual signal(s) at earlier slots t' ($< t$) within the residual buffer. This enables further intra-slot SICs on these signals.

The above process is repeated until no further packets in the buffer can be decoded. Upon successful recovery of packets, AP sends ACKs to the respective devices. The proposed intra- and inter-SIC process is summarized in Algorithm 1.

C. Markov Process of Two-Device SA with Multi-Antenna

This subsection examines the modeling of the spatio-temporal SIC decoding of the two-device SA with multi-antenna reception as a Markov process, specifically constraining the number of devices to $K_\mathrm{T} = 2$.

Before describing the states of the residual buffer, it is essential to define the *potentially-decodable* (PD) packet. In the context of residual signals, an unrecovered packet $\boldsymbol{x}_k$ qualifies as PD, if there exists at least an antenna, i.e., Ant# i, such that its SNR exceeds the decoding threshold η_0. In a two-collision scenario, each of the L residual signals can have zero, one, or two PD packets. In contrast, for a single-packet slot (one-collision), storing an un-decodable packet in the buffer is redundant, hence no PD packets are present. The buffer is designated with q ($q = 2, 1, 0$) PD packets when q is the highest number of PD packets found in the L residual signals.

The decoding process involving intra- and inter-slot SICs is modeled as a Markov process, whose state-transition diagram is shown in Fig. 2. The state set of the two-device residual signal buffer is defined by $\mathcal{S} = \{S_0, S_1, S_2\}$, where S_0 denotes no PD packets, and S_1 (S_2) denotes one (two) PD packet(s).

Let $\Pr\{s_t, d_t | s_{t-1}\}$ be a probability of a state transition from s_{t-1} to s_t and let d_t packets be recovered along with this state transition. A state-transition probability polynomial is defined as $p_{s_{t-1},s_t}(x) \triangleq \sum_{d=0}^{2} \Pr\{s_t, d_t = d | s_{t-1}\} x^d$ [12]. A state transition is triggered by decoding in slot t, with the possible decoding events summarized in Table I. Moreover, the state transition polynomial matrix is expressed in (2) [13].

Note that the state-transition diagram in Fig. 2 remains valid for the multiple-antenna case, where the proposed inter-antenna SIC is integrated into the intra-slot SIC process, preserving the state transitions. Although the diagram is unchanged, computing the corresponding transition probabilities remains nontrivial.

The first-order derivative $\mathrm{d}p_{s',s}(x)/\mathrm{d}x$ evaluated in $x = 1$ indicates the expected number of packets retrieved during the transition from state s' to state s. Consequently, matrix $P'(x = 1) = \mathrm{d}P(x)/\mathrm{d}x|_{x=1}$, which measures number of packets during transition, is then

$$\mathrm{P}'(1) = \begin{pmatrix} P\{E_1^1\} + P\{E_1^2\} + 2P\{E_2^2\} & 0 & 0 \\ 2P\{E_{\mathrm{nop}}^{\mathrm{rec}}\} + P\{E_{\mathrm{pot}}^{\mathrm{rec}}\} & 0 & 0 \\ 2P\{E_1^1\} + 2P\{E_2^2\} + 2P\{E_1^2\} & 0 & 0 \end{pmatrix}. \quad (3)$$

Let a stationary distribution of the Markov process be $\boldsymbol{w} = (w_0, w_1, w_2)^{\mathrm{T}}$, satisfying $\boldsymbol{w}^{\mathrm{T}} P(x = 1) = \boldsymbol{w}^{\mathrm{T}}$ and $\boldsymbol{w}^{\mathrm{T}}\mathbf{1} = 1$, where $\mathbf{1} = (1, 1, 1)^{\mathrm{T}}$. The normalized throughput of the two-device SIC-based SA with L-antenna, as $t \to \infty$, is given by $T(p, R, \bar{\gamma}, L) = \boldsymbol{w}^{\mathrm{T}} P'(1)\mathbf{1}$, which represents the average number of packets successfully decoded per slot with L antennas. The sum rate, defined as the expected amount of information recovered per received symbol, is given by

$$R_{\mathrm{s}} \triangleq R \cdot T(p, R, \bar{\gamma}, L) \text{ bits/channel use.} \quad (4)$$

Algorithm 1 Spatio-Temporal SIC Decoding

Initialize: Let t be the index of the current slot
if any packets at slot t are m-recovery with $m > 0$ **then**
  for Ant# $i = 1$ to L **do**
    Intra-antenna SIC in Ant# i.
    Inter-antenna SIC in slot t.
  end for
  Inter-slot SIC (if retransmissed packets are recovered).
  Store residual signals, each per antenna, into the buffer.
  while any buffer slot $t'(< t)$ is m'-recovery ($m' > 0$) **do**
    for Ant# $i = 1$ to L **do**
      Intra-antenna SIC in Ant# i.
      Inter-antenna SIC in slot t'.
    end for
    Inter-slot SIC (if retransmitted packets are recovered).
  end while
  ACK feedback for each recovered packet.
else
  Store received signals, each per antenna, into the buffer.
end if

