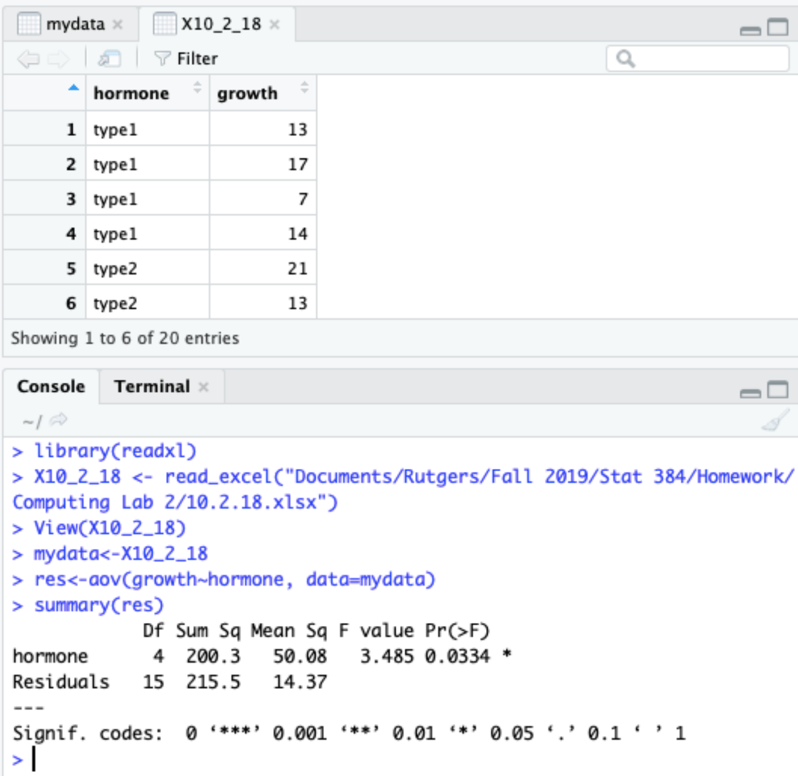
**10.2.18** Consider the accompanying data on plant growth after the application of five different types of growth hormone.

| | | | | |
|---|---|---|---|---|
| 1: | 13 | 17 | 7 | 14 |
| 2: | 21 | 13 | 20 | 17 |
| 3: | 18 | 15 | 20 | 17 |
| 4: | 7 | 11 | 18 | 10 |
| 5: | 6 | 11 | 15 | 8 |

(a) Perform an $F$ test at level $\alpha = .05$.

$H_0$: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ versus $H_a$: at least 2 $\mu_i$'s differ. $\alpha = .05$, $I = 5$, $J = 4$.




Conclusion: Because $P$-value = .0334 < .05, we reject the null hypothesis. The means of the five populations are not all equal. At least one of the means is different.

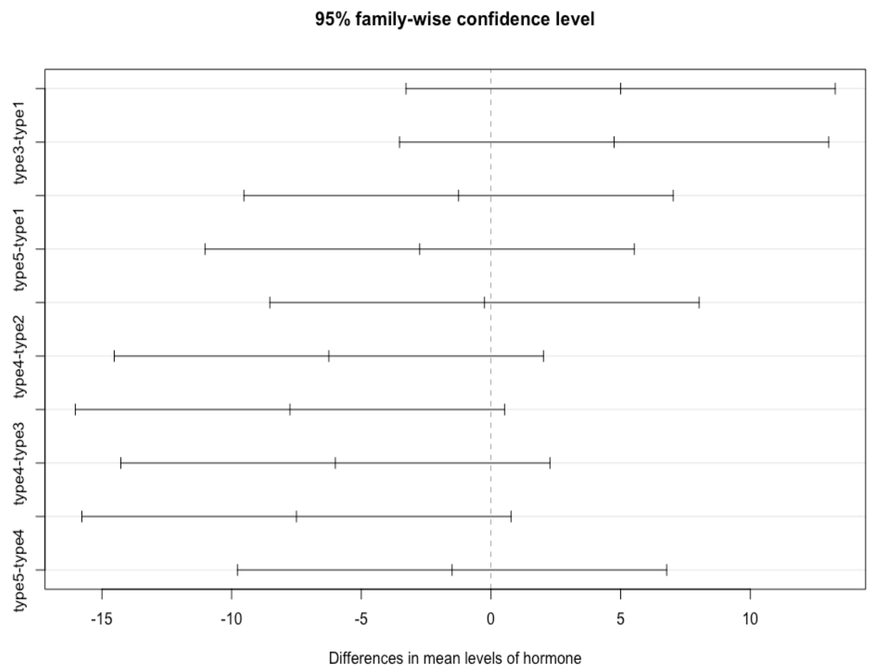
(b) What happens when Tukey's procedure is applied?

