8.3 35a. Section 1: Statement of the Problem

The article **"Uncertainty Estimation in Railway Track Life-Cycle Cost" (*J. of Rail and Rapid Transit*, 2009)** presented the following data on time and repair (min) a rail break in the high rail on a curved track of a certain railway line.

159 120 480 249 270 547 340 43 228 202 240 218

A normal probability plot of the data shows a reasonably linear pattern, so it is plausible that the population distribution of repair time is at least approximately normal. The sample mean and standard deviation are 249.7 and 145.1, respectively.

Question: Is there compelling evidence for concluding that true average repair time exceeds 200 min? Carry out a test of hypotheses using a significance level of .05.

Section 2: Solution of the problem

The hypotheses are H_0: $\mu = 200$ and H_a: $\mu > 200$. With the data provided from textbook, $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \frac{249.7-200}{145.1/\sqrt{12}} = \frac{49.7}{41.89} \approx 1.2$. df = 12 – 1 = 11

P-value = .128 (Appendix Table A.8) or .1304 (R Studio)

In either case, the P-value is $> .05$, so H_0 is not rejected at $\alpha = .05$ level. We have insufficient evidence to conclude that the true average repair time exceeds 200 minutes.

Appendix 1: Computer Printout (R Studio)

Console | Terminal

```
~/
> x<-c(159, 120, 480, 149, 270, 547, 340, 43, 228, 202, 240, 218)
> t.test(x, mu=200, alternative="greater")

        One Sample t-test

data:  x
t = 1.1853, df = 11, p-value = 0.1304
alternative hypothesis: true mean is greater than 200
95 percent confidence interval:
 174.4174      Inf
sample estimates:
mean of x
 249.6667
```

Environment | History | Connections

Import Dataset | List

Global Environment

```
Values
  x                 num [1:12] 159 120 480 149 270 547 340 43…
```

9.2 23 Section 1: Statement of the Problem

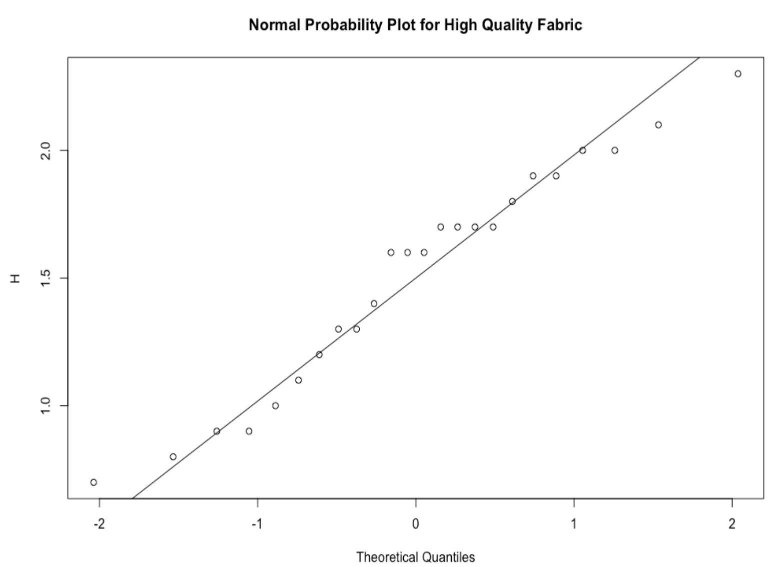
Fusible interlinings are being used with increasing frequency to support outer fabrics and improve the shape and drape of various pieces of clothing. The article **"Compatibility of Outer and Fusible Interlining Fabrics in Tailored Garments" (*Textile Res. J.*, 1997: 137–142)** gave the accompanying data on extensibility (%) at 100 gm/cm for both high-quality (H) fabric and poor-quality (P) fabric specimens.

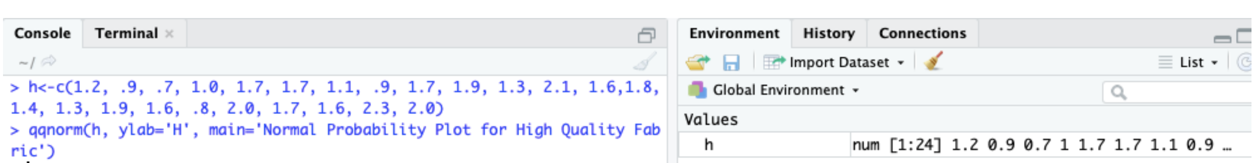
H	1.2	.9	.7	1.0	1.7	1.7	1.1	.9	1.7
	1.9	1.3	2.1	1.6	1.8	1.4	1.3	1.9	1.6
	.8	2.0	1.7	1.6	2.3	2.0			
P	1.6	1.5	1.1	2.1	1.5	1.3	1.0	2.6	

a. Construct normal probability plots to verify the plausibility of both samples having been selected from normal population distributions.
b. Construct a comparative boxplot. Does it suggest that there is a difference between true average extensibility for high-quality fabric specimens and that for poor-quality specimens?
c. The sample mean and standard deviation for the high-quality sample are 1.508 and .444, respectively, and those for the poor-quality samples are 1.588 and .530. Use the two-sample *t* test to decide whether true average extensibility differs for the two types of fabric.

