Radiation

Notes based on practice, tutorial and past year examination papers

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Typical Assumptions

1. Steady heat transfer
2. Constant properties
3. No heat generation
4. Negligible conduction and convection (please check!)
5. Radiation exchange with large surroundings (please check!)
6. View factor = 1 (or some value, please check!)
7. Gray surfaces (emissivity < 1) and Kirchhoff's law applies.
8. Diffuse surfaces where radiation is uniform in all directions
9. Infinite surface (please check!)
10. Black (emissivity = 1, please check!)
11. Negligible conduction
12. Thermal circuits is valid for problem analysis
13. Properties are obtained by linear interpolation.
14. No heat loss to surroundings (please check!).

Basics

TABLE 12.1 Radiative fluxes (over all wavelengths and in all directions)

Flux (W/m^2)	Description	Comment
Emissive power, E	Rate at which radiation is emitted from a surface per unit area	$E = \varepsilon \sigma T_s^4$
Irradiation, G	Rate at which radiation is incident upon a surface per unit area	Irradiation can be reflected, absorbed, or transmitted
Radiosity, J	Rate at which radiation leaves a surface per unit area	For an opaque surface $J = E + \rho G$
Net radiative flux, $q''_{\mathrm{rad}} = J - G$	Net rate of radiation leaving a surface per unit area	For an opaque surface $q''_{\mathrm{rad}} = \varepsilon \sigma T_s^4 - \alpha G$

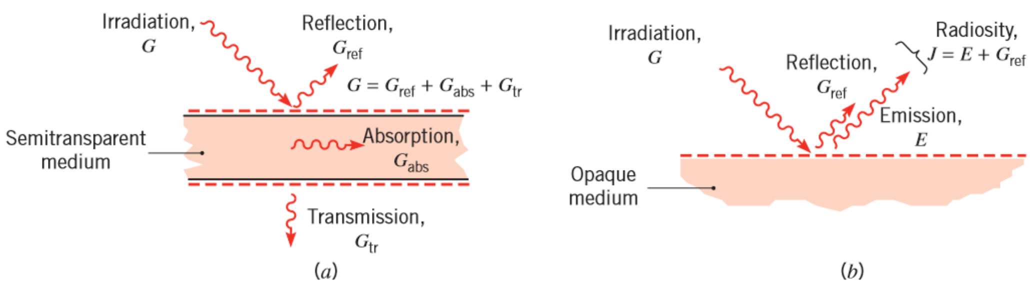


FIGURE 12.5 Radiation at a surface. (*a*) Reflection, absorption, and transmission of irradiation for a semitransparent medium. (*b*) The radiosity for an opaque medium.

View Factor

The view factor from a surface i to a surface j is denoted by $F_{i \to j}$ or just F_{ij}, and is defined as

F_{ij} = *the fraction of the radiation leaving surface i that strikes surface j directly*

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_j \, dA_i \qquad (13.1)$$

1 The Reciprocity Relation

The view factors $F_{i \to j}$ and $F_{j \to i}$ are *not* equal to each other unless the areas of the two surfaces are. That is,

$$
\begin{aligned}
F_{j \to i} &= F_{i \to j} \quad \text{when} \quad A_i = A_j \\
F_{j \to i} &\neq F_{i \to j} \quad \text{when} \quad A_i \neq A_j
\end{aligned}
$$

We have shown earlier that the pair of view factors $F_{i \to j}$ and $F_{j \to i}$ are related to each other by

$$A_i F_{i \to j} = A_j F_{j \to i} \tag{13–11}$$

This relation is referred to as the **reciprocity relation** or the **reciprocity rule**, and it enables us to determine the counterpart of a view factor from a knowledge of the view factor itself and the areas of the two surfaces. When determining the pair of view factors $F_{i \to j}$ and $F_{j \to i}$, it makes sense to evaluate first the easier one directly and then the more difficult one by applying the reciprocity relation.

2 The Summation Rule

The radiation analysis of a surface normally requires the consideration of the radiation coming in or going out in all directions. Therefore, most radiation problems encountered in practice involve enclosed spaces. When formulating a radiation problem, we usually form an *enclosure* consisting of the surfaces interacting radiatively. Even openings are treated as imaginary surfaces with radiation properties equivalent to those of the opening.

The conservation of energy principle requires that the entire radiation leaving any surface i of an enclosure be intercepted by the surfaces of the enclosure. Therefore, *the sum of the view factors from surface i of an enclosure to all surfaces of the enclosure, including to itself, must equal unity.* This is known as the **summation rule** for an enclosure and is expressed as (Fig. 13–9)

$$\sum_{j=1}^{N} F_{i \to j} = 1 \tag{13–12}$$

where N is the number of surfaces of the enclosure. For example, applying the summation rule to surface 1 of a three-surface enclosure yields

$$\sum_{j=1}^{3} F_{1 \to j} = F_{1 \to 1} + F_{1 \to 2} + F_{1 \to 3} = 1$$

The summation rule can be applied to each surface of an enclosure by varying i from 1 to N. Therefore, the summation rule applied to each of the N surfaces of an enclosure gives N relations for the determination of the view factors. Also, the reciprocity rule gives $\frac{1}{2}N(N-1)$ additional relations. Then the total number of view factors that need to be evaluated directly for an N-surface enclosure becomes

$$N^2 - [N + \tfrac{1}{2}N(N-1)] = \tfrac{1}{2}N(N-1)$$

The first relation concerns the additive nature of the view factor for a subdivided surface and may be inferred from Figure 13.7. Considering radiation from surface i to surface j, which is divided into n components, it is evident that

$$F_{i(j)} = \sum_{k=1}^{n} F_{ik} \tag{13.5}$$

Equation 13.5 by A_i and applying the reciprocity relation, Equation 13.3, to each of the resulting terms, it follows that

$$A_j F_{(j)i} = \sum_{k=1}^{n} A_k F_{ki} \tag{13.6}$$

or

$$F_{(j)i} = \frac{\sum_{k=1}^{n} A_k F_{ki}}{\sum_{k=1}^{n} A_k} \tag{13.7}$$

To obtain a relation for the view factor $F_{(2,3)\to 1}$, we multiply Eq. 13–13 by A_1,

$$A_1 F_{1\to(2,3)} = A_1 F_{1\to 2} + A_1 F_{1\to 3}$$

and apply the reciprocity relation to each term to get

$$(A_2 + A_3) F_{(2,3)\to 1} = A_2 F_{2\to 1} + A_3 F_{3\to 1}$$

or

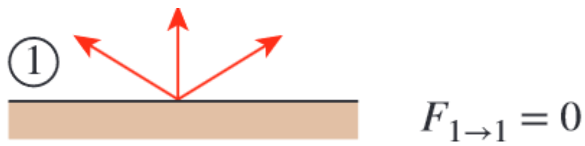
$$F_{(2,3)\to 1} = \frac{A_2 F_{2\to 1} + A_3 F_{3\to 1}}{A_2 + A_3} \tag{13–14}$$

Areas that are expressed as the sum of more than two parts can be handled in a similar manner.

