NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2023-2024

MH1100 – Calculus I

December 2023 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SEVEN (7)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **CLOSED** book exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1 **(16 marks)**

Evaluate the limits

(a)

$$\lim_{x\to 0}\frac{\sin 2023x\cdot\sin 2x}{x^2}.$$

(b)

$$\lim_{x\to\infty}\frac{2022x^2-1}{x^2+x-2023}.$$

QUESTION 2 **(16 marks)**

Use the ϵ-δ definition to prove the following limit

$$\lim_{x \to 1} \frac{x^2 - 1}{2x^2 - x - 1} = \frac{2}{3}.$$

QUESTION 3 **(16 marks)**

Use Newton's method to approximate the root of the following equation

$$x^3 + 3x + 1 = 0.$$

Please start with $x_0 = 0$, and find the second approximation x_2.

QUESTION 4 **(16 marks)**

Suppose that y is an implicit function of x satisfying that

$$x^y = y^x,$$

find y'.

QUESTION 5 **(12 marks)**

Evaluate the limit

$$\lim_{n\to\infty}(1+\frac{1}{n}-\frac{1}{n^2})^n.$$

QUESTION 6 **(12 marks)**

Show that the function $y = Ae^{-x} + Bxe^{-x}$ (with A and B two arbitrary constants) satisfies the differential equation

$$y'' + 2y' + y = 0.$$

QUESTION 7 **(12 marks)**

A number a is called a fixed point of a function $f(x)$ if $f(a) = a$. Show that if $f'(x) \neq 1$ for all real numbers, then f has at most one fixed point.

END OF PAPER