NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2024-2025

MH1100 – Calculus I

December 2024 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SEVEN (7)** questions and comprises **THREE (3)** printed pages.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. Answer each question beginning on a **FRESH** page of the answer book.
4. This is a **CLOSED** book exam.
5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1 **(16 marks)**

Evaluate the limits

(a)

$$\lim_{x\to 1^+}[\ln(x^7-1)-\ln(x^5-1)].$$

(b)

$$\lim_{x\to\infty}\left(1+\frac{2024}{x}\right)^{2x}.$$

QUESTION 2 **(11 marks)**

Use the ϵ-δ definition to prove if we have $\lim_{x\to a} f(x) = f(a)$ and $\lim_{x\to a} g(x) = g(a)$, then we have $\lim_{x\to a}(f(x) - g(x)) = f(a) - g(a)$.

QUESTION 3 **(16 marks)**

Suppose that $f(x)$ and $g(x)$ are continuous functions on $[a, b]$ and differentiable on (a, b). Suppose also that $f(a) = g(a)$ and $f'(x) < g'(x)$ for $a < x < b$. Prove that $f(b) < g(b)$.

QUESTION 4 **(16 marks)**

If $xy + y^3 = 1$, find the value of y'' at the point where $x = 0$.

QUESTION 5 **(12 marks)**

Show that the sum of the $x-$ and $y-$intercepts of any tangent line to the curve $\sqrt{x} + \sqrt{y} = \sqrt{c}$ is equal to c. (Note that number c is a positive constant value.)

QUESTION 6 **(12 marks)**

Suppose that a function $f(x)$ is differentiable on $[a, b]$. Prove that there exists a number ξ such that

$$2\xi[f(b) - f(a)] = (b^2 - a^2)f'(\xi).$$

QUESTION 7 **(12 marks)**

Suppose that a function f is continuous on $[0, 1]$ and $f(0) = f(1)$. Prove that for any positive integer number n, there exists ξ such that

$$f(\xi + \frac{1}{n}) = f(\xi).$$

END OF PAPER