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THE ADJUSTMENT OF BETA FORECASTS

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THE BETA COEFFICIENT of the market model has gained wide acceptance as a relevant measure of risk in portfolio and security analysis. An essential prerequisite for using beta to assess future portfolio risk and return is a reasonable degree of predictability over future time periods. If the portfolio manager cannot predict future beta coefficients, the applicability of this phase of modern capital-market theory is somewhat restricted.

Attempts to predict betas using extrapolative models have met with only limited success, especially for individual securities. Blume [1] and Levy [2] found that single security beta coefficients of one period were not good predictors of the corresponding betas in the subsequent period. However, as portfolio size was increased, the stationarity of extrapolated betas improved significantly. A major problem for both single security and portfolio betas was the tendency for relatively high and low beta coefficients to overpredict and underpredict, respectively, the corresponding betas for the subsequent time period. Thus, forecasting accuracy grew progressively worse as beta levels departed significantly from the average.

The objectives of this note are to investigate the source of forecast errors of extrapolated beta coefficients and three adaptive procedures recommended by others for improving beta forecasts.

I. SOURCES OF PREDICTION ERRORS

The tests of beta forecast accuracy which follow make use of the mean square error as a measure of forecast error. Mean square forecast error (hereafter MSE) is defined as follows:

m: # of stocks

PRESS →

$$\mathrm{MSE} = \frac{1}{m}\sum_{j=1}^{m}(A_j - P_j)^2 \tag{1}$$

where m is the number of predictions contained in the forecast, P_j is the prediction of the beta coefficient of security j, and A_j is the estimated beta coefficient of security j. In terms of the beta forecast, P_j represents the computed beta for the current period used as the predictor of beta for the subsequent period and A_j is the corresponding estimated beta for the subsequent period.

MSE was chosen over alternative measures of forecast error because of its statistical tractability and because it can be easily partitioned into

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TO SHOW THIS DECOMPOSITION BEGIN WITH $\frac{1}{m}\sum[A_j - P_j \pm \bar{A} \pm \bar{P}]^2$

USE $S_P^2 = \frac{\sum(P_j-\bar{P})^2}{m}$ AND $S_A^2 = \frac{\sum(A_j-\bar{A})^2}{m}$

three components of forecast error as follows:

$$\mathrm{MSE} = (\bar{A} - \bar{P})^2 + (1 - \beta_1)^2 S_p^2 + (1 - r_{AP}^2) S_A^2 \quad (2)$$

where $\bar{A}$ and $\bar{P}$ are the means of the realizations and predictions, respectively; β_1 is the slope coefficient of the regression of A on P; S_P^2 and S_A^2 are the sample variances in P and A, respectively; and r_{AP}^2 is the coefficient of determination for P and A.[1] The first term in equation (2) represents bias, the second term inefficiency, and the final term is the random disturbance component of MSE.

THREE COMPONENTS OF FORECAST ERROR

Bias in a forecast indicates that the average prediction was either over or under the average realization. Inefficiency in the forecast represents a tendency for the prediction errors to be positive at low values of P_j and negative at high values of P_j as measured in equation (1). Note that Blume [1] and Levy's [2] observation that beta extrapolations have a tendency to regress toward the mean was evidence of inefficiency in the forecasts. Finally, the remaining component of MSE is the random disturbance element which contains those forecast errors not related to the value of the predictor, P_j, or the predicted, A_j.

The tests of beta forecast accuracy involved beta coefficients computed using the familiar market model:

$$\tilde{R}_{jt} = \alpha_j + \beta_j(\tilde{R}_{mt}) + \tilde{\epsilon}_{jt} \quad (3)$$

SINGLE INDEX MODEL

where $\tilde{R}_{jt}$ is the return on security or portfolio j in month t, $\tilde{R}_{mt}$ is the temporally corresponding market return, α_j is a parameter whose value is such that $E[\epsilon_{jt}] = 0$, β_j is defined as $\mathrm{Cov}(\tilde{R}_j, \tilde{R}_j)/\mathrm{Var}(\tilde{R}_m)$, ϵ_{jt} is a random error term, and the tildes denote random variables.

Beta coefficients were computed using monthly returns obtained from the CRSP Investment Return File and Fisher's Investment Performance Index over all successive, nonoverlapping five-year periods from July 1947 through June 1972.

Beta coefficients computed in one five-year period were used to predict beta for the subsequent five-year period. Table 1 contains the MSEs of the beta forecasts for portfolios of one to ten securities. Portfolio beta coefficients were computed by first ranking in descending order the individual security beta coefficients for each period. Next, the ranked securities were selected sequentially for portfolios containing n = 3, 5, 7, and 10 securities. The total number of securities which had complete data for two consecutive five-year periods varied from 785 to 843.