$Q_{.05,5,15} = 4.37$, MSE = 14.37. $w = 4.37\sqrt{\frac{14.37}{4}} = 8.28$.

```
> TukeyHSD(res, "hormone", ordered=TRUE)
  Tukey multiple comparisons of means
    95% family-wise confidence level
    factor levels have been ordered

Fit: aov(formula = growth ~ hormone, data = mydata)

$hormone
            diff        lwr       upr     p adj
type4-type5 1.50 -6.7761753  9.776175 0.9789688
type1-type5 2.75 -5.5261753 11.026175 0.8395387
type3-type5 7.50 -0.7761753 15.776175 0.0849318
type2-type5 7.75 -0.5261753 16.026175 0.0717704
type1-type4 1.25 -7.0261753  9.526175 0.9892929
type3-type4 6.00 -2.2761753 14.276175 0.2185546
type2-type4 6.25 -2.0261753 14.526175 0.1884779
type3-type1 4.75 -3.5261753 13.026175 0.4235109
type2-type1 5.00 -3.2761753 13.276175 0.3754811
type2-type3 0.25 -8.0261753  8.526175 0.9999807
```




Sample means less than 8.28 apart will belong to the same underscored set. After rearranging the five sample means in increasing order:

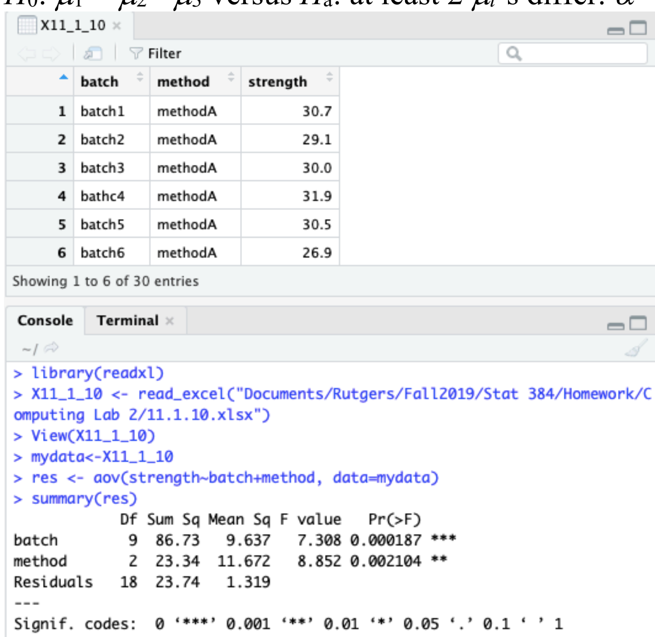
$$\begin{array}{ccccc} \bar{x}_5 & \bar{x}_4 & \bar{x}_1 & \bar{x}_3 & \bar{x}_2 \\ \underline{10} & \underline{11.5} & \underline{12.75} & \underline{17.5} & \underline{17.75} \end{array}$$

Conclusion: There are no significant differences.

**11.1.10** The strength of concrete used in commercial construction tends to vary from one batch to another. Consequently, small test cylinders of concrete sampled from a batch are "cured" for periods up to about 28 days in temperature and moisture-controlled environments before strength measurements are made. Concrete is then "bought and sold on the basis of strength test cylinders" (ASTM C 31 Standard Test Method for Making and Curing Concrete Test Specimens in the Field). The accompanying data resulted from an experiment carried out to compare three different curing methods with respect to compressive strength (MPa). Analyze this data.