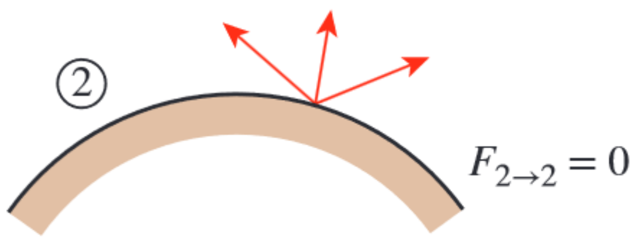
4 The Symmetry Rule

The determination of the view factors in a problem can be simplified further if the geometry involved possesses some sort of symmetry. Therefore, it is good practice to check for the presence of any *symmetry* in a problem before attempting to determine the view factors directly. The presence of symmetry can be determined *by inspection,* keeping the definition of the view factor in mind. Identical surfaces that are oriented in an identical manner with respect to another surface will intercept identical amounts of radiation leaving that surface. Therefore, the **symmetry rule** can be expressed as *two (or more) surfaces that possess symmetry about a third surface will have identical view factors from that surface* (Fig. 13–13).

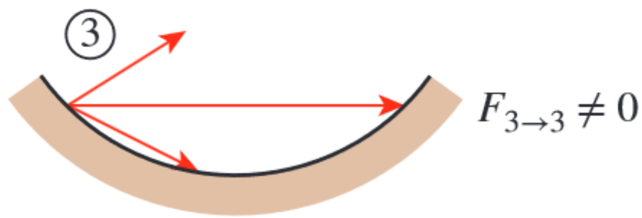
The symmetry rule can also be expressed as *if the surfaces j and k are symmetric about the surface i then* $F_{i\to j} = F_{i\to k}$. Using the reciprocity rule, we can show that the relation $F_{j\to i} = F_{k\to i}$ is also true in this case.



(a) Plane surface



(b) Convex surface



(c) Concave surface

Crossed Strings Method

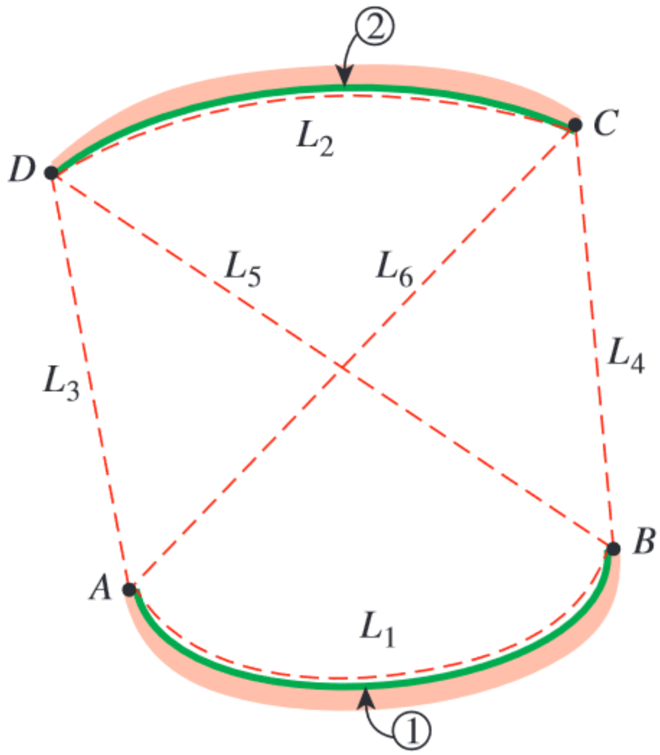


FIGURE 13–16
Determination of the view factor $F_{1 \rightarrow 2}$ by the application of the crossed-strings method.

and connect them to each other with tightly stretched strings, which are indicated by dashed lines. Hottel has shown that the view factor $F_{1 \to 2}$ can be expressed in terms of the lengths of these stretched strings, which are straight lines, as

$$F_{1 \to 2} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1} \tag{13–16}$$

Note that $L_5 + L_6$ is the sum of the lengths of the *crossed strings,* and $L_3 + L_4$ is the sum of the lengths of the *uncrossed strings* attached to the end points. Therefore, Hottel's crossed-strings method can be expressed verbally as

$$F_{i \to j} = \frac{\sum(\text{Crossed strings}) - \sum(\text{Uncrossed strings})}{2 \times (\text{Length of surface } i)} \tag{13–17}$$

The crossed-strings method is applicable even when the two surfaces considered share a common edge, as in a triangle. In such cases, the common edge can be treated as an imaginary string of zero length. The method can also be applied to surfaces that are partially blocked by other surfaces by allowing the strings to bend around the blocking surfaces.

