Monthly Meeting on October

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1. Previous work
2. Progress
3. Next step

Last month

1. searched an (polynomial) algorithm to partition two matroids into their common bases
2. found an algorithm to find all common bases in two matroids in $O(n(n^2 + t)\lambda)$

References

Komei Fukuda, Makoto Namiki: “Finding all common bases in two matroids”, Discrete Applied Mathmatics 56 (1995) 231-243

Finding all common bases in two matroids

Main result

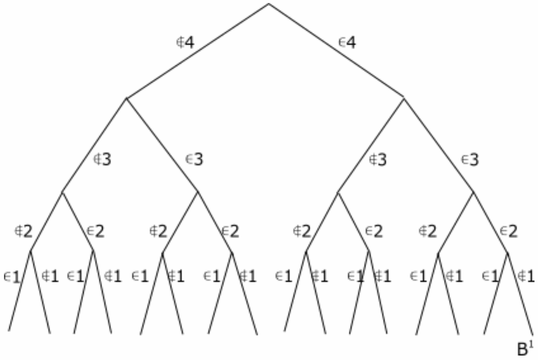
Given two matroids $M_1 = (E, \mathcal{B}_1)$, $M_2 = (E, \mathcal{B}_2)$, and a common base $B^1 \in \mathcal{B}_1 \cap \mathcal{B}_2$, there is an algorithm finding all common bases of them in $O(n(n^2 + t)\lambda)$ where λ is number of the bases and t is the time to make one pivot operation.

Finding all common bases in two matroids

- The algorithm use enumeration tree whose height is $|E| = n$ and the height is indexed in descending order from the root
- In the beginning, the initial comon base B^1 is placed at right most position
- n-dimentional vector $P \in \{L, R\}^n$ is used to indicate the current position
- also current branch is indicated by $f \in E$
- Each one step of algorithm, B^k goes left updating B^k to B^{k+1} if exists

Finding all common bases in two matroids

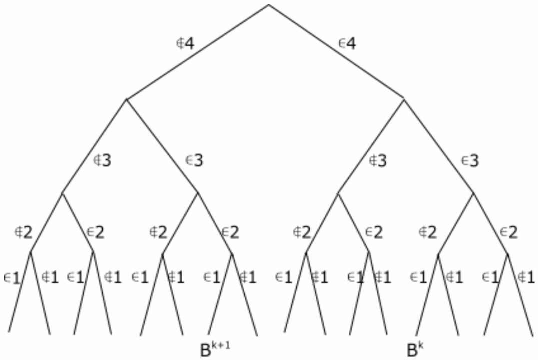
initial state:



$$B^1 = \{2, 3, 4\}$$
$$P = (R\,R\,R\,R),\ f = 1$$

Finding all common bases in two matroids

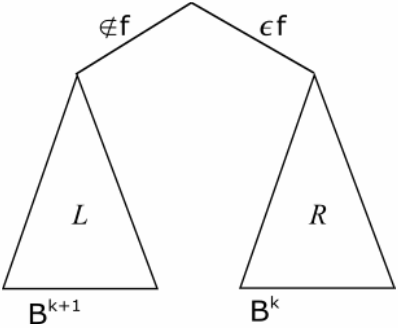
example:



when the current base $B^k = \{1, 3, 4\}$ is placed like above,
$P = (L\,L\,R\,R)$ and $f = 3$

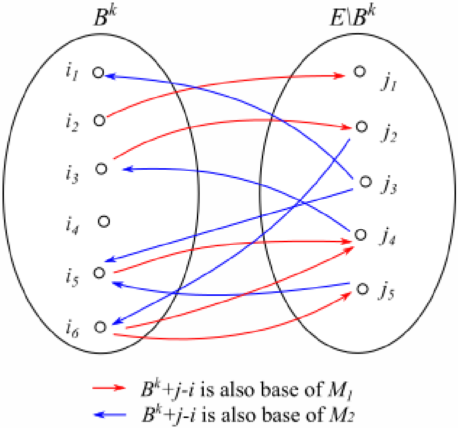
Finding all common bases in two matroids

