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① $m(a+bX) = a + b \cdot m(X)$

$Y = a + bX$

$m(Y) = \frac{1}{n}\sum_{i=1}^{n} Y_i$

$m(Y) = \frac{1}{n}\sum_{i=1}^{n} (a + bX_i)$

$m(Y) = \frac{1}{n}\sum_{i=1}^{n} a + \frac{1}{n}\sum_{i=1}^{n} bX_i$

$m(Y) = \frac{na}{n} + \frac{b}{n}\sum_{i=1}^{n} X_i$ $\quad m(Y) = a + b \cdot m(X)$

$\boxed{m(a+bX) = a + b \cdot m(X)}$

② $cov(X, a+bY) = b \cdot cov(X,Y)$

$Z = a + bY$

$cov(X,Z) = \frac{1}{N}\sum_{i=1}^{n} (x_i - m(X))(z_i - m(Z))$

$(x_i - m(X))(a + by_i - m(a+bY))$

$a + by_i - a - b \cdot m(Y)$

$\frac{1}{N}\sum_{i=1}^{n} (x_i - m(x))(by_i - bm(Y))$

$\frac{b}{N}\sum_{i=1}^{n} (x_i - m(x))(y_i - m(Y))$

$cov(X, Z) = b \cdot cov(X,Y)$

$\boxed{cov(X, a+bY) = b \cdot cov(X,Y)}$

③ $cov(a+bX, a+bX) = b^2 cov(X,X)$

$cov(a+bx, a+bx) = \frac{1}{N}\sum_{i=1}^{N} (a + bx_i - m(a+bx_i))(a + bx_i - m(a+bx_i))$

$(a + bx_i - a - bm(X))^2$

$\frac{1}{N}\sum_{i=1}^{N} (bx_i - bm(x))^2$

$\frac{1}{N}\sum_{i=1}^{N} b^2(x_i - m(X))^2$

$\frac{b^2}{N}\sum_{i=1}^{N} (x_i - m(x))^2$

$\boxed{cov(a+bx, a+bx) = b^2 cov(X,X)}$

$\boxed{cov(X,X) = \frac{1}{N}\sum_{i=1}^{N} (x_i - m(X))^2 = s^2}$

④ Non-decreasing transformation of median is not median of transformed variable. If $g(x)$ is constant, the transformation may be distorted. This applies to any quantile, IQR, and range.

⑤ Not true. ex. $g(x) = x^2$

$$X = [-3, -1, 2, 4]$$

$$m(X) = \frac{-1+2}{2} = \frac{1}{2}$$

$$g(m(X)) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$g(X) = [(-3)^2, (-1)^2, 2^2, 4^2]$$

$$[9, 1, 4, 16]$$

$$m(g(X)) = \frac{13}{2}$$

$$\frac{1}{4} \neq \frac{13}{2}$$