

Power Law Visualisation

<https://www.youtube.com/watch?v=XhTHG3QmVwM>

The information is only valid in the tails for power law class.

The rare event contributes to most of the properties.



Below α , the tail determines the properties.

Pull out the inverse survival function of the Pareto distribution.

Survival function is the probability of exceeding with given parameters.

You can pull out the threshold with closed form inverse survival function.

```
In[ ]:= solK = Solve[FullSimplify[SurvivalFunction[ParetoDistribution[L,  $\alpha$ ], K], K > L] == p, K]
```

 **Solve:** Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. 

Out[]:=

```
{ {K -> L p-1/ $\alpha$  } }
```

This is the contribution above that threshold.

```
In[ ]:= mk = Integrate[x PDF [ParetoDistribution[L,  $\alpha$ ], x], {x, K,  $\infty$ }, Assumptions -> K > L >  $\alpha$  > 1] /  
Simplify[Mean[ParetoDistribution[L,  $\alpha$ ]], Assumptions -> K > L >  $\alpha$  > 1] /.  
solK[[1]] // PowerExpand // FullSimplify
```

Out[]:=

$p^{\frac{-1+\alpha}{\alpha}}$

For example, the tail exponent of 1.16 gives the Pareto 80/20 rule.

The top 20% contributes 80% under $\alpha = 1.16$.

This is one way you can find how much does the top percentage contribute to all the properties.

```
In[ ]:= mk /.  $\alpha \rightarrow 1.16$  /. p -> 0.2 // N
```

Out[]:=

0.800922

Under 80/20 rule, the top 1% contributes 53% of the properties.



```
In[ ]:= mk /.  $\alpha \rightarrow 1.16$  /. p -> 0.01 // N
```

Out[]:=

0.529832

Of course, you can use Solve to determine the tail exponent associated with particular proportions.

```
In[*]:= Solve[(mk /. p → 0.2) == 0.8, α]
```

 **Solve:** Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. 

```
Out[*]=  
{ {α → 1.16096} }
```

```
In[*]:= mk /. α → 1.01 /. p → 0.01 // N
```

```
Out[*]=  
0.955428
```