Homework 5

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Exer.1

Denote the original r-s-flow and s-t-flow as f_{rs} and f_{st} respectively. We prove $f_{rt} = k$ by contradiction.

Suppose $f_{rt} < k$, denote $f_{rt} = p$. By Max-Flow-Min-Cut THM, there is a cut C with $cap(C) = p < k$. If $s \in C$, we know that C is also a r-s-cut. But by Max-Flow-Min-Cut THM, min $cap(r - s - cut) = k > cap(C) = p$, which leads to a contradiction. If $s \notin C$, the proof is similar. So $f_{rt} \geq k$, so there is a flow of value k in between r, t.

Exer.2

First we prove the easy part, these two cases will not occur simultaneously.Prove it by contradiction.

Suppose both cases are true simultaneously, then there are k vertex disjoint paths $p_1, ..., p_k$ from s to t. and there exists $v_1, ..., v_{k-1}$ such that $G - \{v_1, ..., v_{k-1}\}$ contains no s-t-path.

For any $v_1, ..., v_{k-1}$, the can take place in at most $k-1$ paths in $p_1, ..., p_k$ since $p_1, ..., p_k$ are disjoint vertex paths. Then we know there must be at least one path from $p_1, .., p_k$ left, which connects s and t, leading to a contradiction.

Next we prove that one of them must occur.

The second case is obvious since we can just remove all vertices except s, t from the V, then obviously there're no s-t-path now.

For the first case, we construct such a network-graph with all edge in the original graph assigned capacity 1. In such a network, the value of a

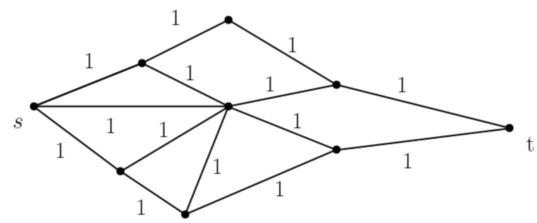

图 1: network

flow is the number of vertex disjoint paths in such a flow. Since the capacity of each edge is 1, we can know for sure that no two s-t-path cross with each other, otherwise there must be a vertex with units bigger than 1. Based on that, we see k, the value of a flow is the number of paths in it. Thus leading to k disjoint paths.