Thermal Circuits

Notes based on practice, tutorial, and past year examination papers

Typical Assumptions

1. Steady state conditions exist
2. Constant properties
3. No heat generation.
4. 1D heat transfer
5. Negligible convection/radiation (please check!)
6. No contact resistance (unless otherwise specified)

Do add other assumptions depending on problem

Examples

Now consider the combined series-parallel arrangement shown in Fig. 3–20. The total rate of heat transfer through this composite system can again be expressed as

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \tag{3–32}$$

where

$$R_{\text{total}} = R_{12} + R_3 + R_{\text{conv}} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{\text{conv}} \tag{3–33}$$

and

$$R_1 = \frac{L_1}{k_1 A_1} \quad R_2 = \frac{L_2}{k_2 A_2} \quad R_3 = \frac{L_3}{k_3 A_3} \quad R_{\text{conv}} = \frac{1}{hA_3} \tag{3–34}$$

Once the individual thermal resistances are evaluated, the total resistance and the total rate of heat transfer can easily be determined from the relations above.

The result obtained is somewhat approximate, since the surfaces of the third layer are probably not isothermal, and heat transfer between the first two layers is likely to occur.

Two assumptions commonly used in solving complex multidimensional heat transfer problems by treating them as one-dimensional (say, in the x-direction) using the thermal resistance network are (1) any plane wall normal to the x-axis is *isothermal* (i.e., assumes the temperature varies in

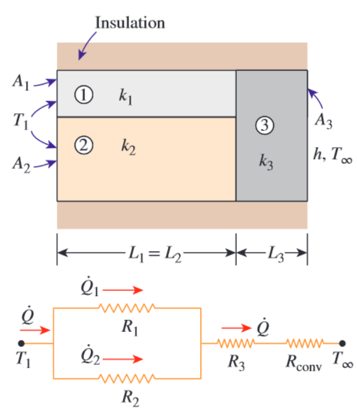


FIGURE 3–20
Thermal resistance network for combined series-parallel arrangement.

