Q1. Logistic Regression with Newton's Method

a) $$\nabla_w J = 2\lambda w - \sum_{i=1}^{n} \frac{\partial S_i}{\partial w}\left(\frac{y_i}{S_i} - \frac{1-y_i}{1-S_i}\right)$$

$$= 2\lambda w - \sum_{i=1}^{n} S_i(1-S_i)\left[\frac{(1-S_i)y_i - (1-y_i)S_i}{S_i(1-S_i)}\right] x_i$$

$$= 2\lambda w - \sum_{i=1}^{n}(y_i - S_i)x_i$$

$$= 2\lambda w - X^T(Y-S)$$

b) $$\nabla_w^2 J = 2\lambda + \sum_{i=1}^{n} x_i S_i(1-S_i) x_i$$

$$= 2\lambda + \sum_{i=1}^{n} S_i(1-S_i) x_i x_i^T$$

$$= 2\lambda + X^T \Omega X \quad \text{where } \lambda = \underset{3\times 3}{\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}}, \quad \Omega = \underset{4\times 4}{\begin{bmatrix} S_1(1-S_1) & 0 & 0 & 0 \\ 0 & S_2(1-S_2) & & 0 \\ 0 & & \ddots & 0 \\ 0 & 0 & 0 & S_4(1-S_4) \end{bmatrix}}$$

c) $$w^{n+1} \leftarrow w^n - (\nabla_w^2 J)^{-1} \nabla_w J$$

$$\leftarrow w^n - (2\lambda + X^T\Omega X)^{-1}(2\lambda w - X^T(Y-S))$$

d) See ipython notebook

Scanned by CamScanner

```
In [1]: import numpy as np
```

Q1

```
In [7]: # 1-4-a
        x = np.matrix('0 3 1; 1 3 1; 0 1 1; 1 1 1')
        w_0 = np.matrix('-2;1; 0')
        s_0 = 1/(1+np.exp(-x*w_0))
        s_0

Out[7]: matrix([[ 0.95257413],
                [ 0.73105858],
                [ 0.73105858],
                [ 0.26894142]])
```

```
In [8]: # 1-4-b
        y = np.matrix('1;1;0;0')
        lam = 0.07
        DJ = 2*lam*w_0 - x.T*(y - s_0)
        lamM = np.diag(np.array([lam, lam, 0]))
        HJ = 2*lamM + x.T * np.diag(np.array(s_0).reshape(1,4)[0]) * x
        w_1 = w_0 - np.linalg.inv(HJ)*DJ
        w_1

Out[8]: matrix([[-1.4288194 ],
                [ 1.46502203],
                [-1.51608461]])
```

```
In [9]: # 1-4-c
        s_1 = 1/(1+np.exp(-x*w_1))
        s_1

Out[9]: matrix([[ 0.94679758],
                [ 0.81002338],
                [ 0.48723713],
                [ 0.18544525]])
```

```
In [10]: # 1-4-d
         DJ = 2*lam*w_1 - x.T*(y - s_1)
         HJ = 2*lamM + x.T * np.diag(np.array(s_1).reshape(1,4)[0]) * x
         w_2 = w_1 - np.linalg.inv(HJ)*DJ
         w_2

Out[10]: matrix([[-1.06301781],
                 [ 1.6171155 ],
                 [-2.12744786]])
```

```
In [ ]:
```

```
In [ ]:
```

Q 2 l_1 - and - l_2 - Regularization

a) $J(w) = |Xw - y|^2 + \lambda \|w\|_1$

$= (Xw - y)^T (Xw - y) + \lambda \|w\|_1$

$= [(Xw)^T - y^T] (Xw - y) + \lambda \|w\|_1$

$= (Xw)^T (Xw) - 2(Xw)^T y + y^T y + \lambda \|w\|_1$

$= w^T X^T X w - 2y^T (Xw) + y^T y + \lambda \|w\|_1$

$= \left(n \sum_{i=1}^d w_i^2 - 2 \sum_{i=1}^d y^T x_{*i} w_i + \lambda \sum_{i=1}^d |w_i| \right) + y^T y$

