Brownian Bridge

https://www.youtube.com/watch?v=zLs6ggd3BPo&feature=youtu.be

https://en.m.wikipedia.org/wiki/Brownian_bridge

A Markov process that is pinned at both ends.
Suppose we know what it was at period t_0 in the past.
We know what it is at period t_1,
We can see the various sample paths

In[∘]:=
```
? BrownianBridgeProcess
```
Out[∘]=

Symbol

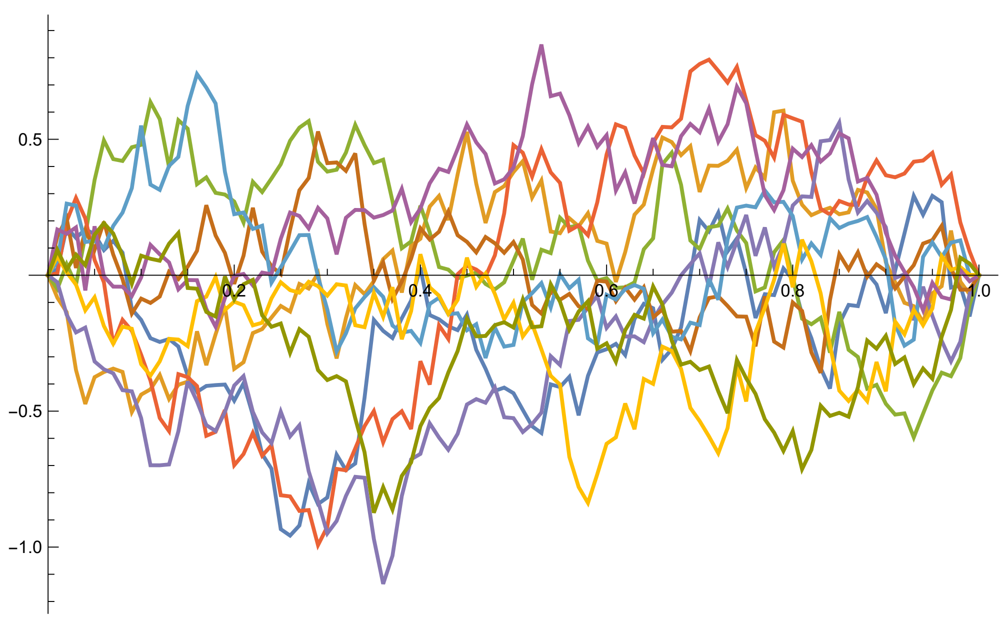
BrownianBridgeProcess[σ, {t_1, a}, {t_2, b}] represents the
Brownian bridge process from value a at time t_1 to value b at time t_2 with volatility σ.

BrownianBridgeProcess[{t_1, a}, {t_2, b}] represents the
standard Brownian bridge process from value a at time t_1 to value b at time t_2.

BrownianBridgeProcess[t_1, t_2] represents the standard Brownian bridge process pinned at 0 at times t_1 and t_2.

BrownianBridgeProcess[] represents the standard Brownian bridge process pinned at 0 at time 0 and at time 1.

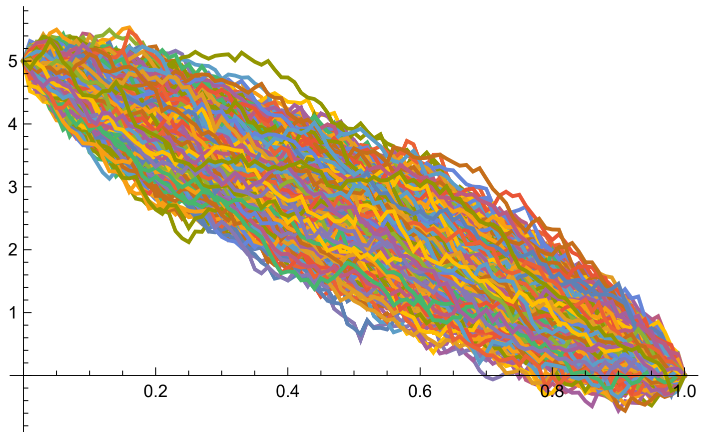
In[∘]:=
```
ListLinePlot[RandomFunction[BrownianBridgeProcess[], {0, 1, 0.01}, 10]]
```
Out[∘]=



Need to figure out a way to take the average path?

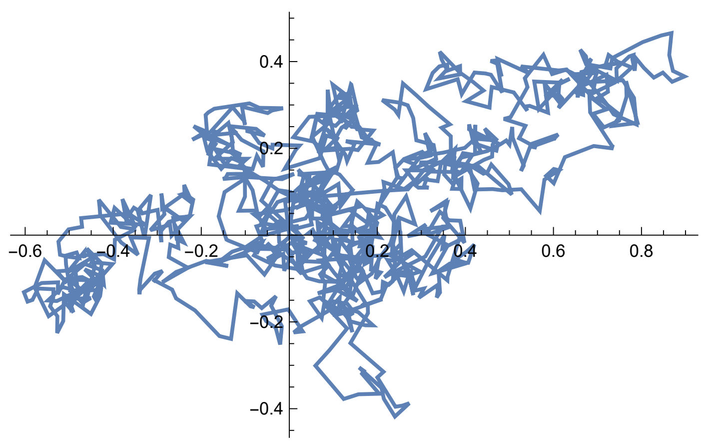
In[ ]:=
```
ListLinePlot[RandomFunction[BrownianBridgeProcess[{0, 5}, {1, 0}], {0, 1, 0.01}, 1000]]
```

Out[ ]=



In[ ]:=
```
proc = BrownianBridgeProcess[];
sample = RandomFunction[proc, {0, 1, 0.001}, 2]["ValueList"];
ListLinePlot[Transpose@sample]
```

Out[ ]=



Path Histogram

Simulate 500 paths from Brownian bridge.