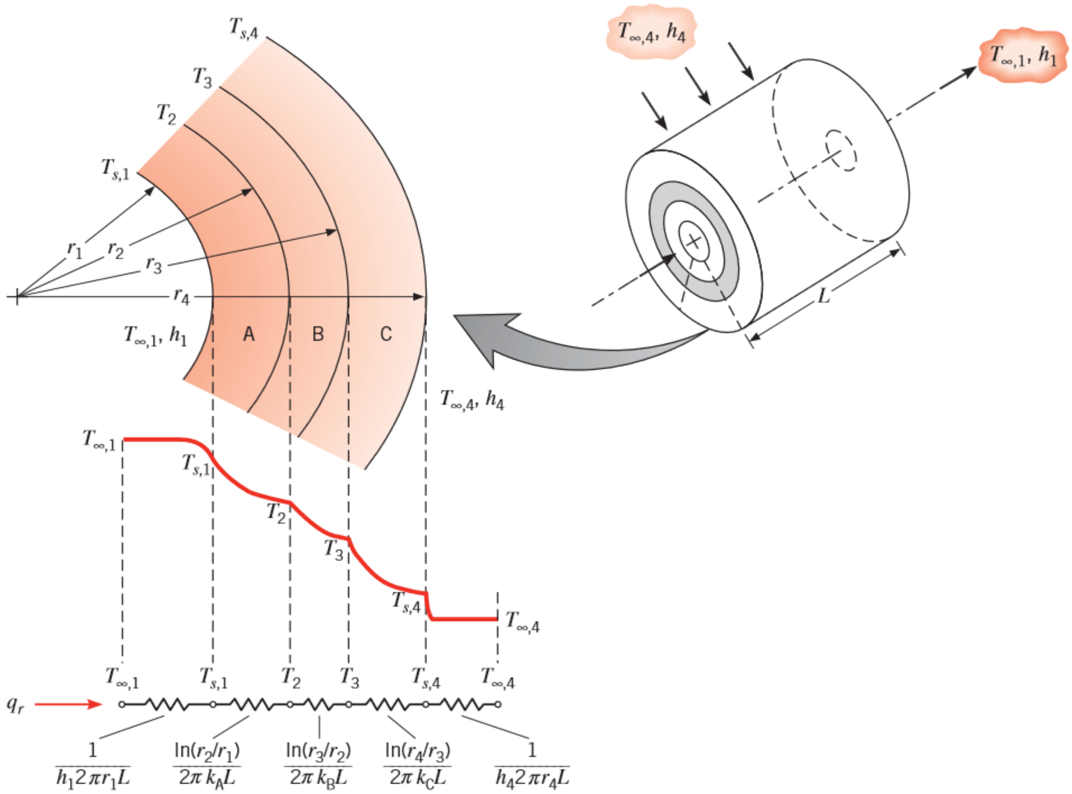
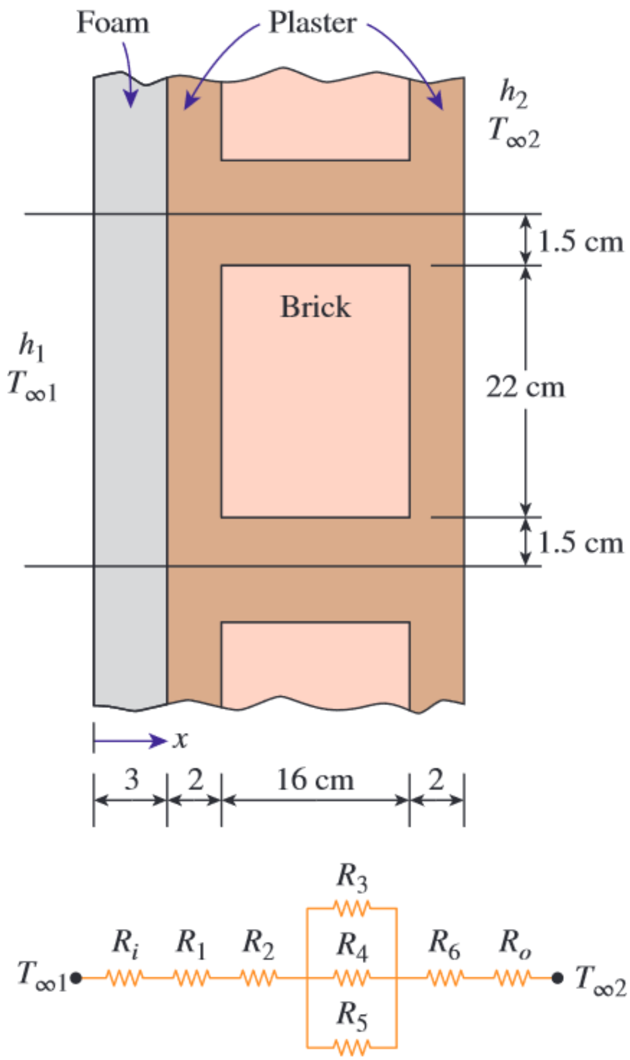


FIGURE 3.8 Temperature distribution for a composite cylindrical wall.



If U is defined in terms of the inside area, $A_1 = 2\pi r_1 L$, Equations 3.34 and 3.35 may be equated to yield

$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{r_1}{k_A}\ln\frac{r_2}{r_1} + \frac{r_1}{k_B}\ln\frac{r_3}{r_2} + \frac{r_1}{k_C}\ln\frac{r_4}{r_3} + \frac{r_1}{r_4}\frac{1}{h_4}} \tag{3.36}$$

This definition is *arbitrary,* and the overall coefficient may also be defined in terms of A_4 or any of the intermediate areas. Note that

$$U_1A_1 = U_2A_2 = U_3A_3 = U_4A_4 = (\Sigma R_t)^{-1} \tag{3.37}$$

and the specific forms of U_2, U_3, and U_4 may be inferred from Equations 3.34 and 3.35.

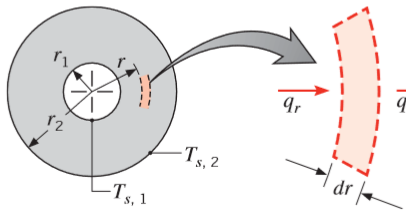


FIGURE 3.9 Conduction in a spherical shell.