$g(y) = y^T y, \quad f(X_{*i}, w_i, y, \lambda) = \sum_{i=1}^d (n w_i^2 - 2y^T x_{*i} w_i + \lambda |w_i|)$

b) $\frac{\partial J(w_i)}{\partial w_i} = 2n w_i - 2y^T x_{*i} + \lambda = 0$

$w_i^* = \frac{2y^T x_{*i} - \lambda}{2n}$

c) $\frac{\partial J(w_i)}{\partial w_i} = 2n w_i - 2y^T x_{*i} w_i - \lambda = 0$

$w_i^* = \frac{2y^T x_{*i} + \lambda}{2n}$

d) If $w_i^* = 0$, it means this w_i^* is the optimal for both b and c scenarios

$\frac{2y^T x_{*i} - \lambda}{2n} = \frac{2y^T x_{*i} + \lambda}{2n} = 0$

$\lambda = 0$

e) $J(w) = |Xw - y|^2 + \lambda \|w\|^2$

$= \sum_{i=1}^d (n w_i^2 - 2y^T x_{*i} w_i + \lambda w_i^2) + y^T y$

$= \underbrace{\sum_{i=1}^d [(n+\lambda) w_i^2 - 2y^T x_{*i} w_i]}_{f} + y^T y$

f - no absolute term, so don't need to separate into positive and negative scenorios

$\frac{\partial J(w_i)}{\partial w_i} = 2(n+\lambda) w_i - 2y^T x_{*i} = 0$

$w_i^* = \frac{y^T x_{*i}}{n+\lambda}$

if $w_i^* = 0$, $\frac{y^T x_{*i}}{n+\lambda} = 0$

$y^T x_{*i} = 0$

Q3. Regression and Dual Solutions

a)

$$|w|^4 = (w_1^2 + w_2^2 \cdots + w_n^2)^2$$

$$\frac{\partial |w|^4}{\partial w} = \begin{bmatrix} \frac{\partial |w|^4}{\partial w_1} \\ \vdots \\ \frac{\partial |w|^4}{\partial w_n} \end{bmatrix} = \begin{bmatrix} 4(w_1^2 + w_2^2 \cdots + w_n^2) w_1 \\ 4(w_1^2 + w_2^2 \cdots + w_n^2) w_2 \\ \vdots \\ 4(w_1^2 + w_2^2 \cdots + w_n^2) w_n \end{bmatrix} = 4|w|^2 w$$

$$\nabla_w |Xw - y|^4 = 4|Xw-y|^2 X^T(Xw-y)$$

b)

$$L = |Xw - y|^4 + \lambda |w|^2$$

$$\nabla L = |Xw-y|^2 X^T(Xw-y) + 2\lambda w = 0$$

$$w^* = \frac{|Xw-y|^2}{-2\lambda} X^T(Xw-y)$$

$$= \frac{|Xw-y|^2}{\cancel{-2\lambda}} X^T(Xw-y) = \sum_{i=1}^n a_i X_i$$

$$a = \frac{|Xw-y|^2}{2\lambda}(Xw-y)$$

c)

$$L = \frac{1}{n}\sum_{i=1}^n L(w^T X_i, y_i) + \lambda |w|^2$$

According to the hint on piazza, for any vector w,

$$w = \sum_i a_i X_i + v \quad \text{s.t.} \quad v^T X_i = 0$$

So, $L = \frac{1}{n}\sum_{i=1}^n L((\sum_i a_i X_i + v)^T X_i, y_i) + \lambda |\sum_i a_i X_i + v|^2$

$$= \frac{1}{n}\sum_{i=1}^n L((\sum_i a_i X_i)^T X_i + v^T X_i, y_i) + \lambda |\sum_i a_i X_i + v|^2$$

$$= \frac{1}{n}\sum_{i=1}^n L((\sum_i a_i X_i)^T X_i, y_i) + \lambda |\sum_i a_i X_i + v|^2$$

If we want L to be minimum, v should be a zero vector

so $w^* = \sum_{i=1}^n a_i X_i$

This is true no matter whether L is convex or not, because L doesn't involve v. And here we only need to prove that v is zero vector in order to express w^* in that form.