TABLE I: Events Occurring in Current Slot (Two-Device SA) [13]

event	interpretation
E_m^n	n-collision with m-recovery, ($m \le n$, $n = 0, 1, 2$)
$E_{\mathrm{nop}}^{\mathrm{rec}}$	a packet is recovered, whose replica in the buffer is not PD
$E_{\mathrm{pot}}^{\mathrm{rec}}$	only a packet is recovered, whose replica in the buffer is PD
E_j^{Col}	j PD packets in 2-collision ($j = 0, 1, 2$)
$E_{\mathrm{new}}^{\mathrm{Col}}$	two-collision occurs with a new PD packet
$E_{\mathrm{old}}^{\mathrm{Col}}$	two-collision occurs without a new PD packet

TABLE II: Events occurring at a single antenna, where P1 (P2) is the packet sent by device 1 (2). (Dec: decodable, Und: undecodable, PD: potential decodable)

region	event	region	event
A, Ă	P1 and P2: Dec	D	P1: PD, P2: Und and not PD
B	P1: Dec, P2: Und and not PD	D̆	P1: Und and not PD, P2: PD
B̆	P1: Und and not PD, P2: Dec	E, Ĕ	P1 and P2: Und and not PD
C, Č	P1 and P2: PD		

III. Event Probabilities Over Fading Channels

In this section, we provide explicit expressions of the event probabilities in Table I for multi-antenna reception.

Before proceeding, Table II summarizes the probabilities of possible events at a single-antenna case. The received SNRs from devices 1 and 2 at Ant# i are denoted as γ_{1i} and γ_{2i}, respectively. The random variable (RVs) $\gamma_{1i}, \gamma_{2i}, \gamma_{1j}$, and γ_{2j} ($i, j \in \mathcal{L}, i \neq j$) are independent. The integral regions illustrated in Fig. 3 are introduced to clarify the computation. For example, the probability associated with the region A_i is denoted as P_{A_i}. Due to symmetry, we have $P_{\mathsf{R}_i} = P_{\breve{\mathsf{R}}_i} \triangleq P_{\mathsf{R}}$, $\mathsf{R} \in \{\mathsf{A}, \mathsf{B}, \mathsf{C}, \mathsf{D}, \mathsf{E}\}$. In Appendix A, we derive the closed-form expressions of these probabilities, which serve as the basis for calculations in the multi-antenna case.

In deriving the results for the multi-antenna case, we rely on the following fundamental facts:

(i) RVs γ_{ji} are i.i.d.
(ii) The inclusion-exclusion principle for computing the probability of the union of L sets $A_1, \ldots, A_L$ [14]:

$$\Pr(\cup_{\ell=1}^{L} A_\ell) = \sum_{\emptyset \neq \mathcal{S} \subseteq \{1,\ldots,L\}} (-1)^{|\mathcal{S}|+1} \Pr(\cap_{\ell \in \mathcal{S}} A_\ell). \quad (5)$$

$$P(x)=\begin{pmatrix} P\{E_0^0\}+P\{E_1^1\}x+P\{E_0^1\}+P\{E_2^2\}x^2+P\{E_1^2\}x+P\{E_0^{\mathrm{Col}}\} & P\{E_1^{\mathrm{Col}}\} & P\{E_2^{\mathrm{Col}}\} \\ P\{E_{\mathrm{nop}}^{\mathrm{rec}}\}x^2+P\{E_{\mathrm{pot}}^{\mathrm{rec}}\}x & P\{E_0^0\}+P\{E_0^1\}+P\{E_{\mathrm{old}}^{\mathrm{Col}}\} & P\{E_{\mathrm{new}}^{\mathrm{Col}}\} \\ P\{E_1^1\}x^2+P\{E_2^2\}x^2+P\{E_1^2\}x^2 & 0 & P\{E_0^0\}+P\{E_0^1\}+P\{E_0^2\} \end{pmatrix} \tag{2}$$

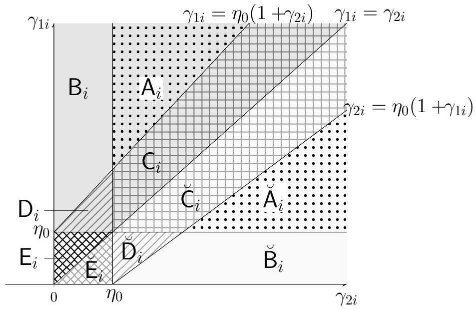


Fig. 3: The SIC feasible region in a two-device SA.

