Exploring the Eco Impacts of Lamprey Sex Variability through an Enhanced Lotka-Volterra Model

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Abstract

The parasitic lamprey plays a significant role in the ecosystem. In order to examine the topic of lamprey gender ratios and their effects on the ecosystem, this study has developed a comprehensive model that explores the interplay between lamprey gender ratio variations and ecosystem responses. The model was analyzed utilizing genetic algorithms and differential equations, leading to a deeper understanding of how changes in lamprey gender ratios influence the ecosystem.

Problem 1: To examine the effect of shifts in lamprey sex ratio on **broader ecosystems**, this study initially develops a Logistic model to simulate the fluctuating dynamics within the lamprey population. Subsequently, an enhanced **Lotka-Volterra model** that incorporates sex ratio variations is formulated, and computational simulations are performed to examine the reciprocal influences between changes in sex ratio and factors such as food availability, spatial resources, and the quantity of prey.

Problem 2: To investigate the advantages and disadvantages of adaptive sex ratio variation in lampreys, four individual models were combined into a **multi-objective optimization model**. Utilizing **genetic algorithms** for iterative solutions, we determined the most favorable sex ratio to be **0.387 females**. This ratio demonstrated significant benefits in terms of reproductive success rate and population resilience; however, it also posed challenges related to resource allocation and genetic diversity throughout the ecosystem.

Problem 3: To evaluate its influence on **ecosystem stability**, factors that affecting the stability were classified into two categories, which are species diversity and environmental resource sustainability. By employing the **Shannon diversity index** to gauge species diversity, we developed a thorough ecosystem stability model. The findings suggested that alterations in gender ratio contributed to heightened species diversity and diminished resources, leading to an increase in ecosystem stability for lamprey populations.

Problem 4: To explore whether lampreys confer benefits to other species within the ecosystem, we initially analyzed their effects on both prey and predators. Subsequently, we concentrated on their function as hosts for parasites. Utilizing a revised version of the Lotka-Volterra model for studying **Parasitism-Host interactions**, we carried out simulations that demonstrated a steady rise in parasite populations within lampreys affected by infections.

Lastly, a **sensitivity analysis** is performed on the developed model, revealing that it possesses robust sensitivity characteristics. The paper culminates with a discussion section and an overall assessment of the constructed models.

Keywords: Lamprey; Sex variation; GA-MOP Model; Lotka-Volterra Model; Ecosystem

Contents

1 Introduction

1.1 Problem Background

Sea lampreys, sometimes inaccurately called lamprey eels, are fascinating aquatic creatures found in diverse environments. They possess a distinctive circular, toothed mouth used for parasitic feeding[1]. Sea lampreys exhibit adaptive sex ratio variation, adjusting their gender ratios based on food availability during larval growth. This intriguing phenomenon raises questions about its ecological impact, advantages, and disadvantages[2]. This adaptive sex ratio variation may have profound implications for the ecological role of sea lampreys and their interactions with other species[3]. This study aims to explore the ecological effects of sea lampreys adjusting their gender ratios under different resource availability conditions through model development.

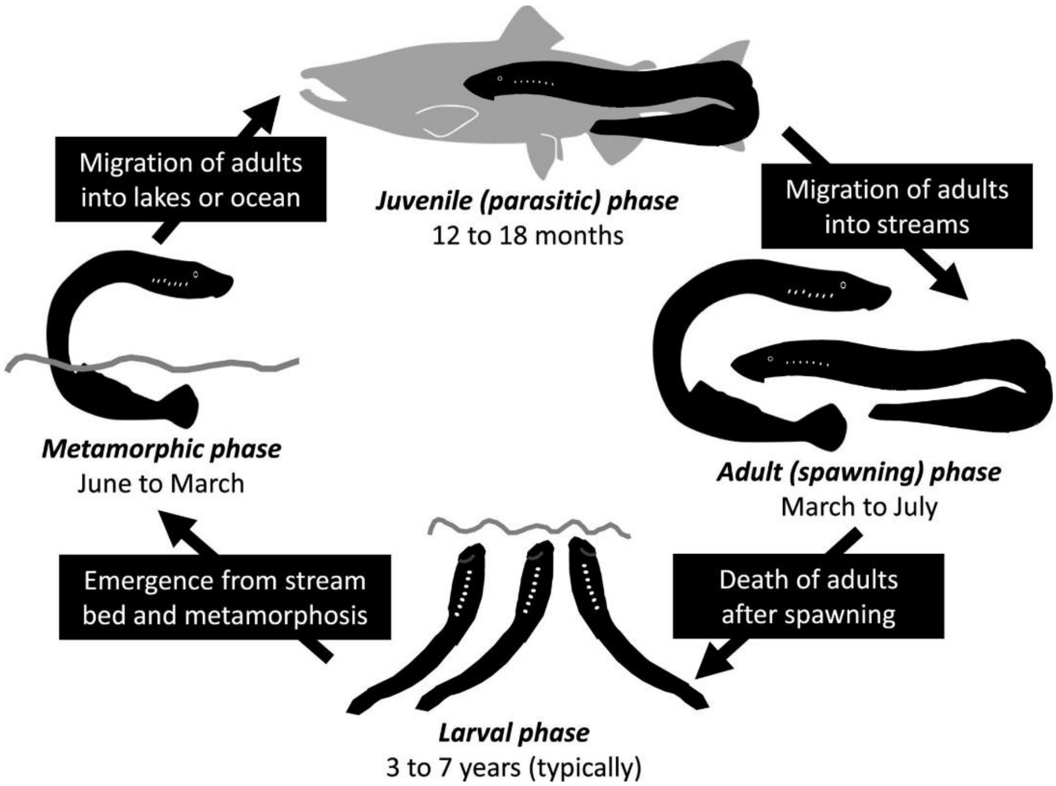

Figure 1: The sea lamprey life cycle annotated with approximate duration and timing of each life stage.

1.2 Restatement of the Problem

This paper aims to establish an appropriate model to investigate the multidimensional impact of gender changes in the lamprey population due to resource availability on the ecosystem.

Given the background information and problem statements defined under specific constraints, we need to address the following objectives:

Initially, we must develop a model to analyze the influence of lampreys being able to alter their gender ratios on a **broader ecosystem**.

Subsequently, an analysis is required to assess how the lamprey population's adjustment of gender ratios affects its own **advantages and disadvantages**.

Expanding on our prior models, it's crucial to adapt them to intricate environments, factoring in the variations in sex ratios of lampreys and their diverse impacts on the stability of the ecosystem.

Ultimately, it is essential to assess if a lamprey population with fluctuating sex ratios can confer benefits to other species within the ecosystem, including parasites.

1.3 Our Work

We began by establishing a Logistic model to simulate the dynamic changes in the lamprey population. Following that, we developed an improved **Lotka-Volterra model** that incorporates gender ratios to quantify the specific effects.

Next, we developed individual single-objective models for each of the four aspects related to gender ratio changes. We then combined these models to create a **multi-objective optimization model** that simultaneously considers the impact of gender ratio changes on these four aspects. We utilized a genetic algorithm to solve for the optimal gender ratio that achieves a balance among these different factors.

Then we based the ecosystem stability model on various population dynamics models and the **Shannon diversity index**, with variables representing species diversity and environmental resources.

Finally, we extended the Lotka-Volterra model to investigate the dominance of lampreys, particularly concerning other species, especially parasites, based on the parasitic-host relationship.