In [2]:
```
import math
import numpy as np
import scipy.io as sio
import matplotlib
from matplotlib import pyplot as plt
from random import randint
%matplotlib inline
```

In [8]:
```
d = 12
n = 5000
data = sio.loadmat("./data.mat")
trainX = data['X']
testX = data['X_test']
trainY = data['y']
ValidateX = trainX[n:]
ValidateY = trainY[n:]
trainX = trainX[:n]
trainY = trainY[:n]
```

Q4. Logistic Regression on Wine Dataset

Logistic:

$$h(x; w, \alpha) = s(w \cdot x + \alpha)$$

Loss:

$$L(z, y) = -ylnz - (1 - y)ln(1 - z)$$

Cost:

$$J(h) = \frac{1}{n}\Sigma_{i=1}^{n} L(h(X_i), y_i) + \lambda\|w\|^2$$

$$\frac{dJ}{dw} = -\frac{1}{n}X^T(Y - S) + 2\lambda w$$

Dimension: λ is (D+1)x(D+1), S is NxN, X is Nx(D+1), w is (D+1)x1

In [243]:
```
class Logistic_Regression:
    def __init__(self, lam):
        self.w = None
        self.lam = lam

    def train(self, x, y, alpha, step, gd_type='batch', change_alpha=False):
        temp = np.array([[1]*n])
        x = np.matrix(np.concatenate((x, temp.T), axis=1)) # N x (D+1)
        self.w = np.matrix([[0]]*(d+1)) # (D+1) * 1
        for i in range(step):
            if gd_type == 'batch':
                self.w = self.w - alpha * deriv_cost(x, self.w, y, self.lam)
            else:
                if change_alpha:
                    alpha = 1 / step
                self.w = self.w - alpha * stochastic_deriv_cost(x, self.w, y
            c = cost(self.w, x, y, self.lam)
#             print('step {}: cost = {}'.format(i, c))

        c = cost(self.w, x, y, self.lam)
        return c

    def predict(self, x):
        temp = np.array([[1]*len(x)])
        x = np.matrix(np.concatenate((x, temp.T), axis=1)) # N x (D+1)
        prob = s(self.w, x)
        y_hat = np.greater_equal(prob, 0.5)
        return y_hat.astype(int)

    def plot(self, steps, alpha, gd_type, change_alpha):
        s = []
        costs = []
        for i in range(steps):
            c = self.train(trainX, trainY, alpha, i, gd_type=gd_type, change
            costs.append(c)
            s.append(i)
            print('Number of Steps {}: cost = {}'.format(i, c))

        fig, ax = plt.subplots()
        ax.plot(s, costs, color="black")
        ax.set_xlabel('Number of Iterations')
        ax.set_ylabel('Cost')
        plt.show()
```

1. Batch gradient descent

Gradient Descent Update:

$$w^{n+1} = w^n - \alpha \cdot \Sigma_{i=1}^{n}(\frac{dJ}{dw})$$
$$= w^n - \alpha \cdot \Sigma_{i=1}^{n}(\frac{dJ}{dw})$$

In [6]:
```
def deriv_cost(x, w, y, lam):
    """
    x: N * (D+1)
    w: (D+1)*1
    y: N * 1
    lam: double
    """
    lamM = np.matrix(np.diag(np.array([lam]*d+[0]))) #(D+1)*(D+1)
    regularization = 2 * lamM * w # (D+1)*1
    log_cost = -x.T*(y - s(w, x))
    cost_mean = 1/n * log_cost
    return cost_mean + regularization

def s(w, x):
    """
    w: (D+1)*1
    x: N * (D+1)
    return: N * 1
    """
    return 1/(1 + np.exp(-x*w))

def cost(w, x, y, lam):
    z = s(w, x)
    z = np.array(z).reshape(1,n)[0]
    y = np.array(y).reshape(1,n)[0]
    loss = -y * np.log(z) - (1 - y) * np.log(1 - z)
    regularization = lam * np.linalg.norm(w)**2
    return np.mean(loss) + regularization

def evaluate(w, x, y):

    temp = np.array([[1]*len(x)])
    x = np.matrix(np.concatenate((x, temp.T), axis=1)) # N x (D+1)
    prob = s(w, x)
    y_hat =np.greater_equal(prob, 0.5)
    return np.mean(np.equal(y_hat, y))
```

