NANYANG TECHNOLOGICAL UNIVERSITY

SPMS/DIVISION OF MATHEMATICAL SCIENCES

2023/24 Sem 1 MH5100 Advanced Investigations into Calculus I Week 4

Problem 1. *Is there a number a such that*

$$\lim_{x\to -2}\frac{3x^2+ax+a+3}{x^2+x-2}$$

exists? If so, find the value of a and the value of the limit.

Problem 2. *Use the precise definition to prove the following limit does not exist.*

$$\lim_{x\to 0} x\tan\frac{1}{x}$$

Problem 3. *If* $\lim_{x\to c}[f(x)+g(x)]=3$ *and* $\lim_{x\to c}[f(x)-g(x)]=-1$ *find* $\lim_{x\to c} f(x)\,g(x)$

Problem 4. *Evaluate* $\lim_{x\to 0}\frac{\sqrt[3]{1+cx}-1}{2x}$,*where c is a constant.*

Problem 5. *Find the global maximum of the function*

$$f(x)=\frac{1}{1+|x-2|}+\frac{1}{1+|x+6|}.$$

Problem 6. *If* $f(x)=\cosh x=\frac{1}{2}(e^x+e^{-x})$*, prove that the inverse function of $f(x)$, as the principal value of the inverse function, is* $\cosh^{-1}x=\ln(x+\sqrt{x^2-1}), x\geq 1$.