Internal Forced Convection

Notes based on practice, tutorial and past year examination papers

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Typical Assumptions

For examination style problems, we typically assume the following:

1. Steady heat transfer
2. 1D heat transfer
3. No external convection or radiation exchange (please check!)
4. Constant properties
5. Flow is incompressible.
6. Flow is laminar or turbulent or mixed (please check!)
7. Velocity does not vary in direction perpendicular to the flow
8. Fully (thermal or hydrodynamic) developed flow or (thermal or hydrodynamic) developing (please check!)
9. No heat generation (from frictional heating)
10. No work done by viscous forces. (please check!)
11. No body forces on fluid (please check!)
12. Negligible heat conduction in direction perpendicular to the flow (please check!)
13. Smooth surface (please check!)
14. Gas (if there is any) is standard and calorically perfect.
15. Tube wall has no contact resistance or thermal resistance (please check!)
16. Negligible kinetic energy and potential energy change in fluid (example fluid)
17. Fluid in question has similar properties to air or water (example: blood)
18. Properties are obtained by linear interpolation.
19. No fouling.
20. No heat loss to surroundings.
21. Flow is inviscid (please check!)

21\. Average velocity is constant

Internal Flow Definition

Internal flow is characterized by the fluid being completely confined by the inner surfaces of the tube. The mean or average velocity and temperature for a circular tube of radius R are expressed as

$$V_{\text{avg}} = \frac{2}{R^2}\int_0^R u(r)rdr \qquad \text{and} \qquad T_m = \frac{2}{V_{\text{avg}}R^2}\int_0^R u(r)T(r)rdr$$

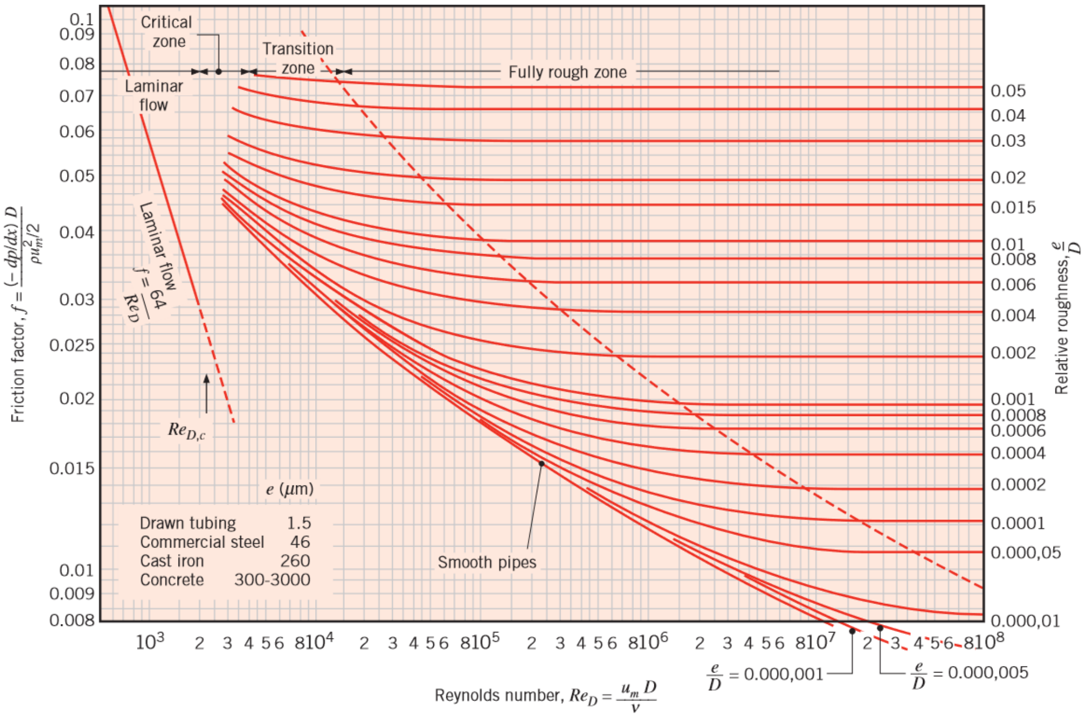
Reynolds and Regions

The Reynolds number for internal flow and the hydraulic diameter are defined as

$$\text{Re} = \frac{\rho V_{\text{avg}} D}{\mu} = \frac{V_{\text{avg}} D}{\nu} \qquad \text{and} \qquad D_h = \frac{4A_c}{p}$$

The flow in a tube is laminar for $\text{Re} < 2300$, turbulent for about $\text{Re} > 10{,}000$, and transitional in between.

