Free Convection

Notes based on practice, tutorial and past year examination papers

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Typical Assumptions

For examination style problems, we typically assume the following:

1. Steady heat transfer
2. 1D heat transfer
3. No forced convection or radiation exchange (please check!)
4. Constant properties
5. Flow is incompressible.
6. Flow is laminar or turbulent or mixed (please check!)
7. No heat generation
8. No work done by viscous forces. (please check!)
9. No body forces on fluid (please check!)
10. Negligible heat conduction in direction perpendicular to the flow (please check!)
11. Fluid is standard and calorically perfect .
12. Fluid is stagnant or quiescent (please check!)
13. Idealised geometry (example: vertical part idealised as thin vertical plate, please check!)
14. Properties are obtained by linear interpolation.
15. Fluid in question has similar properties to air or water (example: blood)
16. No heat loss to surroundings.

NOTE: radiation is usually coupled with free convection, so also check:

1. Radiation exchange with large surroundings (please check!)
2. View factor = 1 (or some value, please check!)
3. Gray surfaces (emissivity < 1) and Kirchhoff's law applies.
4. Diffuse surfaces where radiation is uniform in all directions
5. Infinite surface (please check!)
6. Black (emissivity = 1, please check!)
7. Negligible conduction
8. Thermal circuits is valid for problem analysis

For velocity profiles:

1. No slip at surface.

2. Fluid at zero velocity far away from wall.
3. Boundary layer is laminar from the leading edge of the plate until some distance away where it becomes turbulent.

Check Free Convection

If you have time, please check!:::

TABLE 9.1 Free, forced, and mixed convection processes, and the corresponding correlation forms

Process	Measure of buoyancy relative to inertial forces	Form of correlation	
Forced convection	$Gr_L/Re_L^2 \ll 1$	$\overline{Nu}_L = f(Re_L, Pr)$	(6.50)
Free convection	$Gr_L/Re_L^2 \gg 1$	$\overline{Nu}_L = f(Gr_L, Pr)$	(9.11)
Mixed convection	$Gr_L/Re_L^2 \approx 1$	$\overline{Nu}_L = f(Re_L, Gr_L, Pr)$	(9.12)

Inverse Temperature

$$\beta_{\text{ideal gas}} = \frac{1}{T} \qquad (1/\text{K})$$

Grashof and Rayleigh Numbers

In this chapter, we have considered natural convection heat transfer where any fluid motion occurs by natural means such as buoyancy. The volume expansion coefficient of a substance represents the variation of the density of that substance with temperature at constant pressure, and for an ideal gas, it is expressed as $\beta = 1/T$, where T is the absolute temperature in K or R.

The flow regime in natural convection is governed by a dimensionless number called the *Grashof number,* which represents the ratio of the buoyancy force to the viscous force acting on the fluid and is expressed as

$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

where L_c is the *characteristic length,* which is the height L for a vertical plate and the diameter D for a horizontal cylinder. The correlations for the Nusselt number $\mathrm{Nu} = hL_c/k$ in natural convection are expressed in terms of the *Rayleigh number* defined as

$$\mathrm{Ra}_L = \mathrm{Gr}_L \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \mathrm{Pr}$$

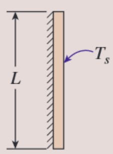
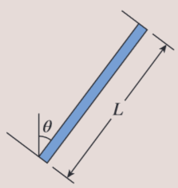
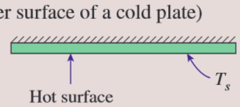
g = gravitational acceleration, m/s^2
β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C
L_c = characteristic length of the geometry, m
ν = kinematic viscosity of the fluid, m^2/s

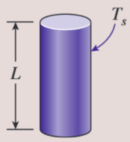
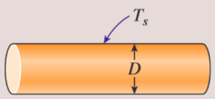
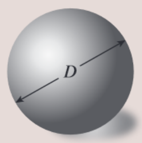
Nusselt Relations

Simple relations for the average Nusselt number for various geometries are given in Table 9–1, together with sketches of the geometries. Also given in this table are the characteristic lengths of the geometries and the ranges of Rayleigh number in which the relation is applicable. All fluid properties are to be evaluated at the film temperature $T_f = \frac{1}{2}(T_s + T_\infty)$.