Black Surface Heat Transfer

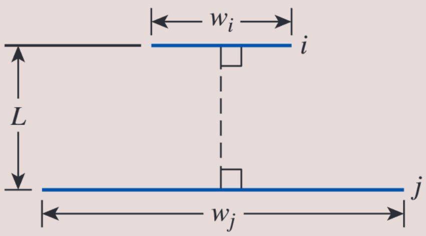
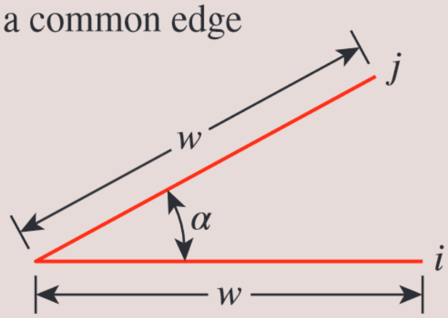
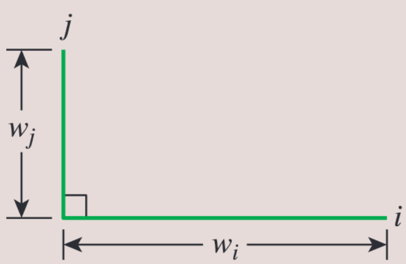
The rate of *net* radiation heat transfer between two *black* surfaces is determined from

$$\dot{Q}_{1 \to 2} = A_1 F_{1 \to 2} \sigma (T_1^4 - T_2^4)$$

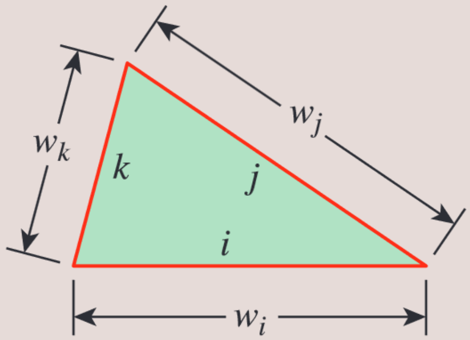
The *net* radiation heat transfer from any surface *i* of a *black* enclosure is determined by adding up the *net* radiation heat transfers from surface *i* to each of the surfaces of the enclosure:

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i \to j} = \sum_{j=1}^{N} A_i F_{i \to j} \sigma (T_i^4 - T_j^4)$$

View Factor Tables

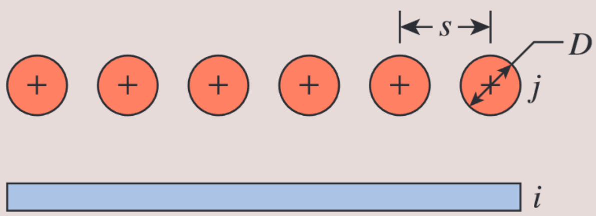
Geometry	Relation
Parallel plates with midlines connected by perpendicular line 	$W_i = w_i/L$ and $W_j = w_j/L$ $F_{i \to j} = \dfrac{[(W_i + W_j)^2 + 4]^{1/2} - [(W_j - W_i)^2 + 4]^{1/2}}{2W_i}$
Inclined plates of equal width and with a common edge 	$F_{i \to j} = 1 - \sin \dfrac{1}{2}\alpha$
Perpendicular plates with a common edge 	$F_{i \to j} = \dfrac{1}{2}\left\{1 + \dfrac{w_j}{w_i} - \left[1 + \left(\dfrac{w_j}{w_i}\right)^2\right]^{1/2}\right\}$

Three-sided enclosure



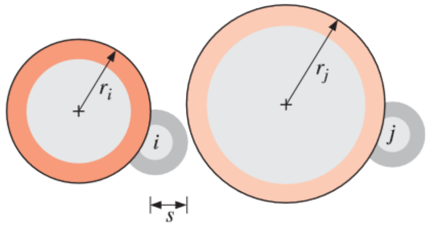
$$F_{i \to j} = \frac{w_i + w_j - w_k}{2w_i}$$

Infinite plane and row of cylinders



$$F_{i \to j} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{1/2} + \frac{D}{s}\tan^{-1}\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}$$

Parallel Cylinders of Different Radii

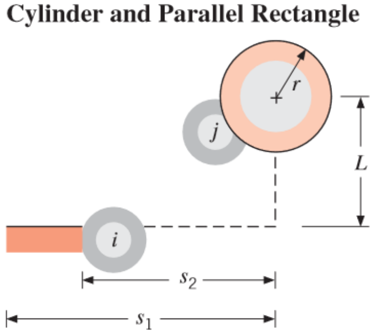


$$F_{ij} = \frac{1}{2\pi}\Big\{\pi + [C^2 - (R+1)^2]^{1/2} - [C^2 - (R-1)^2]^{1/2} + (R-1)\cos^{-1}\left[\left(\frac{R}{C}\right) - \left(\frac{1}{C}\right)\right] - (R+1)\cos^{-1}\left[\left(\frac{R}{C}\right) + \left(\frac{1}{C}\right)\right]\Big\}$$

$R = r_j/r_i$, $S = s/r_i$

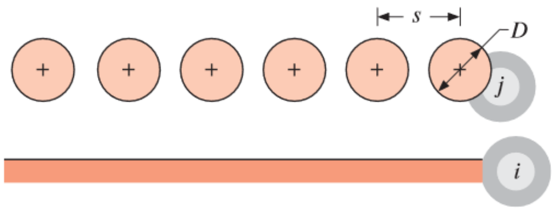
$C = 1 + R + S$

Cylinder and Parallel Rectangle

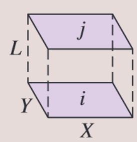
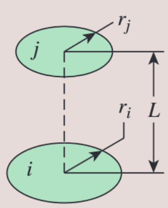
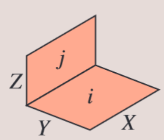