Moody Chart



For *fully developed turbulent flow with rough surfaces,* the friction factor f is determined from the Moody chart or

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \approx -1.8 \log\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

Hydraulic Diameters

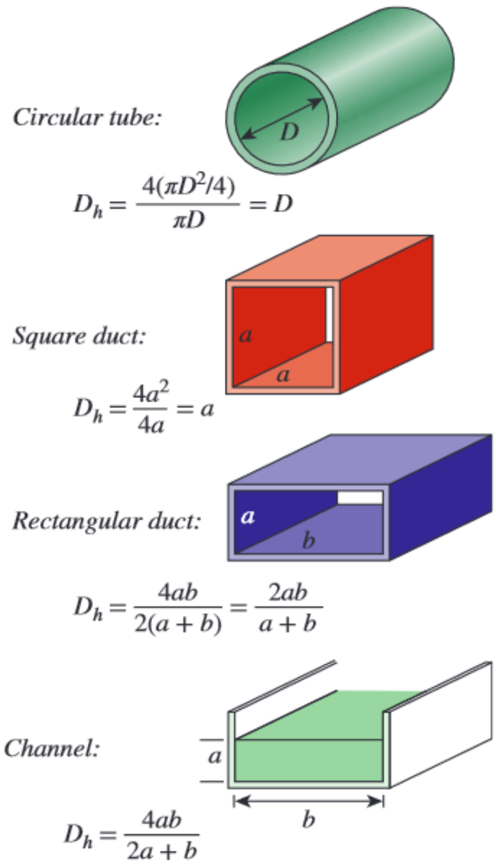


FIGURE 8–4
The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes. When there is a free surface, such as in open-channel flow, the wetted perimeter includes only the walls in contact with the fluid.

For a *concentric annulus,* the hydraulic diameter is $D_h = D_o - D_i$, and the Nusselt numbers are expressed as

$$\mathrm{Nu}_i = \frac{h_i D_h}{k} \quad \text{and} \quad \mathrm{Nu}_o = \frac{h_o D_h}{k}$$

where the values for the Nusselt numbers are given in Table 8–4.

Entry Lengths

Hydrodynamically fully developed: $\frac{\partial u(r, x)}{\partial x} = 0 \longrightarrow u = u(r)$ **(8–7)**

Thermally fully developed: $\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)}\right] = 0$ **(8–8)**

The length of the region from the tube inlet to the point at which the flow becomes fully developed is the *hydrodynamic entry length* L_h. The region beyond the entrance region in which the velocity profile is fully developed is the *hydrodynamically fully developed region.* The length of the region of flow over which the thermal boundary layer develops and reaches the tube center is the *thermal entry length* L_t. The region in which the flow is both hydrodynamically and thermally developed is the *fully developed flow region.* The entry lengths are given by

$$L_{h,\text{ laminar}} \approx 0.05\,\text{Re}\,D$$

$$L_{t,\text{ laminar}} \approx 0.05\,\text{Re}\,\text{Pr}\,D = \text{Pr}\,L_{h,\text{ laminar}}$$

$$L_{h,\text{ turbulent}} \approx L_{t,\text{ turbulent}} = 10D$$

Circular Tube

TABLE 8.4 Summary of convection correlations for flow in a circular tube[a,b,e]

