Name: _____ Tutorial group: ____

Matriculation number: | | | | | | | | | |

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I 2018/19

MH1100 & SM2MH1100 – Calculus I

28 September 2018 Midterm Test 90 minutes

INSTRUCTIONS

1. Do not turn over the pages until you are told to do so.
2. Write down your name, tutorial group, and matriculation number.
3. This test paper contains **FIVE (5)** questions and comprises **SIX (6)** printed pages.
4. Answer **all** questions. The marks for each question are indicated at the beginning of each question.

For graders only	Question	1	2	3	4	5	Total
	Marks						

QUESTION 1. **(5 marks)**

Find the limits if exist.

(a) $\lim_{x\to 0}(x^2+1)(2+\cos x)$ (b) $\lim_{x\to -2}\dfrac{\sqrt{x^2+5}-3}{x+2}$ (c) $\lim_{x\to 2}\dfrac{1}{|x-2|}$

(d) $\lim_{x\to 0^+}\left[x\sin\left(\dfrac{1}{x^2}\right)\right]$ (e) If $\lim_{x\to 0^+}[8g(x)]^{\frac{1}{3}}=3$, find $\lim_{x\to 0^+}g(x)$.

QUESTION 2. **(3 marks)**

Find the derivatives of the functions.

(a) $y = \dfrac{x^2}{x+1}$ (b) $w = (z-2)(z+2)(z^2+4)$ (c) $s = \cos\left(\sqrt{2t+1}\right)$

QUESTION 3. **(4 marks)**

(a) Use the ϵ-δ definition to prove that $\lim_{x \to 2}(3x - 4) = 2$.

(b) Show that the function $f(x) = 3x - 4$ is continuous in its domain.

QUESTION 4. **(4 marks)**

Find the values of a and b that make the following function differentiable for all x-values.

$$g(x) = \begin{cases} ax + b, & x < 1; \\ bx^2 - 3, & x \geq 1. \end{cases}$$

QUESTION 5. **(4 marks)**

Let $f(x) = x + 2\sin\left(\frac{1}{x}\right)$.

(a) Find the domain of $f(x)$.

(b) Show that $f(x)$ is an odd function in its domain.

(c) Prove that the equation $f(x) = 0$ has a root in its domain.