Problem 1.

We wish to evaluate a new textbook for a statistics class. There are five course sections; three are chosen at random to receive the new book (N), two receive the old book (O). At the end of the semester, student evaluations show the following percentages of students rate the textbook as "very good" or "excellent":

Section	1	2	3	4	5
Book	O	N	N	O	N
Rating	37	51	59	32	62

Find the <u>one-sided randomization p-value</u> for testing the null hypothesis that the two books are equivalent versus the alternative that the new book is better (receives higher scores)

$H_0: \mu_N = \mu_O, H_a: \mu_N > \mu_O$ (μ_N = average rating of new books; μ_O = average rating of old books).

Possible numbers of arrangements: 5!/(3!2!) = 5(4) / 2 = 10

$\bar{y}_N = (51 + 59 + 62)/3 = 57.333; \bar{y}_O = (37 + 32)/2 = 34.5, \bar{y}_N - \bar{y}_O = 57.333 - 34.5 = 22.833$

Unit	1	2	3	4	5	
Rating	37	51	59	32	62	$\bar{y}_N - \bar{y}_O$
1	N	N	O	N	O	–20.500
2	N	O	N	N	O	–13.833
3	N	O	O	N	N	–11.333
4	O	N	N	N	O	–2.167
5	O	N	O	N	N	0.333
6	N	N	N	O	O	2.000
7	N	N	O	O	N	4.500
8	O	O	N	N	N	7.000
9	N	O	N	O	N	11.167
10	O	N	N	O	N	22.833

An absolute mean difference of 22.833 or larger occurs with a frequency of 1/10, yielding a significance level of 0.100. The null hypothesis can be rejected at $\alpha = 0.100$.

Ans: *p*-value = 0.100

Problem 2.

The tensile strength of Portland cement was under experimentation using four different mixing techniques in a laboratory at Rutgers University. This is a Completely Randomized Design (CRD). The following data was collected:

Mixing Technique	Tensile Strength (lb/in^2)			
1	3129	3000	2865	2890
2	3200	3300	2975	3150
3	2800	2900	2985	3050
4	2600	2700	2600	2765

The Analysis of Variance method generated the following results:

SoV	df	SS	MS	F_0	p-value
Trt	3	4.8974E+05	.	12.73	0.0005
Error	12	1.5390E+05	12825.69		
Total	15	6.4364E+05			

(a) Calculate the MS for treatment in the above ANOVA table
MSTr = F(MSE) = 12.73(12825.69) = 163,271.034
(b) What is the conclusion you can obtain from this ANOVA table?
The fact that p-value = 0.0005 demonstrates that mixing technique has an effect.
(c) The estimated treatment differences in pairwise comparisons gave the following results:

(1) – (2) = -185.25
(1) – (3) = 37.25 (2) – (3) = 222.50
(1) – (4) = 304.75 (2) – (4) = 490.00 (3) – (4) = 267.50

Obtain 90% confidence intervals for these differences, and indicate which ones are significant, if any. Use the overall significance level 0.10, but adjust each CI using Bonferroni method (Show your work).

1 – 0.10/6 = 0.983. Significant pairs below the R code are in **boldface**.

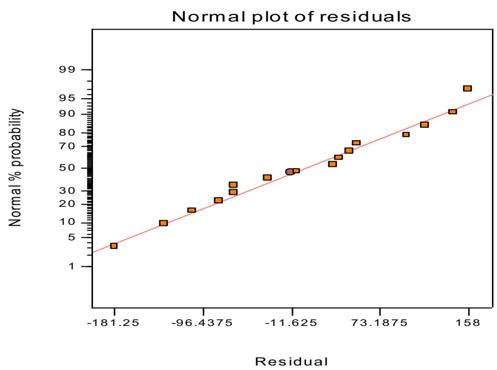
```
> portland1 <- c(3129,3000,2865,2890)
> portland2 <- c(3200,3300,2975,3150)
> portland3 <- c(2800,2900,2985,3050)
> portland4 <- c(2600,2700,2600,2765)
> portland <- cbind(portland1, portland2, portland3, portland4)
> portland.col <- c(portland1, portland2, portland3, portland4)
> fac1 = rep("tech1",4)
> fac2 = rep("tech2",4)
> fac3 = rep("tech3",4)
> fac4 = rep("tech4",4)
> fac <- c(fac1,fac2,fac3,fac4)
> portland.df <- data.frame(portland.col, fac = fac)
> portland.aov = aov(portland.col~fac, data=portland.df)
```

```
> TukeyHSD(portland.aov, "fac", ordered=TRUE, conf.level=0.983)
  Tukey multiple comparisons of means
    98.3% family-wise confidence level
    factor levels have been ordered

Fit: aov(formula = portland.col ~ fac, data = portland.df)

$fac
              diff        lwr      upr     p adj
tech3-tech4 267.50  -19.73317 554.7332 0.0261838
tech1-tech4 304.75   17.51683 591.9832 0.0115923
tech2-tech4 490.00  202.76683 777.2332 0.0002622
tech1-tech3  37.25 -249.98317 324.4832 0.9652776
tech2-tech3 222.50  -64.73317 509.7332 0.0693027
tech2-tech1 185.25 -101.98317 472.4832 0.1493561
```

