# Higher-Order Functions

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Slides adapted from Berkeley CS61a

# Higher-Order Functions

Functions are first-class, meaning they can be manipulated as values

A higher-order function is:

1. A function that takes a function as an argument

*and/or*

1. A function that returns a function as a return value

# Designing Functions

# Describing Functions

A function's *domain* is the set of all inputs it might possibly take as arguments.

A function's range is the set of output values it might possibly return.

A pure function's behavior is the relationship it creates between input and output.

```
def square(x):
      """Return X * X"""
```

*x is a number*

*square returns a non-negative real number*

*square returns the square of x*

# A Guide to Designing Function

Give each function exactly one job, but make it apply to many related situations

```
>>> round(1.23)
1
```

```
>>> round(1.23, 1)
1.2
```

```
>>> round(1.23, 0)
1
```

```
>>> round(1.23, 5)
1.23
```

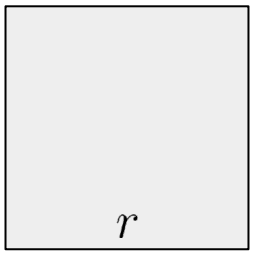
Don’t repeat yourself (DRY). Implement a process just once, but execute it many times.

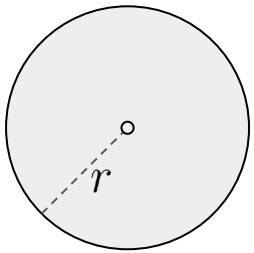
# Generalization

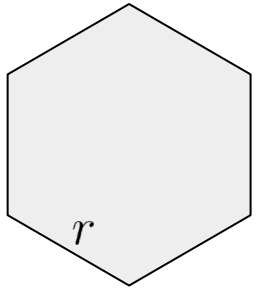
# Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

Shape:




Area:

$r^2$

$\pi \cdot r^2$

$\frac{3\sqrt{3}}{2} \cdot r^2$

Finding common structure allows for shared implementation

Demo

# Higher-Order Functions

# Generalizing Over Computational Processes

The common structure among functions may be a computational process, rather than a number.

$$\sum_{k=1}^{5} k = 1 + 2 + 3 + 4 + 5 = 15$$

$$\sum_{k=1}^{5} k^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225$$

$$\sum_{k=1}^{5} \frac{8}{(4k-3)\cdot(4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} = 3.04$$

Demo

# Summation Example

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Function of a single argument *(not called "term")*