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic Length L_c	Range of Ra	Nu	
Vertical plate 		10^4–10^9	$\text{Nu} = 0.59\,\text{Ra}_L^{1/4}$	(9–19)
		10^9–10^{13}	$\text{Nu} = 0.1\,\text{Ra}_L^{1/3}$	(9–20)
	L	Entire range	$\text{Nu} = \left\{0.825 + \dfrac{0.387\,\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)	(9–21)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g with $g\cos\theta$ for $0 < \theta < 60°$	
Horizontal plate (surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate) 	A_s/p	10^4–10^7	(a) $\text{Nu} = 0.54\,\text{Ra}_L^{1/4}$	(9–22)
		10^7–10^{11}	(a) $\text{Nu} = 0.15\,\text{Ra}_L^{1/3}$	(9–23)
(*b*) Lower surface of a hot plate (or upper surface of a cold plate) 		10^5–10^{11}	(b) $\text{Nu} = 0.27\,\text{Ra}_L^{1/4}$	(9–24)

Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$	
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{0.6 + \frac{0.387\,Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$	**(9–25)**
Sphere 	D	$Ra_D \leq 10^{11}$ ($Pr \geq 0.7$)	$Nu = 2 + \frac{0.589\,Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$	**(9–26)**

Vertical Flat Plate, Constant Surface Temperature

$$\alpha = \frac{k}{\rho c_p} \qquad \overline{Nu}_L = \frac{\overline{h}_L L}{k} = C Ra_L^n \qquad (6)$$

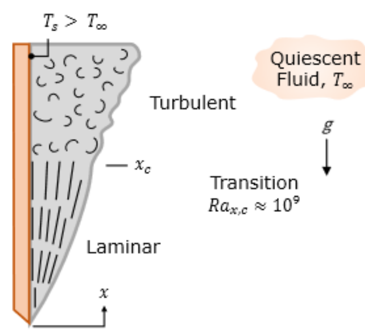
$$Pr = \frac{\mu c_p}{k} = \frac{\nu}{\alpha} \qquad Ra_L = Gr_L\,Pr = \frac{g\beta|T_w - T_\infty|L^3}{\nu\alpha}$$

Flow	Ra_L	C	n
Laminar	10^4-10^9	0.59	1/4
Turbulent	10^9-10^{13}	0.10	1/3

L: Height of vertical flat plate

a) Laminar flow b) Turbulent flow

Isotherms in natural convection over a hot plate in air.



(Continuation)

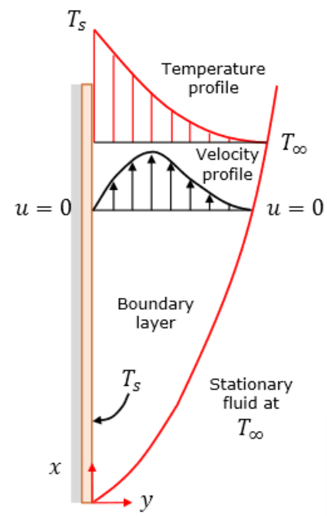
$$\overline{\mathrm{Nu}}_L = \frac{\bar{h}_L L}{k} = 0.68 + \frac{0.67\mathrm{Ra}_L^{1/4}}{\left[1+\left(\frac{0.492}{\mathrm{Pr}}\right)^{9/16}\right]^{4/9}} \qquad (6)$$

$Ra_L \leq 10^9$

Laminar
by Churchill and Chu
(1975)

$$\overline{\mathrm{Nu}}_L = \left\{0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{\left[1+\left(\frac{0.492}{\mathrm{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 \qquad (7)$$

Entire Ra range
by Churchill and Chu
(1975)



Typical velocity and temperature profiles for natural convection flow over a hot vertical plate at temperature T_s inserted in a fluid at temperature T_∞.

Properties evaluated at **film temperature** $T_f = \frac{1}{2}(T_w + T_\infty)$

For gases, $\beta = \frac{1}{T_f}$ where T_f is in Kelvins