Carnegie Mellon University

Master Thesis

Dynamic Energy Mapping

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in the

Building Performance and Diagnostics
School of Architecture

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Declaration of Authorship

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Abstract

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Master of Science

Dynamic Energy Mapping

by Yujie Xu

The project aims at implementing a Dynamic Energy Demand Map that holds, visualizes and analyzes high spatial (each building, building group and the community) temporal (hourly) resolution energy demand data of a community with the focus on creating a highly integrated visualization and an interface is designed to serve this purpose. The target user of the interface is researchers in energy related fields with the basic abilities of map reading, understanding moderately complicated map legend and data plot and understanding building energy performance attributes and their implications. A general purpose design is created to provide both qualitative and quantitative information and to suit different research interest of this user group.

The approach is demonstrated with a conceptual community model created in CityEngine based on the land use pattern of a mixed-use redevelopment project at Lower Hill District, Pittsburgh, PA. The hourly energy demand profile is retrieved from the simulation of DOE Commercial Prototype Building ASHRAE90.1-2013.

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Abbreviations

CMU	**C**arnegie **M**ellon **U**niversity
GHG	**G**reen**H**ouse**G**as
DOE	the U.S. **D**epartment **O**f**E**nergy
HTPR	**H**eat**T**o**P**ower **R**atio
EPM	**E**nergy**P**otential**M**apping
EMIOFN	**E**nergy **M**apping to **I**dentify **O**pportunities for **F**uture

Symbols

THR	condenser total heat of rejection	Btuh
RE	net refrigeration effect	Btuh
f	Heat Rejection Factor	1

Chapter 1

Interface Design

This section provides detailed illustration of the non-interactive (map animation) and interactive dynamic energy map implementation and design choices regarding the interactive dynamic energy map. The section starts with a general overview that explains possible approaches to add the time dimension in an energy map. Then the non-interactive energy map (map animation) approach is presented. For the non-interactive animation, the advantages and disadvantages between different symbol or color representation on the effectiveness of conveying information was briefly discussed.

Next a detailed documentation of the dynamic energy map interface is presented. The layout and functions of each components of the interface is explained and the design of each component based on literature studies of dynamic map design is discussed. The use of dynamic energy map to identify energy recovery opportunities and to help design and size a district energy system is demonstrated.

1.1 Overview

Dorling and Openshaw pointed out that a dynamic map provides new potential and possibilities for data analysis but also poses a great challenge as a result of the less developed theory in space-time pattern detection and measurement [1]. In order to better conduct a space-time visualization of the space-time energy demand information in the dynamic energy map,literature studies on space-time map visualization were used to design the dynamic energy map.

Brownrigg mentions several methods of representing time on a map: 1) a graph or chart that represents a function over time or a time line for displaying chronological events 2) a sequence of snapshots displayed over time (animated map) 3) small-multiples of snapshots of changing states [2].

Based on the classification above, method 1) and 2) were applied to represent time for the dynamic energy map. The dynamic plot of temporal time series uses method 1) to anchor the quantitative information. The sequential map image display is using method 2). The researcher did not use the small multiple method (method 3)). The choice is based on the following points mentioned by Brownrigg: 1) the number of snapshots in one display is limited and the finer the detail per snapshot, the less snapshots one can contain in one display. Since the 3D representation is chosen as one of the major map display methods (2D map is also available), the level of details per image is relatively high. This will result in a very small number of multiples per display [2] 2) the subtle changes are easier to be noticed in the form of animation than with small-multiples [2]. Both drawbacks of small-multiple method will impair the ability to convey the rapid temporal changes of community energy behavior, hence is not suitable for the current project.

1.2 (Non-interactive) Map Animation

Map animation was introduced to cartography in 1930s [3]. Its major application include: 1) demonstrating the dynamic process of geographic events (weather maps in weather forecasting is such an example) 2) assisting pattern recognition and knowledge development for scientific researches. The study by Dorling and Openshaw is an example of application 2), where they discovered new leukaemia hotspots through animated maps [1]. Animated maps are proven to be more powerful in conveying the spatial-temporal pattern than static maps [4].

The level of user control of playback behavior of animated maps is debatable. Providing the full freedom of adjusting the playback can enhance pattern understanding [5], but it might also reduce time animation to still images and impair its ability in conveying temporal changes [6]. In the current dynamic map project. The researcher observed that the non-interactive map animation is especially helpful in conveying the dynamic energy changing and the non-coincident peak arriving time of different buildings in the community. Resch et al. suggest that the interface for general public should "ensure that the amount of information shown to the users at any given time, and its complexity, are reduced" [7]. The non-interactive animated map could be one of the potential choices for an interface of a dynamic energy map with general public as the target user group.