In [92]:
```
# Tune lambda, alpha and step
lam = 0.008
alpha = 0.002
step = 300
model = Logistic_Regression(lam)
model.train(trainX, trainY, alpha, step, gd_type='batch')
```

Out[92]: 0.21799329071181534

In [93]:
```
evaluate(model.w, ValidateX, ValidateY)
```

Out[93]: 0.93600000000000005

In [94]:
```
# Plot
model = Logistic_Regression(lam)
model.plot(step, alpha, gd_type='batch', change_alpha=False)
```

```
Number of Steps 0: cost = 0.6931471805599454
Number of Steps 1: cost = 1.1784201100299996
Number of Steps 2: cost = 0.8924606109624518
Number of Steps 3: cost = 0.6242382060460319
Number of Steps 4: cost = 0.40884453174717095
Number of Steps 5: cost = 0.362013200167675
Number of Steps 6: cost = 0.3703464142431162
Number of Steps 7: cost = 0.3578253575998675
Number of Steps 8: cost = 0.36923070985428486
Number of Steps 9: cost = 0.3509429065432557
Number of Steps 10: cost = 0.3606810195432305
Number of Steps 11: cost = 0.3442887658249516
Number of Steps 12: cost = 0.35223773654019463
Number of Steps 13: cost = 0.33801854812493537
Number of Steps 14: cost = 0.3443384766720468
Number of Steps 15: cost = 0.3320999968074172
Number of Steps 16: cost = 0.3369483953274167
Number of Steps 17: cost = 0.326509927767318
Number of Steps 18: cost = 0.33004842685797
Number of Steps 19: cost = 0.3212288344180104
Number of Steps 20: cost = 0.32362206305382113
Number of Steps 21: cost = 0.3162407873433375
Number of Steps 22: cost = 0.3176547628530933
Number of Steps 23: cost = 0.3115334511167727
Number of Steps 24: cost = 0.3121335081144863
Number of Steps 25: cost = 0.30709799669000626
Number of Steps 26: cost = 0.30704607268938633
Number of Steps 27: cost = 0.3029287769374386
Number of Steps 28: cost = 0.3023798970372846
Number of Steps 29: cost = 0.299022608025318
Number of Steps 30: cost = 0.29812055000106774
Number of Steps 31: cost = 0.29537750819932834
Number of Steps 32: cost = 0.294249909997733
Number of Steps 33: cost = 0.29199084821412646
Number of Steps 34: cost = 0.2907444268781303
Number of Steps 35: cost = 0.28885710656947294
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Number of Steps 37: cost = 0.28596576449157524
Number of Steps 38: cost = 0.28470255371300635
Number of Steps 39: cost = 0.2833001429086134
Number of Steps 40: cost = 0.2820894299720818
Number of Steps 41: cost = 0.28083789217284244
Number of Steps 42: cost = 0.27969339683300615
Number of Steps 43: cost = 0.27855323846776664
Number of Steps 44: cost = 0.2774761752309929
