10.5 Wildlife biologists wish to estimate the total size of the bobwhite quail population in a section of southern Florida. A series of 50 traps is used. In the first sample, $t = 320$ quails are caught. After being captured, each bird is removed from the trap and tagged with a metal band on its left leg. All birds are then released. Several months later, a second sample of $n = 515$ quail is obtained. Suppose $s = 91$ of these birds have tags. Estimate N and place a bound on the error of estimation.

```
> rm(list=ls())
> n=515
> t=320
> s=91
> Nhat=n*t/s
> Nhat
[1] 1810.989
> Vhat=t^2*n*(n-s)/s^3
> Vhat
[1] 29672.14
> B=2*sqrt(Vhat)
> B
[1] 344.5121
```

Ans: $\hat{N} = 1811; B = 344.51$

10.10 A zoologist wishes to estimate the size of the turtle population in a given geographical area. She believes that the turtle population is between 500 and 1000; hence, an initial sample of 100 (10%) appears to be sufficient. The $t = 100$ turtles are caught, tagged, and released. A second sample is begun one month later, and she decides to continue sampling until $s = 15$ tagged turtles are recaptured. She catches 160 turtles before obtaining 15 tagged turtles ($n = 160$, $s = 15$). Estimate N and place a bound on the error of estimation.

keyword for inverse sampling is "until"

```
> rm(list=ls())
> n=160
> t=100
> s=15
> Nhat=n*t/s
> Nhat
[1] 1066.667
> Vhat=t^2*n*(n-s)/(s^2*(s+1)) Equation (10.4)
> Vhat
[1] 64444.44
> B=2*sqrt(Vhat)
> B
[1] 507.7182
```

Ans: $\hat{N} = 1067; B = 507.7182$

10.16 Cars passing through an intersection are counted during randomly selected ten-minute intervals throughout the working day. Twenty such samples show an average of 40 cars per interval. Estimate, with a bound on the error, the number of cars that you expect to go through the intersection in an eight-hour period.

```
> rm(list=ls())
> m=40
> n=20
> N=8*6
> M=N*m
> M
[1] 1920
> lambda=m/a
> lambda
[1] 16.66667
> Vhat=A^2*lambda/(a*n)
> Vhat
[1] 4608
> B=2*sqrt(Vhat)
> B
[1] 135.7645
```

Ans: $\widehat{\boldsymbol{M}} = \mathbf{1920}; \boldsymbol{B} = \mathbf{135.7645}$

10.19 The data in the accompanying table show the number of bacteria colonies observed in 240 microscopic fields. Estimate, with a bound on the error of estimation, the density of colonies per field. What assumptions are necessary for this procedure?

Colonies per field	Number of fields
0	11
1	37
2	64
3	55
4	37
5	24
6	12

```
> rm(list=ls())
> a=1
> n=240
> m=c(rep(0:6,c(11,37,64,55,37,24,12)))
> mbar=mean(m)
> mbar
[1] 2.791667
```

10.5	**10.7**
> sm2=sum((m-mbar)^2)/(n-1) > sm2 [1] 2.266039 > Vhat_10.5=sm2/(a^2*n) > Vhat_10.5 [1] 0.009441829 > B=2*sqrt(Vhat_10.5) > B [1] 0.1943382	> lambda=mbar/a > lambda [1] 2.791667 > Vhat=lambda/(a*n) > Vhat [1] 0.01163194 > B=2*sqrt(Vhat) > B [1] 0.215703

Ans: $\hat{\boldsymbol{\lambda}} = \mathbf{2.792}; \hat{\boldsymbol{V}}(\hat{\boldsymbol{\lambda}}_{\mathbf{10.5}}) = \mathbf{0.00944}; \hat{\boldsymbol{V}}(\hat{\boldsymbol{\lambda}}_{\mathbf{10.7}}) = \mathbf{0.0116}; \boldsymbol{B}_{\mathbf{10.5}} = \mathbf{0.194}; \boldsymbol{B}_{\mathbf{10.7}} = \mathbf{0.216}$

11.3 A retail store wants to estimate the average amount of all past-due accounts. The available list of past-due accounts is outdated because some accounts have since been paid. Because drawing up a new list would be expensive, the store uses the outdated list. A simple random sample of 20 accounts is selected from the list, which contains 95 accounts. Of the 20 sampled accounts, four have been paid. The 16 past-due accounts contain the following amounts (in dollars): 3.65, 15.98, 40.70, 2.98, 50.00, 60.31, 67.21, 14.98, 10.21, 14.32, 1.87, 32.60, 19.80, 15.98, 12.20 and 15.00. Estimate the average amount of past-due accounts for the store and place a bound on the error of estimation.

```
> rm(list=ls())
> N=95
> n=20
> n1=16
> y1=c(3.65,15.98,40.7,2.98,50,60.31,67.21,14.98,10.21,14.32,1.87,32.6,19.8,15.98,12.2,15)
> y1bar=mean(y1)
> y1bar
[1] 23.61188
> si2=sum((y1-y1bar)^2)/(n1-1)
> si2
[1] 419.2938
> Vhat=(N-n)*si2/(N*n1)
> Vhat
[1] 20.68884
> B=2*sqrt(Vhat)
> B
[1] 9.096997
```