(1)-(2) = (–101.983, 427.483); (1)-(3) = (–249.983, 324.483); (2)-(3) = (–64.733, 509.733);

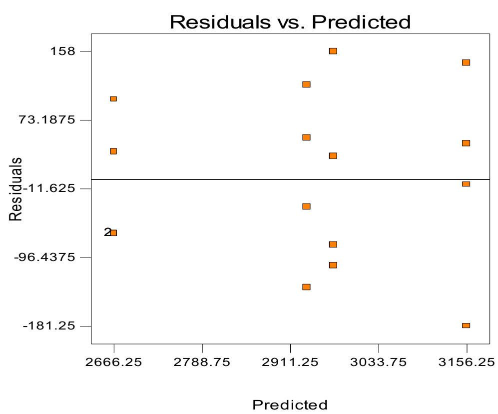
(1)-(4) = (17.51683, 591.9832); **(2)-(4)** = (202.767, 777.233); (3)-(4) = (–19.733, 554.733)

(d) Based on the plot below, do you think the data has a normal distribution?



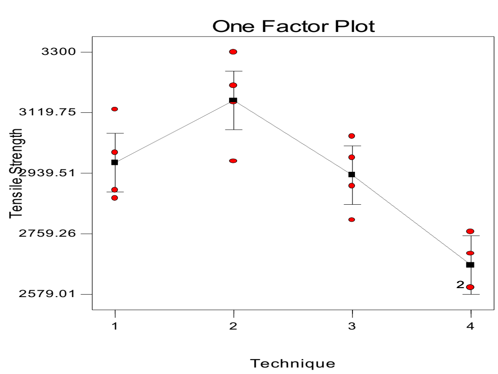
Ans: There is nothing unusual about the normal probability plot of residuals.

e) Using the plot below, do you think there is something unusual about the data: outliers, dependency, constant variance?



Ans: There is nothing unusual about this plot.

f) A scatter plot of all the data by treatment, shows the sample means joint with a line and shows 95% CI for each treatment. Do you have any comments about the data, based on this plot?



Ans: The plot shows the sample average for each treatment and 95% CI on the treatment mean.

Problem 3.
A regional opera company has tried three approaches to solicit donations from 24 potential sponsors. The 24 potential sponsors were randomly divided into three groups of eight, and one approach was used for each group. The dollar amounts of the resulting contributions are shown in the following table.

Approach	**Contributions (in $)**							
1	1000	1500	1200	1800	1600	1100	1000	1250
2	1500	1800	2000	1200	2000	1700	1800	1900
3	900	1000	1200	1500	1200	1550	1000	1100

(a) Do the data indicate that there is a difference in results obtained from the three different approaches? Use $\alpha=0.08$

