Monthly Meeting on October

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1. Previous work
2. Progress
3. Next step

Last month

1. searched an (polynomial) algorithm to partition two matroids into their common bases
2. found an algorithm to find all common bases in two matroids in $O(n(n^2 + t)\lambda)$

1. Previous work
2. Progress
3. Next step

References

Komei Fukuda, Makoto Namiki: “Finding all common bases in two matroids”, Discrete Applied Mathmatics 56 (1995) 231-243

Finding all common bases in two matroids

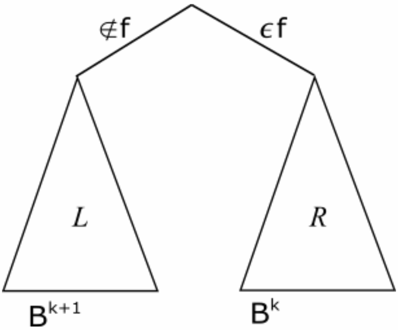
Main result

Given two matroids $M_1 = (E, \mathcal{B}_1)$, $M_2 = (E, \mathcal{B}_2)$, and a common base $B^1 \in \mathcal{B}_1 \cap \mathcal{B}_2$, there is an algorithm finding all common bases of them in $O(n(n^2 + t)\lambda)$ where λ is number of the bases and t is the time to make one pivot operation.

Finding all common bases in two matroids

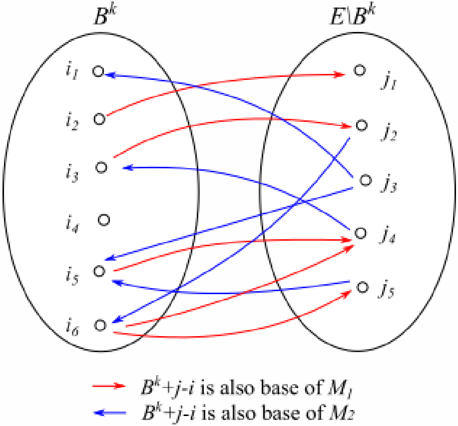
- The algorithm use enumeration tree whose height is $|E| = n$
- In the beginning

Finding all common bases in two matroids



Finding all common bases in two matroids

bipartite graph G



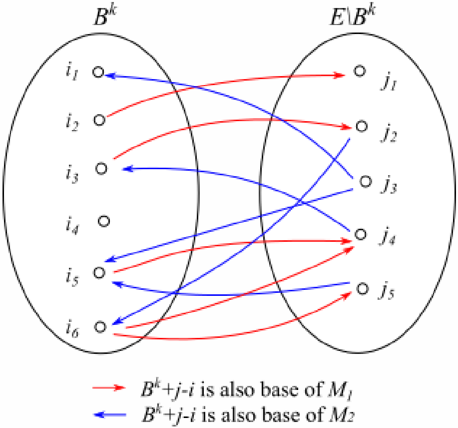
Finding all common bases in two matroids

Theorem 1

There exists a common base B^{k+1} in L if and only if there exists a directed cycle C in G which contains f and consists of elements in E less than or equal to f.

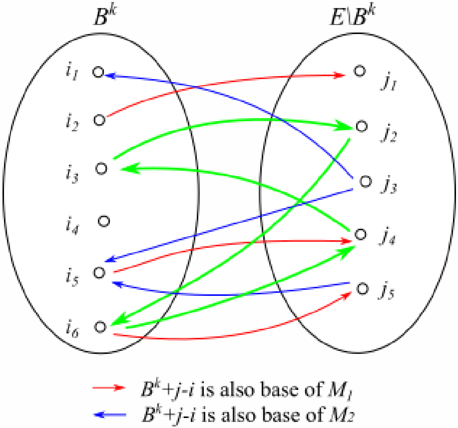
Finding all common bases in two matroids

bipartite graph G
when $f = i_6$...

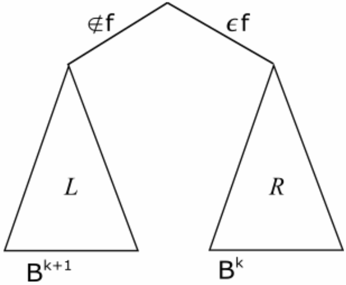


Finding all common bases in two matroids

bipartite graph G
when $f = i_6$...



