

Name: _____

Tutorial group: _____

Matriculation number:

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NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I 2023/24

MH1100– Calculus I

29 September 2023

Midterm Test

90 minutes

INSTRUCTIONS

1. Do not turn over the pages until you are told to do so.
2. Write down your name, tutorial group, and matriculation number.
3. This test paper contains **SIX (6)** questions and comprises **SEVEN (7)** printed pages. Question 6 is optional.
4. The marks for each question are indicated at the beginning of each question.

For graders only	Question	1	2	3	4	5	6	Total
	Marks							

QUESTION 1.

(6 marks)

Find the limits if exist.

(a) $\lim_{x \rightarrow 5} \frac{3}{\sqrt{3x+1}+1}$

(b) $\lim_{x \rightarrow 1} \frac{x^{-1}-1}{x-1}$

(c) $\lim_{x \rightarrow -3} \frac{x^2-9}{\sqrt{x^2+16}-5}$

(d) $\lim_{x \rightarrow 1^-} \frac{\sqrt{2x}(x-1)}{|x-1|}$

(e) $\lim_{t \rightarrow 0} \frac{2t}{\tan t}$

(f) $\lim_{x \rightarrow -1^+} \left(\frac{1}{1+x} - \frac{3}{1+x^3} \right)$

QUESTION 2.**(2 marks)**

Use the precise definition to prove the limit,

$$\lim_{x \rightarrow -2} (x^2 + 2x + 3) = 3.$$

QUESTION 3.

(4 marks)

Show that there is at least one root of the equation

$$x + 3 \cos x = 0.$$

QUESTION 4.**(5 marks)**

Consider

$$f(x) = \begin{cases} -3, & x \leq -1 \\ cx - d, & -1 < x < 1 \\ 3, & x \geq 1 \end{cases}$$

- (a) For what values of c and d is $f(x)$ continuous at every x ?
- (b) With the values of c and d found in (a), find $f'(x)$ and its domain.

QUESTION 5.**(3 marks)**

Evaluate the following derivatives.

$$(a) \quad \frac{d}{dx} \left(\frac{(x+1)(x^2-2x)}{x^4} \right) \quad (b) \quad \frac{d}{dx} \left(\frac{\sin x}{x} + \frac{x}{\sin x} \right) \quad (c) \quad \frac{d^{110}}{dx^{110}} (\sin x + 4 \cos x)$$

QUESTION 6 (Optional).

(1 bonus mark)

If $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} f(x) = M$, then $L = M$.