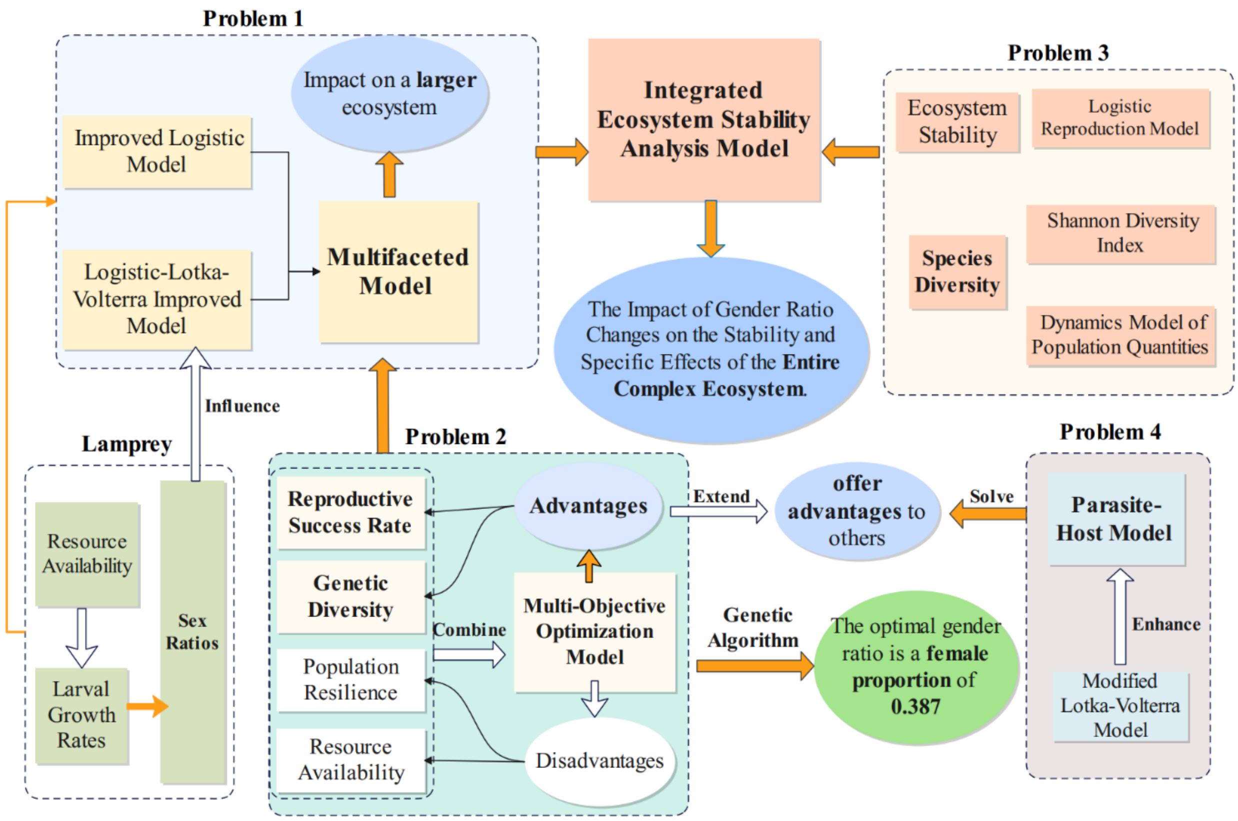

Figure 2: Our Work

2 Assumptions and Justifications

In order to simplify the problem and make it convenient for us to simulate the real conditions, we made the following assumptions, and each of them is properly justified.

- **Assumption 1**: The effect of lampreys as predators on predators is not considered.

Justification: It basically has no natural enemies and is preyed by humans, but has little effect on humans

- **Assumption 2**: The number of species in the community in the ecosystem will not mutate.

Justification: We assume a relatively closed system where no new species are introduced for a short period of time and cannot colonize the sevengill eel community currently under study. [4]

- **Assumption 3**: Species do not mutate, but populations and the resources they control change over time.

Justification: The time scale of the study was relatively short, and genetic changes and mutations that could lead to ecological changes were assumed to be negligible.

- **Assumption 4**: Competitive or mutually beneficial relationships between species in the community remain constant.

Justification: For simplicity the model focuses on the effects of changes in the sex ratio of sevengill eels on the ecosystem because interactions between species in the community are complex and may vary over time due to a variety of factors. [5]

- **Assumption 5**: It is assumed that other environmental factors do not change drastically to interfere, and only the effects of food availability variables are considered.

Justification: This is because food availability has a significant effect on hatchling growth rate, while sex ratio is mainly affected by hatchling growth rate.

3 Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations used in this paper

Symbol	Description
N	Lamprey Population Size
D	Availability of Food Resources
K	Carrying Capacity of the Environment
r_P	Intrinsic Growth Rate of the Population
$p_{(t)}$	Probability of Developing into Females (Gender Ratio)
P	Population of Predators
B_{eff}	Reproductive Success Rate
R_{eff}	Resource Allocation Efficiency
R_t	Resource Quantity
P_a	Parasite Population Quantity
I_t	Number of Lampreys Infected by Parasites

H	Shannon Diversity Index
E_t	Number of Other Predators
b_M, b_F	Efficiency of Predation for Male and Female Lampreys
φ_M, φ_F	Parasite Infection Rate in Male and Female Lampreys

4 Problem 1: Logistic -Lotka-Volterra Improved Model

4.1 Analysis of Factors Affecting the Ecosystem

The changes in the gender ratio of lampreys can indeed have complex and diverse effects on larger ecosystems. First, it can impact their own reproduction. Secondly, it can impact the food chain. Variations in the ratio of male to female lampreys in predator diets may necessitate adjustments in predator numbers and behavioral patterns. Additionally, it can influence the utilization of habitat resources[6]. Based on these impacts, we considered building both Logistic and gender ratio models. Then, we improved upon the Lotka-Volterra model and ultimately proposed a multidimensional model to quantify the specific effects of lamprey gender ratio on the ecosystem.

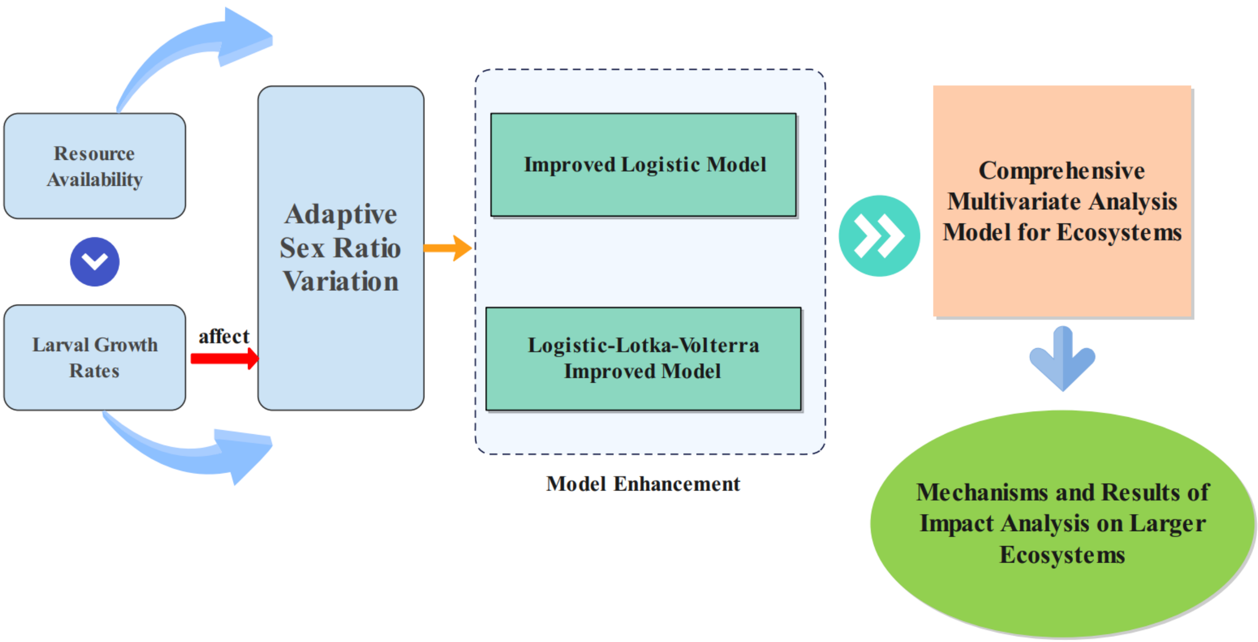

Figure 3: Establishment Process of Model

4.2 The Establishment of Logistic-Lotka-Volterra Improved Model

4.2.1 Improved Logistic Model Considering Food Availability and Gender Ratio Changes

We will construct a comprehensive model, integrating the Logistic growth model with the empirical data of sea lampreys, to elucidate the population dynamics of sea lampreys and its ramifications on the broader ecological system. This model will meticulously incorporate the influence of food availability and gender ratios on population dynamics.

The Logistic model stands as a cornerstone in ecological modeling, conventionally employed to delineate population dynamics within the constraints of finite resources. Its canonical form is expressed as follows:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \tag{1}$$

$$K = \gamma D + \beta S \tag{2}$$

In the provided equation, where N represents the population size of sea lampreys, t denotes time, r signifies the intrinsic growth rate of the population, Drepresents food availability (the number of prey), S denotes spatial resources, and K represents the carrying capacity of the environment. It's worth noting that the carrying capacity of the environment, K, is influenced primarily by food availability and spatial resources. Here, γ represents the impact parameter of food availability on the carrying capacity, and β represents the impact parameter of spatial resources on the carrying capacity.

Firstly, considering a scenario with limited resources, we enhance the original Logistic-based model by incorporating both food availability and resource accessibility. We establish the equation for the Verhulst model as follows:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - \alpha N^2 \tag{3}$$

Here, α represents the saturation parameter for population growth.

Furthermore, we will consider the impact of gender ratios on the population growth of sea lampreys. We will enhance the Logistic model by introducing a gender ratio factor to describe how food supply affects the development rate during the larval stage, consequently influencing gender ratios. The improved model is as follows:

$$p(t) = \frac{1}{1 + e^{-r(D)t}} \tag{4}$$

This model, based on the Logistic function, exhibits an S-shaped curve and is commonly utilized to describe the development or variation of a particular characteristic within a biological population. Considering the gender determination model of sea lampreys may need to account for the influence of food availability on the development rate, the model can be further extended to incorporate environmental factors. Here, food availability D can be considered as a regulating parameter denoted as $r(D)$, signifying the impact of food availability on the development rate. Additionally, $p(t)$ represents the probability of developing into a female (gender ratio), where we can interpret $p(t)$ as the probability of larval development into females. The parameter r signifies the rate of development, assuming that the time when the gender ratio begins to change is at $t=0$.

