NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2016-2017

MH1100 – CALCULUS I

December 2016 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **FOUR (4)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the end of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This **IS NOT an OPEN BOOK** exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1.

(a) State the ϵ-δ definition of

$$\lim_{x \to a} f(x) = L.$$

(b) Use the definition to prove that

$$\lim_{x \to 2} x^2 = 4.$$

(c) Use the definition to prove that

$$\lim_{x \to 2} \frac{1}{x} = \frac{1}{2}.$$

(d) Use the definition to prove that

$$\lim_{x \to 2} \left(x^2 + \frac{1}{x} \right) = \frac{9}{2}.$$

(18 marks)

QUESTION 2.

(a) State the definition of the derivative of a function $f(x)$ at a point $x = a$.

(b) Let $f(x)$ be a function which is differentiable at a point $x = a$. Let $g(x)$ denote the function

$$g(x) = \frac{1}{(f(x))^2 + 1}.$$

Using the definition of derivative you gave in Part (a), determine the derivative of the function $g(x)$ at $x = a$ in terms of the derivative $f'(a)$ of $f(x)$ at $x = a$ and the value $f(a)$. Justify your calculations using standard facts about limits introduced in the course. (Do not use the differentiation rules.)

(18 marks)

QUESTION 3.

Let $f(x)$ be a function with domain $\mathbb{R}$ and assume that

- $f(0) = 0$ and $f(x) > 0$ if $x \neq 0$,
- f is differentiable at every point,
- $f'(0) = 0$ and $f'(x) \neq 0$ if $x \neq 0$.

Let $b \in \mathbb{R}$ be a constant, and define a second function $g(x)$ by the rule

$$g(x) = \begin{cases} \dfrac{\sin\left(f(x)\right)}{2^{f(x)} - 1} & x \neq 0 \\ b & x = 0. \end{cases}$$

Is it possible to choose the constant b so that $g(x)$ is continuous at zero? If so, state a correct b. Justify your answer by carefully applying standard theorems of calculus introduced in the course.

(14 marks)

QUESTION 4.

Apply standard theorems of calculus introduced in the course to carefully show that

(a) No matter what value we set the real constant c, there is at most one real number x from the interval $[-1, 1]$ where

$$x^{2016} - (2016)x + c^{2016} = 0.$$

(b) For all $x > 0$,

$$1 - \frac{x^2}{2} \leq \cos x.$$

(18 marks)

QUESTION 5.

Evaluate the following indefinite integrals:

(a) $$\int e^x \cos x \, dx.$$

(b) $$\int (\sin(\ln x) + \cos(\ln x)) \, dx.$$

(18 Marks)

QUESTION 6.

Let $f\colon (r,s) \to \mathbb{R}$ be a function, with domain an open interval $(r,s) \subset \mathbb{R}$, and satisfying the properties

- $\lim_{x\to r^+} f(x) = -\infty$,
- $\lim_{x\to s^-} f(x) = \infty$,
- f is continuous at every point of (r,s).

(a) Give two different examples of functions satisfying these properties.

(b) State the definitions of $\lim_{x\to r^+} f(x) = -\infty$ and $\lim_{x\to s^-} f(x) = \infty$.

(c) Use these definitions, and standard theorems of calculus introduced in the course, to carefully show that there exists a real number $c \in (r,s)$ where $f(c) = 0$.

(14 Marks)

END OF PAPER

MH1100 CALCULUS I

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.