NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2021-2022

MH1100 – Calculus I

December 2021 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SEVEN (7)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **CLOSED** book exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1 (16 marks)

(a) Evaluate the limit

$$\lim_{x\to 1}\frac{\sqrt{x}-1}{x^2-1}.$$

(b) Use L'Hospital's Rule to evaluate the limit

$$\lim_{x\to 0^+}(1+\sin 3x)^{\cot x}.$$

QUESTION 2 (16 marks)

Use the ϵ-δ definition to prove the limit

$$\lim_{x\to 1} x^2 - 1 = 0.$$

QUESTION 3 (16 marks)

Find the derivatives of the following functions. (**You do not need to simplify the answers**)

(a)

$$g(x) = \cos(x^5 + \sqrt{\tan x}).$$

(b)

$$f(x) = (\sin x)^{\ln x}.$$

QUESTION 4 (12 marks)

Prove that the function

$$f(x) = x^{201} + x^{101} + x + 1,$$

has neither a local maximum nor a local minimum.

QUESTION 5 **(12 marks)**

Suppose $f(x)$ has a third order derivative on $[0,1]$ (that is $f'''(x)$ exists on $[0,1]$), and $f(0) = f(1) = 0$. Show that for the function $F(x) = x^2 f(x)$, there exists a number $c \in (0,1)$, such that $F'''(c) = 0$.

QUESTION 6 **(12 marks)**

Show that the tangent line to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

at the point (x_0, y_0) is

$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1.$$

QUESTION 7 **(16 marks)**

Suppose we have

$$f(x) = \begin{cases} e^{-\frac{1}{x^2}}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Prove that

$$f''(0) = 0.$$

END OF PAPER

MH1100 CALCULUS I

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.