NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2019-2020

MH1100 – Calculus I

December 2019 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **EIGHT (8)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **CLOSED** book exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1 **(13 marks)**

Evaluate the limits (You can use l'Hospital's Rule.)

(a)

$$\lim_{x\to 0}\frac{|x|}{x^3};$$

(b)

$$\lim_{x\to \pi}\frac{e^{\sin x}-1}{x-\pi}.$$

QUESTION 2 **(13 marks)**

Find the derivatives of the following functions. **You do not need to simplify.**

(a)

$$h(t) = \frac{1}{1 + \frac{1}{1+\frac{1}{t}}};$$

(b)

$$g(x) = \sin(x^2 + \sqrt{\tan x}).$$

QUESTION 3 **(13 marks)**

Suppose $f(x)$ and $g(x)$ are differentiable functions such that $f(g(x)) = x$ and $f'(x) = 1 + [f(x)]^2$. Show that

$$g'(x) = \frac{1}{1+x^2}.$$

QUESTION 4 **(13 marks)**

Suppose that $f(x)$ is differentiable in the whole domain $\mathbf{R}$ and have N distinct roots (Note that $N > 1$ and N is an integer). Show that $f'(x)$ has at least $N-1$ distinct roots.

QUESTION 5 **(13 marks)**

Find the equations of both tangent lines to the ellipse $x^2 + 4y^2 = 36$ that pass through the point (12, 3).

QUESTION 6 **(13 marks)**

A stone was dropped off a cliff and hit the ground with a speed of 98 m/s (meter/second). What is the height of the cliff? How long does it take for the stone to hit the ground? (Hint: the acceleration is -9.8 m/s^2)

QUESTION 7 **(11 marks)**

Use the ϵ-δ definition to prove if we have $\lim_{x \to a} f(x) = f(a)$ and $\lim_{x \to a} g(x) = g(a)$, then we have $\lim_{x \to a} (f(x) + g(x)) = f(a) + g(a)$.

QUESTION 8 **(11 marks)**

We have an infinite sequence 1, $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[4]{4}$, $\sqrt[5]{5}$, ..., $\sqrt[N]{N}$,.... Find out and prove which term has the largest value in this sequence.

END OF PAPER