10.1.5. Consider the following summary data on the modulus of elasticity (× 10^6 psi) for lumber of three different grades [in close agreement with values in the article **"Bending Strength and Stiffness of Second-Growth Douglas-Fir Dimension Lumber"** ***(Forest Product J.,*** **1991: 35–43),** except that the same sizes that were larger]:

Grade	J	$\bar{x}_{i\cdot}$	s_i
1	10	1.63	.27
2	10	1.56	.24
3	10	1.42	.26

Use this data and a significance level of .01 to test the null hypothesis of no difference in mean modulus of elasticity for the three grades.

μ_i = true mean modulus of elasticity for grade i (i = 1, 2, 3). $H_0: \mu_1 = \mu_2 = \mu_3$ vs H_a: at least two μ_i's are different. Grand mean = $\frac{1.63(10)+1.56(10)+1.42(10}{30} = 1.5367$.

$$\text{MSTr} = \frac{J}{I-1}[(\bar{X}_{1\cdot} - \bar{X}_{\cdot\cdot})^2 + (\bar{X}_{2\cdot} - \bar{X}_{\cdot\cdot})^2 + (\bar{X}_{3\cdot} - \bar{X}_{\cdot\cdot})^2]$$
$$= \frac{10}{3-1}[(1.63 - 1.5367)^2 + (1.56 - 1.5367)^2 + (1.42 - 1.5367)^2]$$
$$= 5[(0.0933)^2 + (0.0233)^2 + (-0.1167)^2] = .1143$$
$$\text{MSE} = \frac{S_1^2 + S_2^2 + S_3^2}{I} = \frac{.27^2 + .24^2 + .26^2}{3} = .0660, f = \frac{\text{MSTr}}{\text{MSE}} = \frac{.1143}{.0660} = 1.73$$

At df = (2, 27), 1.73 < 2.51, so the P-value is more than .10. Therefore we fail to reject H_0.

Ans: The three grades do not appear to differ significantly

10.1.7. An experiment was carried out to compare electrical resistivity for six different low-permeability concrete bridge deck mixtures. There were 26 measurements on concrete cylinders for each mixture; these were obtained 28 days after casting. The entries in the accompanying ANOVA table are based on information in the article **"In-Place Resistivity of Bridge Deck Concrete Mixtures" (*ACI Materials J.*, 2009: 114–122)**. Fill in the remaining entries and test appropriate hypotheses.

Source	df	Sum of Squares	Mean Square	f
Mixture	5	3575.065	715.013	51.3
Error	150	2089.350	13.929	
Total	155	5664.415		

Let μ_i denote the true mean electrical resistivity for the ith mixture (i = 1, …, 6). $H_0: \mu_1 = \cdots = \mu_6$ vs H_a: at least two of the μ_i's are different. There are I = 6 different mixtures and J = 26 measurements for each mixture. SSE $= I(J-1)MSE = 6(26-1)(13.929) = 2089.350$. $\text{SSTr} = \text{SST} - \text{SSE} = 5664.415 - 2089.350 = 3575.065.$ $\text{MSTr} = \text{SSTr}/(I-1) = 3575.065/(6-1) = 715.013, f = \text{MSTr}/\text{MSE} = 715.013/13.929 = 51.3$.

10.2.14. Use Tukey's procedure on the data in Example 10.3 to identify differences in true average bond strengths among the five protocols.

Treatment	1	2	3	4	5
Sample mean	10.5	14.8	15.7	16.0	21.6
Sample SD	4.5	6.8	6.5	6.7	6.0

From Example 10.3: $\alpha = .05$, $I = 5$, $J = 10$, grand mean = 15.7, MSTr = 156.875, MSE = 37.926.
$Q_{\alpha,I,I(j-1)} = Q_{0.05,5,5(10-1)} = Q_{0.05,5,45} = 4.018$ (I Google'd it since Table A.10 does not provide $\upsilon = 45$). https://www.stat.purdue.edu/~xbw/courses/stat512/q-table.pdf

$$w = Q_{\alpha,I,I(j-1)} \cdot \sqrt{\frac{\text{MSE}}{J}} = 4.018\sqrt{\frac{37.926}{10}} = 7.825$$