Correlation		Conditions
$f = 64/Re_D$	(8.19)	Laminar, fully developed
$Nu_D = 4.36$	(8.53)	Laminar, fully developed, uniform q_s''
$Nu_D = 3.66$	(8.55)	Laminar, fully developed, uniform T_s
$\overline{Nu}_D = 3.66 + \dfrac{0.0668\, Gz_D}{1 + 0.04\, Gz_D^{2/3}}$	(8.57)	Laminar, thermal entry (or combined entry with $Pr \gtrsim 5$), uniform T_s, $Gz_D = (D/x)\, Re_D\, Pr$
$\overline{Nu}_D = \dfrac{\dfrac{3.66}{\tanh[2.264\, Gz_D^{-1/3} + 1.7\, Gz_D^{-2/3}]} + 0.0499\, Gz_D \tanh(Gz_D^{-1})}{\tanh(2.432\, Pr^{1/6} Gz_D^{-1/6})}$	(8.58)	Laminar, combined entry, $Pr \gtrsim 0.1$, uniform T_s, $Gz_D = (D/x)\, Re_D\, Pr$
$\dfrac{1}{\sqrt{f}} = -2.0 \log\left[\dfrac{e/D}{3.7} + \dfrac{2.51}{Re_D \sqrt{f}}\right]$	(8.20)[c]	Turbulent, fully developed
$f = (0.790 \ln Re_D - 1.64)^{-2}$	(8.21)[c]	Turbulent, fully developed, smooth walls, $3000 \lesssim Re_D \lesssim 5 \times 10^6$
$Nu_D = 0.023\, Re_D^{4/5}\, Pr^n$	(8.60)[d]	Turbulent, fully developed, $0.6 \lesssim Pr \lesssim 160$, $Re_D \gtrsim 10{,}000$, $(L/D) \gtrsim 10$, $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$
$Nu_D = 0.027\, Re_D^{4/5}\, Pr^{1/3} \left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(8.61)[d]	Turbulent, fully developed, $0.7 \lesssim Pr \lesssim 16{,}700$, $Re_D \gtrsim 10{,}000$, $L/D \gtrsim 10$
$Nu_D = \dfrac{(f/8)(Re_D - 1000)\, Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$	(8.62)[d]	Turbulent, fully developed, $0.5 \lesssim Pr \lesssim 2000$, $3000 \lesssim Re_D \lesssim 5 \times 10^6$, $(L/D) \gtrsim 10$
$Nu_D = 4.82 + 0.0185(Re_D\, Pr)^{0.827}$	(8.64)	Liquid metals, turbulent, fully developed, uniform q_s'', $3.6 \times 10^3 \lesssim Re_D \lesssim 9.05 \times 10^5$, $3 \times 10^{-3} \lesssim Pr \lesssim 5 \times 10^{-2}$, $10^2 \lesssim Re_D\, Pr \lesssim 10^4$
$Nu_D = 5.0 + 0.025(Re_D\, Pr)^{0.8}$	(8.65)	Liquid metals, turbulent, fully developed, uniform T_s, $Re_D\, Pr \gtrsim 100$

[a]The mass transfer correlations may be obtained by replacing Nu_D and Pr by Sh_D and Sc, respectively.
[b]Properties in Equations 8.53, 8.55, 8.60, 8.61, 8.62, 8.64, and 8.65 are based on T_m; properties in Equations 8.19, 8.20, and 8.21 are based on $T_f = (T_s + T_m)/2$; properties in Equations 8.57 and 8.58 are based on $\overline{T}_m = (T_{m,i} + T_{m,o})/2$.
[c]Equation 8.20 pertains to smooth or rough tubes. Equation 8.21 pertains to smooth tubes.
[d]As a first approximation, Equations 8.60, 8.61, or 8.62 may be used to evaluate the average Nusselt number $\overline{Nu}_D$ over the entire tube length, if $(L/D) \gtrsim 10$. The properties should then be evaluated at the average of the mean temperature, $\overline{T}_m = (T_{m,i} + T_{m,o})/2$.
[e]For tubes of noncircular cross section, $Re_D \equiv D_h u_m/\nu$, $D_h \equiv 4A_c/P$, and $u_m = \dot{m}/\rho A_c$. Results for fully developed laminar flow are provided in Table 8.1. For turbulent flow, Equation 8.60 may be used as a first approximation.