4.2.2 Improved Lotka-Volterra Model

We extend the optimized Lotka-Volterra model to consider changes in the gender ratio of sea lampreys and their impact on the number of offspring, as well as the number of predators and resource availability[7]. Firstly, we need to modify the dynamic equations for male and female sea lampreys in the Lotka-Volterra model to account for changes in gender ratios.

$$p(t) = \frac{1}{1 + e^{-rt}} \tag{5}$$

$$\frac{dM}{dt} = r_M M - (d_{M1} + d_{M2})M + b_M aPM \tag{6}$$

$$\frac{F}{M+F} = p(t) \tag{7}$$

$$\frac{dF}{dt} = r_F F - (d_{F1} + d_{F2})F + b_F aPF \tag{8}$$

$$\frac{dP}{dt} = r_P P - aP(M+F) \tag{9}$$

In this model, sea lampreys, as predators, primarily impact the ecosystem through changes in their own population size and the population size of their prey. In the model, M, F, and P represent the population sizes of male sea lampreys, female sea lampreys, and prey species, respectively. Parameters r_M, d_{M1}, and d_{M2} represent the growth rate, natural mortality rate, and mortality rate due to human predation of male sea lampreys, while b_M and b_F represent the predation efficiencies of male and female sea lampreys (efficiency of converting food into population growth). Similarly, parameters r_F, d_{F1}, and d_{F2} represent the growth rate, natural mortality rate, and mortality rate due to human predation of female sea lampreys. The parameter a represents the probability of prey being consumed, and r_P is the natural growth rate of the prey species.

4.3 The Impact Results on a Larger Ecosystem

4.3.1 Results and Analysis of the Logistic Improved Model

For the Logistic optimization model, we utilized Python tools to obtain four different dynamic system graphs depicting the model's variables' changes over time. The time span considered spans 100 breeding seasons, and the results are shown in the following figures:

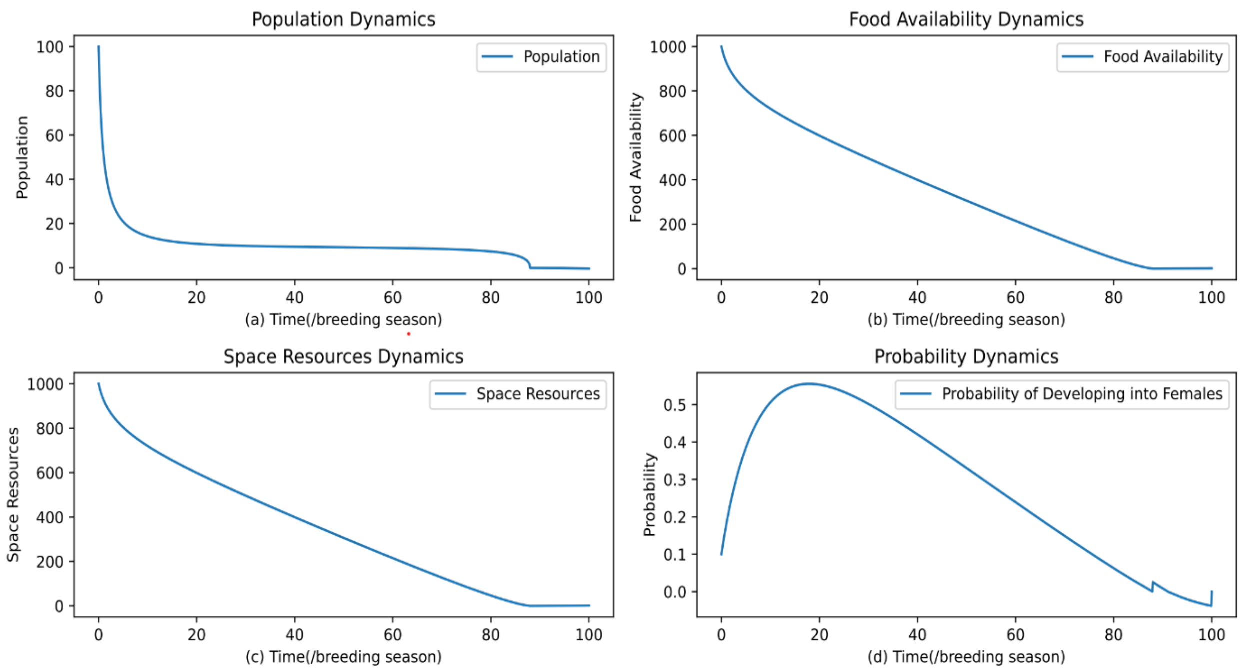

Figure 4: Results of the Logistic Improved Model

Figure (a) depicts the Population Dynamics of sea lampreys, showing that initially, the population of sea lampreys was very high, but over time, it rapidly declined, eventually stabilizing at a lower level, possibly indicating resource depletion. Figures (b) and (c) illustrate the availability of food and spatial resources decreasing steadily over time, with no sharp increases or decreases. This suggests that resources within the ecosystem are being consumed without sufficient replenishment, and the biological population may be facing issues related to limited space or habitat availability, significantly affecting gender ratios. Figure (d) shows the changing trend in the probability of sea lampreys developing into females over time. It indicates that the probability of lampreys developing into females initially rises rapidly to 0.5, and then gradually decreases to 0. In summary, these graphs describe the simulated effects of sea lamprey gender ratio changes on the larger ecosystem in which sea lampreys inhabit.

To verify the accuracy of this model, we visualized the simulation results after altering key parameters as shown in the following figure:

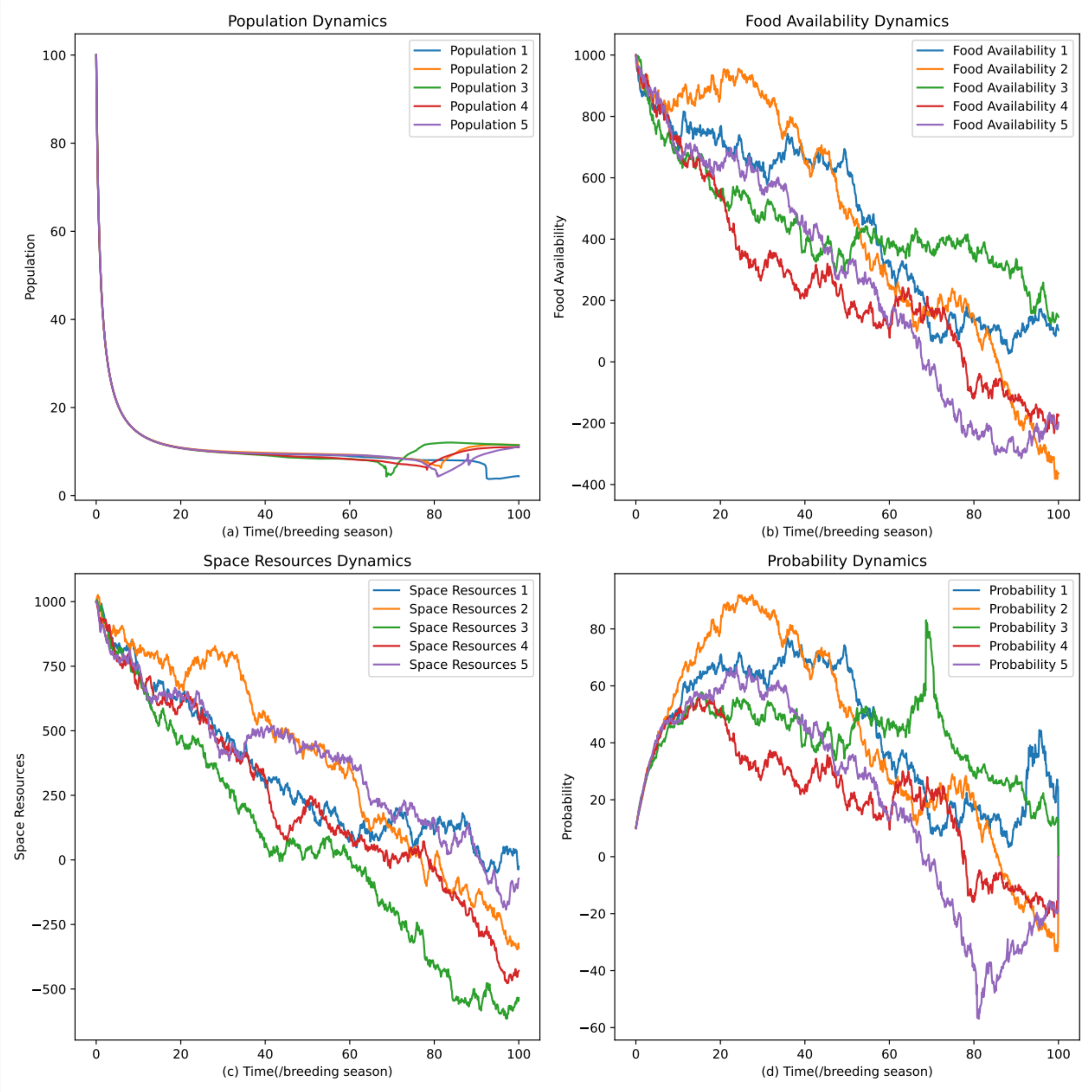

Figure 5: Sensitivity Analysis Results of the Improved Logistic Model

From these figures, we can observe that the lamprey population, food availability, spatial resources, and the proportion developing into females all exhibit systematic changes in response to various factors. This indicates that our model possesses a high level of sensitivity.

In summary, these charts depict the simulation results of how the lamprey population, within a larger ecosystem, is influenced by changes in lamprey gender ratios.