next, we need a theorem to reduce the search space like a backtracking



Finding all common bases in two matroids

bipartite graph $G(B^k)$



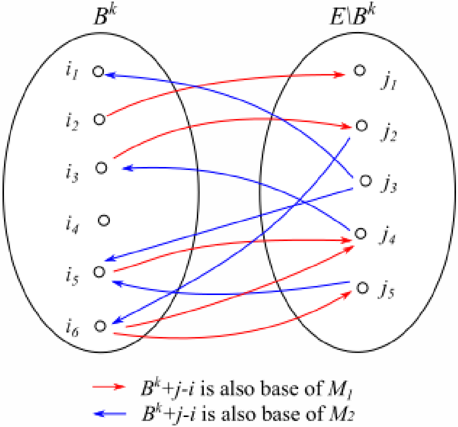
Finding all common bases in two matroids

Theorem 1

There exists a common base B^{k+1} in L if and only if there exists a directed cycle C in $G(B^k)$ which contains f and consists of elements in E less than or equal to f.

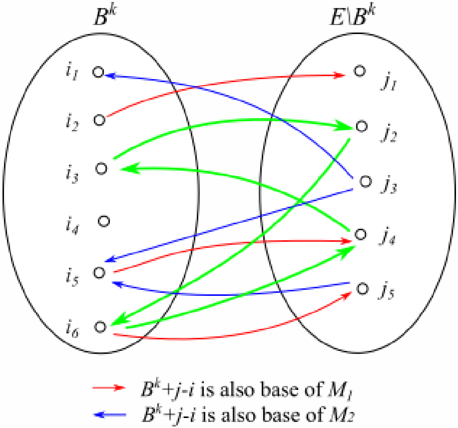
Finding all common bases in two matroids

bipartite graph $G(B^k)$
when $f = i_6$...

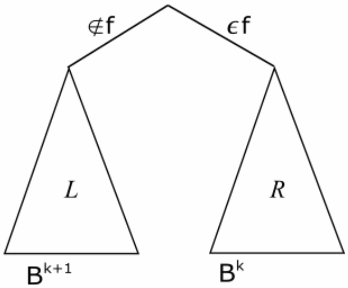


Finding all common bases in two matroids

bipartite graph $G(B^k)$
when $f = i_6$...



Finding all common bases in two matroids



$$\Updownarrow$$

$$B^k \cup \{j_3,\ j_4\} \setminus \{i_3,\ i_6\} \in \mathcal{B}_1 \cap \mathcal{B}_2$$

Finding all common bases in two matroids

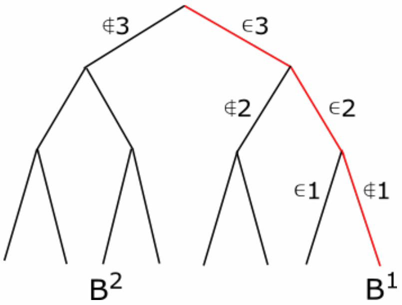
simple algorithm example:

$M_1 = M_2 = (E, \mathcal{B})$

$E = \{1, 2, 3\}, \mathcal{B} = \{\{1, 2\}, \{2, 3\}\}$

$B^1 = \{2, 3\}$

Finding all common bases in two matroids

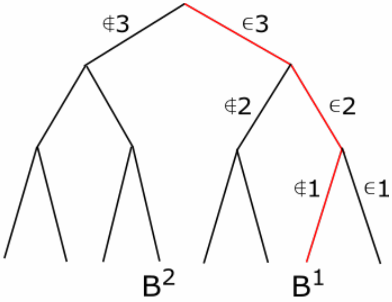


$$B^1 = \{2, 3\}$$

$$P = (R\,R\,R)$$

$$P_1 = R \text{ then } f = 1$$

Finding all common bases in two matroids

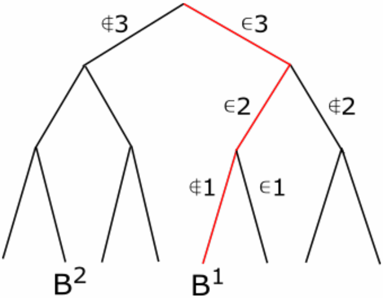


$$B^1 = \{2,\ 3\}$$

$$P = (L\,R\,R)$$

$$P_2 = R \text{ then } f = 2$$

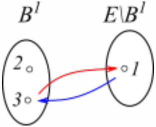
Finding all common bases in two matroids