| Batch | Method A | Method B | Method C |
|---|---|---|---|
| 1 | 30.7 | 33.7 | 30.5 |
| 2 | 29.1 | 30.6 | 32.6 |
| 3 | 30.0 | 32.2 | 30.5 |
| 4 | 31.9 | 34.6 | 33.5 |
| 5 | 30.5 | 33.0 | 32.4 |
| 6 | 26.9 | 29.3 | 27.8 |
| 7 | 28.2 | 28.4 | 30.7 |
| 8 | 32.4 | 32.4 | 33.6 |
| 9 | 26.6 | 29.5 | 29.2 |
| 10 | 28.6 | 29.4 | 33.2 |

$H_0$: $\mu_1 = \mu_2 = \mu_3$ versus $H_a$: at least 2 $\mu_i$'s differ. $\alpha = .05$, $I = 3$, $J = 10$.


X11_1_10

Filter

| | batch | method | strength |
|---|---|---|---|
| 1 | batch1 | methodA | 30.7 |
| 2 | batch2 | methodA | 29.1 |
| 3 | batch3 | methodA | 30.0 |
| 4 | bathc4 | methodA | 31.9 |
| 5 | batch5 | methodA | 30.5 |
| 6 | batch6 | methodA | 26.9 |

Showing 1 to 6 of 30 entries

Console Terminal

```
> library(readxl)
> X11_1_10 <- read_excel("Documents/Rutgers/Fall2019/Stat 384/Homework/C
omputing Lab 2/11.1.10.xlsx")
> View(X11_1_10)
> mydata<-X11_1_10
> res <- aov(strength~batch+method, data=mydata)
> summary(res)
            Df Sum Sq Mean Sq F value   Pr(>F)    
batch        9  86.73   9.637   7.308 0.000187 ***
method       2  23.34  11.672   8.852 0.002104 ** 
Residuals   18  23.74   1.319                     
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Conclusion: Because $P$-value < .05, we reject the null hypothesis. The means of the three methods are not all equal. At least one of the means is different.

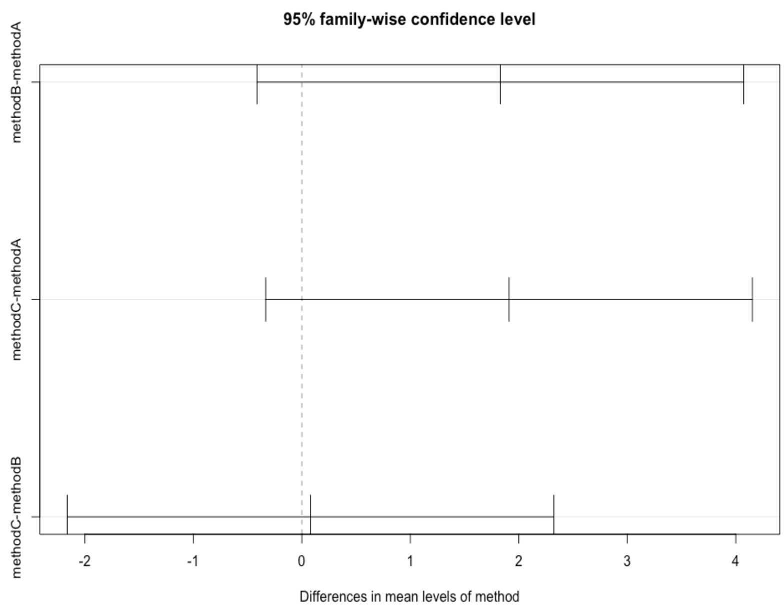
Tukey's procedure:

$Q_{.05,3,18} = 3.61$, MSE = 1.319. $w = 3.61\sqrt{\frac{1.319}{10}} = 1.311$.

