Applications of Information Theory

- *Ergodic theory*. The asymptotic equipartition theorem states that most sample n-sequences of an ergodic process have probability about 2^{-nH} and that there are about 2^{nH} such typical sequences.
- *Hypothesis testing*. The relative entropy D arises as the exponent in the probability of error in a hypothesis test between two distributions. It is a natural measure of distance between distributions.
- *Statistical mechanics*. The entropy H arises in statistical mechanics as a measure of uncertainty or disorganization in a physical system. Roughly speaking, the entropy is the logarithm of the number of ways in which the physical system can be configured. The second law of thermodynamics says that the entropy of a closed system cannot decrease. Later we provide some interpretations of the second law.
- *Quantum mechanics*. Here, von Neumann entropy $S = \mathrm{tr}(\rho \ln \rho) = \sum_i \lambda_i \log \lambda_i$ plays the role of classical Shannon–Boltzmann entropy $H = -\sum_i p_i \log p_i$. Quantum mechanical versions of data compression and channel capacity can then be found.
- *Inference*. We can use the notion of Kolmogorov complexity K to find the shortest description of the data and use that as a model to predict what comes next. A model that maximizes the uncertainty or entropy yields the maximum entropy approach to inference.
- *Gambling and investment*. The optimal exponent in the growth rate of wealth is given by the doubling rate W. For a horse race with uniform odds, the sum of the doubling rate W and the entropy H is constant. The increase in the doubling rate due to side information is equal to the mutual information I between a horse race and the side information. Similar results hold for investment in the stock market.
- *Probability theory*. The asymptotic equipartition property (AEP) shows that most sequences are typical in that they have a sample entropy close to H. So attention can be restricted to these approximately 2^{nH} typical sequences. In large deviation theory, the