For *fully developed laminar flow* in a circular pipe, we have:

$$u(r) = 2V_{\text{avg}}\left(1 - \frac{r^2}{R^2}\right) = u_{\text{max}}\left(1 - \frac{r^2}{R^2}\right)$$

$$f = \frac{64\mu}{\rho D V_{\text{avg}}} = \frac{64}{\text{Re}}$$

$$\dot{V} = V_{\text{avg}} A_c = \frac{\Delta P R^2}{8\mu L}\pi R^2 = \frac{\pi R^4 \Delta P}{8\mu L} = \frac{\pi R^4 \Delta P}{128\mu L}$$

Circular tube, laminar ($\dot{q}_s$ = constant): $\text{Nu} = \dfrac{hD}{k} = 4.36$

Circular tube, laminar (T_s = constant): $\text{Nu} = \dfrac{hD}{k} = 3.66$

For *developing laminar flow* in the entrance region with constant surface temperature, we have

Circular tube: $\text{Nu} = 3.66 + \dfrac{0.065(D/L)\text{Re Pr}}{1 + 0.04[(D/L)\text{Re Pr}]^{2/3}}$

Circular tube: $\text{Nu} = 1.86\left(\dfrac{\text{Re Pr} D}{L}\right)^{1/3}\left(\dfrac{\mu_b}{\mu_s}\right)^{0.14}$

Parallel plates: $\text{Nu} = 7.54 + \dfrac{0.03(D_h/L)\text{Re Pr}}{1 + 0.016[(D_h/L)\text{Re Pr}]^{2/3}}$

For *fully developed turbulent flow with smooth surfaces,* we have

$$f = (0.790 \ln \mathrm{Re} - 1.64)^{-2} \qquad 10^4 < \mathrm{Re} < 10^6$$

$$\mathrm{Nu} = 0.125 f\, \mathrm{Re}\, \mathrm{Pr}^{1/3}$$

$$\mathrm{Nu} = 0.023\, \mathrm{Re}^{0.8}\, \mathrm{Pr}^{1/3} \quad \begin{pmatrix} 0.7 \le \mathrm{Pr} \le 160 \\ \mathrm{Re} > 10{,}000 \end{pmatrix}$$

$\mathrm{Nu} = 0.023\, \mathrm{Re}^{0.8}\, \mathrm{Pr}^{n}$ with $n = 0.4$ for *heating* and 0.3 for *cooling* of fluid

$$\mathrm{Nu} = \frac{(f/8)(\mathrm{Re} - 1000)\mathrm{Pr}}{1 + 12.7 (f/8)^{0.5}(\mathrm{Pr}^{2/3} - 1)} \quad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 3 \times 10^3 < \mathrm{Re} < 5 \times 10^6 \end{pmatrix}$$

The fluid properties are evaluated at the *bulk mean fluid temperature* $T_b = (T_i + T_e)/2$. For liquid metal flow in the range of $10^4 < \mathrm{Re} < 10^6$ we have:

$$T_s = \text{constant:} \qquad \mathrm{Nu} = 4.8 + 0.0156\, \mathrm{Re}^{0.85}\, \mathrm{Pr}_s^{0.93}$$

$$\dot{q}_s = \text{constant:} \qquad \mathrm{Nu} = 6.3 + 0.0167\, \mathrm{Re}^{0.85}\, \mathrm{Pr}_s^{0.93}$$

General Thermal Analysis

Constant Surface Heat Flux ($\dot{q}_s$ = constant)

In the case of $\dot{q}_s$ = constant, the rate of heat transfer can also be expressed as

$$\dot{Q} = \dot{q}_s A_s = \dot{m}c_p(T_e - T_i) \qquad \text{(W)} \tag{8–16}$$

Then the mean fluid temperature at the tube exit becomes

$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m}c_p} \tag{8–17}$$

Note that the mean fluid temperature increases *linearly* in the flow direction in the case of constant surface heat flux, since the surface area increases linearly in the flow direction (A_s is equal to the perimeter, which is constant, times the tube length).

