Jacobs University   October 28, 2019
Fall 2019

Stochastic Methods + Lab

Assignment Sheet 7

Due on November 4, 2019

Problem 1 [12 points]
It is known that the stochastic differential equation

$$\begin{aligned} dS(t) &= \mu\, S(t)\, dt + \sigma\, S(t)\, dW(t), \\ S(0) &= S_0, \end{aligned} \tag{1}$$

is solved by geometric Brownian motion

$$S(t) = S_0\, e^{(\mu - \sigma^2/2)\, t + \sigma\, W(t)}. \tag{2}$$

(a) Use the Euler-Maruyama method to solve (1) with $\mu = 2$, $\sigma = 1$, and $S_0 = 1$ up to final time $T = 1$. Compare the result in a plot pathwise against the exact solution (2).

(b) Find the *strong order of convergence*, i.e., an exponent p such that

$$\mathbb{E}\big[|S_N - S(T)|\big] \le c\,(\Delta t)^p\,,$$

where $S(T)$ denotes true geometric Brownian motion and S_N its Euler-Maruyama approximation at the final time T.

(c) Find the *weak order of convergence*, i.e., an exponent q such that

$$\big|\mathbb{E}[S_N] - \mathbb{E}[S(T)]\big| \le c\,(\Delta t)^q\,.$$

Problem 2 [4 points]
Use the Black-Scholes formula that we discussed in class (see also Problem 2 of Assignment Sheet 4) and plot the call price C against

(a) the stock price S,

(b) the interest rate r,

(c) the volatility σ.

For each plot, use reasonable parameters.

Problem 3 [4 points]
A theoretical exercise: Show that it is never optimal to exercise an American call option on a non-dividend-paying stock before expiration.