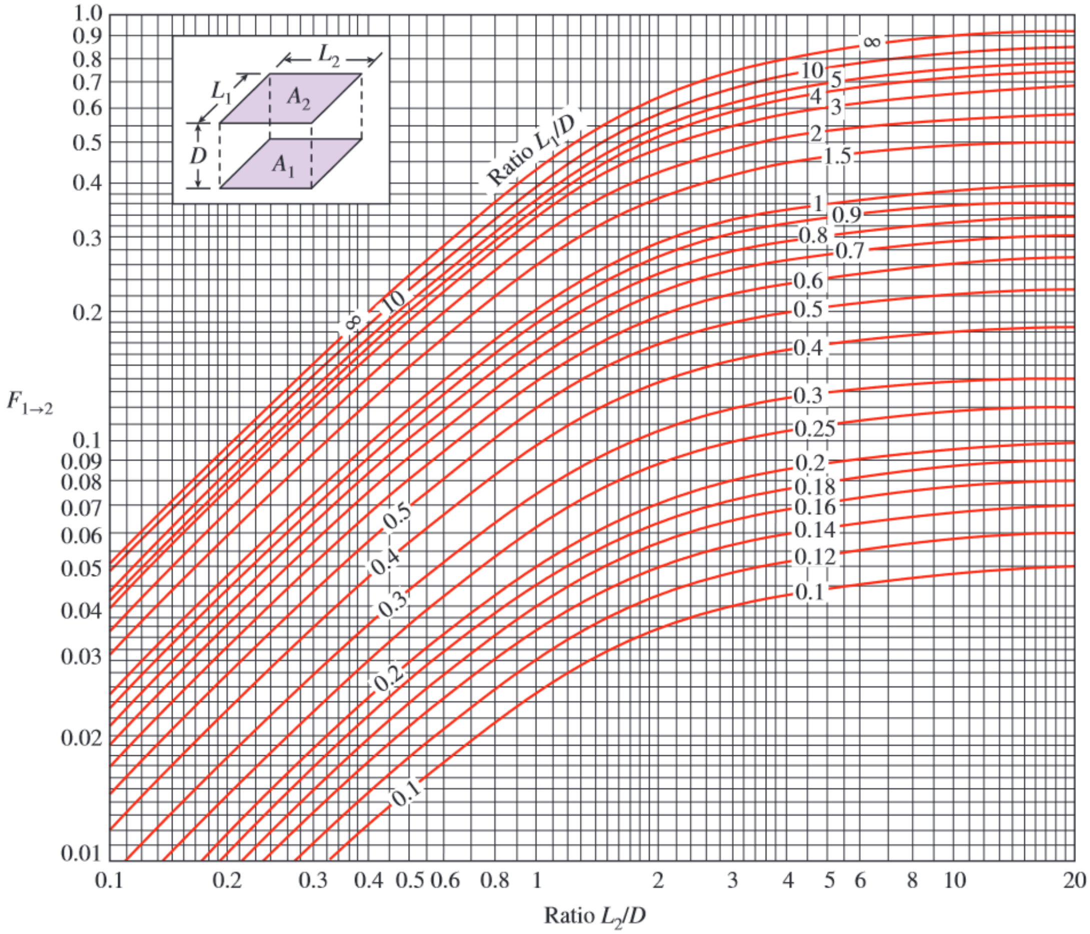
$$F_{ij} = \frac{r}{s_1 - s_2}\left[\tan^{-1}\frac{s_1}{L} - \tan^{-1}\frac{s_2}{L}\right]$$

Infinite Plane and Row of Cylinders

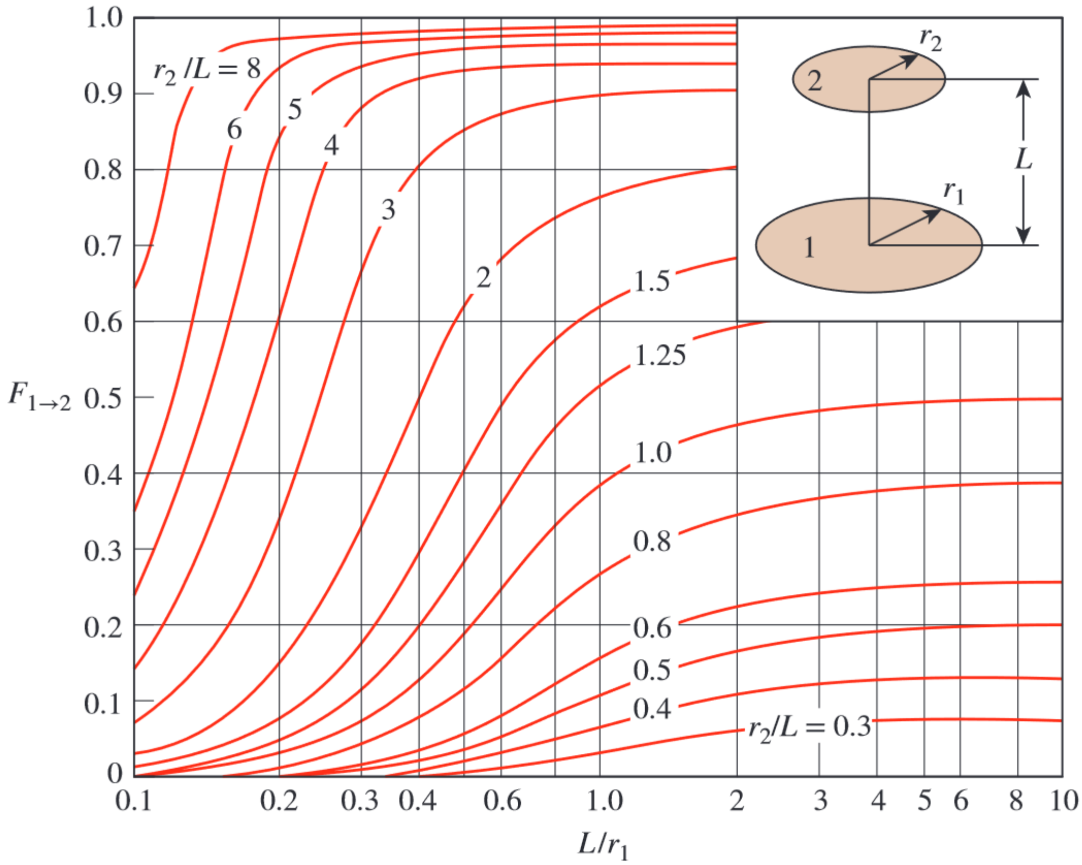


$$F_{ij} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{1/2} + \left(\frac{D}{s}\right)\tan^{-1}\left[\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}\right]$$

