Defining rings

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Definition 0.1. A unital ring is a monoid object in the category of an abelian group. It is a monoid internal to the monoidal category of abelian groups. The monoidal category part is necessary to ensure that the tensor product of abelian groups exist.

A unital ring is a pointed category enriched over the category of abelian groups with a single object.

A unital ring is an abelian group equipped with a neutral element, a bilinear map or a group homomorphism out of the tensor product of abelian groups that is associative and unital.

A ring object in the category of Sets is a ring, for Set is a cartesian monoidal category with a categorical theorectic product being the tensor product, and a neutral element being a terminal object.

A commutative unital ring is a commutative monoid object in the monoidal category of abelian groups.

Example 0.1. One can change the abelian category of abelian groups with a higher category of symmetric monoidal higher groupoids, yielding ring groupoids or symmetric ring groupoids for the commutative case.

You want to drop unitality for analysis for some reason Borcherds explained but I don't understand. Commutativity is a nice plus, there are plenty of examples of noncommutative rings.