# Monthly Meeting on October

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# Last month

- read a paper about an algorithm to find all common bases in two matroids in $O(n(n^2 + t)\lambda)$

# Matroid Intersection Problem

**Problem**

input: $M_1 = (E, \mathcal{I}_1),\ M_2 = (E, \mathcal{I}_2)$ : matroids
output: maximum-cardinality set $S \in \mathcal{I}_1 \cap \mathcal{I}_2$

note: Generally, $(E,\ \mathcal{I}_1 \cap \mathcal{I}_2)$ is not a matroid.

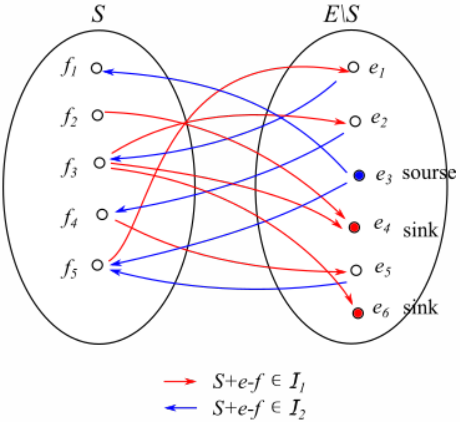
# Matroid Intersection Problem

**Lemma: shortest implies augmenting**

$M_1 := (E, \mathcal{I}_1)$, $M_2 := (E, \mathcal{I}_2)$: matroid
$S \in \mathcal{I}_1 \cap \mathcal{I}_2$
If $P$ is a shortest source-sink dipath in $\mathcal{G}(S)$, then $P$ is augmenting path.

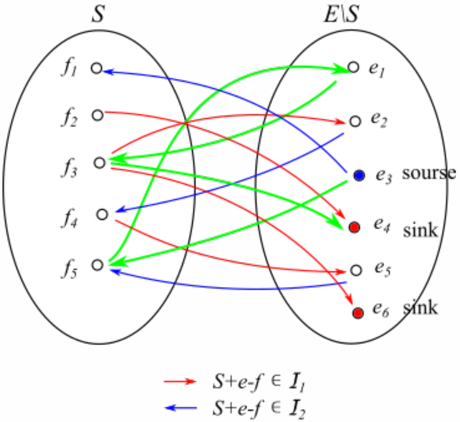
# Matroid Intersection Problem

Figure: $\mathcal{G}(S) := (S + E \setminus S,\ A)$ :bipartite graph



# Matroid Intersection Problem

Figure: source-sink dipath is augmenting path



# Matroid Intersection Problem

## Matroid Intersection Algorithm

input: $M_1 = (E, \mathcal{I}_1),\ M_2 = (E, \mathcal{I}_2)$ : matroids

output: maximum-cardinality set $S \in \mathcal{I}_1 \cap \mathcal{I}_2$

1. Start with $S \in \mathcal{I}_1 \cap \mathcal{I}_2$ such as $S := \emptyset$
2. While $\mathcal{G}(S)$ has a source-sink dipath:
   (1) $P := (e_0, f_1, e_1, ..., f_n, e_n)$ be an augmenting sequense
   (2) $S := S \cup \{e_j \mid 0 \le j \le n\} \setminus \{f_j \mid 1 \le j \le n\}$

- finding $P$ in $\mathcal{G}(S)$ requires $O(n^2)$
- after step 2(2), regenerate $\mathcal{G}(S)$ requires $O(\tau n)$

$\rightarrow$ totally we have $O(\tau n^3)$ as time complexity

# next month

TODO:

- learn about alogrithm for finding k-best perfect matching

# Reference

Jon Lee: "A First Course in Combinatorial Optimization"