In[ ]:=
```
SeedRandom[3]; data = RandomFunction[BrownianBridgeProcess[], {0, 1 / 2, .01}, 500];
```

Take slice at time 0.5 and visualise the distribution

In[ ]:=
```
sd = data["SliceData", 1 / 2];
```

In[ ]:=
```
cf = ColorData["Rainbow"];
sliced = BarChart[Last[#], Axes → False, BarOrigin → Left, AspectRatio → 4,
    ChartStyle → (cf /@ Rescale[MovingAverage[First[#], 2], {Min[sd], Max[sd]}, {0, 1}]),
    ImageSize → 75] &[HistogramList[sd, {Range[-3, 3, .3]}]];
```

Now plot the distribution.

BrownianBridgeProcess

People are rarely static.
When a market goes from 1 to 100, then falls to 50, one has the illusion that people who were in at 1 would be doing well. In fact people tend to get in trouble by being lured to buy close to the top.

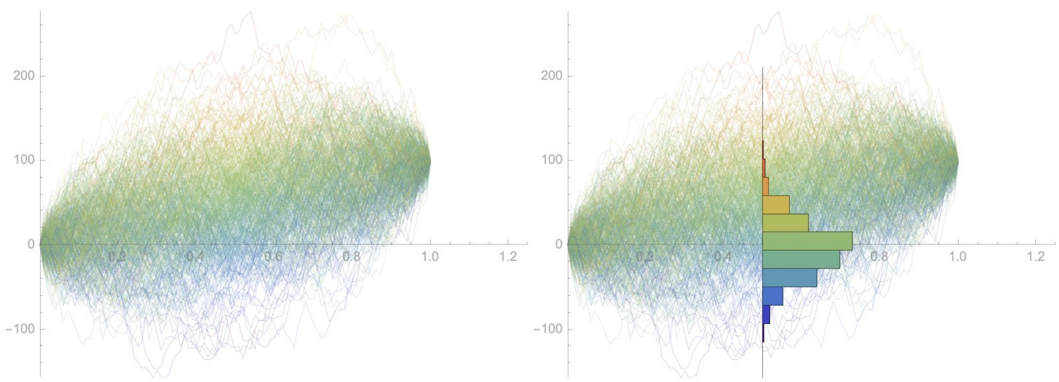
NONDYNAMIC

A- **Early profit takers.** Those who bought at 1 and sold early, between say 2 and 10. No dynamic hedging/trading

DYNAMIC "HEDGERS" (actually traders)

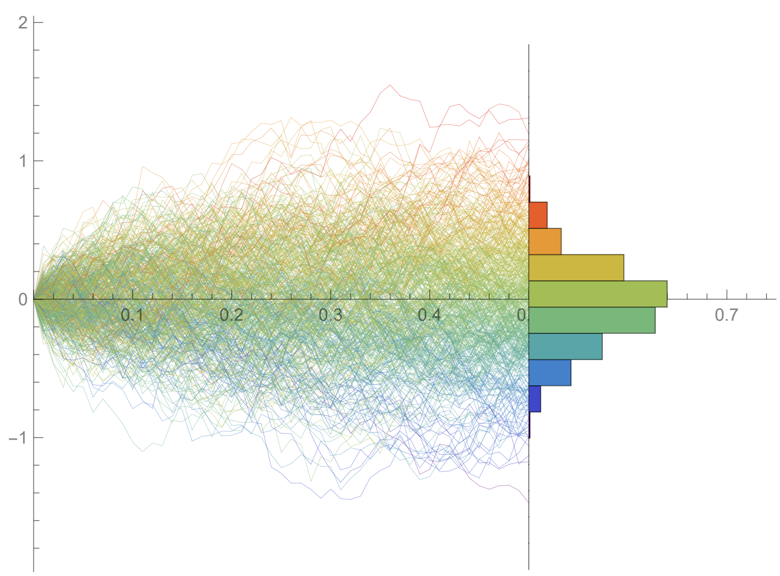
B- **Averaging-pyramiders**. Those who bought at 1 and kept buying all the way up, with an average of about 50-80.

C- **Averaging-scalpers**. Those who bough and sold all the way, with an average (net) of about -50. (If you buy at 1, sell at 11, buy back at 5, your average, depending on qty, can be -5)



In[∘]:=
```
ListLinePlot[data, ImageSize → 400, PlotRange → All,
 AspectRatio → 3 / 4, Epilog → Inset[sliced, {.5, 0}, {0, 10.8}],
 PlotStyle → (cf /@ Rescale[sd]), BaseStyle → Directive[Thin, Opacity[0.5]],
 PlotRangePadding → {{0, .25}, {.5, .5}}]
```

Out[∘]=



My idea is to use the histogram of alternative histories to back model the Brownian bridge. If we ever have a situation where we know the start time and end time, as well as the start state and end state, as well as the histogram distribution of states.