A formal parameter that will be bound to a function

The cube function is passed as an argument value

The function bound to term gets called here

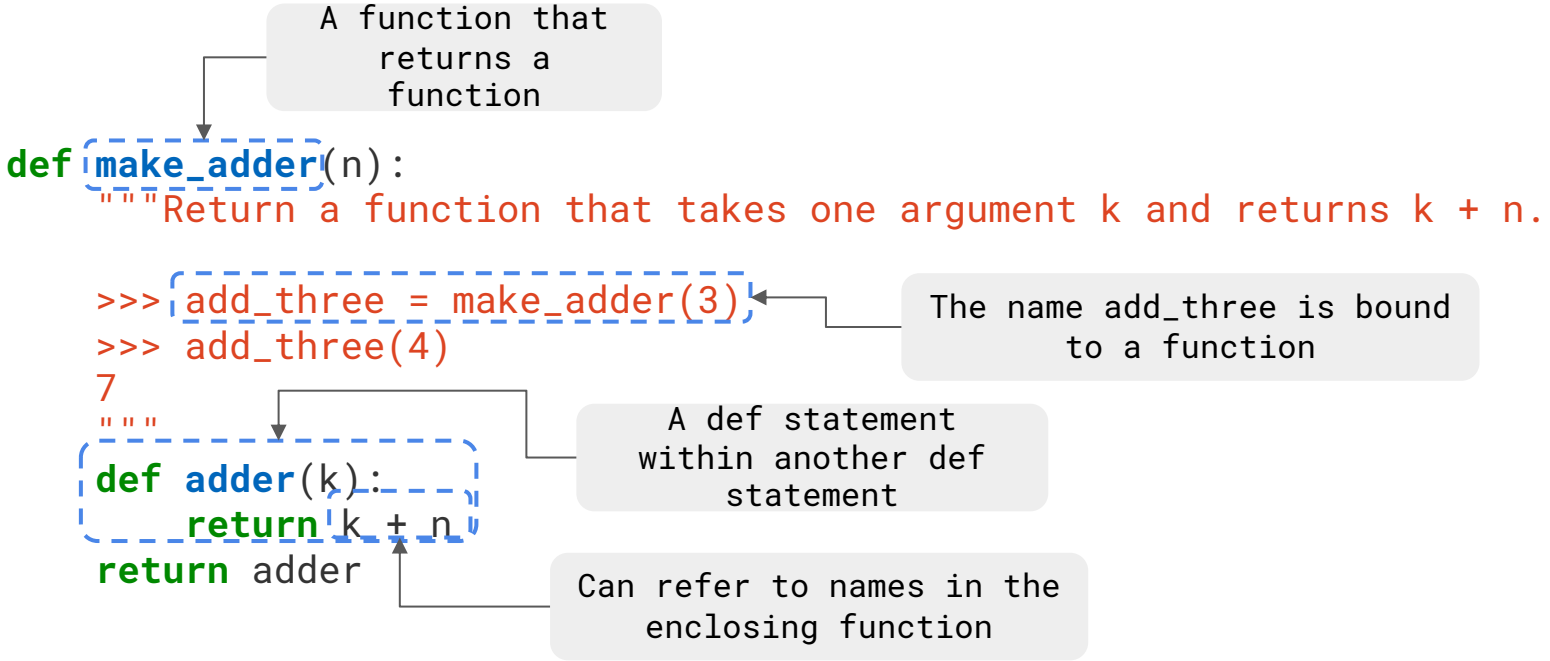
`0 + 1 + 8 + 27 + 64 + 125`

# Functions as Return Values

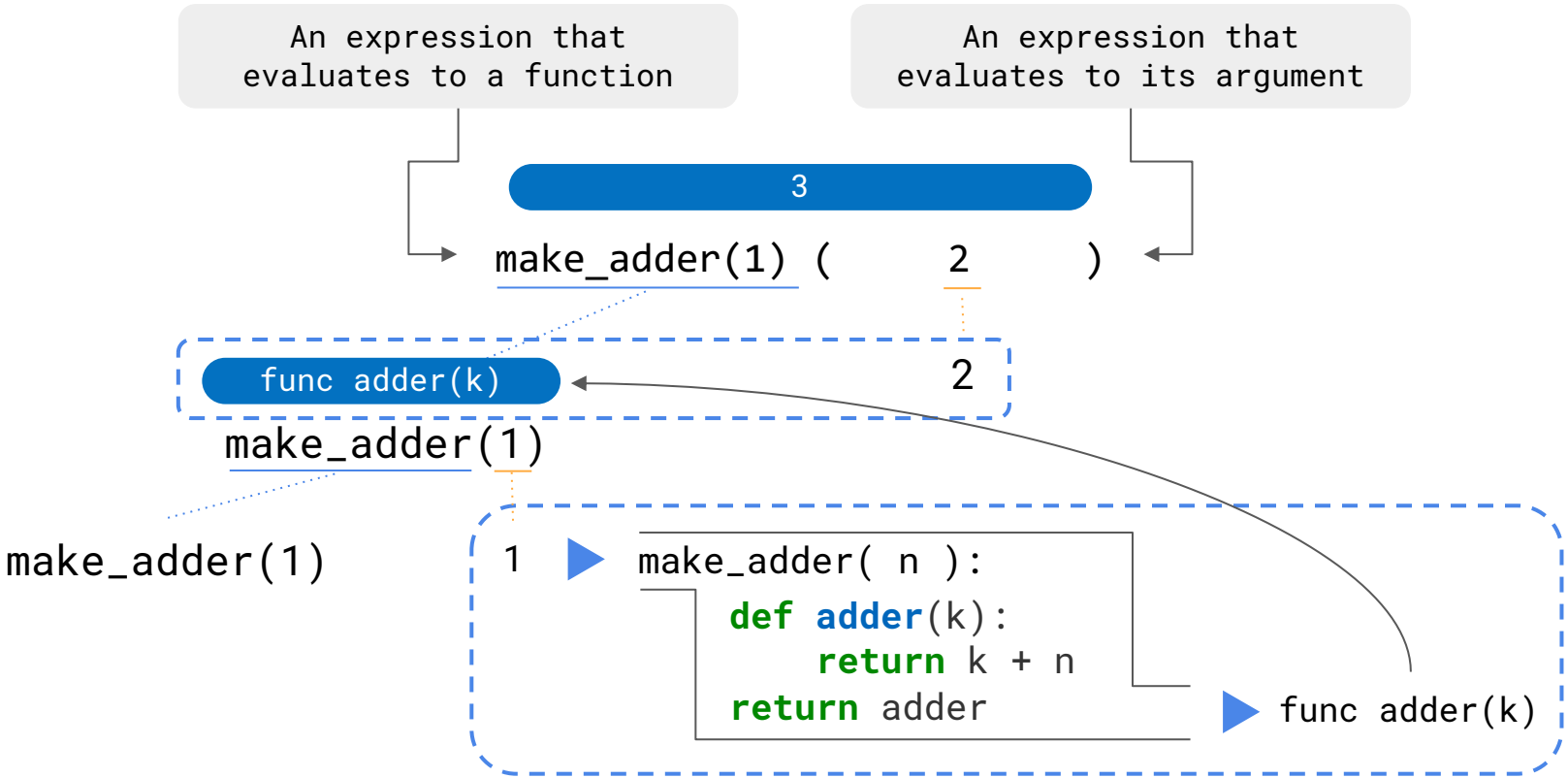
Demo

# Locally Defined Functions

Functions defined within other function bodies are bound to names in a local frame



# Call Expressions as Operator Expressions



# A More Complex Example

```
def make_adder(n):
    """Return a function that takes one argument k and returns k +
n.

    >>> add_three = make_adder(3)
    >>> add_three(4)

    """
    def adder(k):
        return k + n
    return adder

def square(x):
    return x * x

def compose1(f, g):
    def h(x):
        return f(g(x))
    return h
```

```
compose1(square, make_adder(2))(3)
```

# Self Reference

# Returning a Function Using Its Own Name

```
def print_sums(n):
    print(n)
    def next_sum(k):
        return print_sums(n + k)
    return next_sum

print_sums(1)(3)(5)
```

# Summary

- Higher-order function: any function that either accepts a function as an argument and/or returns a function
- Why are these useful?
  - Generalize over different form of computation
  - Helps remove repetitive segments of code
- One use case: summation
  - We generalized over the computation of each term
- We saw nested functions can access variables in outer function (adder) as well as the outer function itself (print_sums)