DIVIDENDS

ASSUME THAT THE DOLLAR AMOUNT OF THE DIVIDEND PAID AND THE TIMING DURING THE LIFETIME OF THE OPTION IS KNOWN.

EX-DIVIDEND DATE IS THE DATE ON WHICH THE DIVIDEND IS PAID. ON THIS DATE THE STOCK PRICE DECLINES BY THE AMOUNT OF THE DIVIDENT.

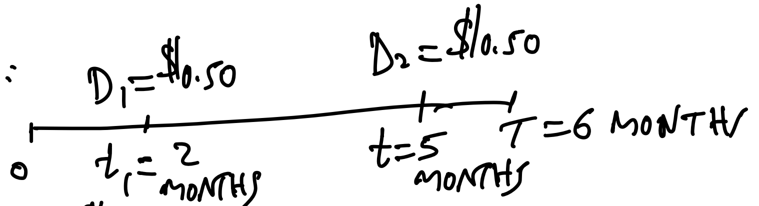
EUROPEAN OPTIONS :

DIVIDEND DISCOUNT MODEL

USE $S_0' = S_0 - \left\{ \begin{array}{l} \text{SUM OF PV OF ALL THE DIVIDENDS} \\ \text{PAID DURING THE LIFETIME} \\ \text{OF THE OPTION} \end{array} \right\}$

AND THEN USE B-S-M MODEL TO PRICE THE OPTION.

EXAMPLE :



$S_0 = \overset{\$}{40}$, $E = \overset{\$}{40}$, $r = 0.09$, $\sigma = 0.30$

PV OF DIVIDENDS $= 0.5\, e^{-0.09 \frac{2}{12}} + 0.5\, e^{-0.09 \frac{5}{12}} = 0.9742$

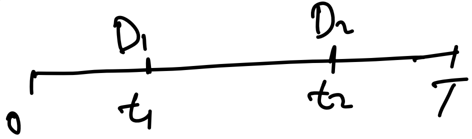
THEN $S_0' = 40 - 0.9742 = 39.0258$

$$d_1 = \frac{\log(39.0258/40) + \left(0.09 + \frac{1}{2} 0.3^2\right) \frac{6}{12}}{0.3 \sqrt{\frac{6}{12}}} = 0.2020 \rightarrow \Phi(d_1) = 0.5800$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.2020 - 0.3\sqrt{\tfrac{6}{12}} = -0.0101 \rightarrow \Phi(d_2) = 0.4960$$

$$C = 39.0258(0.5800) - 40\, e^{-0.09 \frac{6}{12}} (0.4960) = \$3.67$$

AMERICAN OPTIONS :



START BY CONSIDERING THE POSSIBILITY OF EARLY EXERCISE AT THE LAST DIVIDEND (HERE AT TIME t_2).

IF $D_2 > E\left(1-e^{-r(T-t_2)}\right)$ THE OPTION IS EXERCISED.

SIMILARLY, AT TIME t_1 THE OPTION IS EXERCISED IF $D_1 > E\left(1-e^{-r(t_2-t_1)}\right)$

NOTE: PAY ATTENTION TO THE EXPONENT t_2-t_1. THIS IS BECAUSE WHEN WE ARE TIME t_1 POTENTIALLY, THE OPTION MAY EXPIRED AT t_2.

EXAMPLE: USE THE PREVIOUS DATA

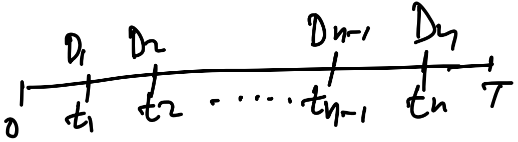
$40\left(1-e^{-0.09\frac{3}{12}}\right] = 0.89$ HERE, $\frac{3}{12}$ IS THE TIME FROM t_1 TO $t_2 = 3$ MONTHS.

SINCE, $D_1 = \$0.50$ IT IS NEVER OPTIMAL TO EXERCISE IMMEDIATELY BEFORE t_1

ALSO $40\left(1-e^{-0.09\frac{1}{12}}\right) = 0.30$ HERE, $\frac{1}{12}$ IS THE TIME FROM t_2 TO $T = 1$ MONTH.

SINCE $D_2 = 0.50 > 0.30$ THE OPTION WILL BE EXERCISED EARLY.

