Name: ________________________________ Tutorial group: ____

Matriculation number: | | | | | | | | | |

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I 2025/26

MH1100 – Calculus I

19 September 2025 Midterm Test 90 minutes

INSTRUCTIONS

1. Do not turn over the pages until you are told to do so.
2. Write down your name, tutorial group, and matriculation number.
3. This test paper contains **SIX (6)** questions and comprises **SEVEN (7)** printed pages. Question 6 is optional.
4. The marks for each question are indicated at the beginning of each question.

For graders only	Question	1	2	3	4	5	6	Total
	Marks							

QUESTION 1. **(3 marks)**

Prove that the function

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0 \end{cases}$$

is continuous at $x = 0$.

QUESTION 2. **(6 marks)**

(a) Evaluate the limits:

$$\lim_{x\to 3} \frac{x^2-9}{x-3}, \qquad \lim_{x\to 0} x^2\, e^{-\frac{1}{x^2}}.$$

(b) Compute the one-sided limits:

$$\lim_{x\to 2^-} \frac{|x-2|}{x-2}, \qquad \lim_{x\to 2^+} \frac{|x-2|}{x-2}.$$

What can you conclude about $\lim\limits_{x\to 2} \frac{|x-2|}{x-2}$?

(c) Find the vertical asymptote(s) of:

$$f(x) = \frac{2x}{x^2-1}.$$

QUESTION 3. **(4 marks)**

(a) Show that $x^3 + x - 1 = 0$ has at least one real root in the interval $(0,\ 1)$.

(b) Show that the equation $e^x = 3x$ has a solution in the interval $[0,\ 2]$.

QUESTION 4. **(4 marks)**

Determine whether the piecewise function

$$f(x) = \begin{cases} x+1, & x < 1, \\ 3-x, & x \geq 1 \end{cases}$$

is continuous at $x = 1$.

QUESTION 5. **(3 marks)**

(a) Show that the absolute value function $F(x) = |x|$ is continuous everywhere.

(b) Prove that if f is a continuous function on an interval, then so is $|f|$.

(c) Is the converse true? That is, if $|f|$ is continuous, does it follow that f is continuous? If so, prove it. If not, find a counterexample.

QUESTION 6 (Optional). **(1 bonus mark)**

Let $\lim_{x \to a} f(x) = L$ and $L \neq 0$. Use the ϵ-δ definition to prove that

$$\lim_{x \to a} \frac{1}{f(x)} = \frac{1}{L}.$$