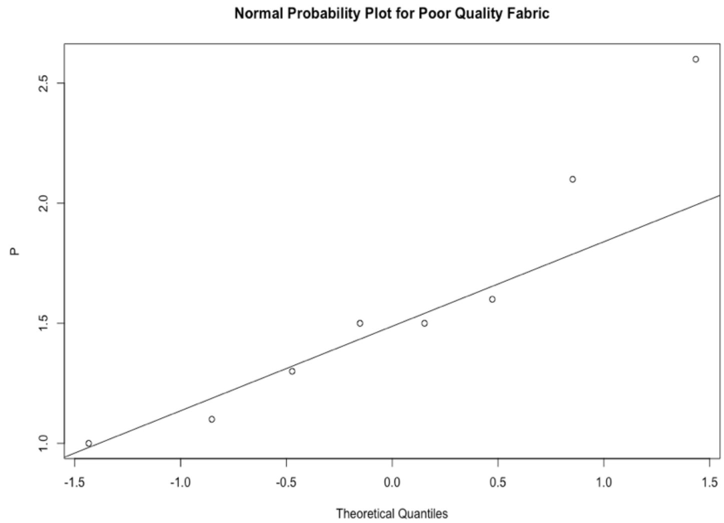
Section 2: Solution of the problem

a. The figure below illustrates the normal probability plot for H.





The figure below illustrates the normal probability plot for P:




```
p (numeric, 112 bytes)
~/
> p<-c(1.6, 1.5, 1.1, 2.1, 1.5, 1.3, 1.0, 2.6)
> qqnorm(p, ylab='P', main='Normal Probability Plot for Poor Quality Fab
ric')
> qqline(p)
```

Environment History Connections

Import Dataset List

Global Environment

Values

p num [1:8] 1.6 1.5 1.1 2.1 1.5 1.3 1 2.6

Neither probability plots contains strong curvature and are roughly linear, so both population distributions are approximately normal. Therefore, it is plausible to use the two-sample t test.

b. The following box shows the sorted data values from smallest to largest:

	H
1	0.7
2	0.8
3	0.9
4	0.9
5	1.0
6	1.1
7	1.2
8	1.3
9	1.3
10	1.4
11	1.6
12	1.6
13	1.6
15	1.7
14	1.7
16	1.7
17	1.7
18	1.8
19	1.9
20	1.9
21	2.0
22	2.0
23	2.1
24	2.3

