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1.

#1

set.seed(123)

expo <- rexp(500, rate = 4)

ggplot(data.frame(expo), aes(expo)) +

geom\_histogram(aes(x=expo, y=after\_stat(density)),

bins=30,

fill="mediumpurple",

colour="black") +

labs(x="Values", y="Density") +

ggtitle(label="Randomly Generated Values",

subtitle="Exponential Distribution with Mean 0.25") +

stat\_function(fun = function(x) dexp(x, rate=4),

color="blue",

linewidth=1)

A graph of values with a line

Description automatically generated

2.

#2

expplot <- function(rate, size, line\_color, fill\_color){

expo <- rexp(size, rate)

ggplot(data.frame(expo), aes(expo)) +

geom\_histogram(aes(x=expo, y=after\_stat(density)),

bins=30,

fill=fill\_color,

colour="black") +

labs(x="Values", y="Density") +

ggtitle(label="Randomly Generated Values",

subtitle=paste("Exponential Distribution with Mean ", 1 / rate, sep="")) +

stat\_function(fun = function(x) dexp(x, rate=4),

color=line\_color,

linewidth=1)

}

expplot(rate=5, size=500, line\_color="red", fill\_color="yellow")

A graph of values with a red line

Description automatically generated

3.

#3

norm.or.exp.opt <- function(n,par1,par2=NULL) {

dist <- sample(c("Exponential", "Normal"), 1, prob = c(0.4, 0.6))

if(is.null(par2)) {

par2 = par1

}

if(dist == "Exponential") {

x1 <- rexp(n,1/par1)

st <- paste("Exponential Distribution with Mean ",par1, sep="")

fn <- function(x) dexp(x, rate = 1/par1)

} else if(dist == "Normal") {

x1 <- rnorm(n,par1,par2)

st <- paste("Normal Distribution with Mean ", par1,

" and Standard Deviation ", par2, sep="")

fn <- function(x) dnorm(x, mean = par1, sd = par2)

}

ggplot(data.frame(x1), aes(x1)) +

geom\_histogram(aes(x=x1, y=after\_stat(density)),

bins=30,

fill="lightblue",

colour="black") +

labs(x="Values", y="Density") +

ggtitle(label="Randomly Generated Values",

subtitle= st) +

stat\_function(fun = fn,

color="black",

linewidth=1)

}

norm.or.exp.opt(1000, 10)

norm.or.exp.opt(500, 10, 4)

A diagram of a normal distribution with Ryugyong Hotel in the background

Description automatically generated

A graph of values and values

Description automatically generated

4.

#4

HighRoll <- function(numDice, numSides, targetValue, numTrials) {

apply(matrix(sample(1:numSides, numDice \* numTrials, replace = TRUE), nrow = numDice), 2, sum) >= targetValue

}

plot\_simulation\_estimate <- function(targetValue, maxTrials) {

num\_trials <- 1:maxTrials

prob\_estimates <- sapply(num\_trials, function(trials) mean(HighRoll(3, 6, targetValue, trials)))

plot\_data <- data.frame(Trials = num\_trials, Probability = prob\_estimates)

ggplot(plot\_data, aes(x = Trials, y = Probability)) +

geom\_line() +

labs(title = paste("Simulation-Based Estimated Probability for Sum ≥", targetValue),

x = "Number of Trials",

y = "Estimated Probability") +

theme\_minimal()

}

plot\_simulation\_estimate(targetValue = 11, maxTrials = 300)

A graph showing a number of trials

Description automatically generated

5.

#5

plot\_gamma\_histogram <- function(reps, N, M) {

random\_values <- replicate(reps, sum(rexp(n = N, rate = M)))

plot\_data <- data.frame(Values = random\_values)

ggplot(plot\_data, aes(x = Values)) +

geom\_histogram(bins = 30, fill = "lightblue", color = "black") +

labs(title = "Histogram of Random Values",

x = "Sum of Exponential Variables",

y = "Frequency") +

stat\_function(fun = function(x) dgamma(x, shape = N, scale = 1/M),color="black",

linewidth=1)

}

plot\_gamma\_histogram(reps = 10000, N = 10, M = 0.2)

A graph of a graph of a graph

Description automatically generated

6.

#6

rate1 <- 0.3

rate2 <- 0.2

num\_replicates <- 500000

x1 <- rexp(num\_replicates, rate = rate1)

x2 <- rexp(num\_replicates, rate = rate2)

min\_values <- pmin(x1, x2)

estimated\_prob <- mean(min\_values < 2)

true\_prob <- pexp(2, rate = rate1 + rate2)

estimated\_prob

true\_prob