Wittgenstein’s Ruler

In the presence of any other distribution, you have to reject to Gaussian.

10 sigma event means it is not Gaussian.
Unless you have a 100% probability that it is Gaussian.
You can use Bayes rule to see the distribution.
Use the ruler to measure the table?
Or do we use the table to measure the ruler?
If someone calls Einstein’s a fraud, then it tells you more about the person than about Einstein.

https://www.youtube.com/watch?v=k_lYeNuBTE8

In[ ]:=
```
? SurvivalFunction
```
Out[ ]=

Symbol

SurvivalFunction[*dist*, *x*] gives the survival function for the distribution *dist* evaluated at x.

SurvivalFunction[*dist*, $\{x_1, x_2, \ldots\}$] gives the

multivariate survival function for the distribution *dist* evaluated at $\{x_1, x_2, \ldots\}$.

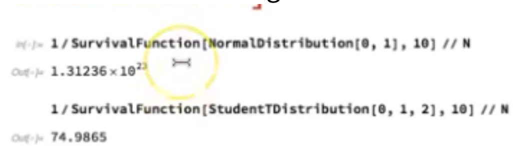
SurvivalFunction[*dist*] gives the survival function as a pure function.

The probability of exceeding 10 sigma is very low. It is a 1 in 10^{23} event.

In[ ]:=
```
1 / SurvivalFunction[NormalDistribution[0, 1], 10] // N
```
Out[ ]=

1.31236×10^{23}

This is extremely strange, the same command typed in into Mathematica by Nassim Taleb gives 75?
Was the definition changed?



```
In[ ]:= 1 / SurvivalFunction[NormalDistribution[0, 1], 10] // N
Out[ ]= 1.31236 × 10^23

        1 / SurvivalFunction[StudentTDistribution[0, 1, 2], 10] // N
Out[ ]= 74.9865
```

In[ ]:=
```
? StudentTDistribution
```
Out[ ]=

Symbol

StudentTDistribution[ν] represents a standard Student t distribution with ν degrees of freedom.

StudentTDistribution[μ, σ, ν] represents a Student t

distribution with location parameter μ, scale parameter σ, and ν degrees of freedom.

If it is StudentTDistribution, it could be 1 in 202.995.

In[ ]:=
```
1 / SurvivalFunction[StudentTDistribution[0, 1, 2], 10] // N
```
Out[ ]=
```
202.995
```

Applying Bayes’ Rule

The advantage of Mathematica is that everything is precise with no ambiguity.

In[ ]:=
```
eq = P[A ⫽ B] == (P[B ⫽ A] * P[A]) / P[B] /.
   P[B] → (P[B ⫽ A] * P[A] + P[B ⫽ nonA] * P[nonA]) // TraditionalForm
```
Out[ ]//TraditionalForm=

$$P(A \mid B) = \frac{P(A)\,P(B \mid A)}{P(A)\,P(B \mid A) + P(\text{nonA})\,P(B \mid \text{nonA})}$$

This is the standard Bayes rule, we can use /. substitution to replace it with the appropriate labels.

In[ ]:=
```
eq1 = eq /. {B → Event, A → Gaussian, nonA → NonGaussian, nonB → NonEvent} /.
  P[NonGaussian] → 1 - P[Gaussian]
```
Out[ ]//TraditionalForm=

$$P(\text{Gaussian} \mid \text{Event}) = \frac{P(\text{Gaussian})\,P(\text{Event} \mid \text{Gaussian})}{(1 - P(\text{Gaussian}))\,P(\text{Event} \mid \text{NonGaussian}) + P(\text{Gaussian})\,P(\text{Event} \mid \text{Gaussian})}$$

Suppose we have a prior that the 10 sigma event is 10^{-23} comparing it to the Student-T distribution.

In[ ]:=
```
eq2 = eq1 /. {P[Event ⫽ Gaussian] → 10^-23} /. {P[Event ⫽ NonGaussian] → 1/203}
```
Out[ ]//TraditionalForm=

$$P(\text{Gaussian} \mid \text{Event}) = \frac{P(\text{Gaussian})}{100\,000\,000\,000\,000\,000\,000\,000 \left(\frac{1}{203}\,(1 - P(\text{Gaussian})) + \frac{P(\text{Gaussian})}{100\,000\,000\,000\,000\,000\,000\,000}\right)}$$

Look, anything short of certain means you must reject the Gaussian compared to the Student-T distribution equivalent.

```
In[•]:= Table[{"P[Gaussian] =", P[Gaussian], eq2},
          {P[Gaussian], {.5, .999, .9999, .99999, .999999, 1}}] // TableForm
```

Out[•]//TableForm=

P[Gaussian] =	0.5	$P(\text{Gaussian} \mid \text{Event}) = 2.03 \times 10^{-21}$
P[Gaussian] =	0.999	$P(\text{Gaussian} \mid \text{Event}) = 2.02797 \times 10^{-18}$
P[Gaussian] =	0.9999	$P(\text{Gaussian} \mid \text{Event}) = 2.0298 \times 10^{-17}$
P[Gaussian] =	0.99999	$P(\text{Gaussian} \mid \text{Event}) = 2.02998 \times 10^{-16}$
P[Gaussian] =	0.999999	$P(\text{Gaussian} \mid \text{Event}) = 2.03 \times 10^{-15}$
P[Gaussian] =	1	$P(\text{Gaussian} \mid \text{Event}) = 1$

You can find this in the Statistical Consequences of Fat Tails