Lognormal Pathology

Nassim Nicholas Taleb @nntaleb · Apr 13, 2019
Absence of ruin reqyires ergodicity. Not the other way

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Ole Peters @ole_b_peters · Apr 13, 2019
I don't think that's what you mean. Ergodic theory doesn't deal with ruin, so first we need to mathematize "ruin."

For example, take geometric Brownian motion. It cannot hit zero (one definition of ruin), but it's not ergodic. So no: absence of ruin doesn't require ergodicity.

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Nassim Nicholas Taleb @nntaleb · Apr 13, 2019
Using the technical definition for a Markov chain = ergodic if no absorbing states.

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Nassim Nicholas Taleb @nntaleb · Apr 13, 2019
And actually a GBM goes to 0 with probabilities approaching 1 as time goes to infinity. The lognormal is a pathology.

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Nassim Nicholas Taleb
@nntaleb

That's the pathology of the GBM/Lognormal: for all \epsilon small but not 0, there is a probability of falling and being stuck below.

```
Limit[CDF[LogNormalDistribution[-1/2 σ^2, σ], ϵ], σ → ∞]
```

$$\begin{cases} 1 & \epsilon > 0 \\ 0 & \text{True} \end{cases}$$

In[∘]:= Limit[CDF[LogNormalDistribution[$\frac{-1}{2}\sigma^2$, σ], ϵ], σ → ∞]

Out[∘]=

$$\begin{cases} 1 & \epsilon > 0 \\ 0 & \text{True} \end{cases}$$

In[∘]:= ? CDF$\left[\text{LogNormalDistribution}\left[\frac{-1}{2}\sigma^2, \sigma\right], \epsilon\right]$

Out[∘]=

$$\text{Information}\left[\begin{cases} \frac{1}{2}\,\text{Erfc}\left[\frac{-\frac{\sigma^2}{2}-\text{Log}[\epsilon]}{\sqrt{2}\,\sigma}\right] & \epsilon > 0 \\ 0 & \text{True} \end{cases}, \text{LongForm} \to \text{False}\right]$$

In[∘]:= ? CDF

Out[∘]=

Symbol

CDF[*dist*, *x*] gives the cumulative distribution function for the distribution *dist* evaluated at *x*.

CDF[*dist*, {x_1, x_2, …}] gives the multivariate cumulative

distribution function for the distribution *dist* evaluated at {x_1, x_2, …}.

CDF[*dist*] gives the CDF as a pure function.

Lognormal at High Variance

In[∘]:= ? LogNormalDistribution

Symbol

In[∘]:=

LogNormalDistribution[μ, σ] represents a lognormal distribution

derived from a normal distribution with mean μ and standard deviation σ.

Out[∘]=

Symbol

LogNormalDistribution[μ, σ] represents a lognormal

distribution derived from a normal distribution with mean μ and standard deviation σ.

In[∘]:=
```
? Clear
```
Out[∘]=

Symbol

Clear[s_1, s_2, …] clears values and definitions for the symbols s_i.

Clear[$patt_1$, $patt_2$, …] clears values and definitions for all symbols whose names textually match any of the arbitrary string patterns $patt_i$.

Clear[{$spec_1$, $spec_2$, …}] clears values and definitions for any symbols that are equal to or whose names match any of the $spec_i$.

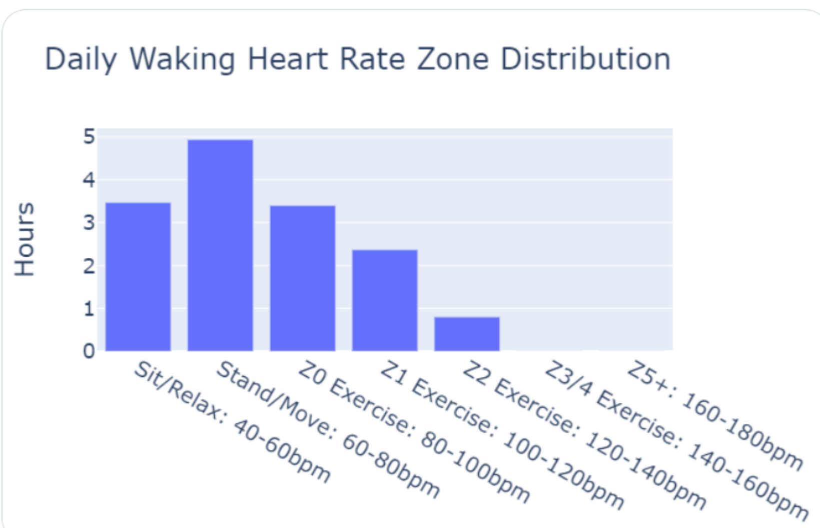
Alan Couzens
@Alan_Couzens

Inspired by @nntaleb to put this together...

The most important chart missing from your "dashboard" & the one that you're probably not going to want to look at...

How much movement do you really have in your day?

Unfortunately, log-normal is anything but normal!



You can see at low variance, the lognormal looks normal.
In reality, it is not.

In[∘]:=
```
Clear[σ]; Clear[x];
```

In[ ]:=
```
Plot[Table[PDF[LogNormalDistribution[1 / 1000, σ], x], {σ, {1 / 2, 1 / 3, 1, 2}}] // Evaluate,
  {x, 0, 4}, Filling → Axis]
```

Out[ ]=