```
> TukeyHSD(res, "method", ordered=TRUE)
  Tukey multiple comparisons of means
    95% family-wise confidence level
    factor levels have been ordered

Fit: aov(formula = strength ~ method, data = mydata)

$method
                 diff        lwr      upr     p adj
methodB-methodA 1.83 -0.4128238 4.072824 0.1259443
methodC-methodA 1.91 -0.3328238 4.152824 0.1063185
methodC-methodB 0.08 -2.1628238 2.322824 0.9956978
```




Sample means less than 1.311 apart will belong to the same underscored set. After rearranging the five sample means in increasing order:

$$\begin{array}{ccc} \bar{x}_{\cdot 1} & \bar{x}_{\cdot 2} & \bar{x}_{\cdot 3} \\ 29.49 & \underline{31.31} & \underline{31.4} \end{array}$$

Conclusion: Method A produces different results from Methods B and C.

**11.2.17b** The article **"Towards Improving the Properties of Plaster Moulds and Castings" (*J. Engr. Manuf.*, 1991: 265–269)** describes several ANOVAs carried out to study how the amount of carbon fiber and sand additions affect various characteristics of the molding process. Here we give data on casting hardness and on wet-mold strength

| Sand Addition (%) | Carbon Fiber Addition (%) | Casting Hardness | Wet-Mold Strength |
|---:|---:|---:|---:|
| 0 | 0 | 61.0 | 34.0 |
| 0 | 0 | 63.0 | 16.0 |
| 15 | 0 | 67.0 | 36.0 |
| 15 | 0 | 69.0 | 19.0 |
| 30 | 0 | 65.0 | 28.0 |
| 30 | 0 | 74.0 | 17.0 |
| 0 | .25 | 69.0 | 49.0 |
| 0 | .25 | 69.0 | 48.0 |
| 15 | .25 | 69.0 | 43.0 |
| 15 | .25 | 74.0 | 29.0 |
| 30 | .25 | 74.0 | 31.0 |
| 30 | .25 | 72.0 | 24.0 |
| 0 | .50 | 67.0 | 55.0 |
| 0 | .50 | 69.0 | 60.0 |
| 15 | .50 | 69.0 | 45.0 |
| 15 | .50 | 74.0 | 43.0 |
| 30 | .50 | 74.0 | 22.0 |
| 30 | .50 | 74.0 | 48.0 |

b. Carry out an ANOVA on the casting hardness observations using $\alpha = .05$.
MATLAB:

```
Command Window
  {0×0 double}

>> hardness = [61, 69, 67; 63, 69, 69; 67, 69, 69; 69, 74, 74; 65, 74, 74; 74, 72, 74]

hardness =

    61    69    67
    63    69    69
    67    69    69
    69    74    74
    65    74    74
    74    72    74

>> [p, tbl] = anova2(hardness, 2)

p =

    0.0297    0.0176    0.8887


tbl =

  6×6 cell array

  Columns 1 through 5

    {'Source'     }    {'SS'      }    {'df'}    {'MS'      }    {'F'        }
    {'Columns'    }    {[ 87.1111]}    {[ 2]}    {[ 43.5556]}    {[  5.3333]}
    {'Rows'       }    {[106.7778]}    {[ 2]}    {[ 53.3889]}    {[  6.5374]}
    {'Interaction'}    {[  8.8889]}    {[ 4]}    {[  2.2222]}    {[  0.2721]}
    {'Error'      }    {[ 73.5000]}    {[ 9]}    {[  8.1667]}    {0×0 double}
    {'Total'      }    {[276.2778]}    {[17]}    {0×0 double}    {0×0 double}

  Column 6

    {'Prob>F'  }
    {[  0.0297]}
    {[  0.0176]}
    {[  0.8887]}
    {0×0 double}
    {0×0 double}
fx
```

```
Workspace
Name ▲       Value
hardness     6x3 double
p            [0.0297,0.0176,0...
tbl          6x6 cell
```

The “2” on the command line [p, tbl] = anova2(hardness, 2) means there are two replications.

Figure 2: Two-way ANOVA

File Edit View Insert Tools Desktop Window Help

**ANOVA Table**

| Source | SS | df | MS | F | Prob>F |
|---|---|---|---|---|---|
| Columns | 87.111 | 2 | 43.5556 | 5.33 | 0.0297 |
| Rows | 106.778 | 2 | 53.3889 | 6.54 | 0.0176 |
| Interaction | 8.889 | 4 | 2.2222 | 0.27 | 0.8887 |
| Error | 73.5 | 9 | 8.1667 | | |
| Total | 276.278 | 17 | | | |

Rows = Fiber; Columns = Sand; Intersection = Sand × Fiber.

Conclusion: There appears to be an effect due to both sand and carbon fiber addition to casting hardness, but not interaction effect.