PRICE OF AMERICAN OPTION
USING BLACK'S APPROXIMATION :



USE THE DIVIDEND DISCOUNT MODEL TO COMPUTE

(a) C_1 USING $S^*_{0n} = S_0 - \sum_{i=1}^{n} PVD_i$

$$C_1 = S^*_{0n}\,\Phi(d_1) - E e^{-rT}\Phi(d_2)$$

(b) C_2 USING $S^*_{0,n-1} = S_0 - \sum_{i=1}^{n-1} PVD_i$

$$C_2 = S^*_{0,n-1}\,\Phi(d_1) - E e^{-rt_n}\Phi(d_2)$$

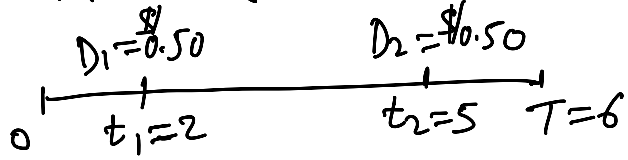
FINALLY, $C = MAX(C_1, C_2)$

THE IDEA IS THE FOLLOWING :

C_1 ASSUMES EXPIRATION DATE AT T

C_2 ASSUMES EXPIRATION DATE AT t_n

EXAMPLE (BLACK'S APPROXIMATION):



$S_0 = 40$, $E = 40$

$r = 0.09$, $\sigma = 0.30$

$C_1 = \$3.67$ (SAME AS THE PREVIOUS EXAMPLE, ASSUMING THAT THE EXPIRATION IS AT $T=6$ MONTHS)

NOW COMPUTE C_2.

$$PVD_1 = 0.5\, e^{-0.09 \frac{2}{12}} \approx 0.4926$$

THEN $S_0^* = 40 - 0.4926 = 39.5076$

$$d_1 = \frac{\log\left(\frac{39.5074}{40}\right) + \left(0.09 + \frac{1}{2} 0.3^2\right)\frac{5}{12}}{0.3\sqrt{\frac{5}{12}}} = 0.2265$$

$$\phi(d_1) = 0.5896$$

$$d_2 = d_1 - \sigma\sqrt{t_2} = 0.2265 - 0.3\sqrt{\frac{5}{12}} = 0.0329$$

$$\phi(d_2) = 0.5131$$

FINALLY,

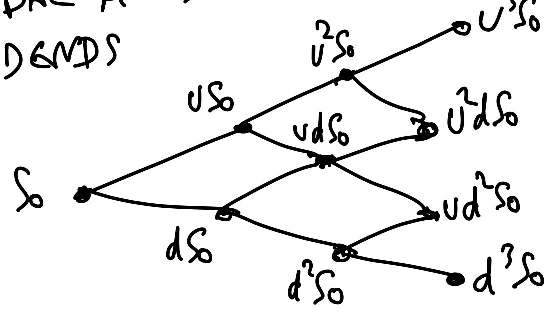
$$C_2 = 39.5074(0.5896) - 40\, e^{-0.09 \frac{5}{12}} (0.5131) = 3.52$$

