NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2018-2019

MH1100 – Calculus I

December 2018 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **EIGHT (8)** questions and comprises **THREE (3)** printed pages.
2. Answer all questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT an OPEN BOOK** exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.
6.

QUESTION 1.

Evaluate the limits

(a)

$$\lim_{x\to\infty} \frac{1}{x} \sin\left(\frac{1}{x}\right).$$

(b)

$$\lim_{n\to\infty} \left(\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{n\cdot(n+1)}\right).$$

(Hint: $\lim_{n\to\infty} \frac{1}{n} = 0$)

(13 marks)

QUESTION 2.

Find the derivatives of the following functions. **You do not need to simplify.**

(a)

$$h(t) = \sin(t^2 + \cos(t));$$

(b)

$$g(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}.$$

(13 marks)

QUESTION 3.

Prove that the equation

$$\sqrt{x+5} = \frac{1}{x+3}$$

has at least one root. **(13 marks)**

QUESTION 4.

Show that the tangent line to the ellipse

$$\frac{x^2}{c^2} + \frac{y^2}{d^2} = 1$$

at the point (x_0, y_0) is

$$\frac{x_0 x}{c^2} + \frac{y_0 y}{d^2} = 1$$

(Note that number c and d are constant value.)

(13 marks)

QUESTION 5.

Hot air is leaking out from a spherical balloon. The volume of the spherical balloon decreases at a rate of $10m^3/s$. How fast is the radius of the balloon decreasing when the diameter is $5m$?

(13 marks)

QUESTION 6.

For a curve defined by $y = x^2+x+1$, we find three points with x coordinates $x_1 = 0, x_2 = -1$ and $x_3 = -\frac{1}{2}$ respectively. prove that the normal lines at these three points intersect with each other at one point.

(13 marks)

QUESTION 7.

Use the $\epsilon - \delta$ definition to prove if we have $\lim_{x\to a} f(x) = f(a) \neq 0$, then we have $\lim_{x\to a} \frac{1}{f(x)} = \frac{1}{f(a)}$.

(11 marks)

QUESTION 8.

Prove that if $f(x)$ and $g(x)$ are continuous on $[a, b]$, and differentiable on (a, b) then there exists a $\xi \in (a, b)$, that satisfies

$$\begin{vmatrix} f(b) - f(a) & g(b) - g(a) \\ f'(\xi) & g'(\xi) \end{vmatrix} = 0$$

(11 marks)

END OF PAPER