5.24 Expert testimony in homicide trials of battered women. Refer to the *Duke Journal of Gender Law and Policy* (Summer 2003) study of the impact of expert testimony on the outcome of homicide trials involving battered woman syndrome, Exercise 5.3 (p. 264). Recall that multiple regression was employed to model the likelihood of changing a verdict from not guilty to guilty after deliberations, y, as a function of juror gender (male or female) and expert testimony given (yes or no).

(a) Write a main effects model for $E(y)$ as a function of gender and expert testimony. Interpret the β coefficients in the model.

Ans: $\boldsymbol{E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2}$,

where x_1 **= {1 if male, 0 if female},** x_2 **= {1 if expert testimony is given, 0 if no}**

$\boldsymbol{\beta_0 = \mu_{11}, \beta_1 = \mu_{2j} - \mu_{1j}, \beta_2 = \mu_{i2} - \mu_{i1}}$

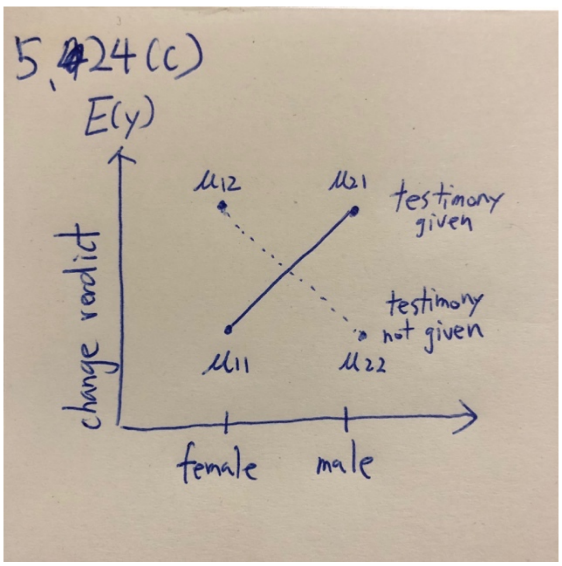
(b) Write an interaction model for $E(y)$ as a function of gender and expert testimony. Interpret the β coefficients in the model.

Ans: $\boldsymbol{E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2}$,

where x_1 **= {1 if male, 0 if female},** x_2 **= {1 if yes, 0 if no}**

$\boldsymbol{\beta_0 = \mu_{11}, \beta_1 = \mu_{21} - \mu_{11}, \beta_2 = \mu_{12} - \mu_{11}, \beta_3 = (\mu_{22} - \mu_{12}) - (\mu_{21} - \mu_{11})}$

(c) Based on data collected on individual juror votes from past trials, the article reported that "when expert testimony was present, women jurors were more likely than men to change a verdict from not guilty to guilty after deliberations." Assume that when no expert testimony was present, male jurors were more likely than women to change a verdict from not guilty to guilty after deliberations. Which model, part (a) or part (b), hypothesizes the relationships reported in the article? Illustrate the model with a sketch.



Ans: part (b) (interaction model)

5.25 Psychological response of firefighters. Refer to the *Journal of Human Stress* study of firefighters, Exercise 5.5 (p. 264). Consider using the qualitative variable, level of social support, as a predictor of emotional stress y. Suppose that four social support levels were studied: none, low, moderate, and high.

(a) Write a model for $E(y)$ as a function of social support at four levels.

Ans: $\boldsymbol{E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3}$**,**
where x_1 **= {1 if low, 0 if not},** x_2 **= {1 if moderate, 0 if not},** x_3 **= {1 if high, 0 if not}**

(b) Interpret the β coefficients in the model.