A. Closed-form Expression of $P\{E_m^n\}$

1) $P\{E_2^2\}$: It represents the probability that both packets are successfully decoded in a 2-collision. Due to the probability that both devices are active in the same slot is p^2, the probability for the event E_2^2 can be described as follows:

$$P\{E_2^2\} = p^2(\Pr\{\varDelta\}+\Pr\{\varXi\}). \tag{6}$$

Here, $\Pr\{\varDelta\}$ is the probability that two packets P1 and P2 are successfully decoded at any element in $\mathcal{L}$, wile $\Pr\{\varXi\}$ is the probability that they are decoded successfully, though not concurrently at an identical antenna element in $\mathcal{L}$. In the following, we deduce the two probabilities.

We first consider the case where both packets are decoded at a single antenna $i\in\mathcal{L}$. The probability of this event is given by $\Pr\{\varDelta_i\}=\Pr\{\mathsf{A}_i\}+\Pr\{\breve{\mathsf{A}}_i\}=2P_{\mathsf{A}}$. Consequently, the probability that P1 and P2 are successfully decoded at any one in $\mathcal{L}$ is $\Pr\{\varDelta\}=\Pr\{\bigcup_{i\in\mathcal{L}}\varDelta_i\}$. With the inclusion-exclusion principle (5), we obtain

$$\begin{aligned}\Pr\{\varDelta\}=\Pr\{\bigcup_{i\in\mathcal{L}}\varDelta_i\} &= \sum_{\emptyset\neq\mathcal{S}\subseteq\mathcal{L}}(-1)^{|\mathcal{S}|+1}\Pr\left(\bigcap_{i\in\mathcal{S}}\varDelta_i\right)\\ &=\sum_{\ell=1}^{L}\sum_{\substack{\mathcal{S}\subseteq\mathcal{L}\\|\mathcal{S}|=\ell}}(-1)^{\ell+1}\Pr\left(\bigcap_{i\in\mathcal{S}}\varDelta_i\right)\\ &=\sum_{\ell=1}^{L}(-1)^{\ell+1}\binom{L}{\ell}(2P_{\mathsf{A}})^{\ell}.\end{aligned}$$

Here, the expression is transformed by noting that, for a fixed $\ell>0$, there are $\binom{L}{\ell}$ subsets $\mathcal{S}\subseteq\mathcal{L}$ such that $|\mathcal{S}|=\ell$, and by using the assumption that the SNRs at the antennas are i.i.d.

Next, we derive the probability $\Pr\{\varXi\}$, that P1 and P2 are decoded successfully, though not concurrently at an identical antenna element in $\mathcal{L}$. To this end, consider the event, represented as $\varXi_{ijp}^{\mathrm{P1}\to\mathrm{P2}}$, in which P1 is only independently decodable at Ant# i, P2 is decodable or in a PD state at Ant# j, and neither is decodable at Ant# p. This implies that 1) P1 and P2 are decoded independently and successfully at Ants# i, and j, respectively, and 2) the initial decoding of P1 at Ant# i and its subsequent removal at Ant# j enable the exclusive decoding of P2. The event can be represented as, for $i\neq j\neq p$,

$$\varXi_{ijp}^{\mathrm{P1}\to\mathrm{P2}}=\mathsf{B}_i\cap\{\breve{\mathsf{B}}_j\cup\mathsf{C}_j\cup\breve{\mathsf{C}}_j\cup\breve{\mathsf{D}}_j\}\cap\overline{\{\mathsf{A}_p\cup\breve{\mathsf{A}}_p\}}. \tag{7}$$

Define $\mathcal{L}_1\subset\mathcal{L}$ as the subset of antenna elements where only P1 can be independently decodable. Define $\mathcal{L}_2\subseteq\mathcal{L}\setminus\mathcal{L}_1$ as the subset where P2 is decodable or in PD state within each antenna element. Further, let $\mathcal{L}_3=\mathcal{L}\setminus(\mathcal{L}_1\cup\mathcal{L}_2)$ as the subset comprising the remaining antenna elements that cannot decode either P1 or P2. Let $\ell_n=|\mathcal{L}_n|$, $n=1,2,3$. Note that $\mathcal{L}_1\cup\mathcal{L}_2\cup\mathcal{L}_3=\mathcal{L}$ and $\mathcal{L}_{n_1}\cap\mathcal{L}_{n_2}=\emptyset$ for $n_1\neq n_2$.