4.3.2 Results and Analysis of the Improved Lotka-Volterra Model

For the modified Lotka-Volterra model tailored to the specific lamprey issue analysis, we executed it using Python tools and obtained the results depicted in the figure below:

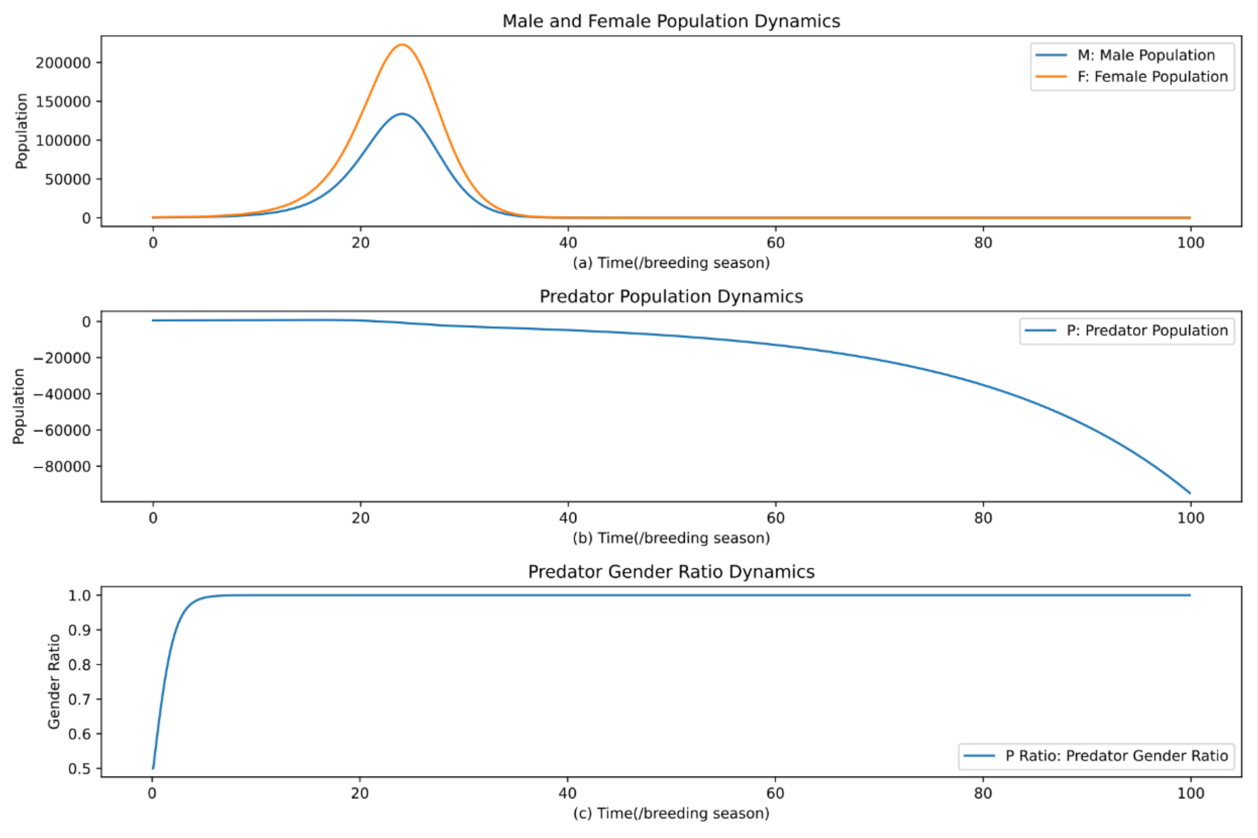

Figure 6: Results of the Improved Lotka-Volterra Model

Figure (a) displays a distinct peak in the number of males and females over the course of time, indicating a rapid increase in population size at specific time points, followed by a decline. This peak might be associated with seasonal reproduction or resource abundance.

Figure (b) demonstrates the continuous decrease in the predator population over time. The decline in the predator population could be due to a reduction in prey resources or the survival pressures on predators themselves. Such a declining trend could lead to ecosystem imbalances since predators play a crucial role in controlling prey populations.

Figure (c) shows the dynamic changes in gender ratios within the predator population, revealing an initial increase from 0.5 followed by stabilization. This may indicate that within the timeframe set by the model, the gender distribution among predators is balanced and stable.

In summary, these result graphs illustrate a complex ecosystem where population dynamics are influenced by multiple factors, including reproductive behavior, resource availability, and predator-prey relationships.

5 Problem 2: Multi-objective Optimization Model Based on Genetic Algorithm

5.1 Factors Influencing Self-Advantage

The changes in the gender ratio of sea lampreys may affect their population's adaptability to environmental changes. To highlight the advantages and disadvantages of altering the gender ratio of sea lampreys in response to environmental factors, we have developed mathematical models focusing on four aspects: reproductive success rate, resource allocation, population resilience, and genetic diversity. These models aim to characterize the implications of gender ratio variations on sea lamprey populations in the face of changing environmental conditions.

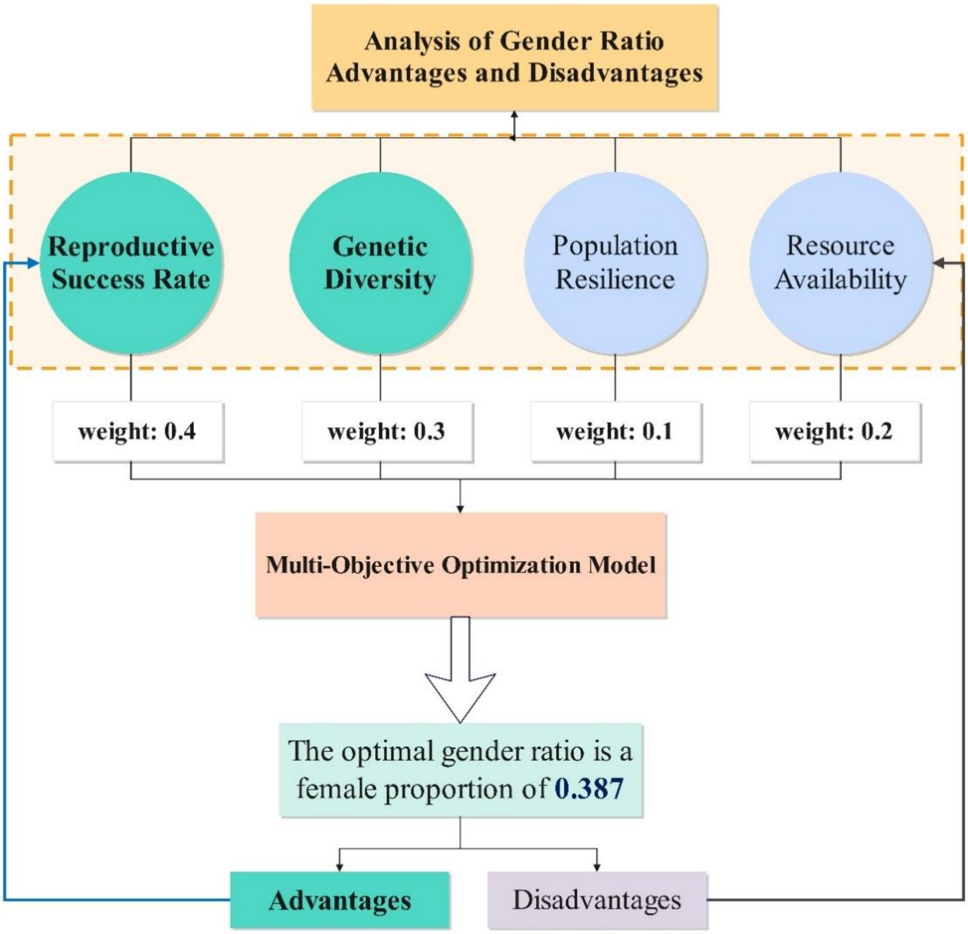

Figure 7: Conceptual Model Analysis Diagram

5.2 Genetic Algorithm Multi-Objective Optimization Model

When characterizing the advantages and disadvantages of the evolution of sea lamprey sex ratio variation, we plan to analyze and model it through both a single model simulation and a combined model approach.

5.2.1 Single Model Simulation

Initially, each model will be simulated distinctly to investigate the repercussions of gender ratio shifts on reproductive success, resource allocation, population resilience, and genetic diversity. This phase involves a detailed assessment, pinpointing both the favorable outcomes and potential drawbacks linked to each aspect.

The first aspect involves establishing the following model to quantify the impact of gender ratio imbalance on reproductive opportunities:

$$B_{eff}=B_i \cdot \min\left(p, \frac{1}{p}\right) \tag{10}$$

This model is used to adjust the basic reproductive rate B_i to reflect the impact of gender ratio imbalance on reproductive opportunities. This adjustment factor ensures that regardless of the gender ratio bias, the reproductive rate is constrained, simulating the potential negative impact of an excessive or insufficient number of a certain gender on reproductive success.