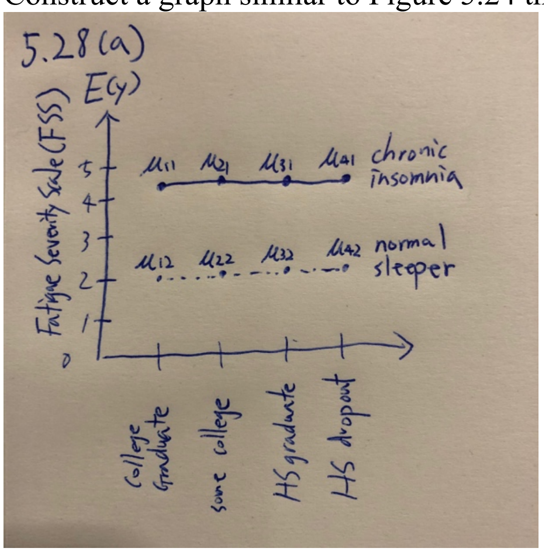
Ans: $\boldsymbol{\beta_0 = \mu_{\text{None}}, \beta_1 = \mu_{\text{Low}} - \mu_{\text{None}}, \beta_2 = \mu_{\text{Mod}} - \mu_{\text{None}}, \beta_3 = \mu_{\text{High}} - \mu_{\text{None}}}$

(c) Explain how to test for differences among the emotional stress means for the four social support levels.

Ans: $\boldsymbol{F}$**-test of** $\boldsymbol{H_0: \beta_1 = \beta_2 = \beta_3 = 0}$

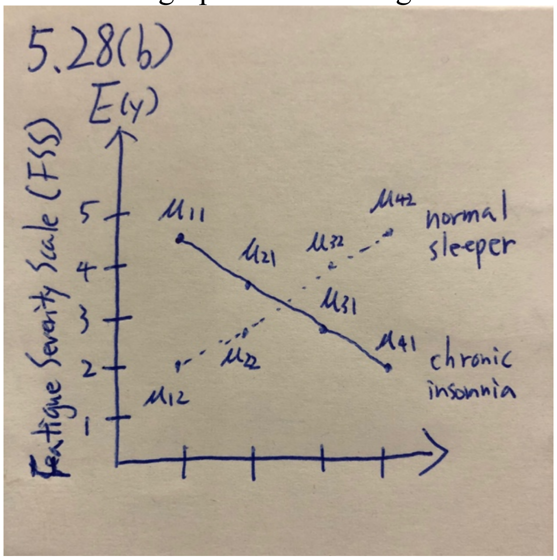
5.28 Insomnia and education. Many workers suffer from stress and chronic insomnia. Is insomnia related to education status? Researchers at the Universities of Memphis, Alabama at Birmingham, and Tennessee investigated this question in the *Journal of Abnormal Psychology* (February 2005). Adults living in Tennessee were selected to participate in the study using a random-digit telephone dialing procedure. In addition to insomnia status (normal sleeper or chronic insomnia), the researchers classified each participant into one of four education categories (college graduate, some college, high school graduate, and high school dropout). The dependent variable (y) of interest to the researchers was a quantitative measure of daytime functioning called the Fatigue Severity Scale (FSS), with values ranging from 0 to 5.

(a) Write a main effects model for $E(y)$ as a function of insomnia status and education level. Construct a graph similar to Figure 5.24 that represents the effects hypothesized by the model.



Ans: $\boldsymbol{E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4}$**,**
where x_1 **= {1 if chronic insomnia, 0 if normal sleeper},**
x_2 **= {1 if college graduate, 0 if not},**
x_3 **= {1 if some college, 0 if not},**
x_4 **= {1 if high school graduate, 0 if not}**

(b) Write an interaction model for $E(y)$ as a function of insomnia status and education level. Construct a graph similar to Figure 5.24 that represents the effects hypothesized by the model.



Ans: $\boldsymbol{E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_1 x_2 + \beta_6 x_1 x_3 + \beta_7 x_1 x_4}$,

where x_1 = {1 if chronic insomnia, 0 if normal sleeper},

x_2 = {1 if college graduate, 0 if not},

x_3 = {1 if some college, 0 if not},

x_4 = {1 if high school graduate, 0 if not}

(c) The researchers discovered that the mean FSS for people with insomnia is greater than the mean FSS for normal sleepers, but that this difference is the same at all education levels. Based on this result, which of the two models best represent the data?

Ans: The main effects model

5.31 Workplace bullying and intention to leave. Refer to the *Human Resource Management Journal* (October 2008) study of workplace bullying, Exercise 5.2 (p. 264). Recall that the researchers employed multiple regression to model a bullying victim's intention to leave the firm (y) as a function of level of bullying (measured on a 50-point scale) and perceived organizational support (measured as "low,", "neutral," or "high").