For any $i\in\mathcal{L}_1, j\in\mathcal{L}_2, p\in\mathcal{L}_3$, the intersection of sets (7) is written as

$$\begin{aligned}\bigcap_{\substack{i\in\mathcal{L}_1,j\in\mathcal{L}_2,\\p\in\mathcal{L}\setminus(\mathcal{L}_1\cup\mathcal{L}_2)}}\varXi_{ijp}^{\mathrm{P1}\to\mathrm{P2}}&=\bigcap_{i\in\mathcal{L}_1}\mathsf{B}_i\bigcap_{j\in\mathcal{L}_2}\{\breve{\mathsf{B}}_j\cup\mathsf{C}_j\cup\breve{\mathsf{C}}_j\cup\breve{\mathsf{D}}_j\}\bigcap_{p\in\mathcal{L}\setminus(\mathcal{L}_1\cup\mathcal{L}_2)}\overline{\{\mathsf{A}_p\cup\breve{\mathsf{A}}_p\}}\\ &=P_{\mathsf{B}}^{\ell_1}(P_{\mathsf{B}}+2P_{\mathsf{C}}+P_{\mathsf{D}})^{\ell_2}(1-2P_{\mathsf{A}})^{L-\ell_1-\ell_2}\\ &\triangleq\varXi_{\mathcal{L}_1\mathcal{L}_2}^{\mathrm{P1}\to\mathrm{P2}}.\end{aligned}$$

Using the inclusion-exclusion principle (5), we have

$$\begin{aligned}\Pr\{\varXi^{\mathrm{P1}\to\mathrm{P2}}\}&=\Pr\{\bigcup_{\substack{i,j,p\in\mathcal{L},\\i\neq j\neq p}}\varXi_{ijp}^{\mathrm{P1}\to\mathrm{P2}}\}\\ &=\sum_{\substack{\mathcal{L}_1\subset\mathcal{L}\\\mathcal{L}_1\neq\emptyset}}\sum_{\substack{\mathcal{L}_2\subseteq\mathcal{L}\setminus\mathcal{L}_1\\\mathcal{L}_2\neq\emptyset}}(-1)^{|\mathcal{L}_1|+|\mathcal{L}_2|+2}\Pr\{\bigcap_{\substack{i\in\mathcal{L}_1,j\in\mathcal{L}_2,\\p\in\mathcal{L}\setminus(\mathcal{L}_1\cup\mathcal{L}_2)}}\varXi_{ijp}^{\mathrm{P1}\to\mathrm{P2}}\}\\ &=\sum_{\ell_1=1}^{L-1}\sum_{\ell_2=1}^{L-\ell_1}(-1)^{\ell_1+\ell_2}\sum_{\substack{\mathcal{L}_1\subset\mathcal{L}\\|\mathcal{L}_1|=\ell_1}}\sum_{\substack{\mathcal{L}_2\subseteq\mathcal{L}\setminus\mathcal{L}_1\\|\mathcal{L}_2|=\ell_2}}\Pr\{\bigcap_{\substack{i\in\mathcal{L}_1,j\in\mathcal{L}_2,\\p\in\mathcal{L}\setminus(\mathcal{L}_1\cup\mathcal{L}_2)}}\varXi_{ijp}^{\mathrm{P1}\to\mathrm{P2}}\}\\ &=\sum_{\ell_1=1}^{L-1}\sum_{\ell_2=1}^{L-\ell_1}(-1)^{\ell_1+\ell_2}\binom{L}{\ell_1}\binom{L-\ell_1}{\ell_2}\Pr\{\varXi_{\mathcal{L}_1\mathcal{L}_2}^{\mathrm{P1}\to\mathrm{P2}}\}.\end{aligned}$$

The term $\binom{L}{\ell_1}\binom{L-\ell_1}{\ell_2}$ accounts for the number of distinct antenna pairs $(\mathcal{L}_1,\mathcal{L}_2)$ of sizes ℓ_1 and ℓ_2, respectively, such that $\mathcal{L}_1$ and $\mathcal{L}_2$ are disjoint sebsets of $\mathcal{L}$.

The symmetrical property of the SNRs for P1 and P2 ensures that $\Pr\{\varXi^{\mathrm{P1}\leftarrow\mathrm{P2}}\}=\Pr\{\varXi^{\mathrm{P1}\to\mathrm{P2}}\}$. The probability that P1 and P2 being independently and successfully decoded at each antenna element in their respective $\mathcal{L}_1$ and $\mathcal{L}_2$ is $\Pr\{\varXi_{\mathcal{L}_1\mathcal{L}_2}^{\mathrm{P1}\to\mathrm{P2}}\cap\varXi_{\mathcal{L}_1\mathcal{L}_2}^{\mathrm{P1}\leftarrow\mathrm{P2}}\}=P_{\mathsf{B}}^{\ell_1+\ell_2}(1-2P_{\mathsf{A}})^{L-\ell_1-\ell_2}$.