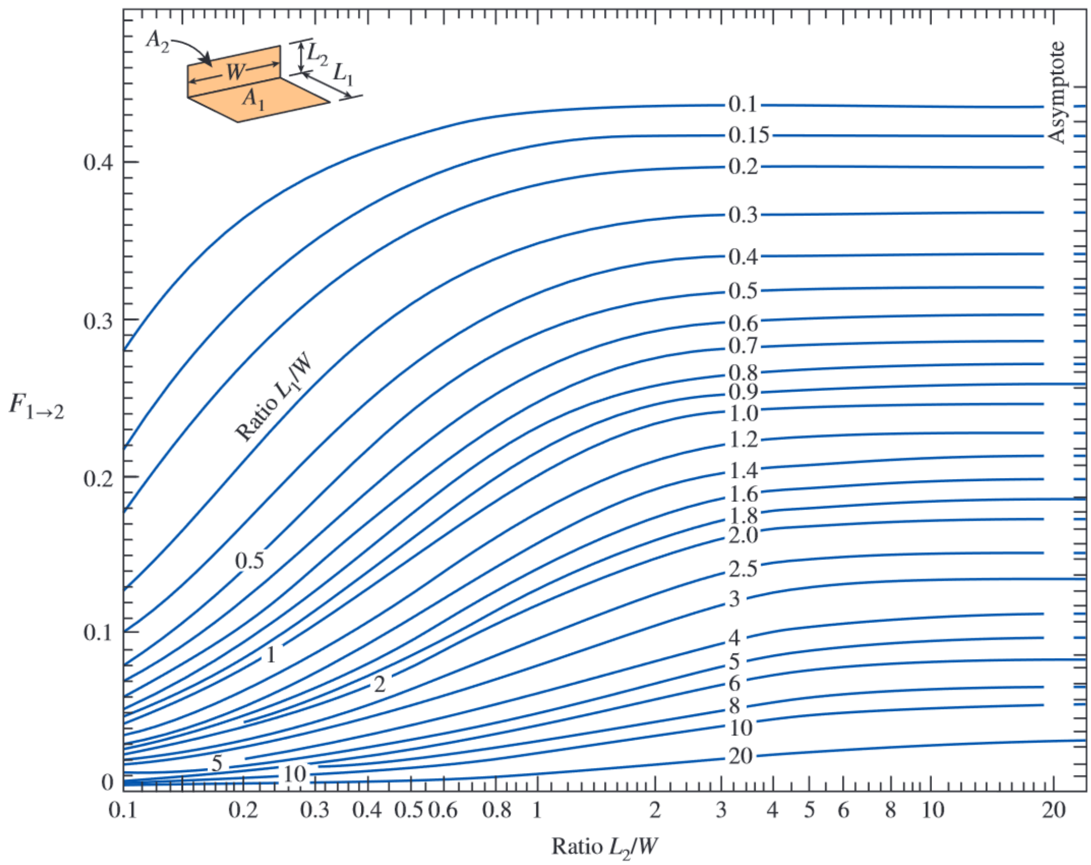
Geometry	Relation
Aligned parallel rectangles 	$\bar{X} = X/L$ and $\bar{Y} = Y/L$ $F_{i\rightarrow j} = \frac{2}{\pi \bar{X}\bar{Y}}\left\{\ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y}\right\}$
Coaxial parallel disks 	$R_i = r_i/L$ and $R_j = r_j/L$ $S = 1 + \frac{1+R_j^2}{R_i^2}$ $F_{i\rightarrow j} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{r_j}{r_i}\right)^2\right]^{1/2}\right\}$ For $r_i = r_j = r$ and $R = r/L$: $F_{i\rightarrow j} = F_{j\rightarrow i} = 1 + \frac{1-\sqrt{4R^2+1}}{2R^2}$
Perpendicular rectangles with a common edge 	$H = Z/X$ and $W = Y/X$ $F_{i\rightarrow j} = \frac{1}{\pi W}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right)$



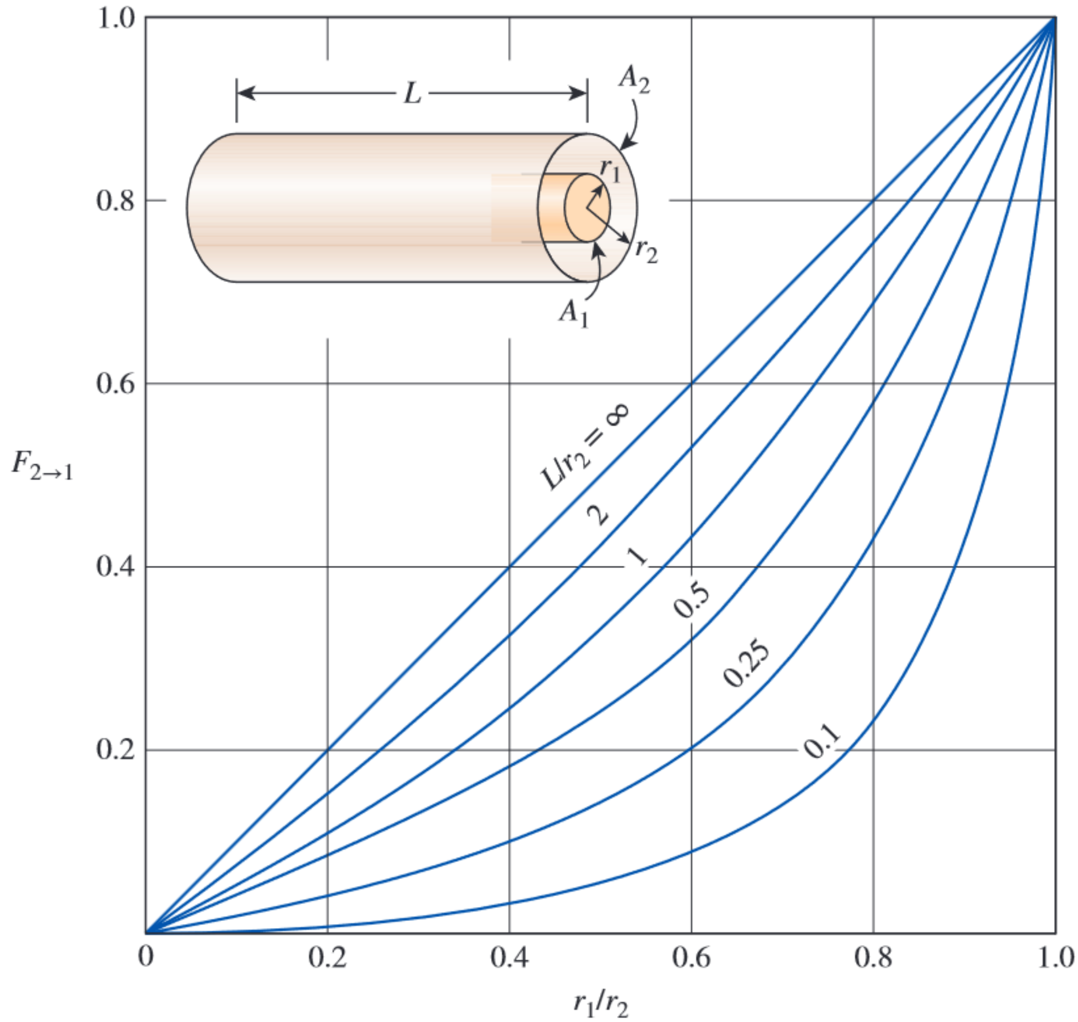
L_2
L_1
A_2
D
A_1
Ratio L_1/D
$F_{1\to 2}$
Ratio L_2/D
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.1
0.2
0.3
0.4
0.5
0.6
0.8
1
2
3
4
5
6
8
10
20
∞
10
5
4
3
2
1.5
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.25
0.2
0.18
0.16
0.14
0.12
0.1