$\bar{x}_{2\cdot} - \bar{x}_{1\cdot} = 14.8 - 10.5 = 4.3 < w$, thus pair (1, 2) should be underlined as a pair.
$\bar{x}_{3\cdot} - \bar{x}_{1\cdot} = 15.7 - 10.5 = 5.2 < w$, thus pair (1, 3) should be underlined as a pair.
$\bar{x}_{4\cdot} - \bar{x}_{1\cdot} = 16.0 - 10.5 = 5.5 < w$, thus pair (1, 4) should be underlined as a pair.
$\bar{x}_{5\cdot} - \bar{x}_{1\cdot} = 21.6 - 10.5 = 11.1 > w$, thus pair (1, 5) should **not** be underlined as a pair.
$\bar{x}_{3\cdot} - \bar{x}_{2\cdot} = 15.7 - 14.8 = 0.9 < w$, thus pair (2, 3) should be underlined as a pair.
$\bar{x}_{4\cdot} - \bar{x}_{2\cdot} = 16.0 - 14.8 = 1.2 < w$, thus pair (2, 4) should be underlined as a pair.
$\bar{x}_{5\cdot} - \bar{x}_{2\cdot} = 21.6 - 14.8 = 6.8 < w$, thus pair (2, 5) should be underlined as a pair.
$\bar{x}_{4\cdot} - \bar{x}_{3\cdot} = 16.0 - 15.7 = 0.3 < w$, thus pair (3, 4) should be underlined as a pair.
$\bar{x}_{5\cdot} - \bar{x}_{3\cdot} = 21.6 - 15.7 = 5.9 < w$, thus pair (3, 5) should be underlined as a pair.
$\bar{x}_{5\cdot} - \bar{x}_{4\cdot} = 21.6 - 16.0 = 5.6 < w$, thus pair (4, 5) should be underlined as a pair.

$\bar{x}_{1\cdot}$	$\bar{x}_{2\cdot}$	$\bar{x}_{3\cdot}$	$\bar{x}_{4\cdot}$	$\bar{x}_{5\cdot}$
10.5	14.8	15.7	16	21.6

Ans: only significant difference is observed on the pair (1, 5)

10.2.16. Reconsider the axial stiffness data given in Exercise 8. ANOVA output from Minitab follows:

```
Analysis of Variance for Stiffness
Source      DF        SS        MS        F        P
Length       4     43993     10998    10.48    0.000
Error       30     31475      1049
Total       34     75468

Level        N      Mean     StDev
4            7    333.21     36.59
6            7    368.06     28.57
8            7    375.13     20.83
10           7    407.36     44.51
12           7    437.17     26.00

Pooled StDev = 32.39
```

```
Tukey's pairwise comparisons
    Family error rate = 0.0500
Individual error rate = 0.00693
Critical value = 4.10
Intervals for (column level mean) - (row level
mean)
               4          6          8         10
6          -85.0
            15.4
8          -92.1      -57.3
             8.3       43.1
10        -124.3      -89.5      -82.4
           -23.9       10.9       18.0
12        -154.2     -119.3     -112.2     -80.0
           -53.8      -18.9      -11.8      20.4
```

(a) Is it plausible that the variances of the five axial stiffness index distributions are identical? Explain.
The largest standard deviation, $s_4 = 44.51$, is only slightly twice as large as the smallest standard deviation, $s_3 = 20.83$, so we can conclude that the population variances are equal.

Ans: Yes

(b) Use the output (without reference to our *F* table) to test the relevant hypotheses.
$H_0: u_i = u_j, i \neq j, H_a$: at least two u_i's are different. *F* = 10.48, *P*-value = 0.000 (both given from Minitab output), hence we reject null hypothesis (i.e., no difference in axial stiffness for different plate lengths.)

Ans: Reject H_0

(c) Use the Tukey intervals given in the output to determine which means differ, and construct the corresponding underscoring pattern.

$\alpha = .05$, $I = 5$, $J = 7$, $Q_{\alpha,I,I(j-1)} = Q_{0.05,5,5(7-1)} = Q_{0.05,5,30} = 4.1$, MSE = 1043 (both given from Minitab output).

$$w = Q_{\alpha,I,I(j-1)} \cdot \sqrt{\frac{\text{MSE}}{J}} = 4.1\sqrt{\frac{1043}{7}} = 50.191$$

