1 Introduction

Given a **population** of interest, which is some sort of collection of individuals or objects, some examples are

1. People in a certain country
2. Students of a certain university
3. Cars in Maryland

Questions about the population

Population: All cars in Maryland Questions about the population:

1. Brands of the cars (how many Toyotas in Maryland?)
2. What proportion of cars are Toyotas?
3. What is the most common brand?
4. What is the average longevity of the brake pedals?
5. Is there a difference in mileage based on transmission type?
6. What is a range of of values for the average mileage of cars?
7. How does the power of a car affect its mileage?

We don't necessarily have to ask questions that just single in on a single value/statistic about our population.

The answers to questions about the population are **"population parameters"**. These parameters remain fixed because our populations do not change.

It is hard to get the data for and calculate exact population parameters. The solution to this is to work with a subset of the population called a **"random sample"**.

The analogue to a parameter for a population is a **statistic** for a sample (the answer to some question). We use the statistic from the sample to draw inferences about the parameter of the population.

Because our samples can always change, the statistic has its own distribution called the **sampling distribution**, because the different random samples lead to different calculated statistics.

Example 1.1 (Coin Toss)
Note: our population was cars in Maryland.
We call a car a H if its a toyota, and T otherwise.
Now: $P(H) = P(\text{choosing a Toyota}) = \text{proportion of Toyotas in Maryland}$

Thus, the question of calculating the proportion of Toyotas is now a probability calculation for a coin toss.
In other words, we want to know $P(H) = p$

Theorem 1.2 (Law of Large Numbers)
If X_i = outcome of the ith independent coin toss, then

$$\bar{X_n} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

The weak law of large numbers states that $\bar{X_n}$ converges to $P(H) = p$, and so $\bar{X_n}$ becomes a good approximation for $P(H)$, which is p.

Suppose we define some **trials** $\{x_1, x_2, \cdots, x_n\}$

The outcome of the coin toss X_i is defined as

$$X_i = \begin{cases} 1 & \text{if H} \\ 0 & \text{if T} \end{cases}$$

And if defined in this way, then

$$\bar{X_n} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

is the proportion of Heads in the sample, which is the sample mean.