Lindy Effect

The Lindy Effect as an Absorbing Barrier

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Write the contents relative both today and a well defined point in the past.
Write it to be relevant 30 years ago.
This will make it stay relevant to 30 years.
Time detects and produces fragility.
One can do negative forecasting. Collapses are easier to predict than what will emerge.

Intuitions

Use a simpler model of arithmetic Brownian motion.
There is an absorbing barrier.
Introduce a drift term of $\mu = 0$. This gives power law with tail exponent $\alpha = 1/2$.
This power law must have an infinite mean.
Use a negative drift. Gives piecewise power law behavior, but is asymptotic, non-power law.

In[2]:=
```
data := RandomFunction[WienerProcess[0, 1], {0, 10, 0.01}][[2]][[1]] // Flatten
```

In[11]:=
```
? WienerProcess
```

Out[11]=

Symbol

WienerProcess[μ, σ] represents a Wiener process with a drift μ and volatility σ.

WienerProcess[] represents a standard Wiener process with drift 0 and volatility 1.

Note that there is no drift.

In[3]:=
```
stoppingtime[H_, X_] :=
 Min[Position[Table[Boole[TrueQ[X[[i]] < H]] 1, {i, 1, Length[data]}], 1]]
```

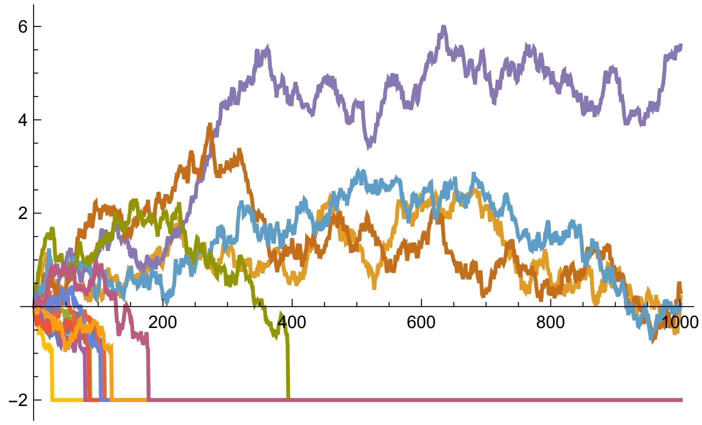
Setting stopping time to be -1 here, then force it to be -2 always if it hits -1.

In[4]:=
```
XL = Table[X = data;
   Table[If[i < stoppingtime[-1, X], X[[i]], -2], {i, Length[data]}], {14}];
```

We can plot the Wiener process as shown.

In[5]:=
```
ListLinePlot[XL]
```

Out[5]=



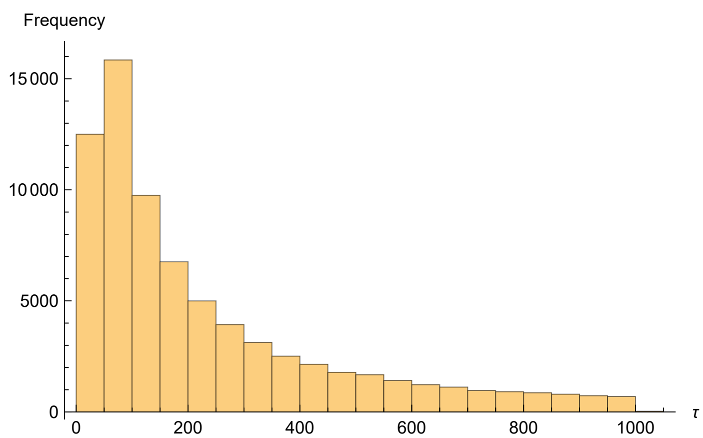
We are not dealing with stopping time τ but Max(τ,T).

The length of the data is T.

Take T $\rightarrow \infty$.

In[13]:=
```
Histogram[Table[stoppingtime[-1, data], {10^5}], AxesLabel → {τ, "Frequency"}]
```

Out[13]=



First Simplified Derivation Assuming Driftless Arithmetic Brownian Motion

Let X_t be a stochastic process with no drift satisfying dX = σ dB.

Let B be a Brownian motion where X_0=0.

We have $X_t = X_0 + \sigma \sqrt{t}\, W_{0,1}$, where W is a standardised normal random variable.

Let L < 0 be the level of the absorbing barrier (constant) hitting from above.

Let τ be the first passage time where $\tau = \inf\{t : X_t = L\}$

We have using the reflection principle the following distribution of τ.

Take the survival function of X above L.

By definition we cannot have a conditional probabiity of hitting L with τ > t.

By symmetry, X is as likely to be above L and below L.
Accordingly the cumulative P(τ<t) = 2 $P(X_t < L)$ = 2 CDF(L). The distribution of τ is $\frac{\partial P(\tau<t)}{\partial \tau}$.