1.0
0.9
0.8
0.7
0.6
$F_{1\to 2}$ 0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.6
1.0
2
3
4
5
6
8
10
L/r_1
$r_2/L = 8$
6
5
4
3
2
1.5
1.25
1.0
0.8
0.6
0.5
0.4
$r_2/L = 0.3$
r_2
r_1
L
2
1

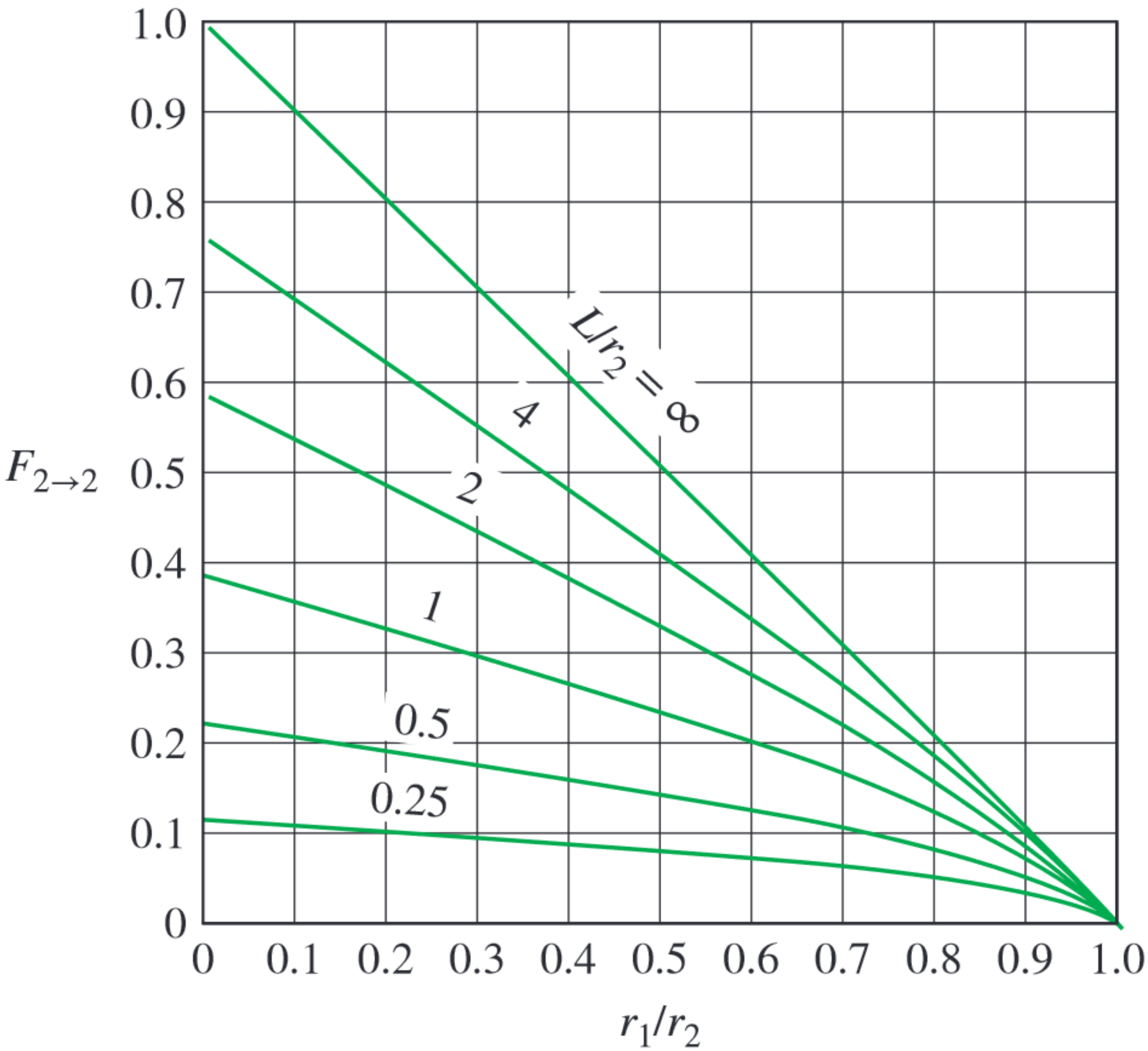


A2
W
L2
L1
A1
Asymptote
Ratio L1/W
F1→2
0.1
0.15
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.2
1.4
1.6
1.8
2.0
2.5
3
4
5
6
8
10
20
0.5
1
2
5
10
0 0.1 0.2 0.3 0.4
0.1 0.2 0.3 0.4 0.5 0.6 0.8 1 2 3 4 5 6 8 10 20
Ratio L2/W



1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1.0
$F_{2\to1}$
r_1/r_2
L
A_2
A_1
r_1
r_2
$L/r_2 = \infty$
2
1
0.5
0.25
0.1

(*a*)



1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$F_{2\to2}$
$L/r_2 = \infty$
4
2
1
0.5
0.25
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
r_1/r_2

Radiation Network

In an electrical analogy to Ohm's law, this equation can be rearranged as

$$\dot{Q}_i = \frac{E_{bi} - J_i}{R_i} \qquad \text{(W)} \tag{13–25}$$

where

$$R_i = \frac{1 - \varepsilon_i}{A_i \varepsilon_i} \tag{13–26}$$

is the **surface resistance** to radiation. The quantity $E_{bi} - J_i$ corresponds to a *potential difference* and the net rate of radiation heat transfer corresponds to *current* in the electrical analogy, as illustrated in Fig. 13–21.