$\bar{x}_{2\cdot} - \bar{x}_{1\cdot} = 368.06 - 333.21 = 34.85 < w$, thus pair (1, 2) should be underlined as a pair.
$\bar{x}_{3\cdot} - \bar{x}_{1\cdot} = 375.13 - 333.21 = 41.92 < w$, thus pair (1, 3) should be underlined as a pair.
$\bar{x}_{4\cdot} - \bar{x}_{1\cdot} = 407.36 - 333.21 = 74.15 > w$, thus pair (1, 4) should **not** be underlined as a pair.
$\bar{x}_{5\cdot} - \bar{x}_{1\cdot} = 437.17 - 333.21 = 103.96 > w$, thus pair (1, 5) should **not** be underlined as a pair.
$\bar{x}_{3\cdot} - \bar{x}_{2\cdot} = 375.13 - 368.06 = 7.07 < w$, thus pair (2, 3) should be underlined as a pair.
$\bar{x}_{4\cdot} - \bar{x}_{2\cdot} = 407.36 - 368.06 = 39.3 < w$, thus pair (2, 4) should be underlined as a pair.
$\bar{x}_{5\cdot} - \bar{x}_{2\cdot} = 437.17 - 368.06 = 69.11 > w$, thus pair (2, 5) should **not** be underlined as a pair.
$\bar{x}_{4\cdot} - \bar{x}_{3\cdot} = 407.36 - 375.13 = 32.23 < w$, thus pair (3, 4) should be underlined as a pair.
$\bar{x}_{5\cdot} - \bar{x}_{3\cdot} = 437.17 - 375.13 = 62.04 > w$, thus pair (3, 5) should **not** be underlined as a pair.
$\bar{x}_{5\cdot} - \bar{x}_{4\cdot} = 437.17 - 407.36 = 29.81 < w$, thus pair (4, 5) should be underlined as a pair.

$\bar{x}_{1\cdot}$	$\bar{x}_{2\cdot}$	$\bar{x}_{3\cdot}$	$\bar{x}_{4\cdot}$	$\bar{x}_{5\cdot}$
333.21	368.06	375.13	407.36	437.17

Ans: significant differences are observed on pairs (1, 4), (1, 5), (2, 5), and (3, 5)

11.1.1. An experiment was carried out to investigate the effect of species (factor *A*, with *I* = 4) and grade (factor *B*, with *J* = 3) on breaking strength of wood specimens. One observation was made for each species—grade combination—resulting in SSA = 442.0, SSB = 428.6, and SSE = 123.4. Assume that an additive model is appropriate.

(a) Test $H_0: \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$ (no differences in true average strength due to species) versus H_a: at least one $\alpha_i \neq 0$ using a level .05 test.

$$f_A = \frac{\text{MSA}}{\text{MSE}} = \frac{\text{SSA}/(I-1)}{\text{SSE}/[(I-1)(J-1)]} = \frac{442.0/(4-1)}{123.4/[(4-1)(3-1)]} = \frac{147.33}{20.567} = 7.16$$

df = (4 – 1, (4 – 1)(3 – 1)) = (3, 6). $F = 4.76 < 7.16 < 9.78$, so *P*-value lies between .01 and .05. We reject H_{0A} at the .05 level.

Ans: At least one $\alpha_i \neq 0$

(b) Test $H_0: \beta_1 = \beta_2 = \beta_3 = 0$ (no differences in true average strength due to grade) versus H_a: at least one $\beta_j \neq 0$ using a level .05 test.

$$f_B = \frac{\text{MSB}}{\text{MSE}} = \frac{\text{SSB}/(J-1)}{\text{SSE}/[(I-1)(J-1)]} = \frac{428.6/(3-1)}{123.4/[(4-1)(3-1)]} = 10.42$$

df = (3 – 1, (4 – 1)(3 – 1) = (2, 6). $F = 5.14 < 10.42 < 10.92$, so P-value lies between .01 and .05. We reject H_{0B} at the .05 level.

Ans: At least one $\boldsymbol{\beta_j \neq 0}$

11.2.16. In an experiment to assess the effect of curing time (factor A) and type of mix (factor B) on the compressive strength of hardened cement cubes, **three** different curing times were used in combination with **four** different mixes, with **three** observations obtained for each of the 12 curing time-mix combinations. The resulting sums of squares were computed to be SSA = 30,763.0, SSB = 34,185.6, SSE = 97.436.8, and SST = 205,966.6.

(a) Construct an ANOVA table.

Source of Variation	**df**	**SS**	**MS**	**f**
Factor A	$I-1$	SSA	MSA	MSA/MSE
Factor B	$J-1$	SSB	MSB	MSB/MSE
Interaction AB	$(I-1)(J-1)$	SSAB	MSAB	MSAB/MSE
Error	$IJ(K-1)$	SSE	MSE	
Total	$IJK-1$	SST		

