Name: ________________________________ Tutorial group: ____

Matriculation number: | | | | | | | | | |

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I 2017/18

MH1100 & SM2MH1100 – Calculus I

13 October 2017 Midterm Test 90 minutes

INSTRUCTIONS

1. Do not turn over the pages until you are told to do so.
2. Write down your name, tutorial group, and matriculation number.
3. This test paper contains **FOUR (4)** questions and comprises **FIVE (5)** printed pages.
4. Answer **all** questions. The marks for each question are indicated at the beginning of each question.

For graders only	Question	1	2	3	4	Total
	Marks					

QUESTION 1. **(7 marks)**

Find the limits if exist.

(a) $\displaystyle\lim_{x\to 2}\frac{(2x+4)(x+2)}{x^2+5x+6}$ (b) $\displaystyle\lim_{x\to 0}\frac{\frac{1}{x-1}+\frac{1}{x+1}}{2x}$ (c) $\displaystyle\lim_{x\to 4}\cos\left(\frac{x-4}{\sqrt{x}-2}\pi\right)$

(d) If $\displaystyle\lim_{x\to 1}\frac{f(x)-5}{x-2}=1$, find $\displaystyle\lim_{x\to 1^+}f(x)$.

QUESTION 2. **(3 marks)**

(a) Let a and L be real numbers. State the ϵ-δ definition of the equation $\lim_{x\to a} f(x) = L$.

(b) Prove that

$$\lim_{x\to 0}\left(-\frac{1}{x^2}\right) = -\infty.$$

QUESTION 3. **(5 marks)**

Let L be a real number. The function $f(x)$ is defined on the real line as

$$f(x) = \begin{cases} x^2 \cos \frac{1}{x}, & x < 0; \\ L, & x = 0; \\ \sqrt{x}, & x > 0. \end{cases}$$

(a) Use the squeeze theorem to prove that $\lim_{x \to 0^-} f(x) = 0$.

(b) Find $\lim_{x \to 0^+} f(x)$.

(c) Based on your conclusions in parts (a) and (b), can you say anything about the limit $\lim_{x \to 0} f(x)$?

(d) Can you say anything about the continuity of $f(x)$ at $x = 0$.

QUESTION 4. **(5 marks)**

Let $f(x) = \sqrt{2x+5} + x^2 - 4$.

(a) Find the domain of $f(x)$.

(b) Use the definition of continuity to show that $f(x)$ is continuous on its domain.

(c) Use the definition to find the derivative function $f'(x)$.

(d) Prove that the equation $f(x) = 0$ has a root in its domain. (Hint: Use the Intermediate Value Theorem.)