Problem 1a Answer

$$\left(\begin{array}{cccccc|c} b_1 & c_1 & & \dots & 0 & d_1 \\ a_1 & b_2 & c_2 & & \vdots & d_2 \\ & a_2 & b3 & \ddots & & d_3 \\ \vdots & & \ddots & \ddots & c_{n-1} & \vdots \\ 0 & \dots & & a_{n-1} & b_n & d_n \end{array}\right) = \left(\begin{array}{ccccc|c} 1 & \frac{c_1}{b_1} & & \dots & 0 & \frac{d_1}{b_1} \\ 0 & b_2 - a_1\frac{c_1}{b_1} & c_2 & & \vdots & \frac{d_2 - a_1\frac{d_1}{b_1}}{b_2 - a_1\frac{c_1}{b_1}} \\ & a_2 & b_3 & \ddots & & d_3 \\ \vdots & & \ddots & \ddots & c_{n-1} & \vdots \\ 0 & \dots & & a_{n-1} & b_n & d_n \end{array}\right)$$

$$= \left(\begin{array}{cccccc|c} 1 & \frac{c_1}{b_1} & & & \dots & 0 & \frac{d_1}{b_1} \\ 0 & 1 & a_2\frac{c_2}{b_2 - a_1\frac{c_1}{b_1}} & & & \vdots & \frac{d_2 - a_1\frac{d_1}{b_1}}{b_2 - a_1\frac{c_1}{b_1}} \\ & 0 & b_3 - a_2\frac{c_2}{b_2 - a_1\frac{c_1}{b_1}} & & \ddots & & d_3 - a_2\frac{d_2 - a_1\frac{d_1}{b_1}}{b_2 - a_1\frac{c_1}{b_1}} \\ \vdots & & & \ddots & \ddots & c_{n-1} & \vdots \\ 0 & \dots & & & a_{n-1} & b_n & d_n \end{array}\right)$$

we can use use $n-1$ steps to eliminate all a_i. In the main diagonal, divide the row by the value of its pivot in n steps. and then we can back substitute $n-1$ steps. After these $3n-2$ steps, the matrix gives the solution. Thus,

$$c'_i = \begin{cases} \dfrac{c_i}{b_i} & i = 1 \\ \dfrac{c_i}{b_i - a_i c'_{i-1}} & i = 2, 3, \dots, n-1 \end{cases}$$

and

$$d'_i = \begin{cases} \dfrac{d_i}{b_i} & i = 1 \\ \dfrac{d_i - a_i d'_{i-1}}{b_i - a_i c'_{i-1}} & i = 2, 3, \dots, n \end{cases}$$

then

$$x_i = \begin{cases} d'_i & i = n \\ d'_i - c'_i x_{x+1} & i = n-1, n-2, \dots, 1 \end{cases}$$