Name: ________________________________ Tutorial group: ____

Matriculation number: | | | | | | | | | |

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I 2019/20

MH1100 & SM2MH1100 – Calculus I

20 September 2019 Midterm Test 90 minutes

<u>INSTRUCTIONS</u>

1. Do not turn over the pages until you are told to do so.
2. Write down your name, tutorial group, and matriculation number.
3. This test paper contains **SIX (6)** questions and comprises **SEVEN (7)** printed pages. Question 6 is optional.
4. The marks for each question are indicated at the beginning of each question.

For graders only	Question	1	2	3	4	5	6	Total
	Marks							

QUESTION 1. **(3 marks)**

Use the ϵ, δ definition of a limit to prove the following statement

$$\lim_{x \to 3} \left(\frac{1}{x} + \frac{1}{3} \right) = \frac{2}{3}.$$

QUESTION 2. **(5 marks)**

Find the limits if exist.

(a) $\lim_{x\to 1} \dfrac{x^4+\sqrt{x}-2}{x^2+\cos x+e^x}$ (b) $\lim_{x\to 0} \dfrac{2-\sqrt{4-x^2}}{2x}$ (c) $\lim_{x\to 2} \dfrac{x^3+x^2+1}{(x-2)^2}$

(d) $\lim_{h\to 0} \left[\dfrac{(x+2h)^2-(x-3h)^2}{5h}\right]$ (e) $\lim_{x\to 1^+} \left(\dfrac{1}{1-x} - \dfrac{3}{1-x^3}\right)$.

QUESTION 3. **(4 marks)**

Show that there is at least one root of the equation

$$\sin x = x + \frac{1}{2}$$

between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

QUESTION 4. **(4 marks)**

Find the value of a that makes the following function continuous for all x-values.

$$f(x) = \begin{cases} x \sin \frac{1}{x}, & x > 0; \\ a + x, & x \leq 0. \end{cases}$$

QUESTION 5. **(4 marks)**

Consider the function

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

(a) Show that $f(x)$ is continuous in its domain.

(b) Find the derivative of $f(x)$ at $x = 0$ if exists.

QUESTION 6 (Optional). **(1 bonus mark)**

Suppose $f(x)$ and $g(x)$ are continuous functions on the interval I. Let

$$F(x) = \max\{f(x), g(x)\} \quad \text{and} \quad G(x) = \min\{f(x), g(x)\}.$$

Show that both $F(x)$ and $G(x)$ are continuous on I.