Number of Steps 45: cost = 0.2764202272823618
Number of Steps 46: cost = 0.2754054695308707
Number of Steps 47: cost = 0.2744156944173009
Number of Steps 48: cost = 0.27345625567620097
Number of Steps 49: cost = 0.27252095472294274
Number of Steps 50: cost = 0.2716103889760142
Number of Steps 51: cost = 0.27072199484326803
Number of Steps 52: cost = 0.26985517574732987
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Number of Steps 53: cost = 0.2690086531671337
Number of Steps 54: cost = 0.26818168757272753
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Number of Steps 68: cost = 0.2583222291100152
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Number of Steps 110: cost = 0.24057145834351323
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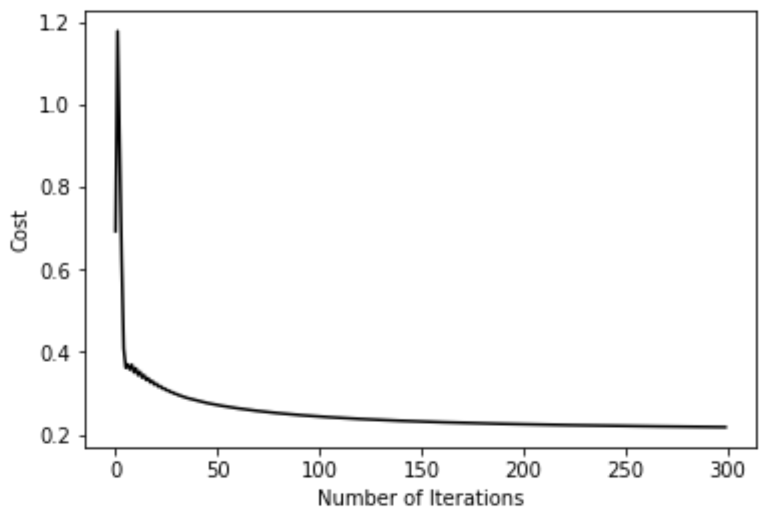

2. Stochastic gradient descent

$$w^{n+1} = w^n - \alpha \cdot \frac{dJ_i}{dw}$$
$$\frac{dJ}{dw} = -\frac{1}{n}(y_i - s_i)x_i + 2\lambda w$$

x_i is (D+1)x1, w is (D+1)x1

In [95]:
```
def stochastic_deriv_cost(x, w, y, lam):
    lamM = np.matrix(np.diag(np.array([lam]*d+[0]))) #(D+1)*(D+1)
    regularization = 2 * lamM * w # (D+1)*1
    rand = randint(0,n-1)
    x = x[rand]
    y = y[rand]
    log_cost = -1/n*(y.item(0) - s(w, x).item(0))*x
    return log_cost.reshape(d+1, 1) + regularization # (D+1)*1
```

In [204]:
```
# Tune lambda, alpha and step
lam = 0.000001
alpha = 1.21
step = 450
model = Logistic_Regression(lam)
model.train(trainX, trainY, alpha, step, gd_type='stochastic')
```

Out[204]: 0.2593286217943202

In [205]:
```
evaluate(model.w, ValidateX, ValidateY)
```

Out[205]: 0.9270000000000005

In [206]:
```
# Plot
model.plot(step, alpha, gd_type='stochastic', change_alpha=False)
```

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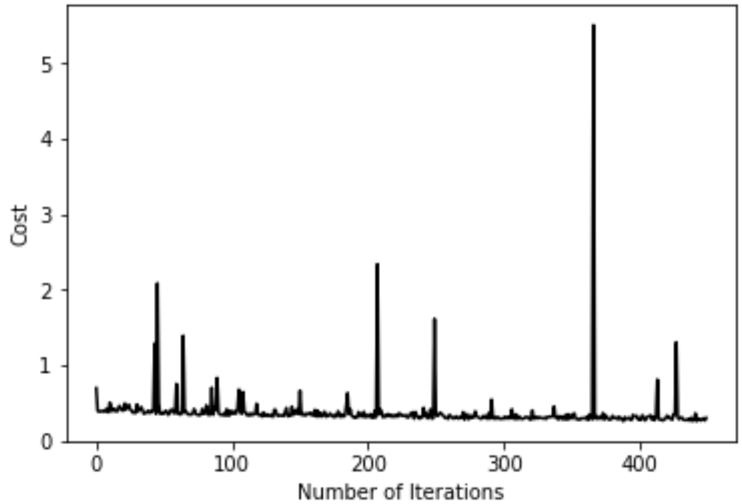

Batch V.S. Stochastic Gradient Descent

From what I observed from my experiment, to achieve similar accuracy, stochastic approach needs a much smaller lambda, a bigger alpha, and a much bigger step size. From the graphs, we can see that stochastic approach fluctuates much more than batch. This makes sense because stochastic update only depends on one point so it's easily influenced by outliers. We have to use a much bigger alpha because one point's value is smaller than the sum of all points, so we need to "push" this one pointer harder in order to see a slightly significant update on the cost
Also, in stochastic I don't see a smooth decrease of cost like batch

