Correlation as Information

https://twitter.com/nntaleb/status/1135116646442590208/photo/1

Correlation Shmorrelation

Let us look at the notion of correlation and how much information it conveys **under the Gaussian distribution**. What does correlation physically mean?
Not much. (Unlike, as we saw, **entropy methods** such as mutual information)

Let X and Y be normalized variables (0 mean, std 1). Consider the ratio of
the probability of both X and Y **exceeding a hurdle** K under a correlation structure ρ,
over the probability of both X and Y exceeding it assuming correlation = 1.
ϕ is the **"proportion of certainty"**

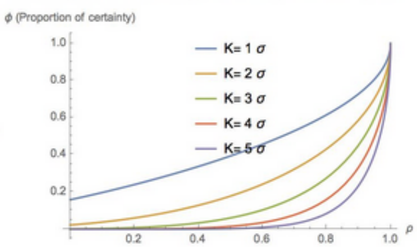
$$\phi(\rho, K) = \frac{P(X > K, Y > K)\mid_{\rho}}{P(X > K, Y > K)\mid_{\rho=1}} = \frac{P(X > K, Y > K)}{P(X > K)}$$

Unfortunately $P(X > K, Y > K) =$

$$\int_K^\infty \left(\int_K^\infty \frac{e^{-\frac{x^2+y^2-2xy\rho}{2(1-\rho^2)}}}{2\pi\sqrt{1-\rho^2}} \, dx \right) dy$$ does not integrate so we need to work with inequalities and numerical tricks.

```
ϕ[ρ_, K_] := SurvivalFunction[BinormalDistribution[ρ], {K, K}] / SurvivalFunction[NormalDistribution[], K] // N
```

```
Plot[Evaluate[Table[Fi[ρ, i], {i, 1, 5}]], {ρ, 0, 1},
 AxesLabel → {ρ, "ϕ (Proportion of certainty)"},
 PlotLegends → Placed[{"K= 1 σ", "K= 2 σ", "K= 3 σ", "K= 4 σ", "K= 5 σ"}, {Center, Top}]]
```



Conclusion
Correlation between X and Y carries disproportionate information for the ordinary, and practically no information for the tails.
Application: assume correlation between "IQ" and outcome (assuming IQ works the way the imbecile psychologists claim). You need something > .98 to "explain" genius.
Gabish?

```
In[ ]:= ϕ[ρ_, K_] := SurvivalFunction[BinormalDistribution[ρ], {K, K}] / SurvivalFunction[NormalDistribution[], K] // N
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Plot[Evaluate[Table[ϕ[ρ, i], {i, 1, 5}]], {ρ, 0, 1},
 AxesLabel → {ρ, "ϕ (Proportion of Certainty)"},
 PlotLegends →
  Placed[{"K = 1σ", "K = 2σ", "K = 3σ", "K = 4σ", "K = 5σ"}, {Center, Top}],
 ImageSize → Large]
```
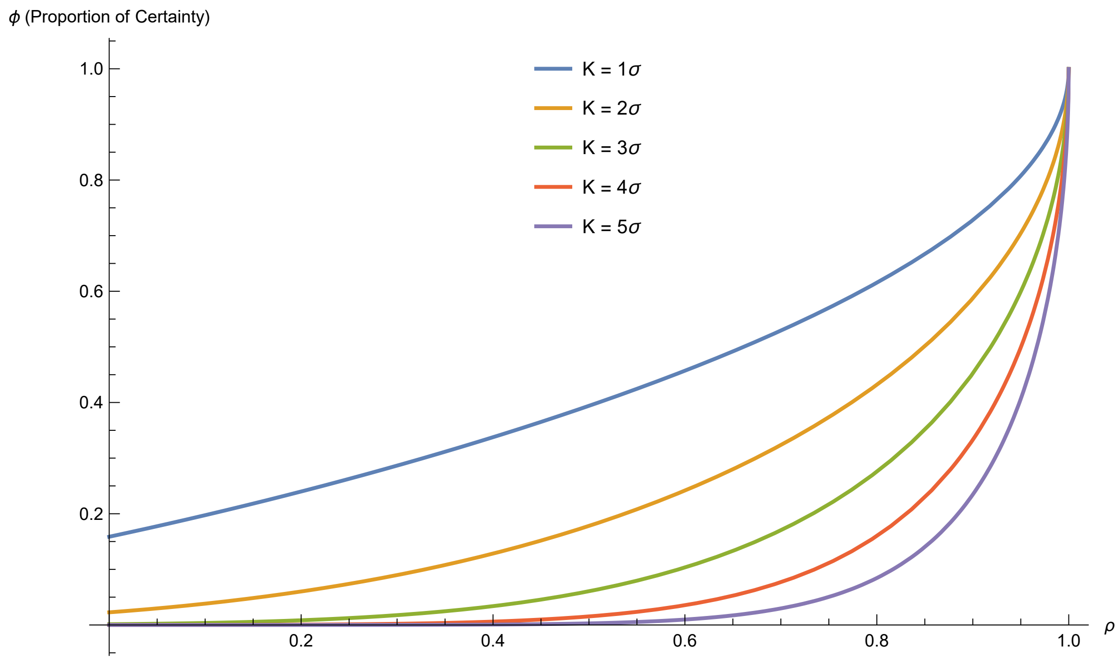

Out[ ]=



Observe only after correlation more than 0.950 you only have 0.408 proportion of certainty for 5-sigma event.

Look at how IQ is normally distributed. N[Mean = 100, SD = 15]. Even at 0.98, proportion of certainty is 0.70. Now the problem, if IQ is not normally distributed, what happens?