Monthly Meeting on October

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1. Previous work
2. Progress
3. Next step
4. Reference

Last month

- read a paper about an algorithm to find all common bases in two matroids in $O(n(n^2 + t)\lambda)$

1 Previous work

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Matroid Intersection Problem

Problem

input: $M_1 = (E, \mathcal{I}_1),\ M_2 = (E, \mathcal{I}_2)$: matroids
output: maximum-cardinality set $S \in \mathcal{I}_1 \cap \mathcal{I}_2$

note: Generally, $(E,\ \mathcal{I}_1 \cap \mathcal{I}_2)$ is not a matroid.

Matroid Intersection Problem

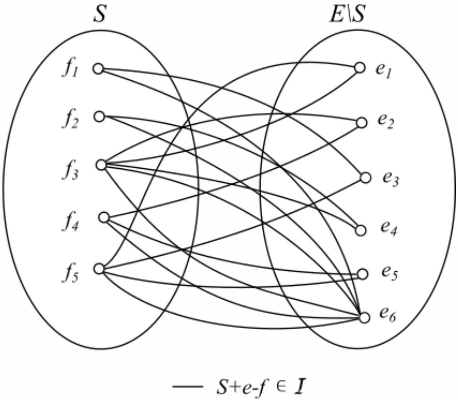
Lemma: unique perfect matching implies exchange

$M_1 := (E, \mathcal{I})$: matroid
$S \in \mathcal{I}$
$\mathcal{G}(S) := (S + E \setminus S,\ A)$: bipartite graph
Suppose that $T \subset E$, $|T| = |S|$, and $\mathcal{G}(S)$ has unique perfect matching X between $T \setminus S$ and $S \setminus T$. Then $T \in \mathcal{I}$.

Matroid Intersection Problem

$$T := \{e_1, e_2, e_3, e_4, f_5\}$$
$$T \setminus S = \{e_1, e_2, e_3, e_4\}$$
$$S \setminus T = \{f_1, f_2, f_3, f_4\}$$

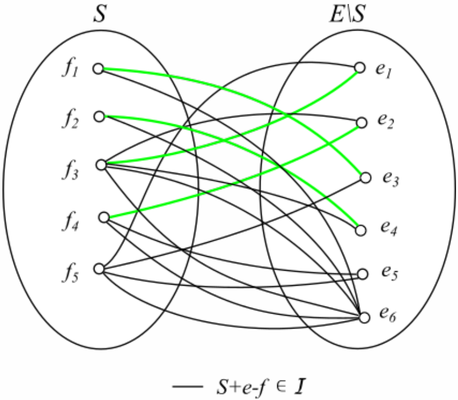
Figure: $\mathcal{G}(S) := (S + E \setminus S,\ A)$



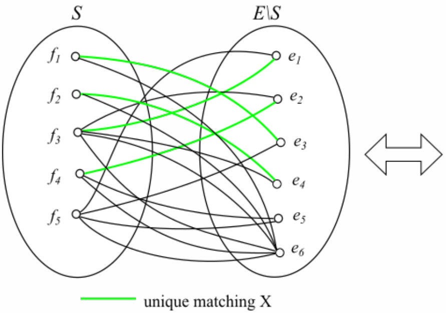
Matroid Intersection Problem

$$T := \{e_1, e_2, e_3, e_4, f_5\}$$
$$T \setminus S = \{e_1, e_2, e_3, e_4\}$$
$$S \setminus T = \{f_1, f_2, f_3, f_4\}$$

Figure: unique perfect matching between $T \setminus S$ and $S \setminus T$



Matroid Intersection Problem



$T \in \mathcal{I}$

Matroid Intersection Problem

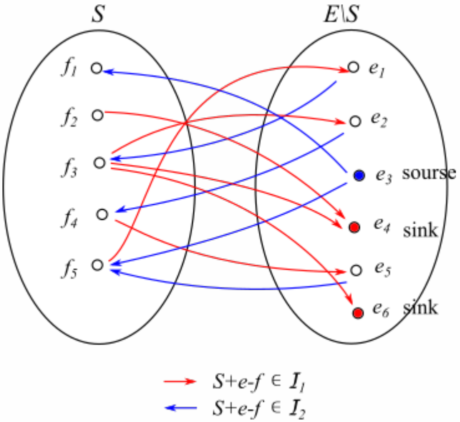
Lemma: shortest implies augmenting

$M_1 := (E, \mathcal{I}_1),\ M_2 := (E, \mathcal{I}_2)$: matroid
$S \in \mathcal{I}_1 \cap \mathcal{I}_2$
$\mathcal{G}(S) := (S + E \setminus S,\ A)$: bipartite digraph
If P is a shortest source-sink dipath in $\mathcal{G}(S)$, then P is augmenting path.

source: $e \in E \setminus S$ s.t. $S + e \in \mathcal{I}_1$
sink: $e \in E \setminus S$ s.t. $S + e \in \mathcal{I}_2$
Note that all edges from source and all edges to sink are omitted in $\mathcal{G}(S)$.

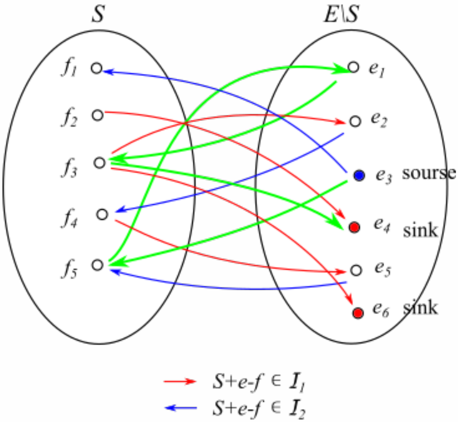
Matroid Intersection Problem

Figure: $\mathcal{G}(S) := (S + E \setminus S,\ A)$:bipartite digraph



Matroid Intersection Problem

Figure: source-sink dipath is augmenting path



Matroid Intersection Problem

Matroid Intersection Algorithm

input: $M_1 = (E, \mathcal{I}_1),\ M_2 = (E, \mathcal{I}_2)$: matroids
output: maximum-cardinality set $S \in \mathcal{I}_1 \cap \mathcal{I}_2$

1. Start with $S \in \mathcal{I}_1 \cap \mathcal{I}_2$ such as $S := \emptyset$
2. While $\mathcal{G}(S)$ has a source-sink dipath:
   (1) $P := (e_0, f_1, e_1, ..., f_n, e_n)$ be an augmenting sequense
   (2) $S := S \cup \{e_j \mid 0 \le j \le n\} \setminus \{f_j \mid 1 \le j \le n\}$

- finding P in $\mathcal{G}(S)$ requires $O(n^2)$
- after step 2(2), regenerate $\mathcal{G}(S)$ requires $O(\tau n)$
  (τ is time complexity of independence oracle)

$\rightarrow$ totally we have $O(\tau n^3)$ as time complexity

1 Previous work

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next month

TODO:

- learn about alogrithm for finding k-best perfect matching

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Reference

Jon Lee: “A First Course in Combinatorial Optimization”