Again in analogy to Ohm's law, this equation can be rearranged as

$$\dot{Q}_{i \to j} = \frac{J_i - J_j}{R_{i \to j}} \qquad \text{(W)} \tag{13–30}$$

where

$$R_{i \to j} = \frac{1}{A_i F_{i \to j}} \tag{13–31}$$

is the **space resistance** to radiation. Again the quantity $J_i - J_j$ corresponds to a *potential difference,* and the net rate of heat transfer between two surfaces corresponds to *current* in the electrical analogy, as illustrated in Fig. 13–22.

The direction of the net radiation heat transfer between two surfaces depends on the relative magnitudes of J_i and J_j. A positive value for $\dot{Q}_{i \to j}$ indicates that net heat transfer is *from* surface *i to* surface *j.* A negative value indicates the opposite.

Methods of Solving Radiation Problems

In the radiation analysis of an enclosure, either the temperature or the net rate of heat transfer must be given for each of the surfaces to obtain a unique solution for the unknown surface temperatures and heat transfer rates. There are two methods commonly used to solve radiation problems. In the first method, Eqs. 13–32 (for surfaces with specified heat transfer rates) and 13–33 (for surfaces with specified temperatures) are simplified and rearranged as

Surfaces with specified net heat transfer rate $\dot{Q}$

$$\dot{Q}_i = A_i \sum_{j=1}^{N} F_{i \to j}(J_i - J_j) \qquad \text{(13–34)}$$

Surfaces with specified temperature T_i

$$\sigma T_i^4 = J_i + \frac{1 - \varepsilon_i}{\varepsilon_i} \sum_{j=1}^{N} F_{i \to j}(J_i - J_j) \qquad \text{(13–35)}$$

Surface Energy Balance

Analogous to Current Law

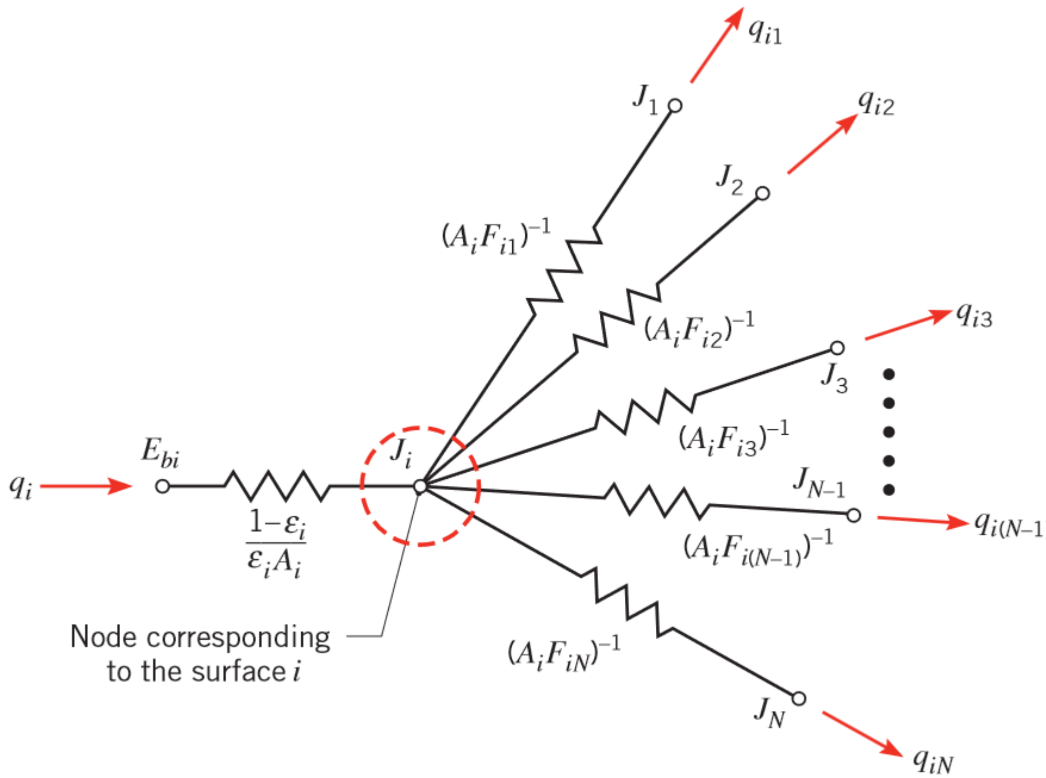


FIGURE 13.10 Network representation of radiative exchange between surface i and the remaining surfaces of an enclosure.

Two-Surface Enclosures

Radiation Heat Transfer in Two-Surface Enclosures

Consider an enclosure consisting of two opaque surfaces at specified temperatures T_1 and T_2, as shown in Fig. 13–24, and we try to determine the net rate of radiation heat transfer between the two surfaces with the network method. Surfaces 1 and 2 have emissivities ε_1 and ε_2 and surface areas A_1 and A_2 and are maintained at uniform temperatures T_1 and T_2, respectively. There are only two surfaces in the enclosure, and thus we can write

$$\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$$

That is, the net rate of radiation heat transfer from surface 1 to surface 2 must equal the net rate of radiation heat transfer *from* surface 1 and the net rate of radiation heat transfer *to* surface 2.