(a) Write a complete second-order model for $E(y)$ as a function of the independent variables.

Ans: $\boldsymbol{E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 x_2 + \beta_4 x_3 + \beta_5 x_1 x_2 + \beta_6 x_1 x_3 + \beta_7 x_1^2 x_2 + \beta_8 x_1^2 x_3}$,

where x_1 = level of bullying, x_2 = {1 if low, 0 if not}, x_3 = {1 if neutral, 0 if not}

(b) In terms of the β's in the model, part (a), give the mean value of intention to leave for a victim who reports a bullying level of 25 points and who perceives organizational support as low.

$$\beta_0 + 25\beta_1 + (25)^2\beta_2 + \beta_3(1) + \beta_4(0) + \beta_5(25)(1) + \beta_6(25)(0) + \beta_7(25)^2(1) + \beta_8(25)^2(0)$$
$$= \beta_0 + 25\beta_1 + 625\beta_2 + \beta_3 + 25\beta_5 + 625\beta_7$$

Ans: $\boldsymbol{\beta_0 + 25\beta_1 + 625\beta_2 + \beta_3 + 25\beta_5 + 625\beta_7}$

(c) How would you test whether the terms in the model that allow for a curvilinear relationship between intent to leave and level of bullying are statistically useful?

Ans: nested F-test of $\boldsymbol{H_0: \beta_2 = \beta_7 = \beta_8 = 0}$

(d) Write a first-order model for $E(y)$ as a function of the independent variables that incorporates interaction.

Ans: $\boldsymbol{E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1 x_2 + \beta_5 x_1 x_3}$

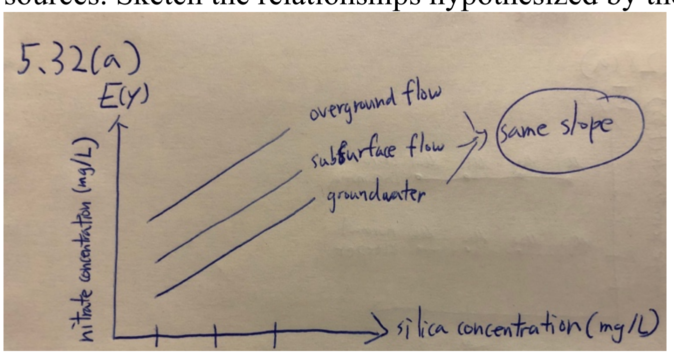
(e) Refer to the model, part (d). Demonstrate that the model graphs out as three nonparallel straight lines, one line for each level of perceived organizational support. As part of your answer, give the slope of each line in terms of the β's.

low $(x_2 = 1, x_3 = 0)$: $\beta_0 + \beta_1 x_1 + \beta_2(1) + \beta_3(0) + \beta_4 x_1(1) + \beta_5 x_1(0)$
$= \beta_0 + \beta_1 x_1 + \beta_2 + \beta_4 x_1 = (\beta_0 + \beta_2) + (\beta_1 + \beta_4)x_1$
neutral $(x_2 = 0, x_3 = 1)$: $\beta_0 + \beta_1 x_1 + \beta_2(0) + \beta_3(1) + \beta_4 x_1(0) + \beta_5 x_1(1)$
$= \beta_0 + \beta_1 x_1 + \beta_3 + \beta_5 x_1 = (\beta_0 + \beta_3) + (\beta_1 + \beta_5)x_1$
high $(x_2 = 0, x_3 = 0)$: $\beta_0 + \beta_1 x_1 + \beta_2(0) + \beta_3(0) + \beta_4 x_1(0) + \beta_5 x_1(0) = \beta_1$
Conclusion: Each of the three levels of perceived organizational support yields a linear function with a different slope.

