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Prelim 1

$$\begin{cases} G_j + R_x \underset{k_{-j}}{\overset{k_{+,j}}{\rightleftharpoons}} (G_j:R_x)_c & (1) \\ (G_j:R_x)_c \xrightarrow{k_{I,j}} (G_j:R_x)_o & (2) \\ (G_j:R_x)_o \xrightarrow{k_{A,j}} R_x + G_j & (3) \\ (G_j:R_x)_o \xrightarrow{k_{E,j}} m_j + R_x + G_j & (4) \end{cases}$$

Q1. a) $r_{x,j} = k_{E,j}(G_j:R_x)_o$

$$\frac{d}{dt}(G_j:R_x)_c = k_{+,j}(G_j)(R_x) - k_{-j}(G_j:R_x)_c - k_{I,j}(G_j:R_x)_c \quad (5)$$

$$\frac{d}{dt}(G_j:R_x)_o = k_{I,j}(G_j:R_x)_c - k_{A,j}(G_j:R_x)_o - k_{E,j}(G_j:R_x)_o \quad (6)$$

proposed total abundance of RNAP $(R_{x,T})$:

$$R_{x,T} = R_x + \underbrace{(G_j:R_x)_c}_{①} + \underbrace{(G_j:R_x)_o}_{②} + \underbrace{\sum_{i=1,j}^{N}\{(G_j:R_x)_c + (G_j:R_x)_o\}}_{③} \quad (7)$$

At steady state, let $\frac{d}{dt}(G_j:R_x)_c = 0$, $\frac{d}{dt}(G_j:R_x)_o = 0$

① $(G_j:R_x)_c \simeq \left(\frac{k_{+,j}}{k_{-j}+k_{I,j}}\right)(G_j)(R_x)$

$(G_j:R_x)_o = \left(\frac{k_{I,j}}{k_{A,j}+k_{E,j}}\right)(G_j:R_x)_c$

let $\begin{cases} \frac{k_{+,j}}{k_{-,j}+k_{I,j}} \equiv K_{x,j}^{-1} \\ \tau_{x,j}^{-1} \equiv \frac{k_{I,j}}{k_{A,j}+k_{E,j}} \end{cases}$

$\Downarrow$

② $(G_j:R_x)_o = (K_{x,j}^{-1})(\tau_{x,j}^{-1})(G_j)(R_x) \quad (8)$

③ $$\sum_{i=1,j}^{N}\{(G_j:R_x)_c + (G_j:R_x)_o\} = \sum_{i=1,j}^{N}\{(K_{x,i}^{-1})(G_i)(R_x) + (K_{x,i}^{-1})(\tau_{x,i}^{-1})(G_i)(R_x)\}$$

$$R_{x,T} = R_x + (K_{x,j}^{-1})(G_j)(R_x) + (K_{x,j}^{-1})(T_{x,j}^{-1})(G_j)(R_x) + \sum_{i=1,j}^{N} \left\{ (K_{x,i}^{-1})(G_i)(R_x) + (K_{x,i}^{-1})(T_{x,i}^{-1})(G_i)(R_x) \right\}$$

plus $K_{x,j}T_{x,j}$ on both sides

$$(K_{x,j}T_{x,j})R_{x,T} = R_x(K_{x,j}T_{x,j}) + (T_{x,j})(G_j)(R_x) + (G_j)(R_X) + \sum_{i=1,j}^{N} \left\{ \frac{K_{x,j}T_{x,j}}{K_{x,i}}(G_i)(R_x) + \frac{K_{x,j}T_{x,j}}{K_{x,i}T_{x,i}}(G_i)(R_X) \right\}$$

$$= R_x \left\{ T_{x,j}K_{x,j} + (T_{x,j}+1)(G_j) + \sum_{i=1,j}^{N} \left(\frac{K_{x,j}T_{x,j}}{K_{x,i}T_{x,i}}\right)(T_{x,i}+1)(G_i) \right\}$$

$$R_x = \frac{R_{x,T}(T_{x,j}K_{x,j})}{T_{x,j}K_{x,j} + (T_{x,j}+1)G_j + \sum_{i=1,j}^{N} \frac{K_{x,j}T_{x,j}}{K_{x,i}T_{x,i}}(1+T_{x,i})G_i}$$

put back into (8)

$$(G_j : R_x)_0 = \frac{R_{x,T}G_j}{T_{x,j}K_{x,j} + (T_{x,j}+1)G_j + \sum_{i=1,j}^{N} \frac{K_{x,j}T_{x,j}}{K_{x,i}T_{x,i}}(1+T_{x,i})G_i}$$

because $r_{x,j} = k_{E,j}(G_j : R_x)_0$

then it can give $r_{x,j} = k_{E,j}R_{x,T}\left(\frac{G_j}{T_{x,j}K_{x,j} + (1+T_{x,j})G_j + \varepsilon_j}\right)$

where $\varepsilon_j = \sum_{i=1,j}^{N} \frac{K_{x,j}T_{x,j}}{K_{x,i}T_{x,i}}(1+T_{x,i})G_i$

b) For the single gene case, the ε_j term vanished

$$r_{x,j} = k_{E,j} R_{X,T} \left(\frac{G_j}{\tau_{x,j} K_{x,j} + (1+\tau_{x,j}) G_j} \right)$$

and with a negligible RNAP abort rate.

$k_{A,j}$ is small compared to both $k_{I,j}$ and $k_{E,j}$

then $\tau_{x,j} = \dfrac{k_{E,j} + k_{A,j}}{k_{I,j}} \approx \dfrac{k_{E,j}}{k_{I,j}}$

when $\tau_{x,j} \ll 1$ $\quad k_{I,j} \gg k_{E,j}$

$$r_{x,j} = k_{E,j} R_{x,T} \left(\frac{G_j}{\tau_{x,j} K_{x,j} + G_j} \right)$$

So the transcription is elongation limited