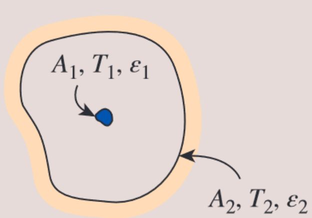
The radiation network of this two-surface enclosure consists of two surface resistances and one space resistance, as shown in Fig. 13–24. In an electrical network, the electric current flowing through these resistances connected in series would be determined by dividing the potential difference between points A and B by the total resistance between the same two points. The net rate of radiation transfer is determined in the same manner and is expressed as

$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2} = \dot{Q}_1 = -\dot{Q}_2$$

or

$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} \quad \text{(W)} \qquad \textbf{(13–36)}$$

Small object in a large cavity

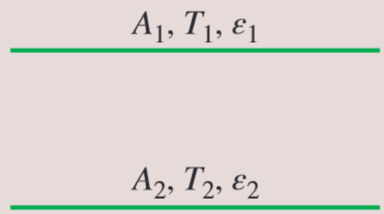


$$\frac{A_1}{A_2} \approx 0$$

$$F_{12} = 1$$

$$\dot{Q}_{12} = A_1\sigma\varepsilon_1(T_1^4 - T_2^4) \qquad \textbf{(13–37)}$$

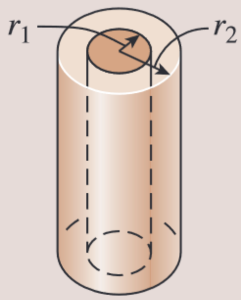
Infinitely large parallel plates



$$A_1 = A_2 = A \qquad F_{12} = 1$$

$$\dot{Q}_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

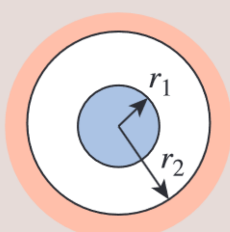
Infinitely long concentric cylinders



$$\frac{A_1}{A_2} = \frac{r_1}{r_2} \qquad F_{12} = 1$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

Concentric spheres

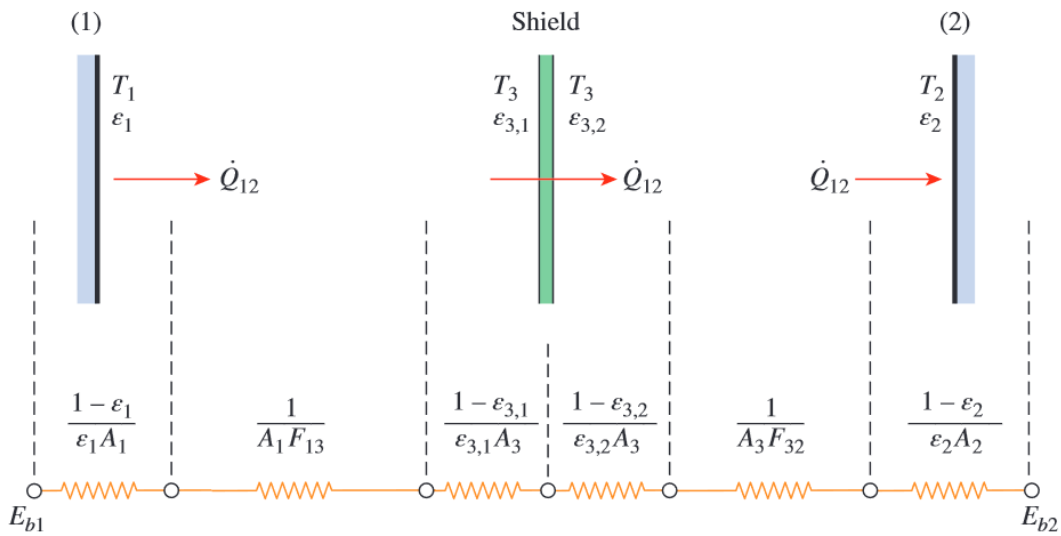


$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2 \qquad F_{12} = 1$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)^2}$$

An example is shown in Figure 13.12*a*, where a thin radiation shield is placed between surfaces 1 and 2. If all surfaces exchange energy only by radiation, and are not otherwise heated or cooled, the rate of radiation transfer must be the same at every surface, that is, $q = q_1 = q_{1s} = q_{s2} = -q_2$. This scenario can be represented by a series radiation network (Figure 13.12*b*) that includes a surface resistance for each of the four surfaces (including both sides of the shield) and a space resistance between each pair of adjacent surfaces. Note that the emissivity of one side of the shield (ε_{s1}) may differ from that of the opposite side (ε_{s2}) and the radiosities will always differ. The resulting heat transfer rate through the system is given by

$$q = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1} + \dfrac{1}{A_1 F_{1s}} + \dfrac{1-\varepsilon_{s1}}{\varepsilon_{s1} A_s} + \dfrac{1-\varepsilon_{s2}}{\varepsilon_{s2} A_s} + \dfrac{1}{A_s F_{s2}} + \dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}} \qquad (13.28)$$



Noting that $F_{13} = F_{32} = 1$ and $A_1 = A_2 = A_3 = A$ for infinite parallel plates, Eq. 13–42 simplifies to

$$\dot{Q}_{12,\text{ one shield}} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)} \quad \textbf{(13–43)}$$

$$\dot{Q}_{12,\ N\text{ shields}} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right) + \cdots + \left(\frac{1}{\varepsilon_{N,1}} + \frac{1}{\varepsilon_{N,2}} - 1\right)} \quad \textbf{(13–44)}$$

If the emissivities of all surfaces are equal, Eq. 13–44 reduces to

$$\dot{Q}_{12,\ N\text{ shields}} = \frac{A\sigma(T_1^4 - T_2^4)}{(N+1)\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right)} = \frac{1}{N+1}\dot{Q}_{12,\text{ no shield}} \quad \textbf{(13–45)}$$

Surfaces with specified net heat transfer rate $\dot{Q}_i$

$$\dot{Q}_i = A_i \sum_{j=1}^{N} F_{i \to j}(J_i - J_j)$$

Surfaces with specified temperature T_i

$$\sigma T_i^4 = J_i + \frac{1-\varepsilon_i}{\varepsilon_i} \sum_{j=1}^{N} F_{i \to j}(J_i - J_j)$$