SPMS / Division of Mathematical Sciences

MH1300 Foundations of Mathematics
2016/2017 Semester 1

MID-TERM EXAM

19 September 2016   TIME ALLOWED: 45 MINUTES

NAME: **Matriculation Number:**

Question	Marks	Question	Marks
1	15	3	10
2	10	4	15

Total:	50

TUTORIAL GROUP (Please tick)

	(T1) 1130–1230, TR11
	(T3) 1130–1230, TR17
	(T5) 1230–1330, TR11
	(T7) 1230–1330, TR17
	(T13) 1530–1630, TR11
	(T15) 1530–1630, TR17

	(T2) 1130–1230, TR14
	(T4) 1130–1230, TR18
	(T6) 1230–1330, TR14
	(T8) 1230–1330, TR18
	(T14) 1530–1630, TR14
	(T16) 1530–1630, TR18

INSTRUCTIONS TO CANDIDATES

1. This test paper contains **FOUR (4)** questions and comprises **EIGHT (8)** printed pages, including this cover page.

2. Answer **ALL** questions. This **IS NOT** an **OPEN BOOK** exam.

3. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. (15 marks)

Determine if the following pairs of statement forms are logically equivalent. If a pair is logically equivalent, prove it. If a pair is not logically equivalent, find truth values for the symbols p, q and r so that the statements forms have different truth values.

(a) Is $p \to (q \lor r) \equiv (p \land \neg r) \to q$?

(b) Is $p \land (\neg q \lor r) \equiv p \lor (q \land \neg r)$?

(c) Is $(p \to q) \to r \equiv p \to (q \to r)$?

QUESTION 1 (Continued).

QUESTION 2 (10 marks)

Let $P(x, y)$ be the predicate
$$x \geq y \rightarrow x^2 > y^2,$$
and the domain $D = \{-2, -1, 0, 1, 2\}$. Determine if each of the following is true. Justify your answer.

(a) $\forall x \in D, \ \exists y \in D, \ P(x, y)$.

(b) $\forall y \in D, \ \exists x \in D, \ P(x, y)$.

QUESTION 3. (10 marks)

Determine if the following are true or false. Justify your answer.

(a) $\{\emptyset\} \subsetneq \{\emptyset, \{\emptyset\}\}$

(b) $\{\emptyset\} \in \{\emptyset, \{\emptyset\}\}$

(c) $\{1, 2\} \subseteq \{x \in \mathbb{R} \mid x^3 - 6x^2 + 11x = 6\}$.

QUESTION 4. (15 marks)

(a) Let $Q(x)$ be the predicate
$$x < \frac{1}{x}.$$

Find the truth set of $Q(x)$ for the domain $\mathbb{R}$ and the truth set of $Q(x)$ for the domain $\mathbb{Z}$. Explain your answer.

(b) Write the negation of the statement form
$$(p \to r) \leftrightarrow (q \to r)$$

without using the symbols $\to$ and $\leftrightarrow$.

QUESTION 4 (Continued).

END OF PAPER