Everything about ES 241 Advanced Elasticity

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You are invited to discuss this course on Twitter: 2021

Lectures: Maxwell Dworkin 123, Tuesday and Thursday 1:30 PM - 2:45 PM (2021)
Office hour: After each class, or appointment by email.

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Office Hour: 4 pm Wednesdays, Pierce Hall 401

Grades
Homework 40%
Final exam 60%
We will have a closed-book final exam. Exam problems will be similar to homework problems.

Class Notes, Problem Sets

Sep 2	Finite deformation: general theory. Division of labor Elastomers and gels Separation of length scales Representative elementary volume Material particle Inhomogeneous deformation Homogeneous deformation Material segment Deformation gradient Basis Components Inner product space	
Sep 7	Homogeneous deformation Deformation tensor Deformation of line, area, and volume	
Sep	Homogeneous deformation	HW 01-04

9	Singular value decomposition of homogeneous deformation.	
Sep 14	Finite deformation: general theory. Area vector Nominal stress Traction Stress-traction relation Balance of moment Work True stress	
Sep 16	Add your tweet comment on linear algebra A review of thermodynamics Energy, space, matter, charge Isolated system Fundamental postulate Sample space Equilibrium Internal variable Subset of sample space Probability for an isolated system to flip among a subset of quantum states	HW 05-08
Sep 21	A review of thermodynamics A falling apple Entropy, extensive, absolute, dimensionless Subset entropy Basic algorithm of thermodynamics (BAT) 1. Construct an isolated system with an internal variable, x. 2. Find subset entropy as a function of the internal variable, S(x). 3. Maximize S(x) to reach equilibrium. 4. Increase S(x) to undergo an irreversible process.	

Sep 23	Add your tweet comment on homogeneous deformation A review of thermodynamics A thermal system is a family of isolated systems. Entropy of a thermal system is a function of energy, S(U). Energy-entropy vector. Features of the curve S(U). BAT on thermal contact Step 1. Identify an isolated system of an internal variable. Step 2. Find the subset entropy as a function of the internal variable. Step 3. Maximize subset entropy to reach thermal equilibrium. Step 4. Increase subset entropy to undergo irreversible process of heat transfer. Definition of Temperature Thermometry Calorimetry Experimentally count the number of quantum states.	HW 09-12
Sep 28	A review of thermodynamics Two units of temperature Fake unit of entropy (Entropy is dimensionless!) Experimentally count the number of quantum states Thermal reservoir A thermal system in a thermal environment Melting Free energy	
Sep 30	A review of thermodynamics Closed system A closed system in a thermomechanical environment Pressure Ideal gas Entropic elasticity	HW 13-16
Oct	A review of thermodynamics	

5	Liquid-gas phase transition Experimental setup TV, PV, PT planes Dome Critical point SUV vector space A homogenous state: a vector. Rule of mixture: convex combination. All mixtures: convex hull. Given U and V, a mixture must maximize S. The liquid and gas share a single smooth function S(U,V). The function is nonconvex. A tangent plane rolls over this nonconvex surface.	
Oct 7	Add a comment to the tweet on the Gibbs surface. Finite deformation: general theory. The problem of rheology. Elasticity, viscosity, viscoelasticity. A rod under uniaxial tension. Inelasticity, irreversibility. Elasticity, reversibility. Thermodynamics of elasticity Entropy inequality. Isothermal deformation. Free energy. Rigid-body rotation.	HW 17-20
Oct 12	Finite deformation: general theory Inhomogeneous deformation. Paths of material particles. Velocity. Acceleration. Deformation gradient. Conservation of mass. Balance of forces. Inertial force.	
Oct	A homework problem in : How	HW 21-24

14	are Steam Tables constructed? Balance of moments Thermodynamic inequality Weak form of the balance of forces Finite element method	
Oct 19	Elasticity of rubber-like materials Stress-stretch curve. 3D stress-stretch relation. Model of elasticity in terms of principal stretches. Incompressibility. Neo-Hookean model.	
Oct 21	Elasticity of rubber-like materials Gent model Use Lagrange multiplier to enforce incompressibility Model of elasticity in terms of a tensor. Invariants of a tensor.	HW 25-28
Oct 26	Neo-Hookean model in tensor form Finite deformation: special cases A metal bar necks, but a rubber band does not YouTube tensile test	
Oct 28	Finite deformation: special cases Blowing a balloon Cavitation: Description A Twitter thread on cavitation	HW 29-32
Nov 2	Finite deformation: special cases Cavitation: Analysis	
Nov 4	Finite deformation: special cases Bifurcation Bifurcation diagram Control variable Proxy for state Branches of states Imperfection	HW 33-36

	Snap through Free energy landscape	
Nov 9	Boltzmann distribution When a thermal system is in contact with a thermal environment for a long time, what is the probability for the thermal system to be in a particular quantum state?	
Nov 11	Freely-jointed chain Rigid link, free joint. Random walk Force competes with temperature, bf/kT. Force-displacement relation.	HW 37-40
Nov 16	Free energy of each chain. Elasticity of rubber-like materials 8-chain model (Arruda-Boyce model) Dielectric elastomer Soft active materials Thermodynamics of dielectric elastomer.	
Nov 18	Dielectric elastomer Ideal dielectric elastomer. Electromechanical instability. Effect of prestretch. Inhomogeneous field in dielectric elastomers	HW 41-44
Nov 23	Polymer gels Condition of thermodynamic equilibrium of a polymer network subject to forces, in a humid environment	
Nov 30	Molecular incompressibility Equivalence between hydrostatic stress and chemical potential A review of thermodynamics Chemical potential: definition on the basis of entropy Chemical potential of a molecular	

	in a pure substance Chemical potential of water molecules in steam Chemical potential of water molecules in liquid water under pressure	
Dec 2	Polymer gels Sec. 5 of this paper contains a derivation of the entropy of mixing of molecules of different sizes. Polymer gels Polymer gels: additional readings: temperature-sensitive gels, pH-sensitive gels, polyelectrolyte gels.	HW 45-48

Background readings

Linear algebra in 3000 bullets

- Number, scalar, vector, linear map
- Symmetric matrix
- Singular value decomposition

An introduction to thermodynamics

- The play of thermodynamics
- Isolated system
- Entropy
- The basic algorithm of thermodynamics
- Thermal system
- Thermal environment
- Chemical potential