Table 1 points out the considerable variation in total MSE as well as the individual components of MSE for the different forecasts. Partitioning the MSE into its bias, inefficiency, and random error components indicated that the largest component for individual securities consisted of random errors. This was especially true in periods one and three when the random error term comprised 86 and 94 per cent, respectively, of the total MSE. However, periods two and four contained substantial inefficiency components, 40 and 25 per cent of MSE, respectively. Since the

1. See Mincer and Zarnowitz [3, pp. 3-46] for a detailed explanation of equation (2).

adjustment procedures to be tested are based on the existence of inefficiency in the forecasts, they should be of more benefit in these periods. The bias component was negligible, less than one per cent of total MSE, in all four periods. Thus, the beta forecasts were found to be unbiased, albeit inefficient.

Increasing portfolio size systematically reduced total MSE, although the percentage reduction varied among the forecasts. Table 1 shows that

TABLE 1
FORECAST ERRORS FOR PORTFOLIOS OF ONE TO TEN SECURITIES

	Portfolio Size (Number of Securities)				
	1	3	5	7	10
Period 1					
(7/47-6/52 vs. 7/52-6/57)					
Mean Square Error (MSE)	.17122	.07484	.05916	.04386	.04182
Portions of MSE due to:					
Bias	.00021	.00014	.00021	.00015	.00021
Inefficiency	.02422	.02178	.02363	.02192	.02320
Random error	.14678	.05291	.03531	.02179	.01839
Period 2					
(7/52-6/57 vs. 7/57-6/62)					
Mean Square Error (MSE)	.18387	.11008	.09196	.08754	.08544
Portions of MSE due to:					
Bias	.00084	.00084	.00095	.00095	.00095
Inefficiency	.07367	.07357	.07343	.07306	.07370
Random errors	.10935	.03566	.01757	.01352	.01078
Period 3					
(7/57-6/62 vs. 7/62-6/67)					
Mean Square Error (MSE)	.12385	.05018	.03574	.02659	.02332
Portions of MSE due to:					
Bias	.00018	.00019	.00018	.00019	.00018
inefficiency	.00730	.00736	.00736	.00730	.00725
Random errors	.11636	.04262	.02818	.01909	.01587
Period 4					
(7/62-6/67 vs. 7/67-6/72)					
Mean Square Error (MSE)	.16122	.08363	.06880	.05982	.05465
Portions of MSE due to:					
Bias	.00093	.00100	.00093	.00097	.00119
Inefficiency	.03992	.03947	.03993	.03975	.03800
Random errors	.12036	.04314	.02792	.01908	.01545

this reduction was due primarily to the random error component as the bias and inefficiency components were virtually unchanged.

II. ADJUSTMENT PROCEDURES

Attempts have been made to correct for inefficiency in beta forecasts by adjusting computed beta coefficients. Blume [1] used a cross sectional regression of security betas computed for two adjacent periods as the basis for adjusting his predictions of beta for the subsequent, nonoverlapping period. The adjusting equation was a simple linear regression of beta for security j in period 2, β_{j2}, on the corresponding coefficient for period 1, β_{j1}:

BLUME METHOD

$$\tilde{\beta}_{j2} = \partial_0 + \partial_1 \tilde{\beta}_{j1} + \epsilon_j \qquad \text{for } j = 1, 2, \ldots, m \tag{4}$$

where m is the number of securities in the cross sectional sample, ∂_0 and ∂_i are least squares regression coefficients and $\tilde{\epsilon}$ is a random disturbance term. Using this adjustment procedure, the adjusted β_{j2} (i.e., $\hat{\beta}'_{j2} = \partial_0 + \partial_i \beta_{j2}$) is used to predict beta for the subsequent nonoverlapping period, β_{j3}.

Merrill Lynch, Pierce, Fenner, & Smith Inc. (MLPFS) makes use of an adjustment procedure which, like Blume's predictor, is based on a cross sectional regression of historical betas for consecutive nonoverlapping time periods. Beta estimates are adjusted toward a mean of one using the following equation:

$$\hat{\beta}''_{j1} = 1.0 + k(\beta_{j1} - 1.0) \tag{5}$$

where β_{j1} is the estimated beta coefficient of security j in period 1, k is a constant common to all stocks, and $\hat{\beta}''_{j1}$ is the adjusted beta used to predict β_{j2}.

