Beta is Correlation for Normalised Variables under Linear Regression

https://en.wikipedia.org/wiki/Beta_(finance)#Technical_aspects

Consider $Y = a + b X + c \epsilon$.

We have ϵ as the standard normally distributed random variable.

This means $\epsilon \approx \mathcal{N}(0,1)$.

Also, X is normally distributed $X \approx \mathcal{N}(m_x, \sigma_x)$

$\mathbb{E}(Y) = m_y = a + b\,\mathbb{E}(X) = a + bm_x$

Consider Pearson's coefficient.

$\rho_{XY} = \dfrac{\mathbb{E}((X - \mathbb{E}(X))\,(Y - \mathbb{E}(Y)))}{\sqrt{\mathbb{V}(X)\,\mathbb{V}(Y)}}$

We have $\mathbb{E}[(X - \mathbb{E}(X))(Y - \mathbb{E}(Y))] = b\sigma_x^2$

Therefore we have:

$\rho_{XY} = \dfrac{b\,\sigma_X}{\sigma_Y}$