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A MATH BEDTIME STORYBOOK

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To my parents.

Introduction

These are my personal notes in mathematics.

Conventions

Set Theory

The Zermelo-Fraenkel axioms of set theory with the axiom of choice by default.

Alternative foundations with homotopy type theory and the univalence axiom may be considered when appropriate in the appropriate sections.

Universes may also be used.

Category theory

We follow these conventions [1] by default.

[1] J. De Jong. *Stacks Project*. Columbia University, 2025

Definition 0.0.1. *A (small) category,* **C** *consists of a set of objects. For each pair of objects there exists a set of morphisms.*

Note that all italicised words can be changed to define enrichment, operads when necessarily.

Algebra

In algebra, a ring is a commutative unitary ring.

In representation theory, we may drop commutativity.

In analysis, we may drop the condition of unitary.

Notation

The natural integers refers to the positive integers, however this will be avoided.

Set Theory

Definitions

Zermelo-Fraenkel Axioms

This follows Jech [2].

[2] Thomas Jech. *Set Theory*. Springer-Verlag Berlin Heidelberg New York, 4th edition, 2006

Definition 0.0.2 (Pairing axiom). *For any two sets X and Y, then there exists a set, denote $\{X, Y\}$, where the set contains exactly $\{X, Y\}$.*

Definition 0.0.3 (Extensionality axiom). *If sets X and Y have the same elements, we define equality where the set X is equal to the set Y.*

Definition 0.0.4 (Union axiom). *The union over elements of a set exists.*

Definition 0.0.5 (Infinity axiom). *An infinite set exists.*

Definition 0.0.6 (Regularity axiom). *All nonempty sets have a membership minimal element.*

Definition 0.0.7 (Separation axiom schema). *If P is a property parameterised by p, then for any set X and parmaeter p, then there exists a set Y that has elements y in X that contains all elements y in X that has property P.*

Definition 0.0.8 (Powerset axiom). *For any set X, there exists the set of all subsets of X called the power set of X, and is denoted by $P(X)$.*

Definition 0.0.9 (Replacement axiom schema). *If a class F is a function, there for any set X, there exists a set called the function set with elements of the form $F(x)$ for an element x in set X, this set is denoted $F(X)$.*

Definition 0.0.10 (Well ordering axiom). *All families of nonempty sets have a choice function.*

Category Theory

Definitions

Definition 0.0.11 (Category). *A category* $\mathbf{C}$ *has a set of objects, denoted* $\mathrm{Ob}(X)$ *or with objects* X.

It has a set of morphisms between objects X, Y *denoted* $\mathrm{Hom}(X, Y)$.

It has a composition map for objects X, Y, Z *where* $\cdot : \mathrm{Hom}(Z, Y) \times \mathrm{Hom}(Y, X) \to \mathrm{Hom}(Z, X)$ *such that for morphism* p *in* $\mathrm{Hom}(Y, X)$ *and morphism* q *in* $\mathrm{Hom}(Z, Y)$ *we have a morphism* $q \cdot p$ *in the set of morphisms* $\mathrm{Hom}(Z, X)$.

These are to satisfy the following rules:

1. *For every object* X *in the set of objects* $\mathrm{Ob}(X)$, *there exists an identity morphism* $i \in Hom_{\mathbf{C}}(X, X)$ *such that it composes with morphisms* p *and* q *where* $p = i \cdot p$ *and* $q \cdot i = q$.

2. *The composition of morphism is associative where* $p \cdot (q \cdot r) = (p \cdot q) \cdot r$.

Bibliography

Thomas Jech. *Set Theory*. Springer-Verlag Berlin Heidelberg New York, 4th edition, 2006.

J. De Jong. *Stacks Project*. Columbia University, 2025.