Ans: $\bar{y}_1 = 23.61188; B = 9.096997$

11.4 For Exercise 11.3, estimate the total amount of past-due accounts for the store and place a bound on the error of estimation.

```
> t=N*sum(y1)/n
> t
[1] 1794.503
> y=c(y1,0,0,0,0)
> y
 [1]  3.65 15.98 40.70  2.98 50.00 60.31 67.21 14.98 10.21 14.32  1.87 32.60 19.80
[14] 15.98 12.20 15.00  0.00  0.00  0.00  0.00
> ybar=mean(y)
> sn2=(sum(y^2)-n*ybar^2)/(n-1)
> sn2
[1] 424.9197
> Vhat=N^2*(1-n/N)*sn2/n
> Vhat
[1] 151377.6
> B=2*sqrt(Vhat)
> B
[1] 778.1456
```

Ans: $\hat{\tau}_1 = 1794.503; B = 778.1456$

11.5 An employee of the store in Exercise 11.3 decides to look through the list of past-due accounts and mark those that have been paid. He finds that only 83 of the 95 accounts are past due. Estimate the total amount of past-due accounts by using this additional information and the data in Exercise 11.3. Place a bound on the error of estimation.

```
> N1=83
> t=N1/n1*sum(y1)
> t
[1] 1959.786
> Vhat=N1^2*(N1-n1)*si2/(N1*n1)
> Vhat
[1] 145730.8
> B=2*sqrt(Vhat)
> B
[1] 763.4941
```

Ans: $\hat{\tau}_1 = 1959.786; B = 763.4941$

10.4 Figures 10.3 through 10.5 show points distributed in planar regions. The objective is to estimate the density of points on the page, which is the same for all three figures. The estimation is to be accomplished through quadrat sampling.

FIGURE 10.3
Planar region I

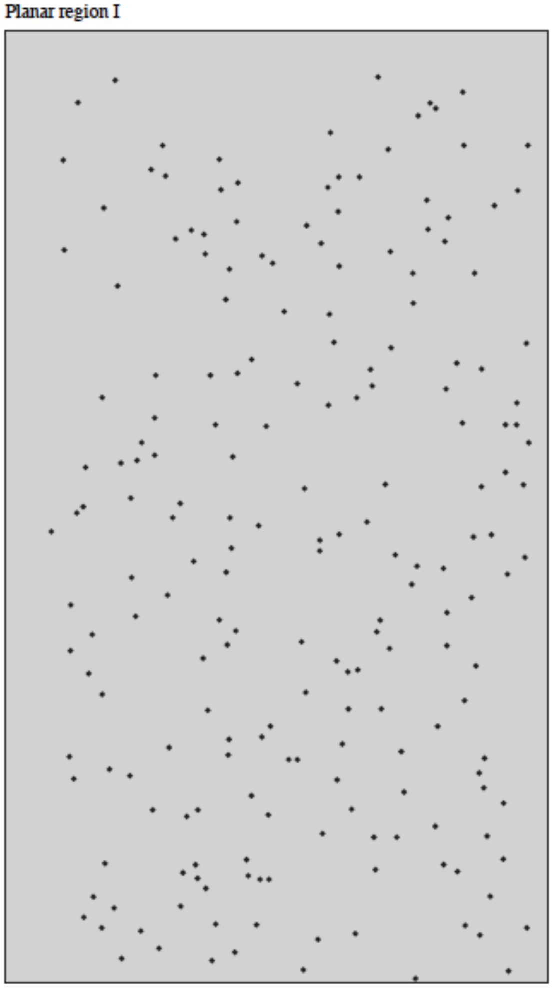
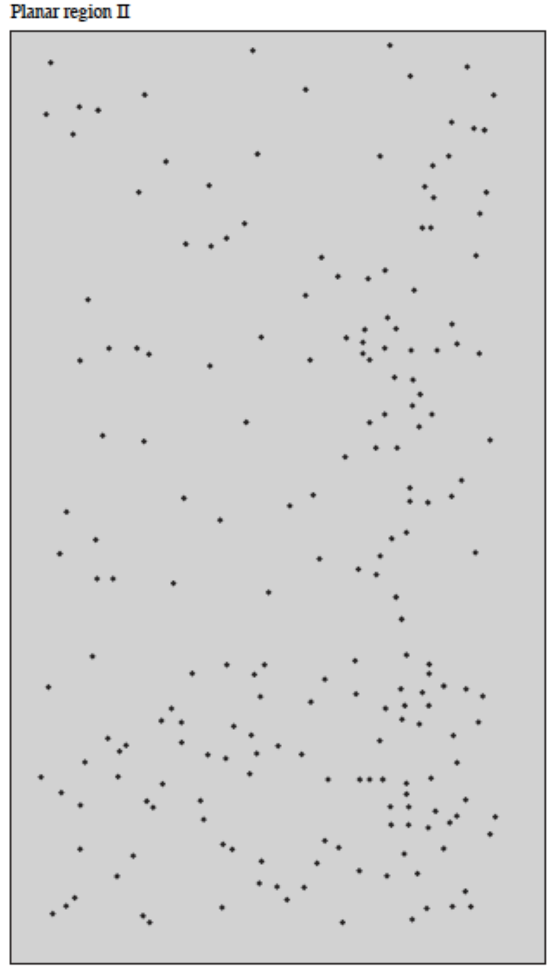


