Name: _____ Tutorial group: ____

Matriculation number: | | | | | | | | | |

NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I 2022/23

MH1100 – Calculus I

23 September 2022 Midterm Test 90 minutes

INSTRUCTIONS

1. Do not turn over the pages until you are told to do so.
2. Write down your name, tutorial group, and matriculation number.
3. This test paper contains **SIX (6)** questions and comprises **SEVEN (7)** printed pages. Question 6 is optional.
4. The marks for each question are indicated at the beginning of each question.

For graders only	Question	1	2	3	4	5	6	Total
	Marks							

QUESTION 1. **(4 marks)**

Use the ϵ, δ definition of a limit to prove that $\lim_{x \to 2} f(x) = 5$ if

$$f(x) = \begin{cases} 2x + 1, & x \neq 2, \\ 0, & x = 2. \end{cases}$$

QUESTION 2. **(4 marks)**

Find the limits if exist.

(a) $\lim\limits_{x\to 2}\dfrac{x^2-4x+4}{x+\frac{1}{x}+\sin(\sqrt{x})}$,

(b) $\lim\limits_{x\to 1}\dfrac{1-\sqrt{x}}{x-1}$,

(c) $\lim\limits_{x\to 2}\dfrac{\sin(x-2)}{x^2-4}$,

(d) $\lim\limits_{x\to -2^+}\left(\dfrac{x}{1+x}\right)\left(\dfrac{2x+5}{x^2+x}\right)$.

QUESTION 3. **(4 marks)**

Find contants a and b so that the following limit is true.

$$\lim_{x \to 0} \frac{\sqrt{ax+b}-1}{x} = 1.$$

QUESTION 4. **(4 marks)**

Suppose that the inequalities

$$\frac{1}{2} - \frac{x^2}{24} < \frac{1 - \cos x}{x^2} < \frac{1}{2}$$

hold for values of x close to zero. What, if anything, does this tell you about

$$\lim_{x \to 0} \frac{1 - \cos x}{x \sin x}.$$

QUESTION 5. **(4 marks)**

Show that the function $f(x) = x|x|$ is differentiable in its domain.

QUESTION 6 (Optional). **(1 bonus mark)**

The function $f(x)$ is a continuous functions on the interval $[a,\, b]$. n is a positive integer. $x_1, \cdots, x_n \in [a,\, b]$. Prove that there exists a $\xi \in [a,\, b]$ such that

$$f(\xi) = \frac{1}{n}\left[f(x_1) + f(x_2) + \cdots + f(x_n)\right].$$