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Statistics C183/C283 **Instructor: Nicolas Christou**

Binomial and Black-Scholes option pricing models - summary

Binomial option pricing formula:

The value C of a European call option at time $t = 0$ is:

$$C = S_0 \sum_{j=k}^{n} \binom{n}{j} p'^j (1-p')^{n-j} - \frac{E}{(1+r)^n} \sum_{j=k}^{n} \binom{n}{j} p^j (1-p)^{n-j}$$

$$u = e^{+\sigma\sqrt{\frac{t}{n}}}, \quad d = e^{-\sigma\sqrt{\frac{t}{n}}} = \frac{1}{u}$$

$$p = \frac{1+r-d}{u-d}, \quad (\text{or } p = \frac{e^{rt}-d}{u-d}), \quad p' = \frac{up}{1+r}.$$

S_0	Price of the stock at time $t = 0$
E	Exercise price at expiration
r	Risk-free interest rate per period
n	Number of periods
σ	Annual standard deviation of the returns of the stock
t	Time to expiration in years

Black-Scholes option pricing formula:

The value C of a European call option at time $t = 0$ is:

$$C = S_0 \Phi(d_1) - \frac{E}{e^{rt}} \Phi(d_2)$$

$$d_1 = \frac{ln(\frac{S_0}{E}) + (r + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{ln(\frac{S_0}{E}) + (r - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}} = d_1 - \sigma\sqrt{t}$$

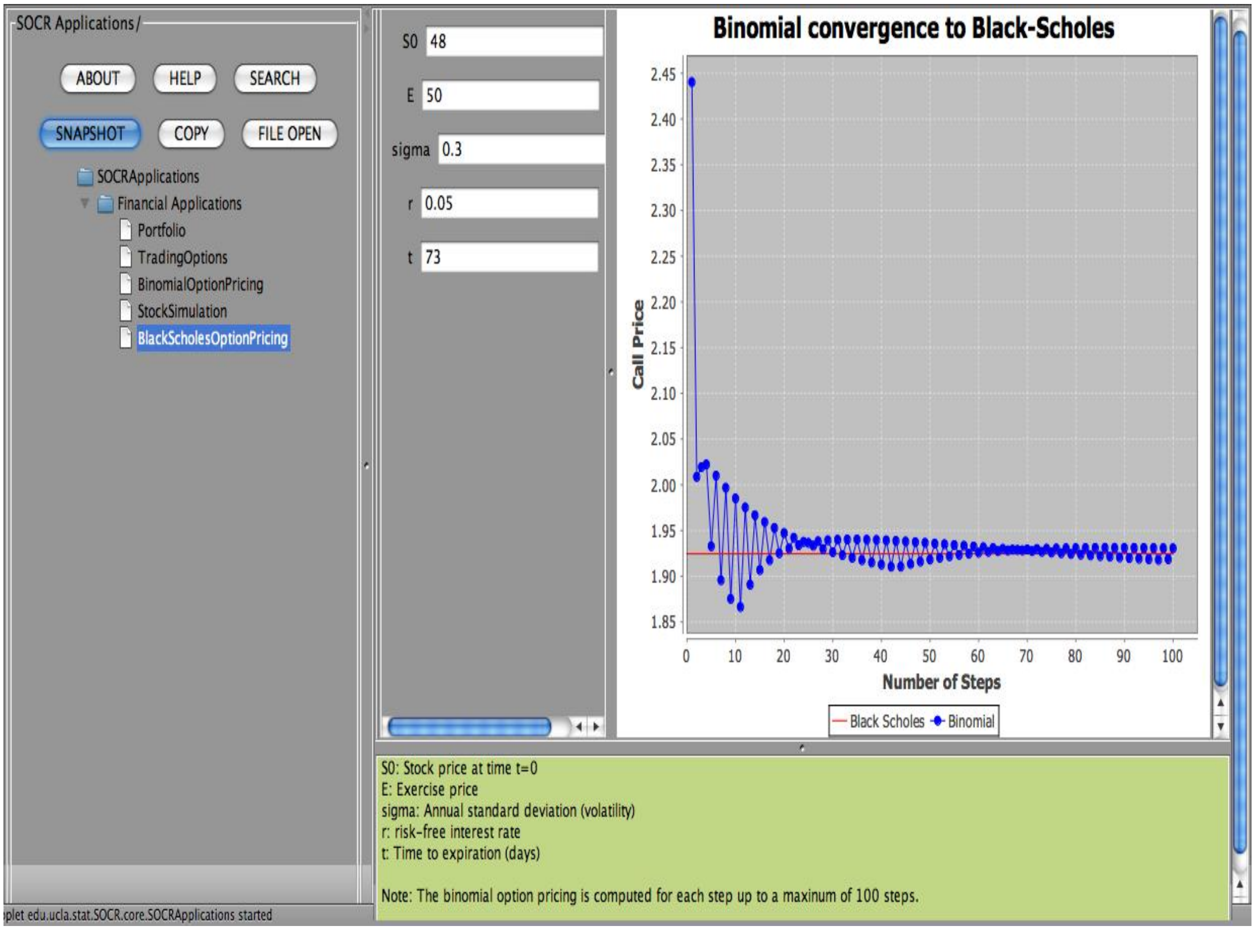
S_0	Price of the stock at time $t = 0$
E	Exercise price at expiration
r	Continuously compounded risk-free interest
σ	Annual standard deviation of the returns of the stock
t	Time to expiration in years
$\Phi(d_i)$	Cumulative probability at d_i of the standard normal distribution $N(0,1)$

Binomial convergence to Black-Scholes option pricing formula:

The binomial formula converges to the Black-Scholes formula when the number of periods n is large. In the example below we value the call option using the binomial formula for different values of n and also using the Black-Scholes formula. We then plot the value of the call (from binomial) against the number of periods n. The value of the call using Black-Scholes remains the same regardless of n. The data used for this example are:

$$S_0 = \$48, \ E = \$50, \ R_f = 0.05, \ \sigma = 0.30, \ \text{Days to expiration} = 73.$$

Using the Statistics Online Computational Resource (SOCR) at `http://www.socr.ucla.edu` we find the results on the next page.


SOCR Applications
ABOUT
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Financial Applications
Portfolio
TradingOptions
BinomialOptionPricing
StockSimulation
BlackScholesOptionPricing
S0 48
E 50
sigma 0.3
r 0.05
t 73
Binomial convergence to Black-Scholes
Call Price
Number of Steps
Black Scholes
Binomial
S0: Stock price at time t=0
E: Exercise price
sigma: Annual standard deviation (volatility)
r: risk-free interest rate
t: Time to expiration (days)
Note: The binomial option pricing is computed for each step up to a maxinum of 100 steps.
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