Instances of X and Y more Correlated than Dependent

https://twitter.com/nntaleb/status/1087341085532307456

QUIZ DU JOUR (this time, difficult)
There are many known examples of fake independence: random variables X & Y can be more dependent than shown in correlation. (x & Y=x^x for instance)

Can you find instances of X & Y that more correlated than dependent?

Simplest Case

1- SIMPLEST CASE:
Correlation with a constant will show a positive correlation.

In[∘]:=
```
{Table[x, {x, 1, n}], Table[(-1)^x 10^-15, {x, 1, n}]} /. n → 10
```

Table: Iterator {x, 1, n} does not have appropriate bounds.

Table: Iterator {x, 1, n} does not have appropriate bounds.

Out[∘]=

$$\left\{\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}, \left\{-\frac{1}{1\,000\,000\,000\,000\,000}, \frac{1}{1\,000\,000\,000\,000\,000}, -\frac{1}{1\,000\,000\,000\,000\,000}, \frac{1}{1\,000\,000\,000\,000\,000}, -\frac{1}{1\,000\,000\,000\,000\,000}, \frac{1}{1\,000\,000\,000\,000\,000}, -\frac{1}{1\,000\,000\,000\,000\,000}, \frac{1}{1\,000\,000\,000\,000\,000}, -\frac{1}{1\,000\,000\,000\,000\,000}, \frac{1}{1\,000\,000\,000\,000\,000}\right\}\right\}$$

In[∘]:=
```
Correlation[Table[x, {x, 1, n}], Table[(-1)^x 10^-15, {x, 1, n}]] /. n → 10 // N
```

Table: Iterator {x, 1, n} does not have appropriate bounds.

Table: Iterator {x, 1, n} does not have appropriate bounds.

Correlation: The arguments to Correlation are not a pair of vectors or a pair of matrices of equal length.

Out[∘]=

0.174078

Nonlinearity

Nassim Nicholas Taleb
@nntaleb

2- Something unknown by *IQ idiots w/sinister theories* (s.a. @charlesmurray @primalpoly): NONLINEARITY. Variables that correlate only ½ the time show ρ >.855 (Gaussian case) but rises to ρ>.92 in absense of tapering of tail values.
These are basic notions.

Probability is hard.

Where we show that correlation at 50 % of the time for Gaussian Variables produces a total correlation $\sqrt{\frac{\pi}{-2+2\pi}}$ ~85%

But without tail tapering (i.e. nonprobabilistic structure) $\sqrt{\frac{\pi}{-2+2\pi}}$ is the lower bound!

First, Non Probabilistic

```
Table[Correlation[Table [x, {x, 1, n}], Table [Boole[x > n / 2] , {x, 1, n}]],
 {n, 100, 1500, 100}]
```

$$\left\{\frac{50}{\sqrt{3333}}, \frac{100}{\sqrt{13\,333}}, 150\sqrt{\frac{3}{89\,999}}, \frac{200}{\sqrt{53\,333}}, \frac{250}{\sqrt{83\,333}}, 300\sqrt{\frac{3}{359\,999}}, \frac{350}{\sqrt{163\,333}}, \frac{400}{\sqrt{213\,333}}, 450\sqrt{\frac{3}{809\,999}}, \frac{500}{3\sqrt{37\,037}}, \frac{550}{\sqrt{403\,333}}, 600\sqrt{\frac{3}{1\,439\,999}}, \frac{650}{\sqrt{563\,333}}, \frac{700}{\sqrt{653\,333}}, 750\sqrt{\frac{3}{2\,249\,999}}\right\}$$

```
% // N
```

```
{0.866069, 0.866036, 0.86603, 0.866028, 0.866027, 0.866027, 0.866026,
 0.866026, 0.866026, 0.866026, 0.866026, 0.866026, 0.866026, 0.866026, 0.866026}
```

