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CS336
Homework 3

Exercise 19.2 Consider a relation R with five attributes $ABCDE$. You are given the following dependencies: $A \to B$, $BC \to E$, and $ED \to A$.

1. List all keys for R.
   Ans: CDE, ACD, BCD
2. Is R in 3NF?
   Ans: R is in 3NF because B, E, and A are all parts of keys.
3. Is R in BCNF?
   Ans: R is not in BCNF because none of A, BC and ED contains a key.

Exercise 19.3 Consider the relation shown in Figure 19.17.

1. List all the functional dependencies that is the relation instance satisfies.
   Ans: R: $Z \to Y$, $X \to Y$, and $XZ \to Y$
2. Assume that the value of attribute Z of the last record in the relation is changed from z_3 to z_2. Now list all the functional dependencies that this relation instance satisfies.
   Ans: Same as part 1. Functional dependency set is unchanged.

X	Y	Z
x_1	y_1	z_1
x_1	y_1	z_2
x_2	y_1	z_1
x_2	y_1	z_3

Figure 19.17 Relation for Exercise 19.3.

Exercise 19.5 Consider the following collection of relations and dependencies. Assume that each relation is obtained through decomposition from a relation with attributes $ABCDEFGHI$ and that all the known dependencies over relation $ABCDEFGHI$ are listed for each question. (The questions are independent of each other, obviously, since the given dependencies over $ABCDEFGHI$ are different.) For each (sub)relation: (a) State the strongest normal form that the relation is in. (b) If it is not in BCNF, decompose it into a collection of BCNF relations.

1. $R1(A, C, B, D, E)$, $A \to B$, $C \to D$
2. $R2(A, B, F)$, $AC \to E$, $B \to F$
3. $R3(A, D)$, $D \to G$, $G \to H$
4. $R4(D, C, H, G)$, $A \to I$, $I \to A$
5. $R5(A, I, C, E)$

Ans: $R1$ is 1NF, BCNF decomposition: AB, CD, ACE. $R2$ is 1NF. BCNF decomposition: AB, BF. $R3$, $R4$, and $R5$ are all BCNF.

Exercise 19.8 Consider the attribute set $R = ABCDEGH$ and the FD set $F = \{AB \to C, AC \to B, AD \to E, B \to D, BC \to A, E \to G\}$.

1. For each of the following attribute sets, do the following: (i) Compute the set of dependencies that hold over the set and write down a minimal cover. (ii) Name the strongest normal form that is not violated by the relation containing these attributes. (iii) Decompose it into a collection of BCNF relations if it is not in BCNF.
   (a) *ABC*
   i. $AB \to C$, $AC \to B$, $BC \to A$. This is already a minimal cover.
   ii. This is in BCNF since *AB*, *AC* and *BC* are candidate keys for $R1$.
   (b) *ABCD*
   i. $AB \to C, AC \to B, B \to D, BC \to A$. This is already a minimal cover.
   ii. The keys are *AB, AC,* and *BC*. *ABCD* is not in BCNF or even 2NF because of the FD, $B \to D$ (*B* is a proper subset of a key). However, it is in 1NF.
   iii. Decompose as in *ABC, BD*. This is a BCNF decomposition.
   (c) *ABCEG*
   i. $AB \to C, AC \to B, BC \to A, E \to G$. This is already a minimal cover.
   ii. The keys are *ABE, ACE,* and *BCE*. This is not even in 2NF since *E* is a proper subset of the keys and there is a FD $E \to G$. *ABCEG* is in 1NF.
   iii. Decompose as in *ABE, ABC, EG*. This is a BCNF decomposition.
   (d) *DCEGH*
   i. $E \to G$. This is in minimal cover already.
   ii. The key is *DCEH*. *DCEGH* is not in BCNF since in the FD $E \to G$, *E* is a subset of the key and is not in 2NF either.
   iii. *DCEGH* is in 1NF. Decompose as in *DCEH*, *EG*.
   (e) *ACEH*
   i. No FDs exist. This is a minimal cover.
   ii. Key is *ACEH* itself and in BCNF form.
2. Which of the following decompositions of $R = ABCDEG$, with the same set of dependencies *F*, is (a) dependency-preserving? (b) lossless-join?
   a. $\{AB, BC, ABDE, EG\}$
      This decomposition is a **lossy decomposition**. This decomposition **does not preserve** the FD, $AB \to C$ (or $AC \to B$).
   b. $\{ABC, ACDE, ADG\}$
      The join of *ABC* and *ACDE* is lossless because their attribute intersection is *AC*, which is a key for *ABCDEG* so this is lossless. The join with *ADG* is also lossless because the attribute intersection is *AD* and $AD \to ADG$. So this step is also a **lossless decomposition**.
      The projection of the FD's of *R* onto *ABC* gives us: $AB \to C, AC \to B,$ and $BC \to A$. The projection of the FD's of *R* onto *ACDE* gives us: $AD \to E$. The projection of the FD's of *R* onto *ADG* gives us $AD \to G$ (by transitivity). The closure of this set of dependencies does not contain $E \to G$ nor does it contain $B \to D$. So this decomposition is **not dependency preserving**.

Exercise 19.10 Suppose you are given a relation $R(A, B, C, D)$. For each of the following sets of FDs, assuming they are the only dependencies that hold for R, do the following: (a) Identify the candidate key(s) for R. (b) State whether or not the proposed decomposition of R into smaller relations is a good decomposition and briefly explain why or why not.

1. $B \rightarrow C, D \rightarrow A$; decompose into BC and AD.
   (a) BD.
   (b) The decomposition is lossy. The join of BC and AD is the cartesian product which could be much bigger than $ABCD$.
2. $AB \rightarrow C$, $C \rightarrow A$, $C \rightarrow D$; decompose into ACD and BC.
   (a) AB, BC.
   (b) The decomposition is lossless since $ACD \cap BC$ (which is C) $\rightarrow ACD$. The projection of the FD's on ACD include $C \rightarrow D$, $C \rightarrow A$ (so C is a key for ACD) and the projection of FD on BC produces no nontrivial dependencies. In particular this is a BCNF decomposition. However, it is not dependency preserving since the dependency $AB \rightarrow C$ is not preserved. So to enforce preservation of this dependency we need to add ABC, which introduces redundancy. So there is some implicit redundancy cross relations (although none inside ACD and BC).
3. $A \rightarrow BC$, $C \rightarrow AD$; decompose into ABC and AD.
   (a) A, C.
   (b) Since A and C are both candidates keys for R, it is already in BCNF. From a normalization standpoint it makes no sense to decompose R. Furthermore, the decomposition is not dependency preserving since $C \rightarrow AD$ can no longer be enforced.
4. $A \rightarrow B$, $B \rightarrow C$, $C \rightarrow D$; decompose into AB and ACD.
   (a) A.
   (b) The projection of the dependencies on AB are $A \rightarrow B$ and those on ACD are $A \rightarrow C$ and $C \rightarrow D$. ACD is not even in 3NF since C is not a superkey, and D is not part of a key. This is a lossless-join decomposition (since A is a key), but not dependency-preserving (since $B \rightarrow C$ cannot be preserved).
5. $A \rightarrow B$, $B \rightarrow C$, $C \rightarrow D$; decompose into AB, AD, and CD.
   (a) A
   (b) This is a lossless BCNF decomposition, but not dependency preserving ($B \rightarrow C$). So it is not free of implied redundancy.