

## Problem 5 Answer

According to the no-arbitrage arguments:

$$\Delta = \frac{f_u - f_d}{S_{0u} - S_{0d}} \quad (1)$$

$$\Delta = \frac{f_u - f_x}{S_{0u} - S_{0x}} \quad (2)$$

$$\Delta = \frac{f_d - f_x}{S_{0d} - S_{0x}} \quad (3)$$

where  $\Delta$  is the number of units of the stock we should hold for each option shorted in order to create a riskless portfolio.  $f$  is the option price.  $S_0$  is the stock price. Subscripts  $u$ ,  $d$ ,  $x$  indicate the stock price goes up, down, or the other possibility.

According to (1) and (2):

$$\frac{f_u - f_d}{S_{0u} - S_{0d}} = \frac{f_u - f_x}{S_{0u} - S_{0x}} \quad (4)$$

Transform (4):

$$\frac{f_u - f_d}{S_{0u} - S_{0d}} = \frac{f_d - f_x}{S_{0d} - S_{0x}} \quad (5)$$

Find (5) is consistent with (1) and (3). Since the no-arbitrage arguments is the only assumption of the binomial tree model, the analysis of two possibilities of the change of the stock price applies to three possibilities.