

Name: _____

Tutorial group: _____

Matriculation number:

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NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I 2024/25

MH1100 – Calculus I

27 September 2024

Midterm Test

90 minutes

INSTRUCTIONS

1. Do not turn over the pages until you are told to do so.
2. Write down your name, tutorial group, and matriculation number.
3. This test paper contains **SIX (6)** questions and comprises **SEVEN (7)** printed pages. Question 6 is optional.
4. The marks for each question are indicated at the beginning of each question.

For graders only	Question	1	2	3	4	5	6	Total
	Marks							

QUESTION 1. **(2 marks)**

Use the ϵ, δ definition of a limit to prove

$$\lim_{x \rightarrow 1} \frac{1}{x+1} = \frac{1}{2}.$$

QUESTION 2.**(4 marks)**

Find the limits, if they exist:

$$(a) \lim_{x \rightarrow 0} \left(\frac{1}{1+2x} - \frac{1}{1-3x} \right),$$

$$(b) \lim_{h \rightarrow 0} \frac{(x-3h)^2 - (x+2h)^2}{\sin h},$$

$$(c) \lim_{x \rightarrow 0^+} \left(\frac{x^2}{2} - \frac{1}{2x} \right),$$

$$(d) \lim_{x \rightarrow 0} \sqrt[3]{x} \sin \left(\frac{1}{x^2} \right).$$

QUESTION 3.**(4 marks)**

(a) Let $f(x) = \cos x - x$. Show that there is a solution to the equation $\cos x = x$ between 0 and $\frac{\pi}{3}$.

(b) Suppose the temperature at 6 AM is 25°C and at 2 PM is 34°C . Assuming the temperature is a continuous function of time, show that at some point during the day, the temperature was exactly 30°C .

QUESTION 4.**(4 marks)**

Compute the derivatives of the functions in (a)-(c):

$$(a) f(x) = (3x^2 + 5)(2x^3 - 4), \quad (b) g(x) = \frac{x^4 - 2x + 1}{x^2 + 3}, \quad (c) h(x) = \sqrt{x + 7},$$

(d) Let $k(x) = x(x+1)(x+2) \cdots (x+2024)$, compute $k'(0)$.

QUESTION 5.**(6 marks)**

(a) Determine if the function is differentiable at $x = 1$:

$$f(x) = \begin{cases} x^2, & \text{if } x \leq 1 \\ 2x + 1, & \text{if } x > 1. \end{cases}$$

(b) Determine if the function is differentiable at $x = 0$:

$$g(x) = \begin{cases} 2x - x^3 - 1, & \text{if } x \leq 0 \\ x - \frac{1}{x+1}, & \text{if } x > 0. \end{cases}$$

QUESTION 6 (Optional).**(1 bonus mark)**

Find the limit

$$\lim_{h \rightarrow 0} \frac{h}{f(a - 2h) - f(a + 3h)},$$

given that $f'(a) = -1$.