Jacobs University
Fall 2019

October 14, 2019

Stochastic Methods + Lab

Assignment Sheet 5

Due on October 21, 2019

Problem 1 [5 points]
Let us investigate the Stirling approximation numerically. In class, we used

$$f(n) = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

to to approximate $n! \approx f(n)$. Now do a logarithmic plot of the relative error $\frac{n!-f(n)}{f(n)}$. Do you obtain a straight line? If so, what is the slope? From that deduce what the next order in the Stirling approximation is.

Problem 2 [5 points]
Plot the binomial distribution

$$b(j,n;p) = \binom{n}{j} p^j\, q^{n-j},$$

where $q = 1-p$, into a coordinate system where the values on the x-axis correspond to j according to

$$x_j = \frac{j-np}{\sqrt{npq}}$$

and the y-values are given by $\sqrt{npq}\, b(j,n;p)$. Compare the graphs for $n = 10$, $n = 100$, and the graph of the standard Gaussian

$$\phi(x) = \frac{1}{\sqrt{2\pi}}\, \mathrm{e}^{-x^2/2}$$

in the same plot. Do one plot with $p = 0.2$ and another one with $p = 0.5$. Comment briefly on what you see.

Problem 3 [5 points]
Generate $N = 10\,000$ samples of the binomial distribution (number of successes in n independent trials). Rescale the samples via

$$X = \frac{J - \mathbb{E}[J]}{\sqrt{\mathrm{Var}[J]}}$$

where you use the sample mean to approximate $\mathbb{E}[J]$ and the sample variance to approximate $\sqrt{\mathrm{Var}[J]}$. (These can be computed via the `numpy`-functions `mean()` and `std()`.) Then generate $N = 10\,000$ samples of the standard normal distribution. Plot the sorted

samples for X vs. the sorted samples for the standard normal distribution. Comment briefly on what you see.

Note: This is called a QQ-plot and is more generally used to empirically compare two probability distributions.

Problem 4 [5 points]

Compute an ensemble (at least 1000) of standard Brownian paths $W(t)$ over the interval $[0, 1]$ partitioned into $N = 500$ time steps. Plot the empirically determined mean and standard deviation of the ensemble as a function of time. In the same figure, plot 10 sample paths.