Exercise 4.1. Largest radix-r number

Proof: $\sum_{i=0}^{N-1} (r-1)\cdot r^i = r^N - 1$ since the largest number by N digits is just put N digits of the greatest r.

$\sum_{i=0}^{N-1} d_i \times r^i = \sum_{i=0}^{N-1} (r-1)\cdot r^i = r^N - 1.$

Case $N = 1$

$(r-1)\cdot r^0 = r-1$

$r^N - 1 = r^1 - 1 = r - 1$

Case $N = n+1$

$\sum_{i=0}^{N-1} (r-1)\cdot r^i = r^N - 1$

$(r-1)\cdot r^n + r^n - 1 = r^{n+1} - 1$

$(r-1+1)\, r^n - 1 = r^{n+1} - 1$

$r\cdot r^n - 1 = r^{n+1} - 1$

$r^{n+1} - 1 = r^{n+1} - 1$

Thus, $\sum_{i=0}^{N-1} (r-1) r^i = r^N - 1$ holds.

Exercise 4.4 2's complement Operation.

Proof: Suppose there is a number just like B with magnitude less than 2^{N-2}

$B = 4 = 0100$ $A = 8 = 1000$. $B - A = -B$. By using 2's complement.

$$\begin{array}{r} \cancel{0100} \\ \cancel{+\,0111} \\ \hline \cancel{1100} \end{array} \qquad \begin{array}{r} 0100 \\ +\,0111 \\ 1 \\ \hline 1100 \end{array} \qquad 1100 = -4$$

Use $-(b_{N-1} \cdots b_0)_2 = (\overline{b_{N-1} \cdots\cdots b_0})_2 + 1$

$$\begin{array}{r} 0011 \\ +\quad 1 \\ \hline 0100 \end{array}$$

$0100 = 4 = -(-4)$

$\cancel{-(1100) = (0011) + 1}$

$-(1100) = (0011) + 1 = 4.$

Thus, 2's complement operation holds.

4.5 Sign extension

Proof: if we use 2's complement to represent a number with N bits.
For instance, represent number 8 in 8 bits will be 00000100.
By using sign extension to extend to 16 bits, it will be 0000000000000100.
To represent −8 in 8 bits will be 11111000, extend to 16 bit by sign extension will be 1111 1111 1111 1000. Both positive and negative number for sign extension are static and correct.
Thus, sign extension is value preserving.

4.6 Shift Operation

Proof: In decimal, when a number is divide by 10 or multiply by 10. We just need to do left or right shift operation. Because a n bits decimal is equal to $a \cdot 10^{n} + b \cdot 10^{n-1} + c * 10^{n-2} + \cdots N * 10^{0}$.
When it's divide or multiply by 10. The number will be $a \cdot 10^{n+1/n-1} + b \cdot 10^{n/n-2} + c^{n-1/n-3} + \cdots N * 10^{0-1}$. It also works with binary since to represent a binary number, The number is $a \cdot 2^{n} + b \cdot 2^{n-1} + c \cdot 2^{n-2} \cdots + N \cdot 2^{0}$. 0001 = 1 0010 = 2 0100 = 4 1000 = 8.
By look at the example above to can see that the number is divide or multiply by 2 just by shift left or right.