Hint: Use the formulas from Montgomery's book, or use the R code or the SAS code attached or try StatDisk..

```
> d1 = c(1000,1500,1200,1800,1600,1100,1000,1250)
> d2 = c(1500,1800,2000,1200,2000,1700,1800,1900)
> d3 = c(900,1000,1200,1500,1200,1550,1000,1100)
> d = cbind(d1,d2,d3)
> don = c(d1,d2,d3)
> fac1=rep("appr1",8)
> fac2=rep("appr2",8)
> fac3=rep("app3r",8)
> fac1
[1] "appr1" "appr1" "appr1" "appr1" "appr1" "appr1" "appr1" "appr1"
> fac2
[1] "appr2" "appr2" "appr2" "appr2" "appr2" "appr2" "appr2" "appr2"
> fac3
[1] "app3r" "app3r" "app3r" "app3r" "app3r" "app3r" "app3r" "app3r"
> f=c(fac1,fac2,fac3)
> f
 [1] "appr1" "appr1" "appr1" "appr1" "appr1" "appr1" "appr1" "appr1" "appr2" "appr2"
[11] "appr2" "appr2" "appr2" "appr2" "appr2" "appr2" "app3r" "app3r" "app3r" "app3r"
[21] "app3r" "app3r" "app3r" "app3r"
```

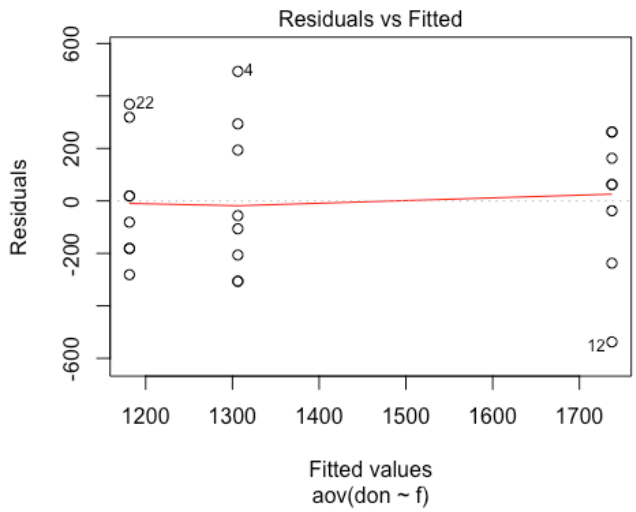
```
> donations = data.frame(don, fac = f)
> donations
    don   fac
1  1000 appr1
2  1500 appr1
3  1200 appr1
4  1800 appr1
5  1600 appr1
6  1100 appr1
7  1000 appr1
8  1250 appr1
9  1500 appr2
10 1800 appr2
11 2000 appr2
12 1200 appr2
13 2000 appr2
14 1700 appr2
15 1800 appr2
16 1900 appr2
17  900 app3r
18 1000 app3r
19 1200 app3r
20 1500 app3r
21 1200 app3r
22 1550 app3r
23 1000 app3r
24 1100 app3r
```

```
> donations.aov = aov(don~f,data=donations)
> summary(donations.aov)
            Df  Sum Sq Mean Sq F value  Pr(>F)
f            2 1362708  681354    9.41 0.00121 **
Residuals   21 1520625   72411
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

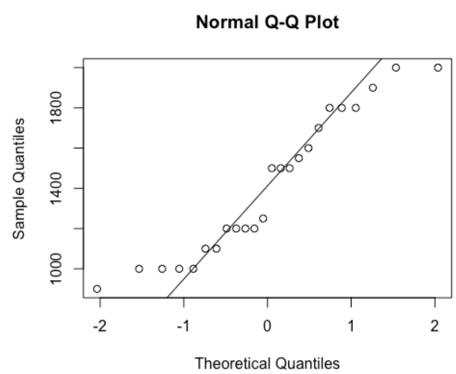
Ans: There is a difference between the approaches ($p = 0.00121 < 0.08$).

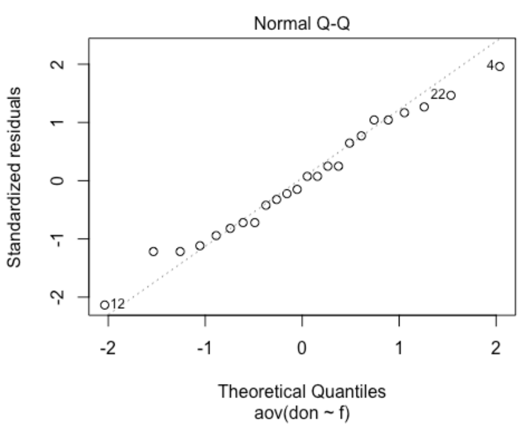
(b) Analyze the residuals from this experiment and comment on model adequacy.
Hint: Use the formulas from Montgomery’s book, or use the R code or the SAS code attached or try StatDisk..

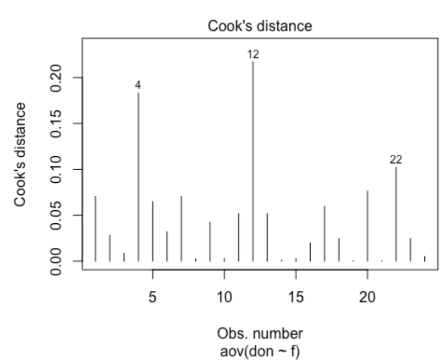
```
> model.tables(donations.aov,"effects")
Tables of effects