$B^1 = \{2,\ 3\}$

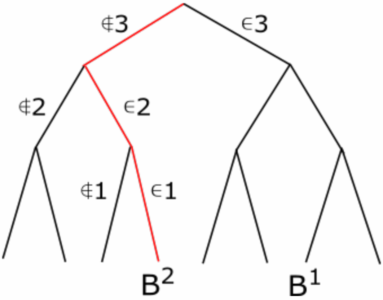
$P = (L\,L\,R)$

$P_3 = R$ then $f = 3$



there exists an directed cycle C in bipartite graph $G(B^1)$
$\rightarrow$make pivot operation along C and get new base $B^2 = \{1,\ 2\}$

Finding all common bases in two matroids

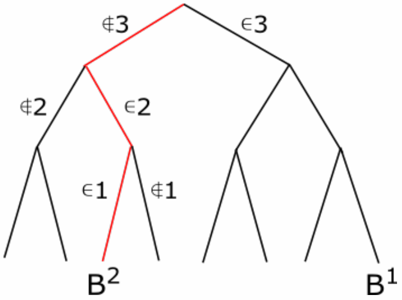


$$B^2 = \{1,\ 2\}$$

$$P = (R\,R\,L)$$

$$P_1 = R \text{ then } f = 1$$

Finding all common bases in two matroids

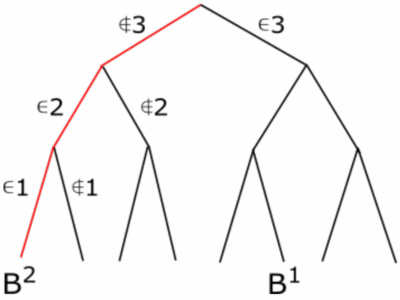


$B^2 = \{1,\ 2\}$

$P = (L\,R\,L)$

$P_2 = R$ then $f = 2$

Finding all common bases in two matroids



$B^2 = \{1,\ 2\}$

$P = (L\,L\,L)$

f doesn't exist

end

Finding all common bases in two matroids

alogrithm “Enumerate”

input: $M_1 := (E, \mathcal{B}_1)$, $M_2 := (E, \mathcal{B}_2)$, $B^1 \in \mathcal{B}_1 \cap \mathcal{B}_2$
output: all common bases of M_1 and M_2

1. $k := 1$, output B^k. $P := (R, ..., R)$. (Here, $P \in \{L, R\}^n$)
2. $f := min\{j \mid P_j = R\}$
3. If f doesn’t exist, stop. Else, create $G(B^k)$ and find directed cycle C satisfying theorem 1 with breadth first search.
4. If C doesn’t exist, goto step 2. Else, $k := k + 1$, make pivot operation along C and obtain new B^k. $P_f := L, \forall j < k; P_j := R$, goto step 2.

Finding all common bases in two matroids

Again, $n = |E|$, $\lambda = |\mathcal{B}_1 \cap \mathcal{B}_2|$, and t is time to generate $G(B^k)$.

time complexity

We evaluate the time from obtaining B^k to obtaining B^{k+1}. Let A_{max} be the maximum number of the arc in $G(B^k)$. Then C in $G(B^k)$ satisfying Theorem 1 can be found in $O(A_{max})$ using BFS. This is iterated at most n times(until finding such C). If such C exists, then we pivot elements in B^k along C to get B^{k+1} and generate $G(B^{k+1})$. This takes $O(C_{max}t)$ where C_{max} is the length of the C.
Since we know $A_{max} \leq (\frac{n}{2})^2 \cdot 2 = \frac{n^2}{2}$ and $C_{max} \leq 2 \cdot \frac{n}{2} = n$, it takes $O(n^3 + nt)$ from B^k to B^{k+1}.
This is iterated λ times, so total time complexity is $O((n^3 + nt)\lambda)$.

(note: t depends on the imput form of the matroid. for example, the matroid is given in a form of a map, $t = O(n^2)$, but matroid is given in a form of a matrix, it's more than n^2)

next month

TODO:

1. learn about matroid intersecrion
2. learn how to partition a matoid into bases
3. tackle two different partition matroids (can be partitioned into their common bases)
4. generalized partition matroid of two different uniform matroids and any matroid