Name: _____   Tutorial group: ____

Matriculation number: | | | | | | | | | |

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I 2021/22

MH1100 – Calculus I

24 September 2021   Midterm Test   90 minutes

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<u>INSTRUCTIONS</u>

1. Do not turn over the pages until you are told to do so.

2. Write down your name, tutorial group, and matriculation number.

3. This test paper contains **SIX (6)** questions and comprises **SEVEN (7)** printed pages.

4. The marks for each question are indicated at the beginning of each question.

QUESTION 1. (2 marks)

Let ϵ be a given positive number. Verify that a possible choice of δ for showing that

$$\lim_{x \to 4} x^2 = 16$$

is

$$\delta = \min\{2, \frac{\epsilon}{10}\}.$$

QUESTION 2. (4 marks)

Find the limits if exist.

(a) $\lim\limits_{t \to 2} \left(\dfrac{t^2 + 6t - 16}{t^2 - 5t + 7} \right)^{100}$,

(b) $\lim\limits_{x \to 0} \dfrac{\sqrt{9 + x} - 3}{3x}$,

(c) $\lim\limits_{x \to 2} \dfrac{x^2 + ax + a + 8}{(x - 2)^2}$, where a is a real number,

(d) $\lim\limits_{x \to 0} \dfrac{|x|}{\sin x}$.

QUESTION 3. (4 marks)

(a) If $\lim\limits_{x \to 0} \frac{f(x)}{x^2} = 10$, find $\lim\limits_{x \to 0} \frac{f(x)}{x}$.

(b) Suppose that $|f(x)| \le x^2$ for all x. Find $\lim\limits_{x \to 0} f(x)$.

QUESTION 4. (3 marks)

Show that there is at least one root of the equation

$$\ln x + \sqrt{4 - x^2} = 1.$$

QUESTION 5. (4 marks)

Let

$$f(x) = \begin{cases} x^2, & x \le 4; \\ ax + b, & x > 4. \end{cases}$$

(a) Find the values of a and b that make $f(x)$ continuous everywhere.

(b) Find the values of a and b that make $f(x)$ differentiable everywhere.

QUESTION 6. (3 marks)

Find equations of the tangent lines to the curve

$$y = \frac{x - 2}{x + 2}$$

that are parallel to the line $x - 4y = 4$.