3. Changing alpha

In [244]:
```
# Tune lambda, alpha and step
lam = 0.000001
alpha = 1.21
step = 450
model = Logistic_Regression(lam)
model.train(trainX, trainY, alpha, step, gd_type='stochastic', change_alpha=
```

Out[244]: 0.46927835612464314

In [245]:
```
evaluate(model.w, ValidateX, ValidateY)
```

Out[245]: 0.74299999999999999

In [246]:
```
# Plot
model.plot(step, alpha, gd_type='stochastic', change_alpha=True)
```

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Number of Steps 320: cost = 0.4623353797349062
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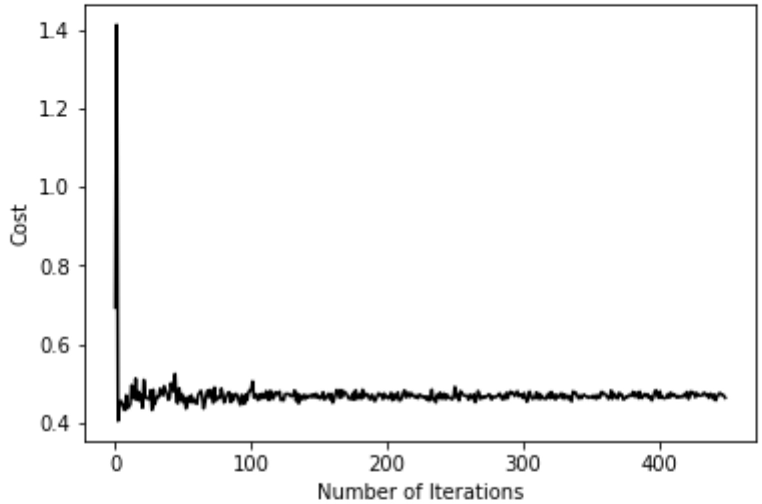

Constant V.S. Dynamic Alpha

I think dynamic alpha should be better than constant because you are learning less and less as you approach the optimal point, so this can avoid jumping around the point without converging. However, in my experiment, using the same other hyperparameters, dynamic alpha seems to be worse than constant alpha. The graph looks more stable than constant though.

4. Kaggle

In [248]:
```
lam = 0.008
alpha = 0.002
step = 300
model = Logistic_Regression(lam)
model.train(trainX, trainY, alpha, step, gd_type='batch')
```

Out[248]: 0.21799329071181534

In [249]:
```
evaluate(model.w, ValidateX, ValidateY)
```

Out[249]: 0.93600000000000005

In [250]:
```
y = model.predict(testX)
```

In [251]:
```
import pandas as pd
df = pd.DataFrame(data = y, columns=["Category"])
df.index.name = "Id"
df.to_csv("./wine.csv")
```

In [ ]:

Kaggle Score: 0.93952
Kaggle name: yika

5 Real World Spam Classification

Adding a timestamp feature makes perfect sense, but different forms of this feature can have totally different impact on its influence. I notice that the range of the feature value within the same class can bump from 0 to $8.64 \cdot 10^7$ (total milliseconds in a day). For instance, spam A comes at 12am and spam B comes at 11:59pm. They are both the midnight spams we want to capture by adding the timestamp feature, but it should be noted that A and B's timestamp values are super far apart (0 and $8.64 \cdot 10^7$). Therefore, we should map the midnight part to the middle of the timestamp value range, and also make the range smaller. One approach is to map all timestamps in a day uniformly onto a [-1, 1] and set 12am(midnight) at the midpoint of the range, which is 0. So, spams around midnight will have similar timestamp values. To make the result better, we can also apply a quadratic kernel.