《计算机程序的构造和解释》
Lab 04: Data Abstraction, Trees, and Mutable Values

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2022 年 10 月 19 日

Anonymous Recursion

Consider this code:

```
def add1(n):
    return n + 1
```

Insight

1 can be *generalized* to an arbitrary integer.

```
def add(n1):
    return lambda n:n + n1
```

Question

how can we use add to define add1 ?

```
add1 = add(1)
```

Anonymous Recursion

Intuition 1

Generalize a term t in a function f yields a higher order function g that is more generalized than f. And f can be recovered from g and t by:

$$f = g(t)$$

Anonymous Recursion

Consider this code:

```
def factor(n):
    return 1 if n == 0 else n * factor(n - 1)
```

Insight

Underlined `factor` can be generalized to an arbitrary function.

```
def factor2(f):
    return lambda n:1 if n == 0 else n * f(n - 1)
```

Question

How can we use `factor2` to define `factor` ?

Anonymous Recursion

Use the Intuition 1:

```
factor = factor2(factor2)
```

But it's not correct.

```
factor2(factor2)
= lambda n:1 if n == 0 else n * factor2(n - 1)
```

`factor2` should be applied to a **function**, not integer.

Question

What is that **function** ?

Anonymous Recursion

Make some fix:

```
def factor3(f):
    return lambda n:1 if n == 0 else n * f(f)(n - 1)

factor = factor3(factor3)
```

Does it work ?

```
factor3(factor3)(3)
= 1 if 3 == 0 else 3 * factor3(factor3)(2)
= 3 * factor3(factor3)(2)
```

Wow, it just work fine !

Anonymous Recursion

We can define a anonymous recursion function which computes factorial.

```
factor3(factor3)
= (lambda f: lambda n:1 if n == 0 else n * f(f)(n - 1))
  (lambda f: lambda n:1 if n == 0 else n * f(f)(n - 1))
```

Anonymous Recursion

Question

How can we use `factor2` to define `factor` ?

Anonymous Recursion

```
def f(n):
    n * n + n
```

Consider this expression:

```
a = 5 * 5 + 5 + 1
```

Insight

Underlined expression can be *folded* into `f(5)`

```
a = f(5) + 1
```

Anonymous Recursion

It seems that ...

```
factor3
= lambda f: lambda n:1 if n == 0 else n * f(f)(n - 1)
= lambda f: factor2(f(f))
```

So,

```
factor
= factor3(factor3)
= (lambda f: factor2(f(f)))(lambda f: factor2(f(f)))
```

Insight

Underlined `factor2` can be generalized to an arbitrary function.

Anonymous Recursion

Now, we get a super power function, `y`. `y` makes every "factor2 like" function to be a recursive function.

```
y = lambda f: (lambda x: f(x(x)))(lambda x: f(x(x)))
```

`y` has a very interesting property:

$$y(f) = f(y(f))$$

In fact,

```
y(factor2) = factor3(factor3)
           = factor2(factor3(factor3))
```

That's why `factor3` works.

Anonymous Recursion

But wait, there's something wrong:

```
y(factor2)
= factor3(factor3)
= (lambda x: factor2(x(x)))(lambda x: factor2(x(x)))
= factor2((lambda x:(factor2(x(x))))
          (lambda x:(factor2(x(x)))))
= factor2(factor3(factor3))
= factor2(factor2(factor3(factor3)))
= ...
```

Question

Why `factor3(factor3)`(the first version) works?

Anonymous Recursion

What will `factor3(factor3)`(the first version) be evaluated to ?

```
factor3(factor3)
= (lambda n: 1 if n == 0 then n * factor3(factor3)(n - 1))
```

Insight

Python will not evaluate anything inside a function.

So, just fold a "smaller" term:

```
factor3
= lambda f: lambda n: 1 if n == 0 else n * f(f)(n - 1)
= lambda f: lambda n: factor2(f(f))(n)
```

Anonymous Recursion

Finally, we get:

```
def y(f):
    a = lambda x: lambda n: f(x(x))(n)
    return a(a)
```

or

```
lambda f: (lambda a: a(a))(lambda x: lambda n: f(x(x))(n))
```

another form: ("Z combinator")

```
lambda f: (lambda a: a(a))(lambda x: f(lambda n: x(x)(n)))
```

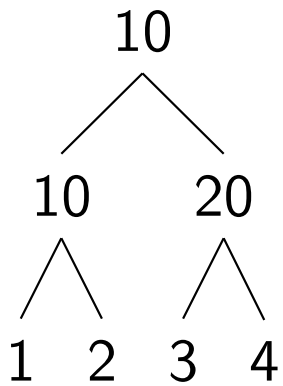
Mobile

图: Mobile(sculpture)

Mobile

25 min to finish.

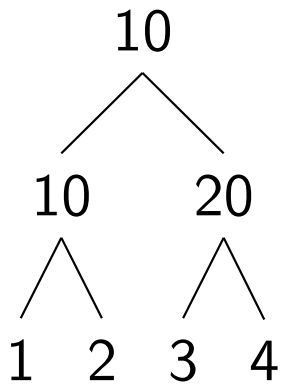
Preorder



See document 2.4(Tree) for more information.

Preorder

Catamorphism, or tree fold, or "recursion left, recursion right, then combine".



15 min to finish.

Insert

Be carful!
15 min to finish.

作业与课程安排

本周的作业

- 10 月 22 日截止：Lab04
- 10 月 25 日截止：Homework04
- 10 月 30 日截止：Project01（还有 11 天！）

Hog Contest 2022

- 不计分，11 月 12 日截止（期中考试后的周六）
- 会有优胜奖品与随机抽奖（仅限 2022 级同学）

期中考试

- 11 月 7 日（周一）14:00 - 16:00
- 不能线下参加的同学请等通知，会进行统一安排

One more thing...

Mock Exam Problem

- 本期主题：Trees
- 2021 年期中考试最后一题（有改动）