THEREFORE,

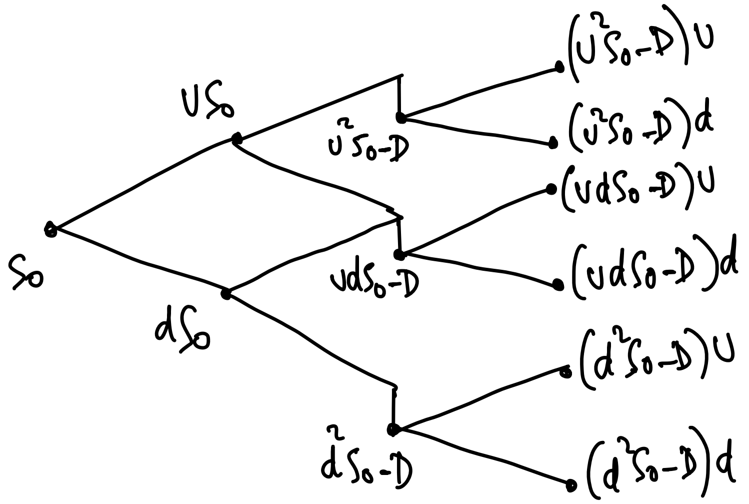
$$C = MAX(C_1, C_2) = \$3.67.$$

BINOMIAL MODEL WHEN THE UNDERLYING STOCK PAYS DIVIDENDS

CONSIDER A 3-STEP BINOMIAL TREE WITHOUT DIVIDENDS



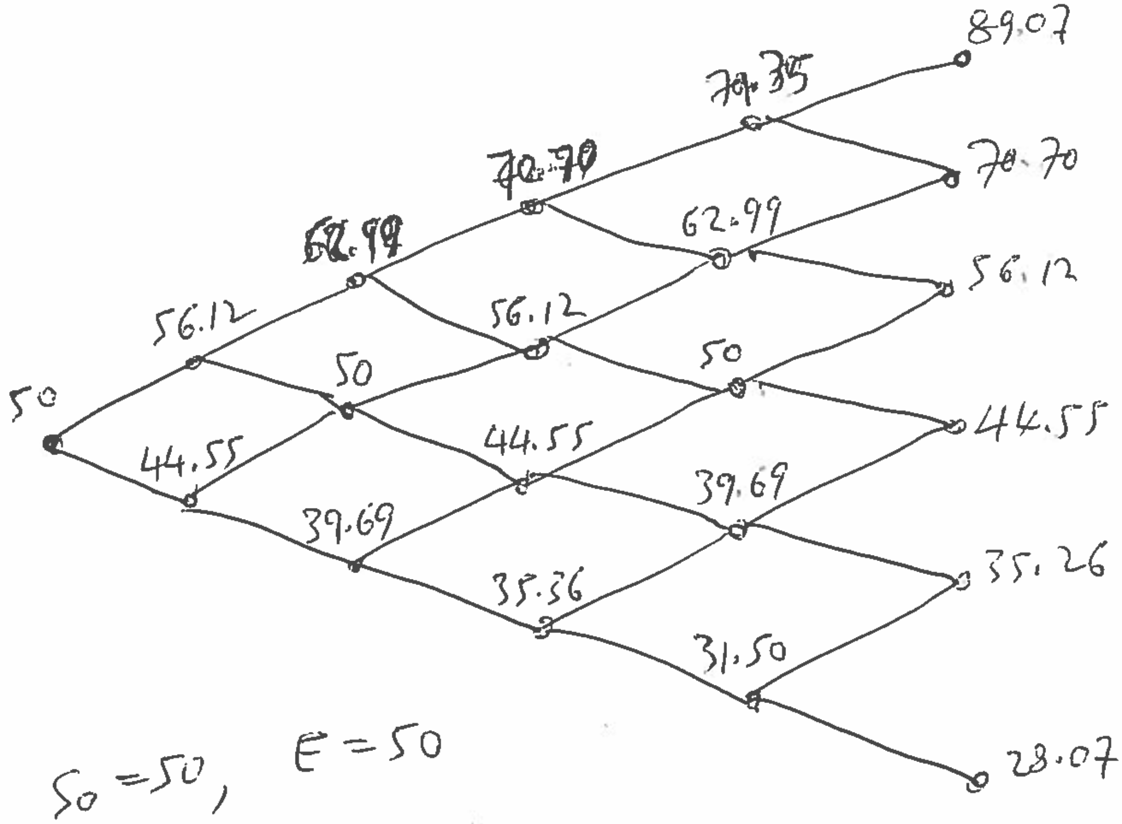
SUPPOSE THE STOCK PAYS A SINGLE DIVIDEND BETWEEN STEP 1 AND STEP 2. HOW DOES THIS CHANGE THE BINOMIAL TREE?



WE CAN SIMPLIFY THIS BINOMIAL TREE BY ADDING TO ALL THE NODES BEFORE THE EX-DIVIDEND DATE THE PRESENT VALUE OF THE DIVIDEND.

(SEE NUMERICAL EXAMPLE NEXT)

EUROPEAN PUT



$S_0 = 50$, $E = 50$

$r = 10\%$, $\sigma = 0.40$

$T = 5$ MONTHS $= \frac{5}{12} = 0.4167$, $n = 5$

$$u = e^{\sigma\sqrt{\frac{t}{n}}} = 1.1224$$

$$d = e^{-\sigma\sqrt{\frac{t}{n}}} = 0.8909$$

$$p = \frac{e^{r\Delta t} - d}{u - d} = 0.5073$$

$$1 - p = 0.4927$$

FROM OPTIONS FUTURES
AND OTHER DERIVATIVES
BY JOHN HULL,
PRENTICE HALL, 6TH EDITION, 2006

USING THE BINOMIAL OPTION PRICING MODEL
WE GET $P = \$4.49$

SUPPOSE NOW $S_0 = 52$, SAME DATA AS BEFORE, AND THE STOCK WILL PAY A SINGLE DIVIDEND OF \$2.06 IN 3.5 MONTHS.

WE FIRST COMPUTE THE PRESENT VALUE OF THE DIVIDEND $2.06\, e^{-0.10 \frac{3.5}{12}} = 2.00$

USE NOW $S_0' = 52 - 2.00 = 50$ AND CONSTRUCT THE SAME BINOMIAL TREE AS BEFORE.

THEN WE ADD THE PRESENT VALUE OF THE DIVIDEND TO ALL THE NODES BEFORE THE EX-DIVIDEND DATE

THEREFORE, $S_0 = 50 + 2 = 52$

NODE 1: $56.12 + 2.06\, e^{-0.10 \frac{2.5}{12}} = 58.14$

NODE 2: $44.55 + 2.06\, e^{-0.10 \frac{2.5}{12}} = 46.56$

NODE 3: $62.99 + 2.06\, e^{-0.10 \frac{1.5}{12}} = 65.02$

$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$

NODE 9: $35.36 + 2.06\, e^{-0.10 \frac{0.5}{12}} = 37.41$

THE OTHER NODES AFTER 3.5 MONTHS WILL BE THE SAME AS THE PREVIOUS FIGURE.