Finally, we have

$$\begin{aligned}\Pr\{\varXi\}&=\Pr\{\varXi^{\mathrm{P1}\to\mathrm{P2}}\}+\Pr\{\varXi^{\mathrm{P1}\leftarrow\mathrm{P2}}\}-\Pr\{\varXi^{\mathrm{P1}\leftarrow\mathrm{P2}}\cap\varXi^{\mathrm{P1}\to\mathrm{P2}}\}\\ &=\sum_{\ell_1=1}^{L-1}\sum_{\ell_2=1}^{L-\ell_1}(-1)^{\ell_1+\ell_2}\binom{L}{\ell_1}\binom{L-\ell_1}{\ell_2}\xi_{\varXi}(\ell_1,\ell_2)\end{aligned}$$

where,

$$\xi_{\varXi}(\ell_1,\ell_2)=(2P_{\mathsf{B}}^{\ell_1}(P_{\mathsf{B}}+2P_{\mathsf{C}}+P_{\mathsf{D}})^{\ell_2}-P_{\mathsf{B}}^{\ell_1+\ell_2})(1-2P_{\mathsf{A}})^{L-\ell_1-\ell_2}.$$

2) $P\{E_1^2\}$: It denotes the probability that either P1 or P2 can be successfully decoded, and can be formulated as

$$P\{E_1^2\} = p^2\left(\Pr\{\varUpsilon^{\mathrm{P1}}\} + \Pr\{\varUpsilon^{\mathrm{P2}}\}\right)$$
$$= 2p^2 \sum_{\ell=1}^{L}(-1)^{\ell+1}\binom{L}{\ell}(P_{\mathsf{B}})^{\ell}(P_{\mathsf{B}}+P_{\mathsf{D}}+2P_{\mathsf{E}})^{L-\ell}. \quad (8)$$

Here, $\Pr\{\varUpsilon^{\mathrm{P1}}\}$ ($\Pr\{\varUpsilon^{\mathrm{P2}}\}$) that P1 (P2) is successfully decoded at any one in $\mathcal{L}$, and we have $\Pr\{\varUpsilon^{\mathrm{P1}}\} = \Pr\{\varUpsilon^{\mathrm{P2}}\}$. The derivation is analogous to (6) and is omitted here for brevity.

3) $P\{E_0^2\}$: Since at Ant# i ($i \in \mathcal{L}$), neither packet can be decoded if the SNR pair $(\gamma_{1i}, \gamma_{2i})$ lies in the region $(\mathsf{C}_i \cup \breve{\mathsf{C}}_i \cup \mathsf{D}_i \cup \breve{\mathsf{D}}_i \cup \mathsf{E}_i \cup \breve{\mathsf{E}}_i)$, the probability that neither packet is successfully decoded at any of the antennas in $\mathcal{L}$ is

$$P\{E_0^2\} = p^2 \Pr\left(\bigcap_{i\in\mathcal{L}}(\mathsf{C}_i \cup \breve{\mathsf{C}}_i \cup \mathsf{D}_i \cup \breve{\mathsf{D}}_i \cup \mathsf{E}_i \cup \breve{\mathsf{E}}_i)\right) = p^2\{2(P_{\mathsf{C}}+P_{\mathsf{D}}+P_{\mathsf{E}})\}^L.$$

4) $P\{E_1^1\}, P\{E_0^1\}, P\{E_0^0\}$: Moreover, when at most one packet is received in the current slot, the SNRs at the antennas are assumed i.i.d. The detailed derivation is omitted due to space limitation; the final expression can be written as

$$P\{E_1^1\} = 2p(1-p)\left(1-(1-e^{-\frac{\eta_0}{\bar{\gamma}}})^L\right),$$
$$P\{E_0^1\} = 2p(1-p)\left(1-e^{-\frac{\eta_0}{\bar{\gamma}}}\right)^L, \; P\{E_0^0\} = (1-p)^2.$$

B. Closed-form Expression of $P\{E_j^{\mathrm{Col}}\}$

1) $P\{E_2^{\mathrm{Col}}\}$: It denotes the probability that both packets P1 and P2 are PD packets, can be formulated as