Vasicek [4] has suggested a Bayesian approach to the adjustment of security and portfolio betas. Information obtained from the cross sectional distribution of beta coefficients is used to adjust sample betas in keeping with a minimum expected loss criterion. This procedure makes use of the prior or historical distribution of beta coefficients. Specifically, the adjusted beta, $\hat{\beta}'''_{j1}$, is found as follows:

VASICEK METHOD

$$\hat{\beta}'''_{j1} = \frac{\bar{\beta}_1/S_{\bar{\beta}1}{}^2 + \beta_{j1}/S_{\beta 1}{}^2}{1/S_{\bar{\beta}1}{}^2 + 1/S_{\beta 1}{}^2} \tag{6}$$

where $\hat{\beta}'''_{j1}$ is the mean of the posterior distribution of beta for security j, $\bar{\beta}_1$ is the mean of the cross sectional distribution of security betas for period 1, $S_{\bar{\beta}1}{}^2$ is the variance of cross sectional betas in period 1, β_{j1} is the estimated beta coefficient for security j in period 1, and $S_{\beta 1}{}^2$ is the variance in the estimate of β_{j1}. A comparison of the Bayesian and MLPFS predictors reveals that the latter assumes $S_{\beta 1}{}^2$ to be the same for all securities [4, p. 1237].

Table 2 contains the MSEs for beta forecasts made using unadjusted and adjusted beta estimates for both individual securities and portfolios of ten securities. Since the Blume and MLPFS adjustment procedures require two full periods of information to make a forecast, the comparisons correspond to the last three periods of Table 1.

The unadjusted MSEs are the results of the extrapolations reported in Table 1. All three adjustment techniques consistently improved upon the unadjusted forecasts as denoted by the reduction in MSE and the inefficiency component. In period two, the MLPFS adjustment technique was most successful, followed closely by Blume's technique and lastly by the

TABLE 2
FORECAST ERRORS OF ADJUSTED VERSUS UNADJUSTED BETA COEFFICIENTS

	Individual Securities				Portfolios (Size Ten)			
	Unadjusted	Bayesian	Blume's	MLPFS	Unadjusted	Bayesian	Blume's	MLPFS
Period 2								
Mean Square Error (MSE)	.18387	.13111	.11123	.11015	.08544	.03460	.01259	.01153
Portion of MSE due to:								
Bias	.00084	.00004	.00183	.00075	.00095	.00006	.00178	.00072
Inefficiency	.07367	.02372	.00004	.00004	.07370	.02355	.00002	.00002
Random error	.10935	.10735	.10936	.10936	.01078	.01100	.01078	.01078
Period 3								
Mean Square Error (MSE)	.12385	.11609	.12207	.12293	.02332	.01356	.02155	.02238
Portion of MSE due to:								
Bias	.00018	.00011	.00000	.00087	.00018	.00011	.00000	.00083
Inefficiency	.00730	.00043	.00571	.00571	.00725	.00047	.00567	.00567
Random error	.11636	.11555	.11636	.11636	.01587	.01298	.01587	.01587
Period 4								
Mean Square Error (MSE)	.16122	.13082	.14660	.14934	.05465	.02018	.04215	.04485
Portion of MSE due to:								
Bias	.00093	.00000	.00263	.00537	.00119	.00000	.00252	.00522
Inefficiency	.03992	.00981	.02361	.02361	.03800	.00980	.02418	.02418
Random error	.12036	.12101	.12036	.12036	.01545	.01037	.01545	.01546

Bayesian adjustment. However, in periods three and four, the Bayesian adjustment achieved the greatest reduction in total MSE.

As previously illustrated in Table 1, the aggregation of securities into a portfolio reduced total MSE as a result of the reduction of the random error component as the inefficiency component was unchanged in moving from single to ten security portfolios. This can be explained by the fact that the adjustments were made on single security betas before aggregating them into portfolios. It should be noted that an attempt was made to adjust portfolio betas directly, as recommended by Vasicek [4]. However, the resulting MSEs were essentially the same as those reported in Table 2.

In conclusion, the accuracy of the simple no-change extrapolative beta forecast can be improved. A combination of the Bayesian predictor and a reasonable portfolio size would appear to make the beta coefficient a highly predictable risk surrogate.

REFERENCES

1. Marshall E. Blume. "On the Assessment of Risk," *Journal of Finance,* 26 (March 1971), 1-10.
2. Robert A. Levy. "On the Short-Term Stationarity of Beta Coefficients," *Financial Analysts Journal,* 27 (November-December 1971), 55-62.
3. Jacob Mincer and Victor Zarnowitz. "The Evaluation of Economic Forecasts," in Jacob Mincer, ed., *Economic Forecasts and Expectations,* National Bureau of Economic Research, 1969.
4. Oldrich A. Vasicek. "A Note on Using Cross-Sectional Information in Bayesian Estimation of Security Betas," *Journal of Finance,* 28 (December 1973), 1233-1239.