SPMS / Division of Mathematical Sciences
MH1300 Foundations of Mathematics
2021/2022 Semester 1

MID-TERM EXAM

11 October 2021 TIME ALLOWED: 50 MINUTES

NAME: **Matriculation Number:**

Question	Marks	Question	Marks
1	**20**	3	**12**
2	**10**	4	**8**

Total:	**50**

TUTORIAL GROUP (Please tick)

	(T1) 1130–1220, TR4 Ng Jeremy
	(T3) 1130–1220, TR10 Goh You Hui
	(T5) 1230–1320, TR4 Ng Jeremy
	(T7) 1230–1320, TR10 Goh You Hui
	(T9) 1330–1420, TR4 Ng Jeremy
	(T11) 1330–1420, TR10 Nguyen Duong Quynh Chi

	(T2) 1130–1220, TR9 Salah Mostafa
	(T4) 1130–1220, TR11 Loh Yi Fong
	(T6) 1230–1320, TR9 Salah Mostafa
	(T8) 1230–1320, TR11 Loh Yi Fong
	(T10) 1330–1420, TR9 Teh Yu Xuan

INSTRUCTIONS TO CANDIDATES

1. This test paper contains **FOUR (4)** questions and comprises **EIGHT (8)** printed pages, including this cover page.
2. Answer **ALL** questions. This **IS NOT** an **OPEN BOOK** exam.
3. You are allowed both sides of one A4 sized helpsheet.
4. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(20 marks)**

Show the following. Justify all of your answers.

(a) Using logical equivalences, deduce whether $p \to (q \leftrightarrow (p \land q))$ is a tautology, contradiction, or neither. You may use the fact that $a \leftrightarrow b \equiv (a \to b) \land (b \to a)$ for every statement forms a, b.

(b) Let $Q(m)$ be the predicate "m is even". Write down predicates $P(m)$ and $R(m)$ such that:

(i) For every $m \in \mathbb{Z}$, $P(m)$ is sufficient for $Q(m)$, but $P(m)$ is not necessary for $Q(m)$ for some $m \in \mathbb{Z}$.

(ii) For every $m \in \mathbb{Z}$, $R(m)$ is necessary for $Q(m)$, but $R(m)$ is not sufficient for $Q(m)$ for some $m \in \mathbb{Z}$.

You need to explain your answers.

QUESTION 1 (Continued).

QUESTION 2. **(10 marks)**

Determine if the following is true or false. Justify your answer.

There are positive integers n and m such that $2m^2 + 3n^2 = 31$.

QUESTION 2 (Continued).

QUESTION 3. **(12 marks)**

Using the definition of $|x|$, prove that for all real numbers x and all positive real numbers d,

$$|x| < d \text{ if and only if } -d < x < d.$$

QUESTION 3 (Continued).

QUESTION 4. **(8 marks)**

Show that the following argument is valid. If you've used any rules of inference, state them.

$$\begin{array}{ll} & \neg p \to (q \to \neg r) \\ & r \to \neg p \\ & (\neg s \lor p) \to \neg\neg r \\ & \neg s \\ \therefore & \neg q \end{array}$$

END OF PAPER