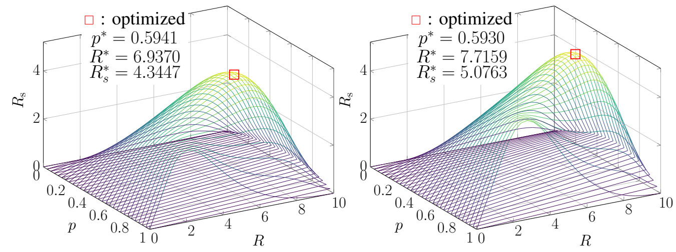
$$P\{E_2^{\mathrm{Col}}\} = p^2\left(\Pr\{\varTheta\} + \Pr\{\varOmega\}\right)$$
$$= p^2\Bigg(\sum_{\ell=1}^{L}(-1)^{\ell+1}\binom{L}{\ell}(2P_{\mathsf{B}})^{\ell}(2P_{\mathsf{C}}+2P_{\mathsf{D}}+2P_{\mathsf{E}})^{L-\ell}$$
$$+ \sum_{\ell_1=1}^{L-1}\sum_{\ell_2=1}^{L-\ell_1}(-1)^{\ell_1+\ell_2}\binom{L}{\ell_1}\binom{L-\ell_1}{\ell_2}$$
$$\cdot (P_{\mathsf{D}})^{\ell_1+\ell_2}(2P_{\mathsf{D}}+2P_{\mathsf{E}})^{L-\ell_1-\ell_2}\Bigg), \quad (9)$$

where $\Pr\{\varTheta\}$ denotes the probability that P1 and P2 are PD packets concurrently at the same antenna element, and $\Pr\{\varOmega\}$ denotes the probability that the packet from one device is a PD packet at one antenna while the packet from the other device is a PD packet at a different antenna. The detail derivations can be found in [15] and are omitted here due to space limitations.

2) $P\{E_1^{\mathrm{Col}}\}$: It is the probability that one of P1 or P2 is in the PD state, and the other is neither decoded nor in the PD state, and can be formulated as

$$P\{E_1^{\mathrm{Col}}\} = p^2\left(\Pr\{\varPhi^{\mathrm{P1}}\} + \Pr\{\varPhi^{\mathrm{P2}}\}\right)$$
$$= \sum_{\ell=1}^{L}(-1)^{\ell+1}\binom{L}{\ell}(P_{\mathsf{D}})^{\ell}(P_{\mathsf{D}}+2P_{\mathsf{E}})^{L-\ell}. \quad (10)$$

Here, $\Pr\{\varPhi^{\mathrm{P1}}\}$ ($\Pr\{\varPhi^{\mathrm{P2}}\}$) denotes the probability that only P1 (P2) is in the PD state while P2 (P1) remains undecodable at any one of the antennas in $\mathcal{L}$, and we have $\Pr\{\varPhi^{\mathrm{P1}}\} = \Pr\{\varPhi^{\mathrm{P2}}\}$. The derivation is omitted here for brevity.



(a) $L = 2$ **(b)** $L = 4$

Fig. 4: For $L = 2$ and 4 antennas, the sum rate R_s is evaluated with different values of p and R.

3) $P\{E_0^{\mathrm{Col}}\}$: The event E_0^{Col} refers to the case where both packets P1 and P2 are neither decodable nor in the PD state. Since P1 and P2 satisfy the condition at Ant# i only when the SNR pair $(\gamma_{1i}, \gamma_{2i})$ lies in the region $(\mathsf{E}_i \cup \breve{\mathsf{E}}_i)$, the probability that neither packet is decodable nor in the PD state at any of the antennas in $\mathcal{L}$ is calculated as

$$P\{E_0^{\mathrm{Col}}\} = p^2 \Pr\left(\bigcap_{i\in\mathcal{L}}(\mathsf{E}_i \cup \breve{\mathsf{E}}_i)\right) = p^2\prod_{i=1}^{L}\left(P_{\mathsf{E}_i}+P_{\breve{\mathsf{E}}_i}\right) = p^2(2P_{\mathsf{E}})^L.$$

C. $P\{E_{\mathrm{pot}}^{\mathrm{rec}}\}$, $P\{E_{\mathrm{nop}}^{\mathrm{rec}}\}$, $P\{E_{\mathrm{new}}^{\mathrm{Col}}\}$, and $P\{E_{\mathrm{old}}^{\mathrm{Col}}\}$

The other probabilities listed in Table I are detailed in [13].

$$P\{E_{\mathrm{pot}}^{\mathrm{rec}}\} = (P\{E_1^1\} + P\{E_1^2\})/2$$
$$P\{E_{\mathrm{nop}}^{\mathrm{rec}}\} = (P\{E_1^1\} + P\{E_1^2\})/2 + P\{E_2^2\}$$
$$P\{E_{\mathrm{new}}^{\mathrm{Col}}\} = P\{E_1^{\mathrm{Col}}\}/2 + P\{E_2^{\mathrm{Col}}\}$$
$$P\{E_{\mathrm{old}}^{\mathrm{Col}}\} = P\{E_1^{\mathrm{Col}}\}/2 + P\{E_0^{\mathrm{Col}}\}.$$

D. Optimization of Probability p and Coding Rate R

Before concluding this section, we aim to optimize transmission probability p and channel coding rate R. This optimization ensures that the sum rate of (4) is maximized for the given average received SNR $\bar{\gamma}$ and the number of antennas L.

$$\begin{aligned} \underset{0<p\le 1, R\ge 1}{\text{maximize}} \quad & R_s = R \cdot T(p, R, \bar{\gamma}, L). \\ \text{subject to} \quad & \bar{\gamma},\ L \text{ is constant.} \end{aligned} \quad (11)$$

Where a complete analysis for $R > 0$, omitted here due to space limitations, can be found in our journal version [15].