Ans: low: $\boldsymbol{\beta_1 + \beta_4}$; neutral: $\boldsymbol{\beta_1 + \beta_5}$; high: $\boldsymbol{\beta_1}$

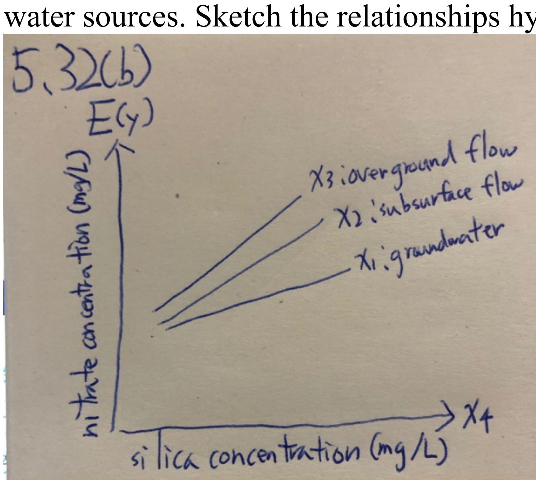
5.32 Chemical composition of rain water. Refer to the *Journal of Agricultural, Biological, and Environmental Statistics* (March 2005) study of the chemical composition of rain water, Exercise 5.4 (p. 264). Recall that the researchers want to model the nitrate concentration, y (milligrams per liter), in a rain water sample as a function of two independent variables: water source (groundwater, subsurface flow, or overground flow) and silica concentration (milligrams per liter).

(a) Write a first-order model for $E(y)$ as a function of the independent variables. Assume that the rate of increase of nitrate concentration with silica concentration is the same for all three water sources. Sketch the relationships hypothesized by the model on a graph.



Ans: $\boldsymbol{E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4}$,
where x_1 = {1 if groundwater, 0 if not},
x_2 = {1 if subsurface flow, 0 if not},
x_3 = {1 if overground flow, 0 if not},
x_4 = milligrams per liter

(b) Write a first-order model for $E(y)$ as a function of the independent variables, but now assume that the rate of increase of nitrate concentration with silica concentration differs for the three water sources. Sketch the relationships hypothesized by the model on a graph.



Ans: $\boldsymbol{E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_1 x_4 + \beta_6 x_2 x_4 + \beta_7 x_3 x_4}$,
where x_1 **= {1 if groundwater, 0 if not},**
x_2 **= {1 if subsurface flow, 0 if not},**
x_3 **= {1 if overground flow, 0 if not},**
x_4 **= milligrams per liter**
`

5.36 Improving SAT scores. Refer to the *Chance* (Winter 2001) study of students who paid a private tutor (or coach) to help them improve their Standardized Assessment Test (SA) scores, Exercise 4.50 (p. 223). Recall that multiple regression was used to estimate the effect of coaching on SAT–Mathematics scores, where

$y = \text{SAT– Math score}$
$x_1 = \text{score on PSAT}$
$x_2 = \{1 \text{ if student was coach, } 0 \text{ if not}\}$

(a) Write a complete second-order model for $E(y)$ as a function of x_1 and x_2.

Ans: $\boldsymbol{E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 x_2 + \beta_4 x_1 x_2 + \beta_5 x_1^2 x_2}$

(b) Give the equation of the curve relating $E(y)$ to x_1 for noncoached students. Identify the y-intercept, shift parameter, and rate of curvature in the equation.

$\beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3(0) + \beta_4 x_1(0) + \beta_5 x_1^2(0) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2$

Ans: $\boldsymbol{x_2 = 0, E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2}$
***y*-intercept:** $\boldsymbol{\beta_0}$**, shift parameter:** $\boldsymbol{\beta_1}$**, curvature coefficient:** $\boldsymbol{\beta_2}$

(c) Repeat part (b) for students who have been coached on the SAT.

$\beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3(1) + \beta_4 x_1(1) + \beta_5 x_1^2(1) = (\beta_0 + \beta_3) + (\beta_1 + \beta_4)x_1 + (\beta_2 + \beta_5)x_1^2$

Ans: $\boldsymbol{E(y) = (\beta_0 + \beta_3) + (\beta_1 + \beta_4)x_1 + (\beta_2 + \beta_5)x_1^2}$,
***y*-intercept:** $\boldsymbol{\beta_0 + \beta_3}$**, shift parameter:** $\boldsymbol{\beta_1 + \beta_4}$**, curvature coefficient:** $\boldsymbol{\beta_2 + \beta_5}$

(d) How would you test to determine if coaching has an effect on SAT–Math scores?

Ans: Nested *F*-test of $\boldsymbol{H_0: \beta_4 = \beta_5 = 0}$