The surface temperature in the case of constant surface heat flux $\dot{q}_s$ can be determined from

$$\dot{q}_s = h(T_s - T_m) \quad \longrightarrow \quad T_s = T_m + \frac{\dot{q}_s}{h} \tag{8–18}$$

In the fully developed region, the surface temperature T_s will also increase linearly in the flow direction since h is constant and thus $T_s - T_m$ = constant (Fig. 8–11). Of course this is true when the fluid properties remain constant during flow.

The slope of the mean fluid temperature T_m on a T-x diagram can be determined by applying the steady-flow energy balance to a tube slice of thickness dx shown in Fig. 8–12. It gives

$$\dot{m}c_p dT_m = \dot{q}_s(pdx) \quad \longrightarrow \quad \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}c_p} = \text{constant} \tag{8–19}$$

where p is the perimeter of the tube.

Substituting this into Eq. 8–14, we obtain

$$\dot{Q} = hA_s\Delta T_{\text{lm}} \tag{8–32}$$

where

$$\Delta T_{\text{lm}} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)} \tag{8–33}$$

is the **log mean temperature difference**. Note that $\Delta T_i = T_s - T_i$ and $\Delta T_e = T_s - T_e$ are the temperature differences between the surface and the fluid at the inlet and the exit of the tube, respectively. This ΔT_{lm} relation appears to be prone to misuse, but it is practically fail-safe, since using T_i in place of T_e and vice versa in the numerator and/or the denominator will, at most, affect the sign, not the magnitude. Also, it can be used for both heating ($T_s > T_i$ and T_e) and cooling ($T_s < T_i$ and T_e) of a fluid in a tube.

The log mean temperature difference ΔT_{lm} is obtained by tracing the actual temperature profile of the fluid along the tube and is an *exact* representation of the *average temperature difference* between the fluid and the surface. It truly reflects the exponential decay of the local temperature difference. When ΔT_e differs from ΔT_i by no more than 40 percent, the error in using the arithmetic mean temperature difference is less than 1 percent. But the error increases to undesirable levels when ΔT_e differs from ΔT_i by greater amounts. Therefore, we should always use the log mean temperature difference when determining the convection heat transfer in a tube whose surface is maintained at a constant temperature T_s.

For $\dot{q}_s =$ constant, the rate of heat transfer is expressed as

$$\dot{Q} = \dot{q}_s A_s = \dot{m}c_p(T_e - T_i)$$

For $T_s =$ constant, we have

$$\dot{Q} = hA_s\Delta T_{\text{lm}} = \dot{m}c_p(T_e - T_i)$$

$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}c_p)$$

$$\Delta T_{\text{lm}} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

Fully Developed Laminar Flow (Different Cross Section)

TABLE 8–1

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $\mathrm{Re} = V_{avg}D_h/\nu$, and $\mathrm{Nu} = hD_h/k$)

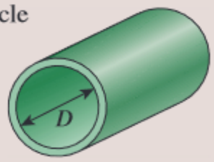
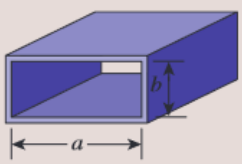
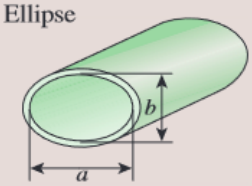
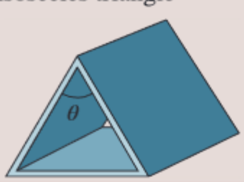
		Nusselt Number		
Tube Geometry	a/b or $\theta°$	T_s = Const.	$\dot{q}_s$ = Const.	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	$\underline{a/b}$			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	$\underline{a/b}$			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Isosceles triangle 	$\underline{\theta}$			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

TABLE 8.2 Nusselt number for fully developed laminar flow in a circular tube annulus with one surface insulated and the other at constant temperature

D_i/D_o	Nu_i	Nu_o	*Comments*
0	—	3.66	See Equation 8.55
0.05	17.46	4.06	
0.10	11.56	4.11	
0.25	7.37	4.23	
0.50	5.74	4.43	
≈1.00	4.86	4.86	See Table 8.1, $b/a \to \infty$

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