IV. Numerical and Simulation Results

We present numerical results with average received SNR per device $\bar{\gamma} = 25$ dB and number of devices $K_{\mathrm{T}} = 2$, unless otherwise specified.

In Fig. 4, the sum rate in (4) is illustrated as a function of R and p at $L = 2$, and 4. The optimal parameters R^* and p^* that maximize R_s in (11) are determined using the Sequential Quadratic Programming (SQP) algorithm [16]. Increasing L from 2 to 4 enhances R_s^* by about 16.84%.

Fig. 5 depicts R_s^*, R^*, and p^* for different $\bar{\gamma}$. R_s^* increases monotonically with $\bar{\gamma}$. A trade-off arises between R^* and p^*. In the low-SNR regime, a low coding rate is chosen to mitigate channel fading, and consequently, $p = 1$ becomes optimal. In contrast, in the high-SNR regime, the effect of

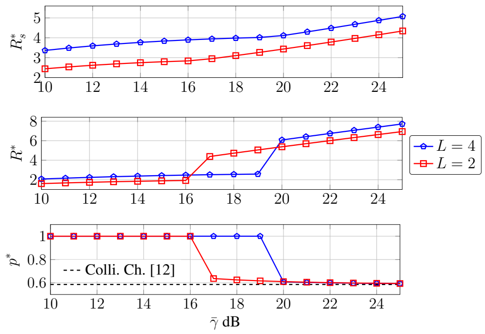


Fig. 5: The maximized sum rate R_s^* and optimized parameters (p^*, R^*) vs. received SNR $\bar{\gamma}$ $(L = 2, 4)$.

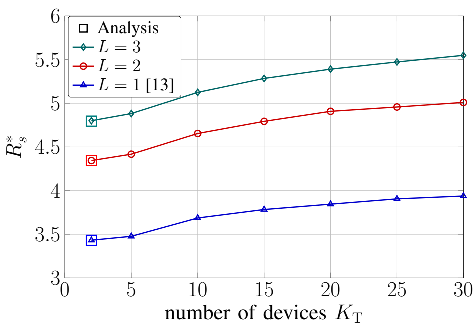


Fig. 6: R_s^* for different K_{T} and L. The simulation results are averaged over 10^2 independent experiments with $n_s = 10^3$.

channel fading is largely mitigated, enabling a higher coding rate. However, the increased coding rate raises the decoding threshold, making collisions more difficult to resolve; thus, collisions become the dominant limiting factor and reduce p^*. Accordingly, as $\bar{\gamma}$ grows large, p^* converges to 0.5858, the value for the collision channel [12], while R^* increases monotonically. Moreover, due to diversity gain, more antennas sustain higher transmission probabilities at higher SNRs.

Since the Markov model becomes intractable for large K_{T}, we obtain the average sum rate R_s^* over n_s slots by simulation, through searching for the optimal (p^*, R^*). Fig. 6 indicates that multiple antennas provide spatial diversity that improves the sum rate with larger device populations. When $K_{\mathrm{T}} = 2$, the simulations and numerical results show an excellent match, which validates the numerical analysis.

V. Conclusion

We studied the spatio-temporal SIC-based SA system with multi-antenna reception over Rayleigh fading, deriving a closed-form sum rate expression validated by simulation. Results confirm that multi-antenna design markedly improves performance. An extension to general fading channels remains a further challenge for future work.

Appendix A
Closed-From Probability Expressions

Fig. 3 illustrates the regions A–E for $\gamma_1 > \gamma_2$, together with their symmetric counterparts Ă–Ĕ for $\gamma_1 < \gamma_2$. For compact notation, we ignore antenna index i in this section, and due to symmetry, we focus on the range where $\gamma_1 > \gamma_2$.

