Monthly Meeting on September

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1. Previous work
2. Progress
3. Next step

matroid

definition

Let $M = (S, \mathcal{I})$ where S is ground set, and $\mathcal{I}$ is a family satisfying $\mathcal{I} \subseteqq 2^S$.
M is called a **matroid** when $\mathcal{I}$ satisfies:

$$\emptyset \in \mathcal{I} \tag{1}$$

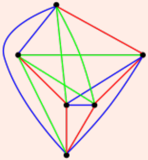
$$I_1 \subset I_2, I_2 \in \mathcal{I} \Rightarrow I_1 \in \mathcal{I} \tag{2}$$

$$I_1, I_2 \in \mathcal{I}, |I_1| < |I_2| \Rightarrow \exists i_2 \in I_2 \setminus I_1 \ ; \ I_1 \cup \{i_2\} \in \mathcal{I} \tag{3}$$

On Disjoint Common Bases in Two Matroids

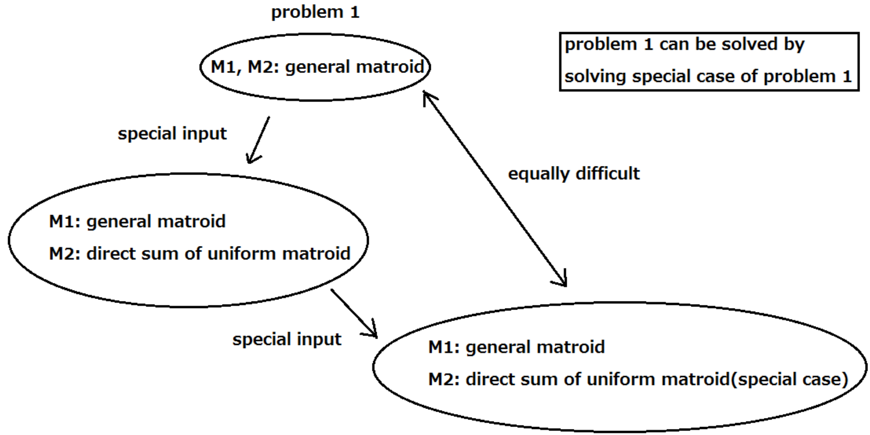
problem 1(open)

input: $M_1 = (S, \mathcal{I}_1)$, $M_2 = (S, \mathcal{I}_2)$ *: matroids*
output: partition of S *into common bases of* M_1 *and* M_2



$\rightarrow$ Can be solved in polynomial time?

On Disjoint Common Bases in Two Matroids



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primitive plan

1. prepare some algorithm to find all common bases
2. check disjoint combination among them

primitive plan

1. prepare some algorithm to find all common bases
2. check disjoint combination among them (NP hard?)

primitive plan

Last month, I came across a paper that suggested a method to find all common bases in two matroids in $O(n(n^2 + t)\lambda)$:
Finding all common bases in two matroids (Fukuda, Namiki 1993)
λ: number of the solution
t: number of pivot

Hall's theorem

$\mathcal{S} = \{S_\lambda \mid \lambda \in \Lambda\}$, S_λ: finite set

transversal

$X = \{x_\lambda \mid \lambda \in \Lambda\}$ is **transversal** of $\mathcal{S}$ when $\forall \lambda \in \Lambda\,;\ x_\lambda \in S_\lambda$.

marriage condition

$\mathcal{S}$ satisfies the **marriage condition** when

$$\forall \mathcal{T} \subseteqq \mathcal{S}\,;\ |\mathcal{T}| \leq \left| \bigcup_{A \in \mathcal{T}} A \right|$$

Hall's theorem

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$\mathcal{S} = \{S_\lambda \mid \lambda \in \Lambda\}$, S_λ: finite set

Then,

$\mathcal{S}$ satisfies marriage condition $\iff$ $\mathcal{S}$ has transversal

Hall's theorem (graph theory)

$G = (X + Y, E)$: bipartite graph ($|X| = |Y|$)

$N_G(A)$: neibourhood of $A \subsetneqq X + Y$

Then,

$\forall W \subsetneqq X$; $|W| \leq |N_G(W)|$ $\iff$ G has perfect matching

$\to$ in original theorem, $\Lambda = X$, $\mathcal{S} = \{N_G(\{x\}) \mid x \in X\}$ then the second follows.

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next month

TODO:

1. continue to read a paper: Finding all common bases in two matroids (Fukuda, Namiki 1993)
2. think about whether $O(n(n^2 + t)\lambda)$ algorithm is applicable

Thank you for your attention.