Jacobs University
Fall 2019

October 7, 2019

Stochastic Methods + Lab

Assignment Sheet 4

Due on October 14, 2019

Problem 1 [**14 points**]

Implement the binomial tree in python via backwards induction. Use only one `for` loop to go from one step to the previous one, and implement all other operations with vectors. In more detail, implement a function

```
binomial_tree(payoff, n, rp, sigma, S, K, T)
```

that returns the price of the option at time $T = 0$. The arguments of the function are:

- `payoff`, a function that takes the stock price `S` (possibly a vector) and strike price `K` as arguments and returns the payoff,
- `n`, the number of steps,
- `rp`, the risk-free period interest rate,
- `sigma`, the volatility,
- `S`, the initial stock price,
- `K`, the strike price,
- `T`, the maturity.

Use the calibration of the model that we discussed in class, i.e., use the parameters

$$u = \frac{1}{d} = \exp\left(\sigma\sqrt{\frac{T}{n}}\right).$$

Test your code by pricing a European call option with strike price $K = 0.8$, risk-free period interest rate $r_p = 0.02$, volatility $\sigma = 0.4$, maturity $T = 1$, and initial stock price $S = 1$, using $n = 1000$ steps.

Problem 2 [**6 points**]

The price of a European Call option with current stock price S, strike price K, annualized volatility σ, annual risk-free interest rate r, and maturity time T can be computed explicitly with the Black-Scholes formula

$$C = S\,\Phi(x) - K\,\mathrm{e}^{-rT}\,\Phi(x - \sigma\sqrt{T})\,,$$

where

$$x = \frac{\ln(S/K) + (r + \sigma^2/2)\,T}{\sigma\sqrt{T}},$$

and Φ denotes the cumulative distribution function of the standard normal distribution with mean zero and variance one. Compare your call option prices from the binomial tree model with n steps against those computed by the Black-Scholes formula. Plot the logarithm of the error vs. n. Do you roughly obtain a straight line? If so, with what power of n does the error scale?

Choose parameters $S = 1$, $K = 1.2$, $\sigma = 0.5$, $T = 1$, and $r = 0.03$ (for which the option price is 0.1410).