We derive the probabilities P_{A} to P_{E} as follows.

$$P_{\mathsf{A}} = \frac{e^{-\frac{1}{\bar{\gamma}}((1+\eta_0)^2-1)}}{1+\eta_0}, \quad P_{\mathsf{B}} = \frac{e^{-\frac{\eta_0}{\bar{\gamma}}}}{1+\eta_0} - \frac{e^{-\frac{1}{\bar{\gamma}}((1+\eta_0)^2-1)}}{1+\eta_0},$$

$$P_{\mathsf{C}} = \frac{1}{2}e^{-\frac{2\eta_0}{\bar{\gamma}}} - \frac{e^{-\frac{1}{\bar{\gamma}}((1+\eta_0)^2-1)}}{1+\eta_0},$$

$$P_{\mathsf{D}} = \frac{\eta_0 e^{-\frac{\eta_0}{\bar{\gamma}}}}{1+\eta_0} - e^{-\frac{2\eta_0}{\bar{\gamma}}} + \frac{e^{-\frac{\eta_0(2+\eta_0)}{\bar{\gamma}}}}{1+\eta_0}, \quad P_{\mathsf{E}} = \frac{1}{2}\left(1 - e^{-\frac{\eta_0}{\bar{\gamma}}}\right)^2.$$

References

[1] G. D. Nguyen, A. Ephremides, and J. E. Wieselthier, "On capture in random-access systems," in *2006 IEEE International Symposium on Information Theory*. IEEE, 2006, pp. 2072–2076.

[2] A. Zanella and M. Zorzi, "Theoretical analysis of the capture probability in wireless systems with multiple packet reception capabilities," *IEEE Transactions on Communications*, vol. 60, no. 4, pp. 1058–1071, 2012.

[3] M. Zorzi, "Mobile radio slotted ALOHA with capture and diversity," in *Proceedings of INFOCOM'95*, vol. 1, 1995, pp. 121–128 vol.1.

[4] M. Zorzi, "Mobile radio slotted ALOHA with capture, diversity and retransmission control in the presence of shadowing," *Wireless Networks*, vol. 4, pp. 379–388, 1998.

[5] M. S. Corson and A. Ephremides, "An analysis of multi-receiver, non-adaptive, slotted ALOHA with capture for wireless communications in factories," in *IEEE INFOCOM'93 The Conference on Computer Communications, Proceedings*. IEEE, 1993, pp. 421–428.

[6] A. Munari, M. Heindlmaier, G. Liva, and M. Berioli, "The throughput of slotted ALOHA with diversity," in *2013 51st Annual Allerton Conference on Communication, Control, and Computing (Allerton)*. IEEE, 2013, pp. 698–706.

[7] A. Munari, F. Clazzer, G. Liva, and M. Heindlmaier, "Multiple-relay slotted ALOHA: Performance analysis and bounds," *IEEE Transactions on Communications*, vol. 69, no. 3, pp. 1578–1594, 2020.

[8] G. Liva, "Graph-based analysis and optimization of contention resolution diversity slotted ALOHA," *IEEE Transactions on Communications*, vol. 59, no. 2, pp. 477–487, 2011.

[9] F. Clazzer, E. Paolini, I. Mambelli, and Č. Stefanović, "Irregular repetition slotted ALOHA over the Rayleigh block fading channel with capture," in *IEEE International Conference on Communications (ICC)*, 2017, pp. 1–6.

[10] Y. Takahashi, G. Song, T. Kimura, and J. Cheng, "Irregular repetition slotted ALOHA over Rayleigh block fading channels: Bounds and threshold saturation via spatial coupling," *IEEE Access*, vol. 11, pp. 106 528–106 543, 2023.

[11] C. R. Srivatsa and C. R. Murthy, "On the impact of channel estimation on the design and analysis of IRSA based systems," *IEEE Transactions on Signal Processing*, vol. 70, pp. 4186–4200, 2022.

[12] G. Song, K. Cai, Y. Chi, and J. Cheng, "Throughput analysis of interference cancellation-based random access with feedback," *IEEE Communications Letters*, vol. 22, no. 12, pp. 2423–2426, 2018.

[13] Z. Zhang, Y. Li, G. Song, C. Yuen, and Y. L. Guan, "Random NOMA with cross-slot successive interference cancelation packet recovery," *IEEE Wireless Communications Letters*, vol. 9, no. 7, pp. 1065–1069, 2020.

[14] S. M. Ross, S. M. Ross, S. M. Ross, S. M. Ross, and E.-U. Mathématicien, *A first course in probability*. Prentice Hall Upper Saddle River, NJ, 1998, vol. 6.

[15] Y. Takahashi, D. Fukui, G. Song, T. Kimura, Z. Liu, and J. Cheng, "Spatio-Temporal SIC-Based Slotted ALOHA with Multi-Antenna Reception." [Online]. Available: https://github.com/yuhei-takahashi/Multi-Antenna_SA/SpatioTemporal_SIC_SA.pdf

[16] P. T. Boggs and J. W. Tolle, "Sequential quadratic programming," *Acta numerica*, vol. 4, pp. 1–51, 1995.