# Probability of Probabilities

## An Alternative to p - values: Probability of Probability, a Simple Trick

**Case (Real World)**: A thoracic surgeon who does mostly cardiac and lung transplants (in addition to emergency bypass and aortic ruptures) operates in a business with ≈5% perioperative mortality. So far in his new position in the U.S. he has done 60 surgeries with 0 mortality.

**What is the probability that is error probability is higher than q?**

Note that there may be selection bias in his unit, which is no problem for our analysis: the probability we get is conditional on being operated on by **that** doctor in **that** unit.

Let $F(x)$ be the Survival function (exceedance probability) of the Binomial Distribution $\mathcal{B}(n, p)$, that is with $n$ trials at $p$ probability of success.

We are looking for, given realization $x$ over $n$ trials, $\{p : F^{-1}(p) = q\}$

For $x = 0$, the result is simple, in closed form : $q = 1 - (1-p)^{\frac{1}{n}}$. And $F^{-1}(.014) = \frac{1}{2}$.

```
TableForm[Table[{q, (1 - q)^60}, {q, .005, .1, .005}],
 TableHeadings → {None, {q, "Probability higher"}}]
```

| q | Probability higher |
|---|---|
| 0.005 | 0.740261 |
| 0.01 | 0.547157 |
| 0.015 | 0.403807 |
| 0.02 | 0.297553 |
| 0.025 | 0.218916 |
| 0.03 | 0.160807 |
| 0.035 | 0.117934 |
| 0.04 | 0.0863523 |
| 0.045 | 0.0631251 |
| 0.05 | 0.0460698 |
| 0.055 | 0.0335667 |
| 0.06 | 0.0244158 |
| 0.065 | 0.0177295 |
| 0.07 | 0.0128522 |
| 0.075 | 0.00930045 |
| 0.08 | 0.00671846 |
| 0.085 | 0.00484469 |
| 0.09 | 0.00348725 |
| 0.095 | 0.00250562 |

A surgeon does transplants operates in a business with 5% perioperative mortality. He has done 60 surgeries with 0 mortality.
What is the probability that the error probability is higher than q?

Let F(x) be the survival function (exceedance probability) of the binomial distribution B(n,p), with n trials and p probability of success.

This is not the probability of his error rate given 0 failures. For that you need a prior. This is the reverse: conditional probability of him achieving 0 failures at each error rate.

In[•]:=
```
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Out[•]//TableForm=

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| 0.09 | 0.00348725 |
| 0.095 | 0.00250562 |
| 0.1 | 0.00179701 |