Acknowledging that q_r is a constant, independent of r, Equation 3.38 may be expressed in the integral form

$$\frac{q_r}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r^2} = -\int_{T_{s,1}}^{T_{s,2}} k(T)\,dT \tag{3.39}$$

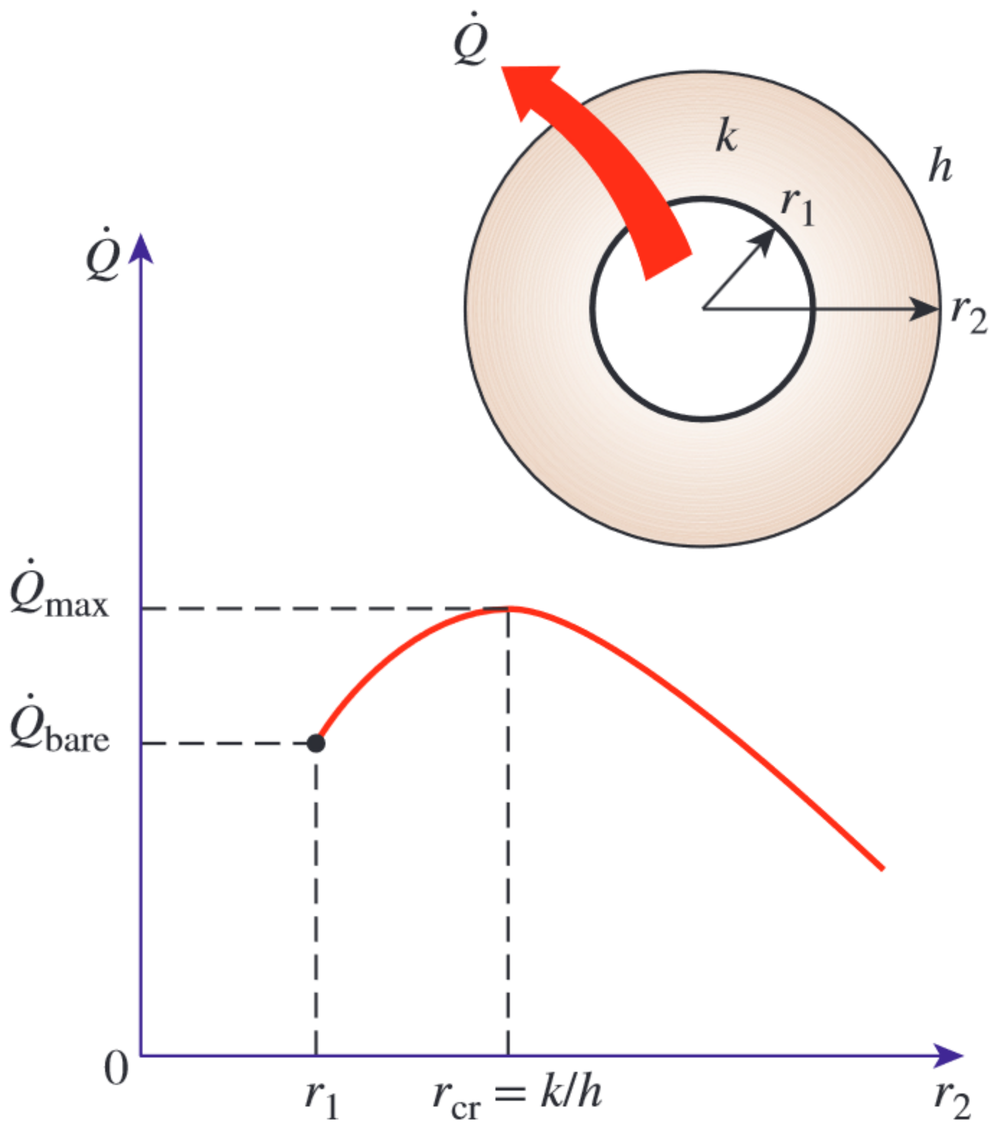
Assuming constant k, we then obtain

$$q_r = \frac{4\pi k(T_{s,1} - T_{s,2})}{(1/r_1) - (1/r_2)} \tag{3.40}$$

Remembering that the thermal resistance is defined as the temperature difference divided by the heat transfer rate, we obtain

$$R_{t,\text{cond}} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{3.41}$$

Critical Radius of Insulation



Consider a cylindrical pipe of outer radius r_1 whose outer surface temperature T_1 is kept constant (Fig. 3–30). The pipe is now insulated with a material whose thermal conductivity is k and whose outer radius is r_2. Heat is lost from the pipe to the surrounding medium at temperature T_∞, with a convection heat transfer coefficient h. The rate of heat transfer from the insulated pipe to the surrounding air can be expressed as (Fig. 3–31)

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{ins}} + R_{\text{conv}}} = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi Lk} + \dfrac{1}{h(2\pi r_2 L)}} \tag{3–49}$$

