SPMS / Division of Mathematical Sciences
MH1300 Foundations of Mathematics
2020/2021 Semester 1

MID-TERM EXAM

12 October 2020 TIME ALLOWED: 50 MINUTES

NAME: **Matriculation Number:**

Question	Marks	Question	Marks
1	14	3	14
2	14	4	8

Total:	50

TUTORIAL GROUP (Please tick)

	(T1) 1130–1230, TR4 Goh You Hui
	(T3) 1130–1230, TR10 Lee Xin Qi
	(T6) 1230–1330, TR9 Salah Mostafa
	(T9) 1330–1430, TR4 Lee Xin Qi
	(T11) 1330–1430, TR10 Inggriany Dwitami

	(T2) 1130–1230, TR9 Salah Mostafa
	(T5) 1230–1330, TR4 Goh You Hui
	(T7) 1230–1330, TR10 Inggriany Dwitami
	(T10) 1330–1430, TR9 Salah Mostafa
	(T13) 1530–1630, TR9 Loo Dong Lin

INSTRUCTIONS TO CANDIDATES

1. This test paper contains **FOUR (4)** questions and comprises **EIGHT (8)** printed pages, including this cover page.
2. Answer **ALL** questions. This **IS NOT** an **OPEN BOOK** exam.
3. You are allowed both sides of one A4 sized helpsheet.
4. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(14 marks)**

Determine if each of the following is true or false. Justify your answer.

(a) For every natural number n, if $n^2 + (n+1)^2 = (n+2)^2$ then $n = 3$.

(b) There is no natural number m such that $(m-1)^3 + m^3 = (m+1)^3$.

(c) For all integers a and b there are integers c and d such that $a = c + d$ and $b = c - d$.

QUESTION 1 (Continued).

QUESTION 2 **(14 marks)**

Determine if each of the following is true or false. Justify your answer.

(a) For every integer n, $n^2 + n$ is even.

(b) There exists an integer b such that for every integer $a \neq 0$, b is divisible by a.

(c) For every integer x there exists an integer $y \neq 0$ such that x is divisible by y.

QUESTION 2 (Continued).

QUESTION 3. **(14 marks)**

Prove each of the following statements.

(a) Suppose that a and b are integers such that $a \mid b$. Then $a^n \mid b^n$ for all positive integers n.

(b) Suppose that c, d, e, x and y are integers such that $c \mid d$ and $c \mid e$. Then $c \mid (dx + ey)$.

(c) Without using the Fundamental Theorem of Arithmetic, prove that if n and m are integers such that $3 \mid mn$, then either $3 \mid m$ or $3 \mid n$.
Hint - The following fact may be useful: $2n = 3n - n$.

QUESTION 3 (Continued).

QUESTION 4. **(8 marks)**

Find non-empty sets D and E such that the statement

$$\forall x \in D,\ \forall y \in D,\ \ (x \neq y \to \forall z \in D,\ (z = x \text{ or } z = y))$$

is true, and where the statement

$$\forall x \in E,\ \forall y \in E,\ \ (x \neq y \to \forall z \in E,\ (z = x \text{ or } z = y))$$

is false. Justify your answer.
Hint: You should take D and E to be sets with very few elements.

END OF PAPER