NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2015-2016

MH1100 - Calculus I

December 2015 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SIX (6)** questions and comprises **FOUR (4)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This **IS NOT** an **OPEN BOOK** exam.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(15 marks)**

Find the equation of the tangent line to the curve

$$y = \ln\left[\frac{\sin(\pi x)}{e^x}\right]$$

at $x = \frac{1}{2}$.

QUESTION 2. **(10 marks)**

Evaluate the limit

$$\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{x^2}{x-1}\right).$$

[You may use any relevant facts from the course, but please state them clearly.]

QUESTION 3. **(15 marks)**

Find some combination of techniques to evaluate the indefinite integral:

$$\int t^3 e^{-t^2}\, dt\ .$$

QUESTION 4. **(20 marks)**

Let $f(x) = \sqrt{|x - 2015|}$. Its domain is $\mathbb{R}$.

(a) (10 marks.) At which values of x is f continuous?

(b) (10 marks.) At which values of x is f differentiable?

[Carefully explain with reference to the definitions. You may use any standard facts or theorems from the course.]

QUESTION 5. **(20 marks)**

[In this question, ∞ denotes positive infinity.]

(i) (5 marks.) State the precise (ϵ-N) definition of the equation

$$\lim_{x \to \infty} f(x) = L$$

(ii) (15 marks.) Let a, b and c be constants, with $c > 0$. Using your definition in part (i), show that

$$\lim_{x \to \infty} \frac{ax + b}{x + c} = a$$

QUESTION 6. **(20 marks)**

(i) (10 marks.) Use the Mean Value Theorem to show that

$$\sqrt{1+x} < 1 + \frac{1}{2}x \qquad \text{if } x > 0.$$

(ii) (10 marks.) Using the result in part (i), show that

$$\sqrt{1+x} > 1 + \frac{1}{2}x - \frac{1}{8}x^2 \qquad \text{if } x > 0.$$

END OF PAPER

MH1100 CALCULUS I

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.