NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2017-2018

MH1100 – Calculus I

December 2017 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **EIGHT (8)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **CLOSED** book exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1 (12 marks)

Evaluate the limits

(a)
$$\lim_{x\to 0}\frac{x^2-3x}{\tan(2x)};$$

(b)
$$\lim_{x\to 1}\frac{\sqrt{3x+6}-3}{x^2-1}.$$

QUESTION 2 (12 marks)

Use the ϵ-δ definition to prove the limit $\lim_{x\to 2}(x^2+2x-7)=1$.

QUESTION 3 (12 marks)

Find the derivatives of the following functions. **You do not need to simplify.**

(a)
$$g(x)=(\frac{1}{x^2}+3x)(\frac{1}{\sqrt{x}}+1);$$

(b)
$$h(t)=7\sec(t)\tan(\frac{3}{t}).$$

QUESTION 4 (12 marks)

(a) State the Intermediate Value Theorem (IVT).

(b) Use the IVT to show that the equation $\sin(x)=x^2-1$ has at least one root. (Remember to state why you can apply the IVT)

QUESTION 5 **(12 marks)**

Find the equation of the tangent line to the curve $y^2 + 13x = x^2y + 13$ at the point $P(4, 3)$.

QUESTION 6 **(12 marks)**

Suppose you throw a stone upward from the top edge of a 100 meter tall building at time $t = 0$ and the stone's height is equal to $h(t) = 5t - 5t^2 + 100$ meters at t seconds.

(a) When will the stone reach the highest point?

(b) With what velocity will the stone hit the ground?

QUESTION 7 **(12 marks)**

Find the horizontal asymptote for the function

$$y = \frac{3x^2 - x - 2}{5x^2 + 4x + 1}.$$

QUESTION 8 **(16 marks)**

Find the values of a, b, c, and d that make f differentiable everywhere.

$$f(x) = \begin{cases} \frac{x^2-4}{x-2}, & \text{if } x < 2; \\ ax^2 + bx + 6, & \text{if } 2 \leq x < c; \\ 2x + d, & \text{if } x \geq c. \end{cases}$$

END OF PAPER

MH1100 CALCULUS I

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.