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Statistics C183/C283 **Instructor: Nicolas Christou**

Dynamic Delta hedging

(From *Options Futures and Other Derivatives* by John Hull, Prentice Hall 6th Edition, 2006)

A financial institution has sold for \$300000 a European call option on 100000 shares of the underlying stock. Expiration time is 20 weeks (or $T = \frac{20}{52} = 0.3846$ years). Assume that $S_0 = \$49, E = \$50, r = 5\%, \sigma = 20\%, \mu = 13\%$. The hedge is to be adjusted weekly for the next 20 weeks (lifetime of the call option). The Black-Scholes-Merton price of this call option is about \$240000.

Initially (at the beginning of the option lifetime) Delta (Δ) is calculated using:

$$d_1 = \frac{ln(\frac{S_0}{E}) + (r + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}} = \frac{ln(\frac{49}{50}) + (0.05 + \frac{1}{2}0.20^2)0.3846}{0.20\sqrt{0.3846}} = 0.05417375.$$

Therefore, $\Delta = \Phi(d_1) = \Phi(0.05417375) = 0.522$. This means that the institution must purchase $100000 \times 0.522 = 52200$ shares. Since each share cost \$49, the institution must borrow $52200 \times 49 = \$2557800$ to buy them. In the meantime, during week 1 of the life of the option the interest cost will be $2557800 \times (exp(0.05\frac{1}{52}) - 1) \approx \$2500$. All these calculations are shown on the first row of the table below.

At the end of week 1, suppose that the price of the stock is \$48.12. The new Δ ratio will be:

$$d_1 = \frac{ln(\frac{S_0}{E}) + (r + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}} = \frac{ln(\frac{48.12}{50}) + (0.05 + \frac{1}{2}0.20^2)\frac{19}{52}}{0.20\sqrt{\frac{19}{52}}} = -0.1054492.$$

Therefore, $\Delta = \Phi(d_1) = \Phi(-0.1054492) = 0.458$. For a hedge position the institution must have $0.458 \times 100000 = 45800$ shares, but since they already have 52200 shares they must sell $52200 - 45800 = 6400$ shares of the underlying stock. From selling 6400 shares the institution will receive $6400 \times 48.12 \approx \$308000$. The total cost by the end of week 2 will be: $\$2557800 + \$2500 - \$308000 = \$2252300$.

Exercise:
Show the calculations for week 3 on the table (next page).

Example 1: The complete table of delta hedging (option closes in the money):

Week	Stock price	Delta	Shares purchased or (sold)	Cost of shares purchased or sold	Total cost including interest	Interest cost
0	49.00	0.522	52200	-2557800	2557800	2500
1	48.12	0.458	(6400)	+308000	2252300	2200
2	47.37	0.400	(5800)	+274700	1979800	1900
3	50.25	0.596	19600	-984900	2966600	2900
4	51.75	0.693	9700	-502000	3471500	3300
5	53.12	0.774	8100	-430300	3905100	3800
6	53.00	0.771	(300)	+15900	3893000	3700
7	51.87	0.706	(6500)	+337200	3559500	3400
8	51.38	0.674	(3200)	+164400	3398500	3300
9	53.00	0.787	11300	-598900	4000700	3800
10	49.88	0.550	(23700)	+1182200	2822300	2700
11	48.50	0.413	(13700)	+664400	2160600	2100
12	49.88	0.542	12900	-643500	2806200	2700
13	50.37	0.591	4900	-246800	3055700	2900
14	52.13	0.768	17700	-922700	3981300	3800
15	51.88	0.759	(900)	+46700	3938400	3800
16	52.87	0.865	10600	-560400	4502600	4300
17	54.87	0.978	11300	-620000	5126900	4900
18	54.62	0.990	1200	-65500	5197300	5000
19	55.87	1.000	1000	-55900	5258200	5100
20	57.25	1.000	0	0	5263300	

Example 2: Option closes out of the money:

Week	Stock price	Delta	Shares purchased or (sold)	Cost of shares purchased or sold	Total cost including interest	Interest cost
0	49.00	0.522	52200	-2557800	2557800	2500
1	49.75					
2	52.00					
3	50.00					
4	48.38					
5	48.25					
6	48.75					
7	49.63					
8	48.25					
9	48.25					
10	51.12					
11	51.50					
12	49.88					
13	49.88					
14	48.75					
15	47.50					
16	48.00					
17	46.25					
18	48.13					
19	46.63	0.007	(17600)	+820700	290000	300
20	48.12	0.000	(700)	+33700	256600	