SPMS / Division of Mathematical Sciences
MH1300 Foundations of Mathematics
2023/2024 Semester 1

MID-TERM EXAM

13 October 2023 TIME ALLOWED: 50 MINUTES

NAME: **Matriculation Number:**

Question	Marks	Question	Marks
1	**15**	3	**12**
2	**15**	4	**8**

Total:	**50**

TUTORIAL GROUP (Please tick)

	(T1) Mon 0930–1020, TR4 Darryl Chan
	(T3) Mon 1130–1220, TR4 Darryl Chan
	(T5) Mon 1530–1620, TR4 Chan Joshua
	(T7) Tues 0930–1020, TR7 Muhd Firaz
	(T9) Tues 1030–1120, TR7 Muhd Firaz
	(T11) Tues 1230–1320, TR4 Madeleine Harlow

	(T2) Mon 0930–1020, TR8 Madeleine Harlow
	(T4) Mon 1130–1220, TR8 Peng Yuhan
	(T6) Mon 1530–1620, TR8 Peng Yuhan
	(T8) Tues 0930–1020, TR8 Damian Tan
	(T10) Tues 1030–1120, TR8 Damian Tan

INSTRUCTIONS TO CANDIDATES

1. This test paper contains **FOUR (4)** questions and comprises **EIGHT (8)** printed pages, including this cover page.
2. Answer **ALL** questions. This **IS NOT** an **OPEN BOOK** exam.
3. You are allowed both sides of one A4 sized helpsheet.
4. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(15 marks)**

Prove each of the following statements.

(a) Let n be an integer. Show that $2 \mid (n^4 - 3)$ if and only if $4 \mid (n^2 + 3)$.

(b) By using the definition of the absolute value function $|x|$, show that for any two real numbers x, y, $|xy| = |x||y|$.

QUESTION 1 (Continued).

QUESTION 2. **(15 marks)**

Determine if each of the following is true or false. Justify your answer.

(a) For every integer a, $5a - 9$ is odd if and only if $3a + 4$ is even.

(b) Let n be a positive integer. If $n^2 + \dfrac{1}{n^2} > 4$ then $n + \dfrac{2}{n} \geq 3$.

(c) There is an integer m such that $m^6 + 2m^4 + m^2 - 5 = 0$.

QUESTION 2 (Continued).

QUESTION 3. **(12 marks)**

Show each of the following statements. Justify your answers.

(a) Write down a biconditional predicate whose negation is logically equivalent to the following predicate: n^3 and $3n + 4$ are both odd or n^3 and $3n + 4$ are both even.

(b) Find all values of the integers a and b (if any) such that $n^2 + an + b$ is odd for every integer n. You need to justify why these are all the values.

QUESTION 3 (Continued).

QUESTION 4. **(8 marks)**

Find two predicates $P(n)$ and $Q(n)$ with domain $E =$ the set of positive even numbers, such that $P(n) \wedge \neg Q(n)$ holds for infinitely many values of $n \in E$, and $\neg P(n) \wedge Q(n)$ holds for infinitely many values of $n \in E$. You should find a single pair of predicates $P(n)$ and $Q(n)$ satisfying both the conditions above. Justify your answer.

END OF PAPER