Sec 3/4 Quadratic Inequality Problem

Problem

Find the value of c and d for which $-5 < x < 3$ is the solution of $x^2 + cx < d$.

This problem is surprisingly difficult. Standard things one typically tries (complete the square, sum-product of roots, quadratic equation, use of discriminants for no real roots) do not work effectively here.

Wolfram Mathematica Solution

In[6]:=
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Reduce[ForAll[x, -5 < x < 3, x^2 + c * x < d], {c, d}, Reals]
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Out[6]= $(c \le 2 \,\&\&\, d \ge 25 - 5c) \,||\, (c > 2 \,\&\&\, d \ge 9 + 3c)$

Handwritten Solution

The hard part was to figure out how to symmetrise the $c > 2$.

$3 > x > -5$

Case 1:

$3(3+c) > x(x+c) > [-5][-5+c]$

$9+3c > 25-5c$

$8c > 16$

$\underline{\underline{c > 2}}$

$d \geq 9+3c > x^2 + cx$

$d \geq 9+3c$

$3 > x > -5$

$\cancel{-3 < -x < 5}$

Case 2:

$-3(3+c) > -x(x+c) > 5(-5+c)$

$-9-3c > -x^2 - cx > -25+5c$

$9+3c < x^2+cx < 25-5c$

$9+3c < 25-5c$

$8c < 16$

$\underline{\underline{c < 2}}$

AND $x^2+cx < 25-5c \leq d$

$\underline{\underline{25-5c \leq d}}$

You can make one of the inequalities weaker (either $c < 2$ becomes $c \leq 2$ or the other) since they join at $d = 15$. This solves the problem for c real.