The variation of $\dot{Q}$ with the outer radius of the insulation r_2 is plotted in Fig. 3–31. The value of r_2 at which $\dot{Q}$ reaches a maximum is determined from the requirement that $d\dot{Q}/dr_2 = 0$ (zero slope). Performing the differentiation and solving for r_2 yields the **critical radius of insulation** for a cylindrical body:

$$r_{\text{cr, cylinder}} = \frac{k}{h} \qquad \text{(m)} \tag{3–50}$$

The discussions above can be repeated for a sphere, and it can be shown in a similar manner that the critical radius of insulation for a spherical shell is

$$r_{\text{cr, sphere}} = \frac{2k}{h} \tag{3–51}$$

where k is the thermal conductivity of the insulation and h is the convection heat transfer coefficient on the outer surface.

Thermal Contact Resistance

where A is the apparent interface area (which is the same as the cross-sectional area of the rods) and $\Delta T_{\text{interface}}$ is the effective temperature difference at the interface. The quantity h_c, which corresponds to the convection heat transfer coefficient, is called the **thermal contact conductance** and is expressed as

$$h_c = \frac{\dot{Q}/A}{\Delta T_{\text{interface}}} \quad (\text{W/m}^2\cdot\text{K}) \tag{3–27}$$

It is related to thermal contact resistance by

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{\text{interface}}}{\dot{Q}/A} \quad (\text{m}^2\cdot\text{K/W}) \tag{3–28}$$

TABLE 3–1

Thermal contact conductance for aluminum plates with different fluids at the interface for a surface roughness of 10 μm and interface pressure of 1 atm

Fluid at the Interface	Contact Conductance, h_c, $\text{W/m}^2\cdot\text{K}$
Air	3640
Helium	9520
Hydrogen	13,900
Silicone oil	19,000
Glycerin	37,700

TABLE 3–2

Thermal contact conductance of some metal surfaces in air (from various sources)

Material	Surface Condition	Roughness, μm	Temperature, °C	Pressure, MPa	h_c,* W/m²·K
Identical Metal Pairs					
416 Stainless steel	Ground	2.54	90–200	0.17–2.5	3,800
304 Stainless steel	Ground	1.14	20	4–7	1,900
Aluminum	Ground	2.54	150	1.2–2.5	11,400
Copper	Ground	1.27	20	1.2–20	143,000
Copper	Milled	3.81	20	1–5	55,500
Copper (vacuum)	Milled	0.25	30	0.17–7	11,400
Dissimilar Metal Pairs					
Stainless steel–				10	2,900
Aluminum		20–30	20	20	3,600
Stainless steel–				10	16,400
Aluminum		1.0–2.0	20	20	20,800
Steel Ct-30–				10	50,000
Aluminum	Ground	1.4–2.0	20	15–35	59,000
Steel Ct-30–				10	4,800
Aluminum	Milled	4.5–7.2	20	30	8,300
				5	42,000
Aluminum-Copper	Ground	1.17–1.4	20	15	56,000
				10	12,000
Aluminum-Copper	Milled	4.4–4.5	20	20–35	12,000

Summary

TABLE 3.3 One-dimensional, steady-state solutions to the heat equation with no generation

	Plane Wall	Cylindrical Wall[a]	Spherical Wall[a]
Heat equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$
Temperature distribution	$T_{s,1} - \Delta T\frac{x}{L}$	$T_{s,2} + \Delta T\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$	$T_{s,1} - \Delta T\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$
Heat flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\,\Delta T}{r\ln(r_2/r_1)}$	$\frac{k\,\Delta T}{r^2[(1/r_1)-(1/r_2)]}$
Heat rate (q)	$kA\frac{\Delta T}{L}$	$\frac{2\pi Lk\,\Delta T}{\ln(r_2/r_1)}$	$\frac{4\pi k\,\Delta T}{(1/r_1)-(1/r_2)}$
Thermal resistance ($R_{t,\text{cond}}$)	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi Lk}$	$\frac{(1/r_1)-(1/r_2)}{4\pi k}$

[a]The critical radius of insulation is $r_{cr} = k/h$ for the cylinder and $r_{cr} = 2k/h$ for the sphere.

Convection resistance: $$R_{\text{conv}} = \frac{1}{hA}$$

Interface resistance: $$R_{\text{interface}} = \frac{1}{h_c A} = \frac{R_c}{A}$$

Radiation resistance: $$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A}$$