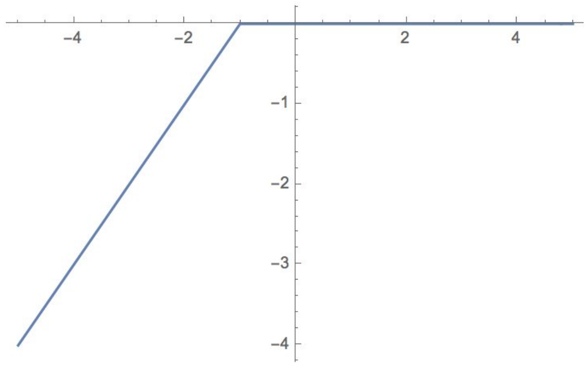
```
Table[Correlation[Table [x, {x, 1, n}], Table [Boole[x > n / 2] x, {x, 1, n}]],
 {n, 100, 1500, 100}]
```

$$\left\{83\sqrt{\frac{101}{807\,411}}, 499\sqrt{\frac{67}{19\,356\,133}}, \frac{749\sqrt{\frac{301}{370\,331}}}{23}, 111\sqrt{\frac{1203}{17\,195\,437}}, 1249\sqrt{\frac{167}{302\,227\,833}}, 1499\sqrt{\frac{601}{1\,566\,625\,199}}, 583\sqrt{\frac{701}{276\,400\,211}}, 1999\sqrt{\frac{267}{1\,237\,704\,533}}, 2249\sqrt{\frac{901}{5\,286\,660\,299}}, \frac{833\sqrt{\frac{1001}{89\,527\,679}}}{3}, 2749\sqrt{\frac{367}{3\,217\,286\,233}}, 2999\sqrt{\frac{1201}{12\,530\,510\,399}}, 361\sqrt{\frac{3903}{590\,044\,337}}, \frac{3499\sqrt{\frac{467}{736\,941\,437}}}{3}, 3749\sqrt{\frac{1501}{24\,472\,675\,499}}\right\}$$

```
N[%]
```

```
{0.928307, 0.928386, 0.928415, 0.92843, 0.928439, 0.928445, 0.92845,
 0.928453, 0.928456, 0.928458, 0.928459, 0.928461, 0.928462, 0.928463, 0.928464}
```

```
Plot[Min[x - k, 0] /. k → -1, {x, -5, 5}]
```



Unconditional Correlation

```
mpayoff =
  Integrate[(Min[x - k, 0]) PDF[NormalDistribution[0, σ], x], {x, -∞, ∞},
   Assumptions → σ > 0] // FullSimplify
```

$$-\frac{e^{-\frac{k^2}{2\sigma^2}}\sigma}{\sqrt{2\pi}}-\frac{1}{2}k\left(1+\mathrm{Erf}\left[\frac{k}{\sqrt{2}\,\sigma}\right]\right)$$

```
Cov = Integrate[((Min[x - k, 0]) - mpayoff) x PDF[NormalDistribution[0, σ], x],
   {x, -∞, ∞}, Assumptions → σ > 0] // FullSimplify
```

$$\frac{1}{2}\sigma^2\left(1+\mathrm{Erf}\left[\frac{k}{\sqrt{2}\,\sigma}\right]\right)$$

```
varpayoff =
  Integrate[((Min[x - k, 0]) - mpayoff)^2 PDF[NormalDistribution[0, σ], x],
   {x, -∞, ∞}, Assumptions → σ > 0] // FullSimplify
```

$$-\frac{e^{-\frac{k^2}{\sigma^2}}\sigma^2}{2\pi}-\frac{e^{-\frac{k^2}{2\sigma^2}}k\,\sigma\,\mathrm{Erf}\left[\frac{k}{\sqrt{2}\,\sigma}\right]}{\sqrt{2\pi}}-\frac{1}{4}\left(-2+\mathrm{Erfc}\left[\frac{k}{\sqrt{2}\,\sigma}\right]\right)\left(2\sigma^2+k^2\,\mathrm{Erfc}\left[\frac{k}{\sqrt{2}\,\sigma}\right]\right)$$

$$\text{corr} = \frac{\text{Cov}}{\sqrt{\text{varpayoff}\ \sigma}} \text{ // Expand // FullSimplify}$$