SPMS / Division of Mathematical Sciences

MH1300 Foundations of Mathematics
2015/2016 Semester 1

MID-TERM EXAM

14 September 2015 TIME ALLOWED: 40 MINUTES

NAME: **Matriculation Number:**

Question	Marks	Question	Marks
1	**22**	3	**11**
2	**17**		

Total:	**50**

TUTORIAL GROUP (Please tick)

	(T1) 1230–1330, TR6
	(T3) 1230–1330, TR9
	(T5) 1330–1430, TR6
	(T7) 1330–1430, TR9
	(T9) 1430–1530, TR6
	(T12) 1430–1530, TR10

	(T2) 1230–1330, TR7
	(T4) 1230–1330, TR10
	(T6) 1330–1430, TR7
	(T8) 1330–1430, TR10
	(T11) 1430–1530, TR9
	(T13) 1530–1630, TR6

INSTRUCTIONS TO CANDIDATES

1. This test paper contains **THREE (3)** questions and comprises **SEVEN (7)** printed pages, including the cover page.
2. Answer **ALL** questions. This **IS NOT** an **OPEN BOOK** exam.
3. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1(a). **(12 marks)**

Use a truth table to determine whether the pair of statement forms are logically equivalent. Include a few words of explanation.

$$(P \to Q) \to R, \text{ and } (P \wedge \neg Q) \vee R.$$

QUESTION 1(b). **(10 marks)**

Without using truth tables, show that the following logical equivalence holds

$$(P \wedge Q) \rightarrow R \quad \equiv \quad (P \wedge \neg R) \rightarrow \neg Q.$$

Supply a reason for each step.

QUESTION 2(a) **(8 marks)**

Let $T = \{3, 17\}, V = \{2, 3, 7, 26\}$. Which of the following quantified statements are true?

You DO NOT need to justify your answer. Circle the correct option.

T	F	$\exists x \in T,\ \ x$ is odd $\rightarrow x > 8$
T	F	$\exists x \in V,\ \ x$ is odd $\rightarrow x > 8$
T	F	$\forall x \in T,\ \ x$ is odd $\rightarrow x > 8$
T	F	$\forall x \in V,\ \ x$ is even $\rightarrow x > 8$
T	F	$\forall x \in T,\ \exists y \in V,\ \ x$ is odd $\rightarrow y > 8$
T	F	$\exists x \in T,\ \forall y \in V,\ \ x$ is odd $\rightarrow y > 8$
T	F	$\forall x \in V,\ \exists y \in T,\ \ x$ is odd $\leftrightarrow xy < 24$

QUESTION 2(b). **(9 marks)**

What can be said about the truth value of Q in each of the following cases:

(i) P is true and $P \rightarrow Q$ is false,

(ii) P is false and $P \leftrightarrow Q$ is true,

(iii) P is true and $P \leftrightarrow Q$ is false,

(iv) P is true and $P \rightarrow (Q \rightarrow \neg P)$ is true,

(v) $(P \leftrightarrow Q) \rightarrow P$ is false,

(vi) R is false and $(P \leftrightarrow Q) \leftrightarrow (\neg P \leftrightarrow R)$ is true.

Your answer should be either "Q is true", or "Q is false" in each case. You do not need to draw a truth table, but you should justify your answer in each case.

QUESTION 2(b) (continued).

QUESTION 3(a). **(6 marks)**

Write down the converse, contrapositive and the negation of the following statement:

For all integers n, $n + 1 > 2$ is necessary for $n^2 > 5$.

QUESTION 3(b). **(5 marks)**

Give an example of a nonempty domain D, and predicates $P(x)$ and $Q(x)$ such that

$$\exists x \in D, \;\; P(x) \vee Q(x) \;\; \text{is true, and}$$

$$(\forall x \in D, P(x)) \vee (\forall x \in D, Q(x)) \;\; \text{is false.}$$

END OF PAPER