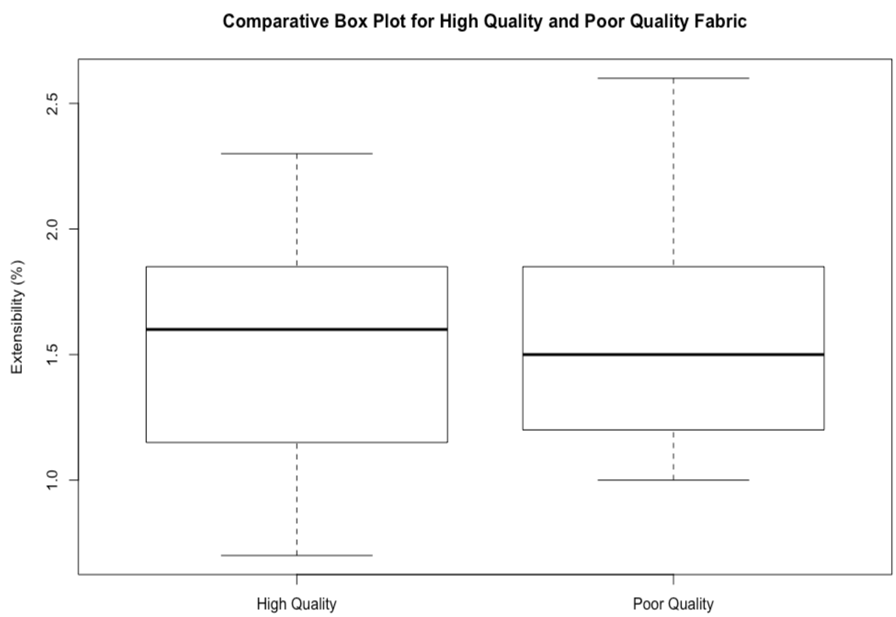
P
1.0
1.1
1.3
1.5
1.5
1.6
2.1
2.6

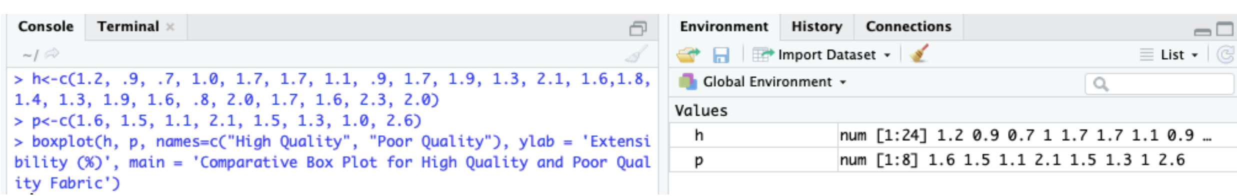
H: The minimum for H is 0.7. Since the number of data values is even, the median is the average of the two middle values of the sorted data set: $(1.6 + 1.6)/2 = 1.6$.

The first quartile is (1.1 + 1.2)/2 = 1.15; and the third quartile is (1.8 + 1.9)/2 = 1.85. The maximum is 2.3.

P: The minimum for P is 1. The median is (1.5 + 1.5)/2 = 1.5; the first quartile is (1.1 + 1.3)/2 = 1.2; and the third quartile is (1.6 + 2.1)/2 = 1.85. The maximum is 2.6.

The figure below shows that there appears to be a small difference between the true average extensibility for high-quality fabric specimens and that of poor-quality specimens. The vertical line in the box of the boxplot is not at roughly the same position in both boxplots.





c. $\bar{x}_1 = 1.508$, $\bar{x}_2 = 1.588$, $s_1 = 0.444$, $s_2 = 0.530$. Assume $\alpha = 0.05$, H_0: $\mu_1 = \mu_2$, and H_a: $\mu_1 \neq \mu_2$. The test statistic is calculated as follows:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2}}} = \frac{1.508 - 1.588}{\sqrt{\frac{0.444^2}{24} + \frac{0.530^2}{8}}} \approx -0.384$$

The degree of freedom is calculated as follows: (rounded down to the nearest integer)

$$\Delta = \frac{\left(\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2}\right)^2}{\frac{({s_1}^2/n_1)^2}{n_1 - 1} + \frac{({s_2}^2/n_2)^2}{n_2 - 1}} = \frac{\left(\frac{0.444^2}{24} + \frac{0.530^2}{8}\right)^2}{\frac{(0.444^2/24)^2}{24 - 1} + \frac{(0.530^2/8)^2}{8 - 1}} \approx 10$$

The P-value is the probability of obtaining the value

```
> h<-c(1.2, .9, .7, 1.0, 1.7, 1.7, 1.1, .9, 1.7, 1.9, 1.3, 2.1, 1.6,1.8,
1.4, 1.3, 1.9, 1.6, .8, 2.0, 1.7, 1.6, 2.3, 2.0)
> p<-c(1.6, 1.5, 1.1, 2.1, 1.5, 1.3, 1.0, 2.6)
> boxplot(h, p, names=c("High Quality", "Poor Quality"), ylab = 'Extensi
bility (%)', main = 'Comparative Box Plot for High Quality and Poor Qual
ity Fabric')
> t.test(h, p)

        Welch Two Sample t-test

data:  h and p
t = -0.38011, df = 10.482, p-value = 0.7115
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -0.5403506  0.3820172
sample estimates:
mean of x mean of y
 1.508333  1.587500
```

Since $P > 0.05$, H_0 is not rejected. There is not sufficient evidence to support the claim that the true average extensibility differs for two types of fabric.

9.3 36 Section 1: Statement of the Problem

Consider the accompanying data on breaking load (kg/25 mm width) for various fabrics in both an unabraded condition and an abraded condition **("The Effect of Wet Abrasive Wear on the Tensile Properties of Cotton and Polyester-Cotton Fabrics,"** ***J. Testing and Evaluation*****, 1993: 84–93)**. Use the paired t test, as did the authors of the cited article, to test H_0: $\mu_D = 0$ versus H_a: $\mu_D > 0$ at significance level .01.

	Fabric							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
U	36.4	55.0	51.5	38.7	43.2	48.8	25.6	49.8
A	28.5	20.0	46.0	34.5	36.5	52.5	26.5	46.5
d	7.9	35	5.5	4.2	6.7	-3.7	-0.9	3.3

Section 2: Solution of the problem

$\bar{d} = 7.25$, $S_D = 11.8628$, $\mu_0 =$ true average difference of breaking load of fabrics. H_0: $\mu_0 = 0$ versus H_a: $\mu_0 > 0$. $t_{0.01,\,7} = 2.998$

$$t = \frac{\bar{d} - \mu_D}{S_D/\sqrt{n}} = \frac{7.25 - 0}{11.8628/\sqrt{8}} = 1.7286$$

Since $1.7286 < 2.998$, H_0 cannot be rejected. The data does not support the claim.