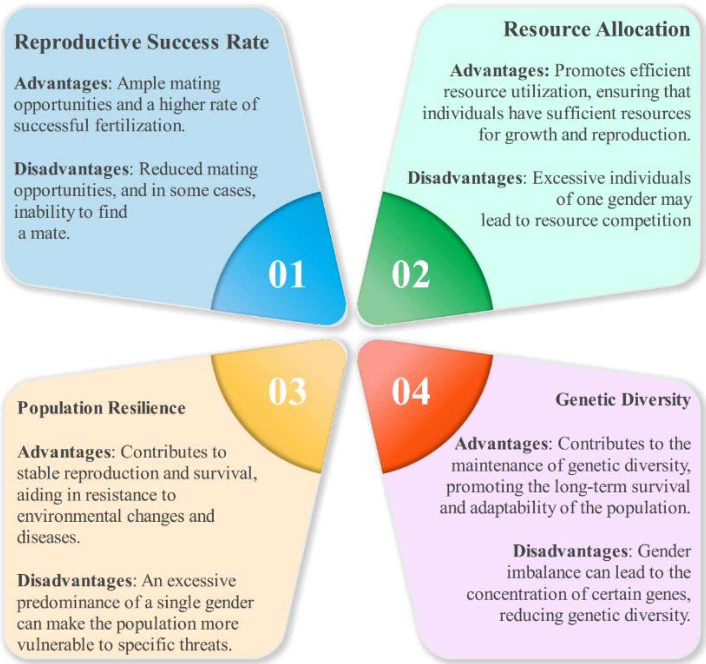

Figure 8: Analysis of the Four Single-Factor Models

In the second aspect, we assume a scenario where resource efficiency varies nonlinearly with the deviation of gender ratio from the optimal value. We use a Gaussian function to combine the basic resource efficiency with gender ratio, and based on the Gaussian function (or normal distribution curve) adjustment, we provide a method to quantify the impact of gender ratio deviation on resource efficiency. This expresses how gender ratio p affects resource allocation efficiency. We establish the following model:

$$R_{eff}=R_i \cdot e^{-\alpha \cdot (p-popt)^2} \tag{11}$$

The core idea of the model is that there exists an optimal gender ratio p_{opt} at which resource efficiency is maximized. When $p = p_{opt}$, $e^{-\alpha\cdot(p-popt)^2}$ =1, indicating that resource efficiency is unaffected and equals Ri. When p deviates from p_{opt}, $e^{-\alpha\cdot(p-popt)^2}$ decreases, representing a decrease in resource efficiency.

For the third aspect, considering that deviations in gender ratio from the optimal ratio can reduce the **population's resilience**, we established the following model:

$$\text{Resilience} = 1 - \delta \cdot |p - p_{\text{opt}}| \tag{12}$$

Where δ represents the impact of gender ratio deviation from the optimal value on population resilience, indicating the rate or extent of the population's resilience reduction when the gender ratio deviates from the optimal value. A larger δ value implies that even slight changes in gender ratio can significantly decrease the population's resilience.

In the fourth aspect, deviations in gender ratio from the optimal ratio will reduce the **genetic diversity** of the population. We establish a model to quantify the impact of gender ratio on genetic diversity as follows:

$$\text{Diversity} = 1 - \sigma \cdot |p - p_{\text{opt}}| \tag{13}$$

Where p_{opt} represents the optimal gender ratio, and σ represents the rate or extent of genetic diversity reduction when the gender ratio deviates from the optimal value. A larger σ value indicates that even slight changes in the gender ratio can significantly reduce the genetic diversity of the population. When $p = p_{\text{opt}}$ resilience reaches its maximum (i.e., 1).

Through simulations of the above individual models, we can quantify the potential advantages and disadvantages of adjusting the gender ratio on various aspects of the sea lamprey population.

5.2.2 Combined Model Simulation

Next, we will combine the four models and consider the overall impact of sex ratio variation on these four aspects. To integrate the four individual models into a comprehensive model, we will create a multi-objective optimization framework, with each model being an objective function. This approach allows us to simultaneously consider the effects of sex ratio variation on these four aspects and find the optimal sex ratio that balances these different factors. Here are the steps for constructing this comprehensive model:

- **Step1. Define Objective Functions**

Firstly, define each aspect's model as an objective function, namely: $f1(p)$ for the reproductive success model, $f2(p)$ for the resource allocation model, $f3(p)$ for the population resilience model, and $f4(p)$ for the genetic diversity model.

- **Step 2. Establish Weight Factors**

Because different objectives may have varying levels of importance, introduce weight factors, denoted as ω_i, to represent the relative importance of each objective in the overall objective:

$$\omega_1, \omega_2, \omega_3, \omega_4 \geq 0 \tag{14}$$

$$\sum_{i=1}^{4} \omega_i = 1 \tag{15}$$

- **Step 3. Construct a Multi-Objective Optimization Problem**

Combine these objective functions into a comprehensive objective function to form a multi-objective optimization problem:

$$F(p) = \omega_1 \cdot f_1(p) + \omega_2 \cdot f_2(p) + \omega_3 \cdot f_3(p) + \omega_4 \cdot f_4(p) \tag{16}$$

The objective is to find an appropriate gender ratio, denoted as p that maximizes the comprehensive objective function $F(p)$.

- **Step 4. Solution and Decision-Making**

We use genetic algorithms through computer simulations with 100 iterations to find the optimal solution. This multi-objective optimization method allows us to comprehensively consider the impact of gender ratios on reproductive success, resource allocation, population resilience, and genetic diversity. Consequently, we aim to identify the gender ratio that is most suitable for the survival of sea lamprey populations $p_{solution}$ and analyze the advantages and disadvantages of altering gender ratios on their own population.

Table 2: The algorithm corresponding to Multi-Objective Optimization Model

Algorithm 2: Genetic Algorithm for Optimal Gender Ratio Solving

```
input: {Beff}, {Reff}, {Resilience}, {Diversity}
    creator.create("Fitness", base.Fitness, weights)
    creator.create("Individual", list, fitness)
      toolbox = base.Toolbox()
      toolbox.register()
  def main():
    pop = toolbox.population(n=100)
    stats = tools.Statistics()
    stats.register()
  pop, logbook = algorithms.eaMuPlusLambda()
output: {gen}, {avg}, {min}, {max}
```

5.3 Optimal Sex Ratio and Advantages and Disadvantages

We tried different weights when combining the four objectives, assuming equal importance for all objectives with equal weights. After conducting **100 iterations** using Python tools, we obtained the optimization results as shown in the following figure:

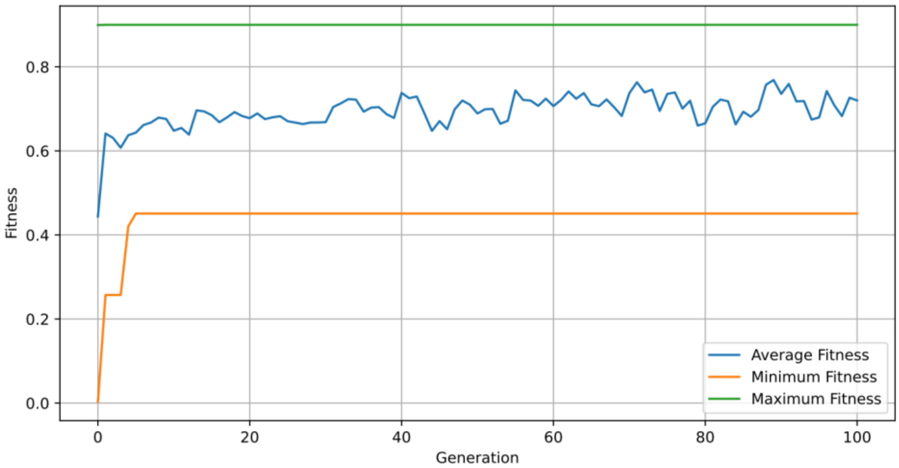

Figure 9: Multi-Objective Optimization Results Using Genetic Algorithm

In the final analysis, as the generations increase in the genetic algorithm optimization process, we observe how the population's fitness changes. The curve for maximum fitness remains relatively flat and consistently at the top, indicating that the fitness of the best individuals remains at a relatively high level throughout the evolution process.

The curve for average fitness rises from an initial low value to a higher stable value. The initial rapid increase suggests that the quality of individuals in the population rapidly improves, followed by a stabilization of fitness, indicating that the population has converged to a relatively stable state, with reduced differences in individual quality.

The curve for minimum fitness quickly rises at the beginning and then stabilizes, suggesting that even the poorest individuals in the population are improving in quality, resulting in an overall increase in population fitness.

Overall, the analysis reveals that with an increase in generations, all three curves tend to stabilize, and the gap between maximum fitness and average fitness is relatively small. This may indicate that individuals in the population have similar qualities, and the population has reached a relatively balanced state, possibly resulting in decreased diversity.The following figure displays the results obtained when simulating the four objectives separately:

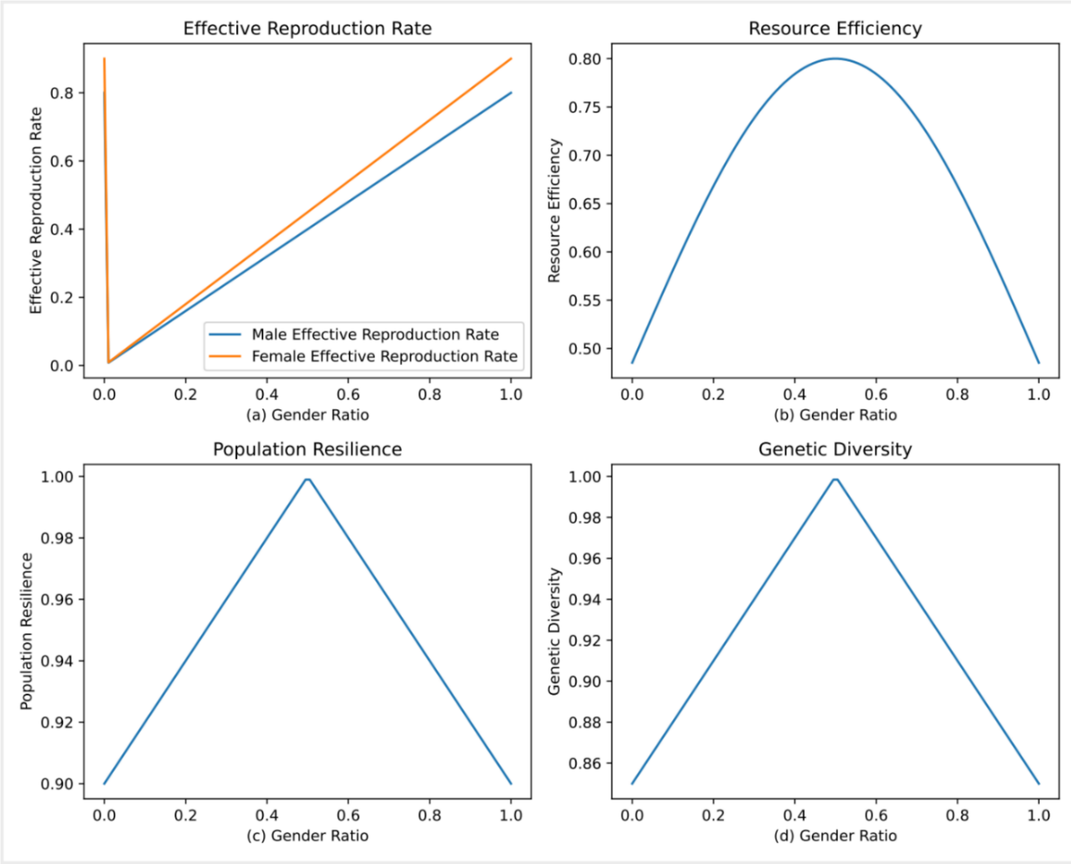

Figure 10: Results of single-objective simulations

Combining the results presented in the single-objective simulations, we can conclude that although the empirical optimal gender ratio is set to 0.5, the optimal gender ratio in the genetic algorithm may likely be associated with the individual corresponding to the maximum fitness, representing the gender ratio of the individual best suited to the environment among all individuals. In other words, it represents the optimal gender ratio under the current model and parameter settings. Thus, we obtain that the optimal gender ratio is when females account for **0.387**.

From this, we can infer that the change in gender ratio of lampreys has a significant advantage in terms of reproductive success and population resistance, but it has a disadvantage in terms of resource allocation within the entire ecosystem and genetic diversity. This may lead to a reduction in species diversity within the ecosystem.

6 Problem 3: Ecosystem Stability Model

6.1 Summary of Factors Affecting Ecosystem Stability

Ecosystem stability is a multidimensional concept encompassing aspects such as species diversity, stability of ecological niches, and the persistence of ecological processes like nutrient cycling. To investigate the impact of sea lamprey gender ratio changes on ecosystem stability, we have developed an ecosystem stability model based on various population dynamics models and the Shannon diversity index.

This model allows for a comprehensive assessment of ecosystem stability by considering both species diversity and environmental resource availability, thus gaining recognition among modeling experts in the field.

6.2 Ecosystem Stability Model Based on Shannon Diversity Index

We conducted separate studies to investigate the effects of sea lamprey gender ratio changes on both species diversity and environmental resources.

6.2.1 Dynamics Model of Population Quantities for Each Species

Taking into account the interactions between the sea lamprey population and other ecological factors such as predators, prey, environmental changes, and their effects on species diversity, we established a comprehensive population dynamic model. Using numerical methods, we simulated the dynamics of the ecosystem and analyzed the long-term effects of gender ratio changes on both the sea lamprey population and the entire ecosystem.

Population dynamics of sea lampreys:

$$\frac{dM}{dt} = r_M M - (d_{M1} + d_{M2})M + b_M aPM + cR_t - dE_t \tag{17}$$

$$\frac{dF}{dt} = r_F F - (d_{F1} + d_{F2})F + b_F aPF + cR_t - dE_t \tag{18}$$

Dynamics model of predators' population quantities:

$$\frac{dP}{dt} = r_P P - aP(M + F) \tag{19}$$

Dynamics model of other predators' population quantities:

$$\frac{dE_t}{dt} = \epsilon M - \tau E_t \tag{20}$$

6.2.2 Measuring Species Diversity Based on Shannon Diversity Index

Based on the aforementioned population dynamic models, we assume that the impact of sea lampreys on the biodiversity of their ecosystem is primarily considered from these three categories of organisms: sea lampreys, the prey of sea lampreys (such as certain small fish), and other predators. Biodiversity is typically assessed in terms of species richness (the number of species) and species evenness (the equitable distribution of species). We utilize the Shannon diversity index because it takes into account not only species richness but also the relative abundance distribution of species, making it suitable for characterizing the species diversity of an ecosystem.

For an ecosystem, the Shannon diversity index H is defined as follows:

$$H = -\sum_{i=1}^{G} p_i \ln(p_i) \tag{21}$$

Applying it to the sea lamprey ecosystem, where there are three types of organisms considered, $G = 3$. The total number of individuals A is given by:

$$A = N + P + E \tag{22}$$

The relative abundances are as follows:

$$p_N = \frac{N}{A}, p_P = \frac{P}{A}, p_E = \frac{E}{A} \tag{23}$$

Plugging these values into the Shannon diversity index formula, we get:

$$H = -(\frac{N}{A}\ln(\frac{N}{A}) + \frac{P}{A}\ln(\frac{P}{A}) + \frac{E}{A}\ln(\frac{E}{A})) \tag{24}$$

This index quantifies the biodiversity of the sea lamprey ecosystem. It's important to note that a higher Shannon index generally indicates higher species diversity and a more even distribution of species in the ecosystem. In practical applications, ecosystem health and stability are often positively correlated with species diversity.

6.2.3 Measuring Environmental Resource Sustainability Based on the Logistic Reproduction Model

Taking into account the effect of resource abundance on the reproduction rate using the Logistic Reproduction Model, where a larger resource quantity leads to a slower reproduction rate until reaching an equilibrium point, we establish the following dynamic resource quantity model:

$$\frac{dR_t}{dt} = \nu R(t)(1 - \frac{R(t)}{K}) - \varepsilon M \tag{25}$$

Different genders of sea lampreys may have distinct ecological roles and resource utilization patterns. Typically, females may rely more on food resources to support reproduction, while males may be more focused on competition and reproductive behavior. Therefore, we are only considering the impact of males on resources.

Finally, to comprehensively assess the impact of sea lamprey gender ratio changes on ecosystem stability, we define an ecosystem stability index Estability(t). This index can be a function involving both aspects of species diversity and environmental rηesources:

$$E_{stability}(t) = \eta_1 \ H(t) + \ \eta_2 \cdot R(t) \tag{26}$$

Table 3: The algorithm of Logistic-Lotka-Volterra Improved Model

Algorithm 1: Population Dynamics Simulation Process

```
input: {dM_dt}, {dF_dt}, {dP_dt}, {dEt_dt}, {dRt_dt}
def dy_dt(t, y):
    M, F, P, Rt, Et = y
      return [dM_dt, dF_dt, dP_dt, dRt_dt, dEt_dt]
        y0 = [M0, F0, P0, Rt0, Et0]
        t_span = [0, 100]
        t_eval = np.linspace(t_span[0], t_span[1], 1000)
        define para.
    sol = solve_ivp(dy_dt, t_span, y0, t_eval=t_eval)
output: yi = sol.y[i]
```

6.3 Impact Results on Ecosystem Stability

6.3.1 Population Dynamics and Environmental Resource Dynamics Results

We obtained the results of population dynamics and environmental resource dynamics simulations using Python tools, as shown in the following figures:

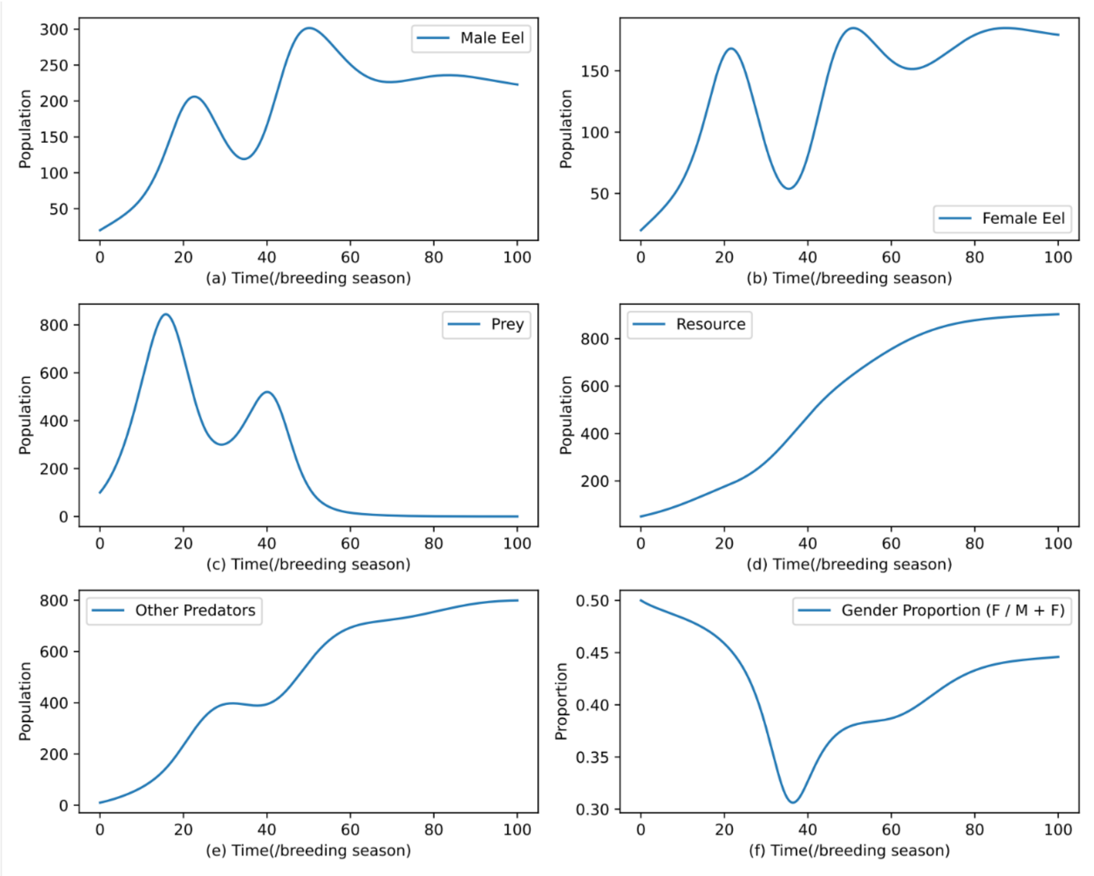

Figure 11: Population Dynamics and Environmental Resource Dynamics

Figures (a) and (b) depict periodic fluctuations in the numbers of male and female lampreys during different breeding seasons. These fluctuations may represent natural population oscillations or variations due to seasonal changes in resource availability. However, the female population exhibits a wider range and smoother fluctuations, suggesting that female individuals may have longer lifespans or more stable reproductive rates.