In[15]:=
```
cum = 2 CDF[NormalDistribution[0, σ Sqrt[τ]], L] // FunctionExpand
```

Out[15]=

$$1 + \text{Erf}\left[\frac{L}{\sqrt{2}\,\sigma\,\sqrt{\tau}}\right]$$

In[14]:=
```
? D
```

Out[14]=

Symbol

D[f, x] gives the partial derivative $\partial f / \partial x$.

D[f, {x, n}] gives the multiple derivative $\partial^n f / \partial x^n$.

D[f, x, y, …] gives the partial derivative $\cdots (\partial / \partial y)(\partial / \partial x) f$.

D[f, {x, n}, {y, m}, …] gives the multiple partial derivative $\cdots (\partial^m / \partial y^m)(\partial^n / \partial x^n) f$.

D[f, {{x_1, x_2, …}}] for a scalar f gives the vector derivative $(\partial f / \partial x_1, \partial f / \partial x_2, \ldots)$.

D[f, {$array$}] gives an array derivative.

Note that this is the distribution of τ.

In[8]:=
```
Φ = D[cum, τ]
```

Out[8]=

$$-\frac{e^{-\frac{L^2}{2\sigma^2\tau}}\,L}{\sqrt{2\pi}\,\sigma\,\tau^{3/2}}$$

In[17]:=
```
hf = -D[Log[1 - cum], τ]
```

Out[17]=

$$\frac{e^{-\frac{L^2}{2\sigma^2\tau}}\,L}{\sqrt{2\pi}\,\sigma\,\tau^{3/2}\,\text{Erf}\left[\frac{L}{\sqrt{2}\,\sigma\,\sqrt{\tau}}\right]}$$

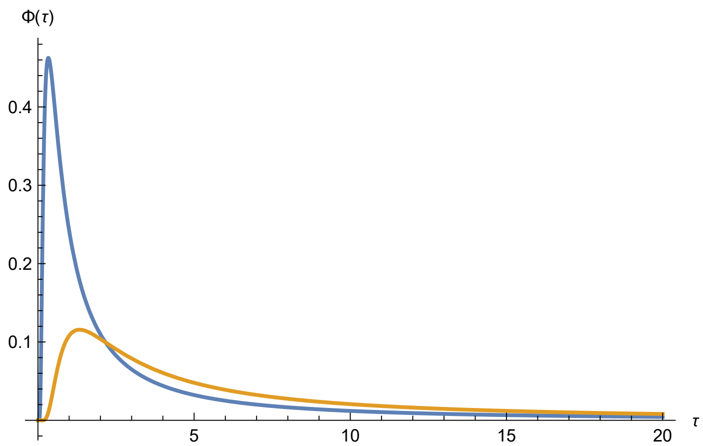
The plot shows how there just a difference in absorbing barrier

In[9]:=
```
Plot[{Φ /. {σ → 1, L → -1}, Φ /. {σ → 1, L → -2}},
  {τ, 0, 20}, PlotRange → All, AxesLabel → {τ, "Φ(τ)"}]
```

General: Exp[-1223.78] is too small to represent as a normalized machine number; precision may be lost.

General: Exp[-4895.1] is too small to represent as a normalized machine number; precision may be lost.

Out[9]=



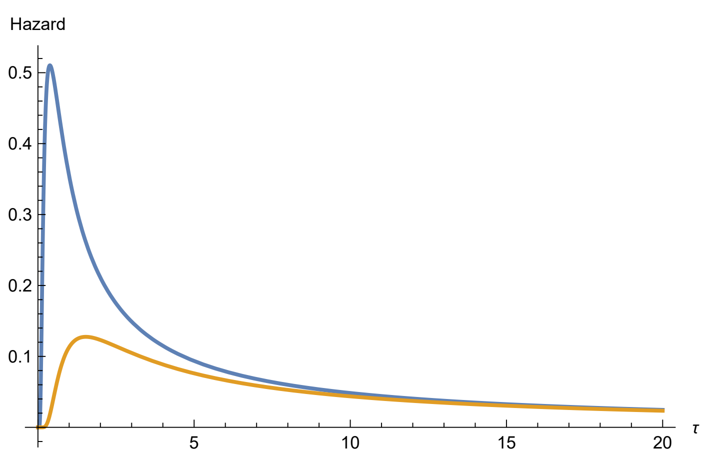
This is a power law with half tail exponent.
Take the logarithm of the survival function.

In[18]:=
```
Plot[{hf /. {σ → 1, L → -1}, hf /. {σ → 1, L → -2}},
  {τ, 0, 20}, PlotRange → All, AxesLabel → {τ, "Hazard"}]
```

General: Exp[-1223.78] is too small to represent as a normalized machine number; precision may be lost.

General: Exp[-4895.1] is too small to represent as a normalized machine number; precision may be lost.

Out[18]=



In[10]:= $\texttt{Limit}\left[\frac{\texttt{Log[1 - cum]}}{\texttt{Log}[\tau]}, \tau \to \texttt{Infinity}, \texttt{Assumptions} \to \sigma > 0\right]$

Out[10]=

$$-\frac{1}{2}$$

Girsanov change of measure for negative drift.