SPMS / Division of Mathematical Sciences
MH1300 Foundations of Mathematics
2024/2025 Semester 1

MID-TERM EXAM

11 October 2024 TIME ALLOWED: 50 MINUTES

FULL NAME (e.g. Tan Ah Kow) on matric card:

MATRICULATION NUMBER (e.g. U2400888D):

INSTRUCTIONS TO CANDIDATES

1. This test paper contains **SIX (6)** questions and comprises **EIGHT (8)** printed pages, including this cover page.
2. Answer **ALL** questions. This **IS NOT** an **OPEN BOOK** exam.
3. You are allowed both sides of one A4 sized helpsheet.
4. Candidates may use calculators. However, they should write down systematically the steps in the workings.
5. This paper will be scanned and marked digitally. Only write your answers within the boxes provided.
6. The last page is an additional page if you run out of space. If you use it, please indicate clearly which question(s) you are answering.

QUESTION 1. **(16 marks)**

Prove that the cube of any integer is of the form $9k, 9k+1$ or $9k+8$ for some integer k.

QUESTION 2. **(16 marks)**

Prove that for any two positive real numbers x and y,

$$\lfloor xy \rfloor \geq \lfloor x \rfloor \cdot \lfloor y \rfloor.$$

Is the above inequality true for all real numbers x, y? Justify your answer.

QUESTION 3. **(16 marks)**

Determine if the following is true or false. Justify your answer.

For every three integers a, b and x, if $a \bmod 3 = b \bmod 3$ then $ax \bmod 3 = bx \bmod 3$.

Is the converse true for all integers a, b and x?

QUESTION 4. **(18 marks)**

Prove that for any real number x, $x - \lfloor x \rfloor < \frac{1}{2}$ if and only if $\lfloor 2x \rfloor = 2\lfloor x \rfloor$.

QUESTION 5. **(18 marks)**

Is the following a tautology, contradiction, or neither? Justify your answer.

$$((p \vee q) \to r) \to ((p \wedge q) \to r).$$

QUESTION 6.

Let $\mathbb{Q}$ be the set of rational numbers and E be the set of even integers. Determine if each of the following is true or false, and justify your answers.

(a) $\forall x \in \mathbb{Q},\ \exists y \in \mathbb{Q},\ \exists z \in E,\ xyz = 3.$ **(8 marks)**

(b) $\exists x \in E,\ \exists y \in \mathbb{Q},\ \forall z \in E,\ x + yz = 4.$ **(8 marks)**

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You may use this page if you run out of space. If you do so, indicate clearly which question you are answering.

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