$I = 3, J = 4, K = 3, df_A = I - 1 = 3 - 1 = 2, df_B = 4 - 1 = 3$

$$df_{AB} = (I-1)(J-1) = (3-1)(4-1) = 6, df_E = IJ(K-1) = 3 \cdot 4(3-1) = 24$$

$$df_T = IJK - 1 = 3 \cdot 4 \cdot 3 - 1 = 35$$

$$SSAB = SST - SSA - SSB - SSE = 205{,}966.6 - 30{,}763.0 - 34{,}185.6 - 97{,}436.8$$
$$= 43581.2$$

$$MSA = \frac{SSA}{df_A} = \frac{30{,}763.0}{2} = 15{,}381.5, MSB = \frac{SSB}{df_B} = \frac{34{,}185.6}{3} = 11{,}395.2$$

$$MSAB = \frac{SSAB}{df_{AB}} = \frac{43{,}581.2}{6} = 7263.53, MSE = \frac{SSE}{df_E} = \frac{97{,}436.8}{24} = 4{,}059.86$$

$$f_A = \frac{MSA}{MSE} = \frac{15{,}381.5}{4{,}059.86} = 3.79, f_B = \frac{MSB}{MSE} = \frac{11{,}395.2}{4059.86} = 2.81$$

$$f_{AB} = \frac{MSAB}{MSE} = \frac{7{,}263.53}{4{,}059.86} = 1.79$$

Source of Variation	**df**	**SS**	**MS**	**f**
Factor A	2	30,763.0	15,381.5	3.79
Factor B	3	34,185.6	11,395.2	2.81
Interaction AB	6	43.581.2	7,263.53	1.79
Error	24	97,436.8	4,059.86	
Total	35	205,966.6		

(b) Test at level .05 the null hypothesis H_{0AB}: all γ_{ij}'s = 0 (no interaction of factors) against H_{aAB}: at least one $\gamma_{ij} \neq 0$.

$X_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ij}$. H_{0AB}: all γ_{ij}'s $= 0$, H_{aAB}: at least one $\gamma_{ij} \neq 0$. $f_{AB} = 1.79$ (from (a)). $F_{\alpha,(I-1)(J-1),IJ(K-1)} = F_{0.05,(3-1)(4-1),3\cdot 4\cdot(3-1)} = F_{0.05,6,24} = 2.51$ (from Table A.9). Because $F_{0.05,6,24} > f_{AB}$, we do not reject null hypothesis H_{0AB}.

Ans: No interaction between factors

(c) Test at level .05 the null hypothesis $H_{0A}: \alpha_1 = \alpha_2 = \alpha_3 = 0$ (factor *A* main effects are absent) against H_{aA}: at least one $\alpha_i \neq 0$.

$f_A = 3.79$ (from (a)). $F_{\alpha,I-1,IJ(K-1)} = F_{0.05,(3-1),3\cdot 4\cdot(3-1)} = F_{0.05,2,24} = 3.40$ (from Table A.9). Because $F_{0.05,2,24} < f_A$, we reject null hypothesis H_{0A}.

Ans: Reject null hypothesis H_{0A}

(d) Test $H_{0B}: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ versus H_{aB}: at least one $\beta_i \neq 0$ using a level .05 test.

$f_B = 2.81$ (from (a)). $F_{\alpha,J-1,IJ(K-1)} = F_{0.05,(4-1),3\cdot 4\cdot(3-1)} = F_{0.05,3,24} = 3.01$ (from Table A.9). Because $F_{0.05,3,24} > f_B$, we do not reject null hypothesis H_{0B}.

Ans: Do not reject null hypothesis H_{0B}

(e) The values of the $\bar{x}_{i..}$'s were $\bar{x}_{1..} = 4010.88$, $\bar{x}_{2..} = 4029.10$, and $\bar{x}_{3..} = 3960.02$. Use Tukey's procedure to investigate significant differences among the three curing times.

$Q_{\alpha,I,IJ(K-1)} = Q_{0.05,3,3\cdot 4\cdot(3-1)} = Q_{0.05,3,24} = 3.53$ (from Table A.10). MSE = 4059.87 (from (a)).

$$w = Q_{\alpha,I,IJ(k-1)} \cdot \sqrt{\frac{\text{MSE}}{JK}} = 3.53\sqrt{\frac{4059.87}{4 \cdot 3}} = 64.93, \bar{x}_{3.} < \bar{x}_{1.} < \bar{x}_{2.}$$

$\bar{x}_{1.} - \bar{x}_{3.} = 4010.88 - 3960.02 = 50.86 < w$, thus pair (3, 1) should be underlined as a pair.
$\bar{x}_{2.} - \bar{x}_{3.} = 4029.10 - 3960.02 = 69.08 > w$, thus pair (3, 2) should **not** be underlined as a pair.
$\bar{x}_{2.} - \bar{x}_{1.} = 4029.10 - 4010.88 = 18.22 < w$, thus pair (1, 2) should be underlined as a pair.

$\bar{x}_{3.}$	$\bar{x}_{1.}$	$\bar{x}_{2.}$
3960.02	4010.88	4029.10