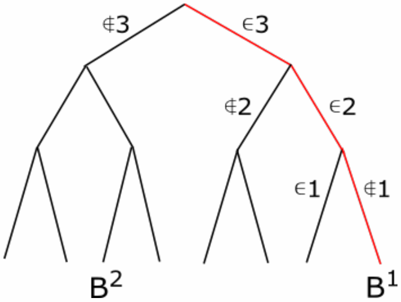
Finding all common bases in two matroids



$$\Updownarrow$$

$$B^k \cup \{j_3,\ j_4\} \setminus \{i_3,\ i_6\} \in \mathcal{B}_1 \cap \mathcal{B}_2$$

Finding all common bases in two matroids

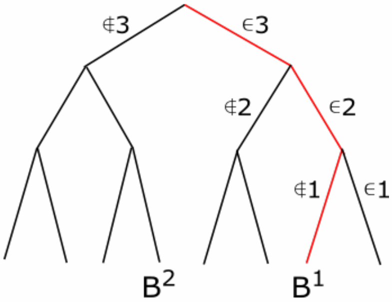


$$P = (R\,R\,R)$$

$$P_1 = R \text{ then } f = 1$$

$$B^1 = \{2,\ 3\}$$

Finding all common bases in two matroids

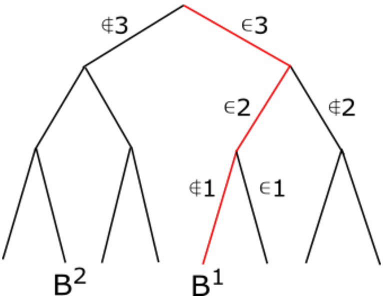


$$P = (L\,R\,R)$$

$$P_2 = R \text{ then } f = 2$$

$$B^1 = \{2,\ 3\}$$

Finding all common bases in two matroids

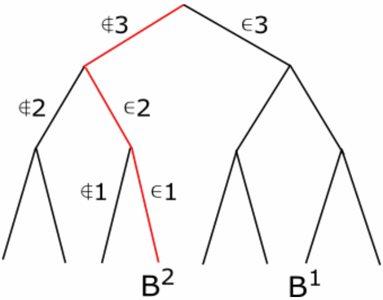


$P = (L\,L\,R)$

$P_3 = R$ then $f = 3$

$B^1 = \{2,\ 3\}$

Finding all common bases in two matroids

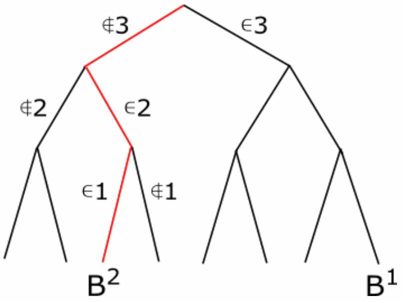


$$P = (R\,R\,L)$$

$$P_1 = R \text{ then } f = 1$$

$$B^2 = \{1, 2\}$$

Finding all common bases in two matroids

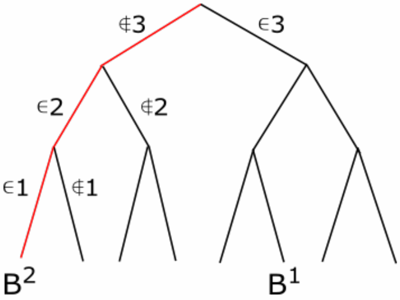


$P = (L\,R\,L)$

$P_2 = R$ then $f = 2$

$B^2 = \{1,\ 2\}$

Finding all common bases in two matroids



$P = (L\,L\,L)$

f doesn't exist

end

Finding all common bases in two matroids

alogrithm “Enumerate”

input: $M_1 := (E, \mathcal{B}_1)$, $M_2 := (E, \mathcal{B}_2)$, $B^1 \in \mathcal{B}_1 \cap \mathcal{B}_2$
output: all common bases of M_1 and M_2

1. $k := 1$, output B^k. $P := (R, ..., R)$. (Here, $P \in \{L, R\}^n$)
2. $f := min\{j \mid P_j = R\}$
3. If f doesn’t exist, stop. Else, create G and find directed cycle C satisfying theorem 1 with breadth first search.
4. If C doesn’t exist, goto step 2. Else, $k := k + 1$, make pivot operation along C and obtain new B^k, $T_{M_1}(B^k)$, and $T_{M_2}(B^k)$. $P_f := L$, $\forall j < k$; $P_j := R$, goto step 2.

1. Previous work
2. Progress
3. Next step

next month

TODO:

1. learn about matroid intersecrion
2. learn how to partition a matoid into bases
3. tackle two different partition matroids (can be partitioned into their common bases)
4. generalized partition matroid of two different uniform matroids and any matroid