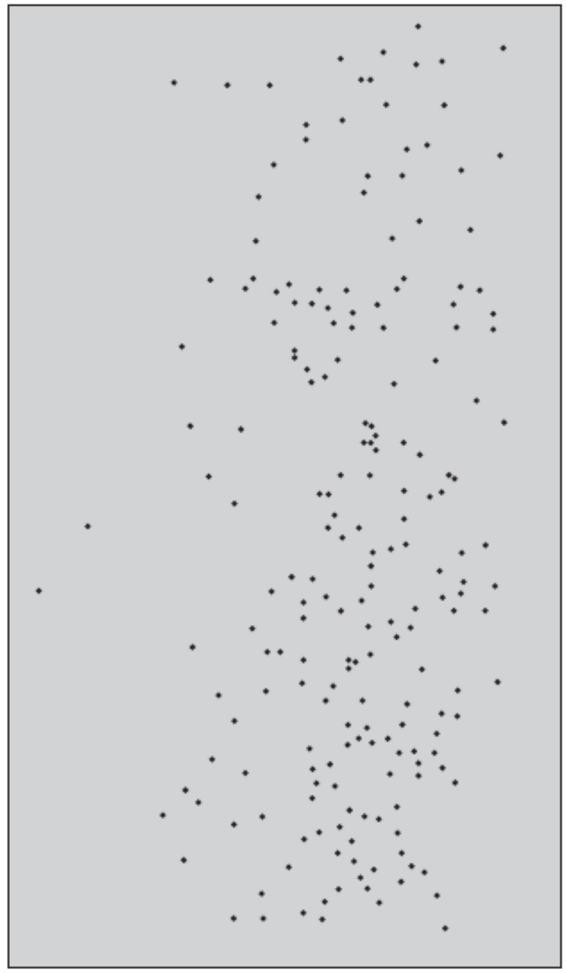
FIGURE 10.4
Planar region II



b. Randomly select six quadrants from Figure 10.3, counting the number of points in each. Use these data to estimate the density of points in Figure 10.3, with a bound on the error of estimation.

```
> rm(list=ls())
> N=35
> a=1
> n=sample(N,6)
> n
[1] 19 16 13 34  6 31
> n=6
> m=c(3,7,6,5,2,7)
> mbar=mean(m)
> mbar
[1] 5
> lambda=mbar/a
> lambda
[1] 5
> Vhat=lambda/(a*n)
> Vhat
[1] 0.8333333
> B=2*sqrt(Vhat)
> B
[1] 1.825742
```

FIGURE 10.5
Planar region III



Ans: $\hat{\lambda} = 5; B = 1.825742$

c. Repeat the instructions in part (b) for Figures 10.4 and 10.5. Compare the results. Which figure produces the largest bound on the error for the estimate of point density?

10.4

```
> rm(list=ls())
> N=35
> a=1
> n=sample(N,6)
> n
[1] 30 15 27 17 12 21
> m=c(3,3,15,3,2,8)
> mbar=mean(m)
> mbar
[1] 5.666667
> lambda=mbar/a
> lambda
[1] 5.666667
> n=6
> Vhat=lambda/(a*n)
> Vhat
[1] 0.9444444
> B=2*sqrt(Vhat)
> B
[1] 1.943651
```

10.5

```
> rm(list=ls())
> N=35
> a=1
> n=sample(N,6)
> n
[1]  9 29 25 17 12  7
> m=c(1,26,17,9,1,0)
> mbar=mean(m)
> mbar
[1] 9
> lambda=mbar/a
> lambda
[1] 9
> Vhat=lambda/(a*n)
> Vhat
[1] 1.5
> B=2*sqrt(Vhat)
> B
[1] 2.44949
```

Conclusion: Figure 10.5 produces the largest bound on the error for the estimate of point density.

Ans: 10.4: $\boldsymbol{\hat{\lambda} = 5.666667; B = 1.943651; 10.5: \hat{\lambda} = 9; B = 2.44949}$