 f
f
 app3r  appr1  appr2
-227.1 -102.1  329.2
> model.tables(donations.aov,"means",se=TRUE)
Tables of means
Grand mean

1408.333

 f
f
 app3r  appr1  appr2
1181.2 1306.3 1737.5

Standard errors for differences of means
            f
        134.5
replic.     8
```

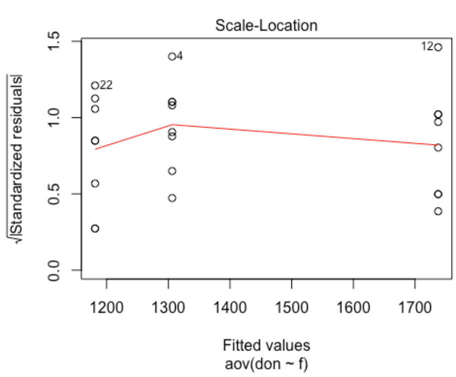



```
> plot(donations.aov,1)
> plot(donations.aov,2)
> qqnorm(don)
> qqline(don)
> plot(donations.aov,3)
> plot(donations.aov,4)
```









Ans: There is nothing unusual about the residuals.

Problem 4.

The response time in milliseconds was determined for three different types of circuits that could be used in an automatic valve shutoff mechanism. The results from a completely randomized experiment are shown in the following table:

Circuit Type	Response Time				
1	9	12	10	8	15
2	20	21	23	17	30
3	6	5	8	16	7

a) Obtain 99% confidence intervals for the mean of each type of circuit

```
> error1 <- qnorm(0.995)*sd(cir1)/sqrt(5)
> left1 <- mean(cir1) - error1
> right1 <- mean(cir1) + error1
> left1
[1] 7.60348
> right1
[1] 13.99652
> error2 <- qnorm(0.995)*sd(cir2)/sqrt(5)
> left2 <- mean(cir2) - error2
> right2 <- mean(cir2) + error2
> left2
[1] 16.59202
> right2
[1] 27.80798
> error3 <- qnorm(0.995)*sd(cir3)/sqrt(5)
> left3 <- mean(cir3) - error3
> right3 <- mean(cir3) + error3
> left3
[1] 3.339298
> right3
[1] 13.4607
```

Ans: 1: (7.60348, 13.99652); 2: (16.59202, 27.90798); 3: (3.339298, 13.4607)

b) Calculate the MSE and get a conclusion about the null hypothesis H_0: $\mu_1 = \mu_2 = \mu_3$

H_a: at least one μ_i is different.

```
> cir1 <- c(9,12,10,8,15)
> cir2 <- c(20,21,23,17,30)
> cir3 <- c(6,5,8,16,7)
> cir <-cbind(cir1,cir2,cir3)
> cir.col <- c(cir1,cir2,cir3)
> fac1 = rep("treat1",5)
> fac2 = rep("treat2",5)
> fac3 = rep("treat3",5)
> fac <- c(fac1,fac2,fac3)
> cir.df <- data.frame(cir.col,fac=fac)
> cir.aov = aov(cir.col~fac,data=cir.df)
> cir.aov
Call:
   aov(formula = cir.col ~ fac, data = cir.df)

Terms:
                  fac Residuals
Sum of Squares  543.6     202.8
Deg. of Freedom     2        12

Residual standard error: 4.110961
Estimated effects may be unbalanced
> summary(cir.aov)
            Df Sum Sq Mean Sq F value   Pr(>F)
fac          2  543.6   271.8   16.08 0.000402 ***
Residuals   12  202.8    16.9
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Conclusion: MSE = 16.9. Reject H_0 in commonly used significance level (p-value = 0.000402).