Figure (c) illustrates a declining trend in the population of lamprey prey over time, indicating a gradual reduction in prey resources. This decline may result from overpredation or other environmental pressures. Figure (d) represents periodic variations in environmental resource quantities, while Figure (e) displays similar periodic fluctuations in the population of other predators. This suggests potential competition between these predators and lampreys for the same resources or prey, or direct predation on lampreys.

Figure (f) shows that, with the passage of time, the proportion of female individuals in the total population gradually increases. This change may be attributed to higher male mortality rates or greater female survival rates, and it could potentially impact reproductive capacity and the long-term stability of the population.

In brief, changes in gender ratios may directly influence the population dynamics of male and female lampreys. An increasing proportion of females may lead to population growth, as females typically control reproductive capacity. The decline in prey populations may result from overpredation, potentially causing a decrease in the number of top predators in the food chain as food resources become limited. The periodic fluctuations in environmental resources could affect the population dynamics of various species, as these resources serve as the foundation for the survival and reproduction of many organisms in the ecosystem.

6.3.2 Integrated Effects Analysis Results

Results of comprehensive impact analysis using Python simulation are presented in the figure below:

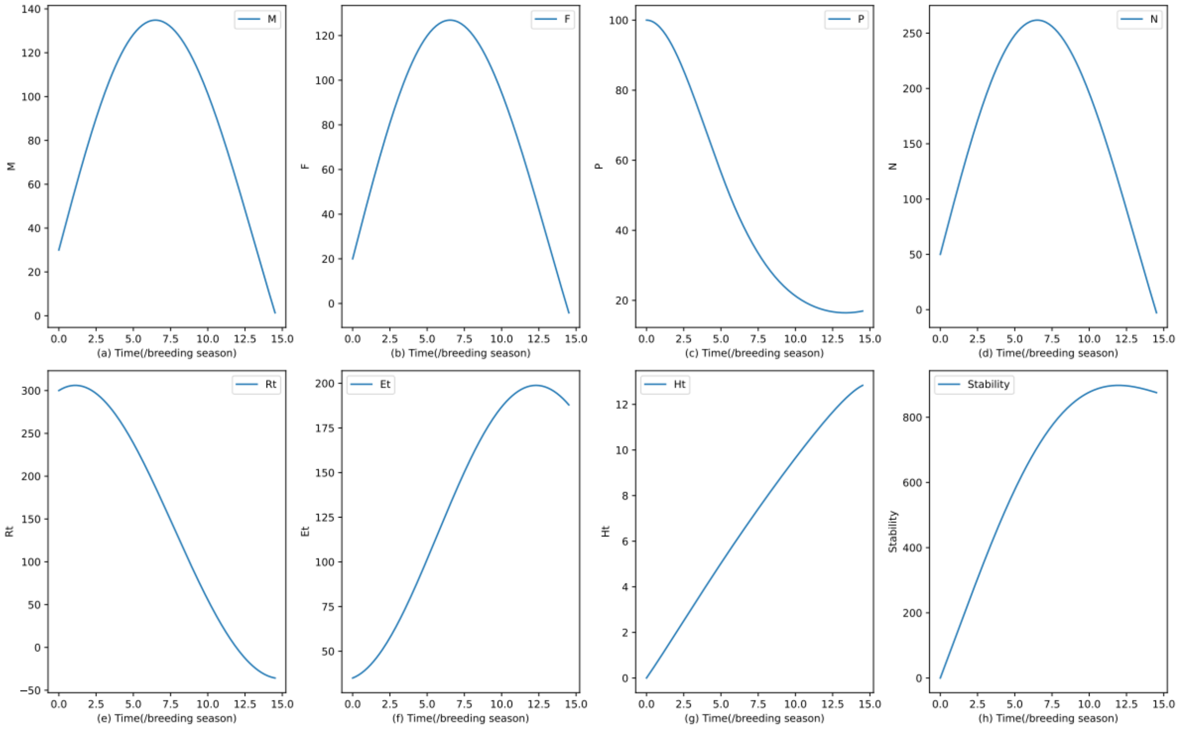

Figure 12: Results of Comprehensive Impact Synthesis

Figures (a), (b) and (d) show that the numbers of male, female, and total lampreys fluctuate over time, displaying unimodal oscillations, which may reflect seasonal breeding cycles or population fluctuations due to resource limitations. However, the peak number of female lampreys is lower than that of males, suggesting either a lower number of females or lower survival rates.

Figure (c) indicates a continuous decrease in the number of prey species over time, possibly due to the increase in lamprey population. Figure (e) demonstrates a significant decline in resource abundance over time, likely attributable to consumption by lampreys and other species.

Figure (g) shows a continuous linear increase in species diversity parameters, potentially indicating ecosystem recovery or enrichment. Figure (h) illustrates an enhancement in the overall ecosystem stability over time. This enhancement may result from interactions and balance between species diversity and environmental resources within the ecosystem.

Analyzing these sub-figures suggests that a decline in female numbers, leading to a decrease in reproductive rates, could indirectly impact the number of prey (P), as lampreys require less food. Simultaneously, the decline in resources could limit further growth of the lamprey population, contributing to ecosystem stability. Increasing species diversity may support more species and a higher lamprey population, thereby enhancing overall ecosystem stability.

In summary, these results suggest that changes in gender ratios may indirectly influence multiple aspects of the ecosystem, including prey numbers, other predator numbers, species diversity, and environmental resource levels, through their effects on reproductive success and population dynamics, ultimately affecting the overall stability of the ecosystem.

7 Problem 4: Parasite-Host Model Based on Modified Lotka-Volterra Model

7.1 Determining Relationships with Other Species

In the previous three questions, we have addressed the impact of gender ratio changes in lampreys on their prey and other predators, as well as their impact on ecosystem stability. This question will consider the effect of lampreys as hosts on their parasites, studying the advantage of lampreys on other species in the ecosystem.

7.2 Parasite-Host Model

Considering that changes in gender ratio can affect population reproductive rates and thus impact the availability of food chains and parasite hosts, we primarily choose a dynamic systems model to analyze the impact of gender ratio changes on the parasite population. We extend the Lotka-Volterra model to include gender ratio as a variable[8]. For the relationship between parasites and hosts, we established the following host-parasite model:

$$\frac{dM}{dt} = (r_M - d_{M1} - d_{M2})M\left(1 - \frac{M+F}{K}\right) - \varphi_M P_a M + \phi I_t \tag{27}$$

$$\frac{dF}{dt} = (r_F - d_{F1} - d_{F2})F\left(1 - \frac{M+F}{K}\right) - \varphi_F P_a F + \phi I_t \tag{28}$$

Parasite population dynamics:

$$\frac{dP_a}{dt} = \theta I_t - \lambda P_a \tag{29}$$

$$\frac{dI_t}{dt} = \varphi_M P_a M + \varphi_F P_a F - (\varsigma_I + \phi) I_t \tag{30}$$

In this model, the lamprey, as the host, and its changing gender ratio may influence the transmission and dominance of parasites. Considering the gender ratio as a key variable for analyzing the dominance of the parasite population, changes in the gender ratio can be simulated by adjusting φ_M and φ_F which reflect the impact of gender ratio on infection rates.

