

# Introducing representations with functors

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This presents a top down view of ordinary representation theory.

**Definition 0.1.** A representation of a representable category in a representing category is a functor from the representable category to the representing category.

**Definition 0.2.** A homomorphism between representations is simply a natural transformation between representations.

**Example 0.1.** The representing category is the category of vector spaces over a field. The functor is a linear representation.

**Example 0.2.** The representable category is the delooping of a group, it has a group representation in the representing category.

**Example 0.3.** The representable category is the free category on a quiver. This is quiver representation.

**Example 0.4.** The representable category is the category of  $G$ -sets for a group  $G$ . The representing category is a category of linear representation. The functor is a permutation representations.