

# Stochastic Methods + Lab

## Assignment Sheet 6

Due on October 28, 2019

### Problem 1 [10 points]

- (a) Compute an ensemble of geometric Brownian paths (at least  $M = 1000$ )

$$S(t) = \exp \left( \left( \mu - \frac{1}{2} \sigma^2 \right) t + \sigma W(t) \right)$$

as a function of time on the interval  $[0, 1]$  partitioned into  $N = 500$  time steps. Then plot the empirical mean and standard deviation and 10 sample paths. Make two plots, one using  $\mu = 0.2$  and  $\sigma = 0.6$ , and the other one using  $\mu = 0.6$  and  $\sigma = 0.2$ , respectively.

- (b) Plot the mean and standard deviation of the stock price paths which underlie the binomial tree model (using the risk-neutral probabilities) with  $N = 500$  time steps calibrated with the same set of parameters  $r = \mu$  and annualized volatility  $\sigma$  as in part (a), together with 10 sample paths (two plots for the different parameters are required here). Use the calibration that we discussed in class and in Assignment Sheet 4.
- (c) Now, for each of the two parameter choices, plot the results of part (a) and (b) in the same figure and describe what you see.

### Problem 2 [5 points]

Use geometric Brownian motion (with  $\mu = 0.05$ ,  $\sigma = 0.3$ ) in a Monte-Carlo valuation of a European call option with strike price  $X = 0.9$ , time to maturity  $T = 1$  and risk free rate  $r = \mu$ . Compare your result against the price obtained from using the Black-Scholes formula by plotting the deviation from the Black-Scholes price against the number of samples in a doubly logarithmic plot.

What is the convergence rate of the Monte-Carlo method as a function of the number of samples?

### Problem 3 [5 points]

For some large  $N$ , approximate the Itô integral

$$I(T) = \int_0^T X(t) dW(t) = \lim_{N \rightarrow \infty} \sum_{i=0}^{N-1} X(t_i) (W(t_{i+1}) - W(t_i))$$

and the Stratonovich integral

$$S(T) = \int_0^T X(t) \circ dW(t) = \lim_{N \rightarrow \infty} \sum_{i=0}^{N-1} X\left(\frac{t_{i+1} + t_i}{2}\right) (W(t_{i+1}) - W(t_i)),$$

where  $W(t)$  denotes standard Brownian motion, and  $t_i = i \Delta t$  with  $\Delta t = T/N$ . As example, choose  $X(t) = W(t)$ .

- (a) Plot one realization of Brownian motion, and the corresponding Itô and Stratonovich integrals.
- (b) For some large  $N$ , look at the difference between the Itô and Stratonovich integrals and describe what you see.