7.3 The Advantage of Parasites

With the assistance of Python tools, the simulated population dynamics of lampreys as hosts and their parasites are depicted in the following figure:

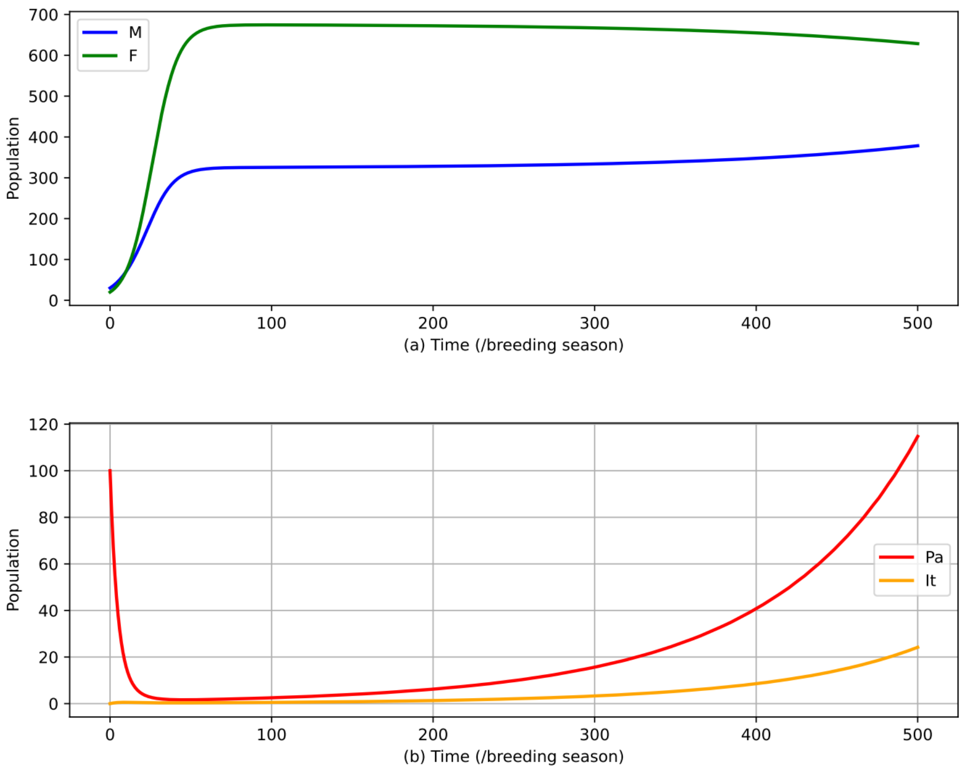

Figure 13: Population Dynamics Results of the Parasitic-Host Model

Figures (a) and (b) show that the number of male and female lampreys initially increases rapidly and then stabilizes over time. This may indicate a phase of rapid population growth initially, followed by reaching the environmental carrying capacity, resulting in population stability. However, the number of females reaches a higher stable value compared to males, suggesting that female lampreys may have higher survival rates or reproductive rates.

Figure (c) indicates that the number of parasites starts off low and gradually increases, showing exponential growth in later stages. This usually suggests that the parasites have found a suitable host environment and started reproducing extensively, possibly because the increasing host population provides more opportunities for infection.

Figure (d) shows that the number of lampreys infected by parasites is initially low but also exhibits exponential growth over time. This suggests that as time progresses, the parasites successfully infect more and more lampreys. This growth is likely directly related to the growth of the parasite population, as more parasites mean higher infection opportunities.

Overall, these graphs may represent a typical dynamic equilibrium between hosts and parasites in an ecosystem. Initially, the rapid growth of the host population provides ample infection targets for parasites, and as the host population stabilizes, the parasites also start to increase, reflecting the dependence of parasites on the host population size. In some cases, if the growth rate of parasites is too high, it may eventually lead to a decrease in the host population, as high infection rates can increase host mortality.

More importantly, we observe that despite the growth of the parasite population shown in the graphs, there is no decrease in the host population numbers. This may indicate that the host population can withstand the current infection pressure from parasites. However, if the trend of parasite growth continues, they may exert increasing pressure on the host population, ultimately affecting the health and survival of the host population.

Therefore, changes in the gender ratio of lampreys when they act as hosts can contribute to increasing ecosystem stability, benefiting their own population reproduction, and providing a stable living environment and reproductive opportunities for their parasites.

8 Sensitivity Analysis

In the previous questions, we have conducted sensitivity analysis on the improved Lotka-Volterra model with gender ratio inclusion . We can observe that the impact of gender ratio changes in sea lampreys on the larger ecosystem shows a good fit with parameter variations.

In various population dynamic models, we have defined a range of variation for each key parameter, including growth rates (r_M, r_F, r_P), mortality rates (d_{M1}, d_{F1}), predation rate (a), resource impact coefficient (c), and other predator impact coefficients (d, ε, τ). Due to the large number of parameters in the model and the need to understand the interactions between parameters on the output of this model, we simultaneously changed multiple parameters and analyzed the simulation output results of each multi-parameter change in comparison to the baseline model (original parameter settings).

The following graph displays the results of five simulations:

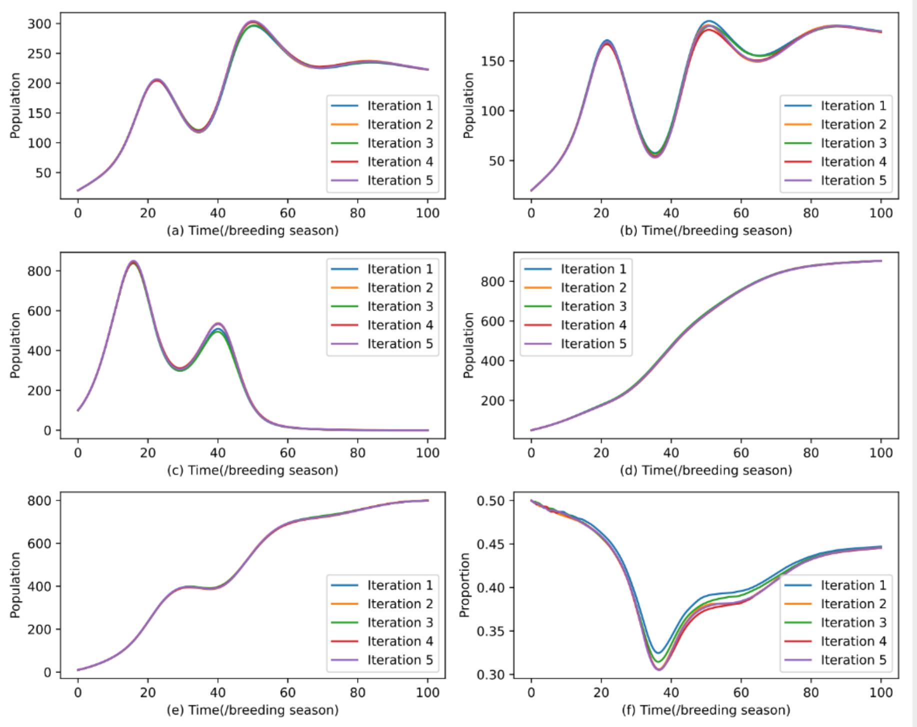

Figure 14: Sensitivity Test Results for Population Dynamics of Various Groups

Taking into account the results of the comprehensive sensitivity analysis, it can be concluded that the populations of male and female lampreys, prey, and resources exhibit systematic variations in response to various factors. Therefore, **our model demonstrates excellent sensitivity.**

9 Model Evaluation and Further Discussion

9.1 Strengths

Flexibility: The model can be adjusted to different ecosystems and species, making it applicable in various contexts.

Novelty: We utilize the Shannon diversity index to measure species diversity and employ a multi-objective optimization model to determine the optimal gender ratio, which is a novel approach to analyzing the effects of lamprey gender ratio changes on their advantages and disadvantages.

Comprehensiveness: Our model integrates various classic and scientific mathematical models, such as differential equation models for calculating species population dynamics and multi-objective optimization models for optimizing gender ratios. We have made adjustments, simplifications, and modifications to these models, ultimately constructing our comprehensive model.

Uniform Simplicity: The basic framework and equations of our model remain consistent, primarily based on the Lotka-Volterra model. When considering more complex factors, we simply introduce parameters into the equations, making them more intricate, yet yielding excellent results. The entire model for the dynamics of population sizes in the lamprey ecosystem is uniform and concise.

9.2 Weaknesses

Our results may have certain limitations and may not cover all possible scenarios. To further investigate the impact of lamprey gender ratio changes on the ecosystem, additional empirical evidence on population sizes of various species is required.

Due to constraints in experimental data, some parameter settings in the model rely on computer simulations, and the weighting of factors in the optimization model is also influenced by subjective experience.

Given time constraints, it was not possible to specify the exact identity of the prey and other predators. Different prey and predators may have slightly different growth patterns.

9.3 Further Discussion

Certainly, scientific studies have indicated that lampreys often display varying preferences for spawning habitats of different sizes. Typically, they exhibit a strong affinity for shallow riffle habitats within stream environments, characterized by fast-flowing water over rocky substrates[9]. These spawning habitats have distinct environmental conditions that may interact with the fluctuations in lamprey gender ratios. In our next phase of research, we intend to incorporate the dynamics of spawning habitat alterations into our model, focusing on critical factors such as water temperature, water depth, and flow velocity to enhance the precision and scientific rigor of our modeling efforts.

10 Conclusion

The population dynamics model simulations reveal that there are complex interactions between changes in lamprey gender ratios and several factors, particularly evident in population reproductive rates, resource availability, and predator-prey relationships.

The optimal gender ratio for lampreys to gain advantages across different factors, including reproductive success, resource allocation, population resistance, and genetic diversity, is determined to be a **female proportion of 0.387**. This is particularly prominent in the context of changes in lamprey gender ratios affecting reproductive success and population resilience, while showing a disadvantage in terms of resource allocation and genetic diversity within the entire ecosystem.

The changes in lamprey gender ratios can indirectly impact various aspects of the ecosystem, including prey abundance, the population of other predators, species diversity, and environmental resource levels, ultimately influencing the overall stability of the ecosystem.

When lampreys undergo gender ratio changes as hosts, it can enhance the stability of the ecosystem, benefiting their own population reproduction and providing a stable living environment and reproductive opportunities for their parasites.

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