For this project a continuous color encoding method was used in the creation of non-interactive animation of a univariate gas heating energy demand map. Similar to the National Heat Map, the Calgary Map and the EMIOFN project (Table **??**) discussed in Section **??**, a red to blue color ramp is used in the map display. Different from the cases above, the current project chose to represent high heat demand with blue and low heat demand with red as a result of a limited survey of potential users. Each color within this red-to-blue color scheme is represented as a real number between 0 and 1 with 0 representing pure red and 1 representing pure blue.

The first approach to calculate the corresponding color for each heating energy value is to calculate the normalized distance between the current value and the maximum value (Equation 1.1).

$$\frac{E(t)-E_{max}}{E_{max}} \tag{1.1}$$

$E(t)$ is the energy consumption for the current time spot t, E_{max} is the maximum energy consumption over the year. The problem for this approach is that the color changing is not visible enough as a result of a extremely right skewed energy data (with each data point representing the hourly energy consumption of a certain building in the community at a certain hour of a year) distribution (Figure 1.1).

By directly applying this normalized color scheme, the color distribution on a map will be very un-even, with most of the buildings colored with the red color for most of the time.

Kolter and Ferreira discovered that the annual total energy consumption of the 6500 buildings in Cambridge MA area follows a "log-normal" distribution [**?**]. By applying similar log scaling for the hourly heating energy data of the community, the researcher

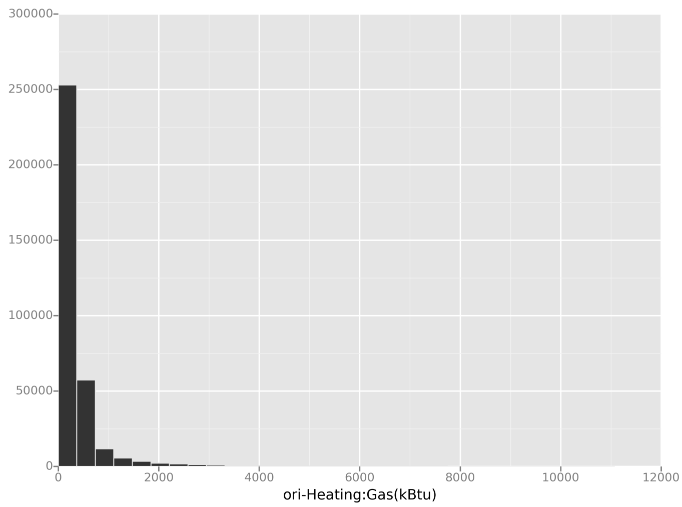

FIGURE 1.1: A histogram of hourly energy consumption per building, for the 68 buildings in the community

found that the hourly heating energy distribution also roughly follows a normal distribution (Figure 1.2). the researcher apply log scaling to flatten the distribution and calculate the color from energy ($E(t)$) as follows:

$$\frac{\ln(E(t)) - \ln(E_{max})}{\ln(E_{max})} \tag{1.2}$$

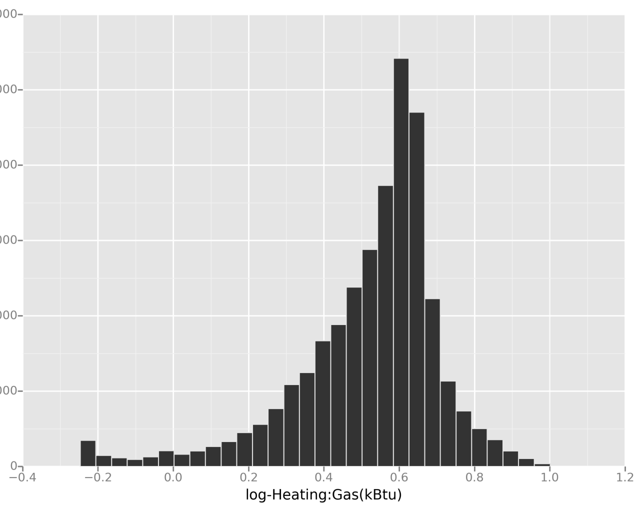

FIGURE 1.2: Heating Demand of Conceptual City

Figure 1.3 is one snapshot of the conceptual urban environment model under the log scaled calculation method in Equation 1.2.

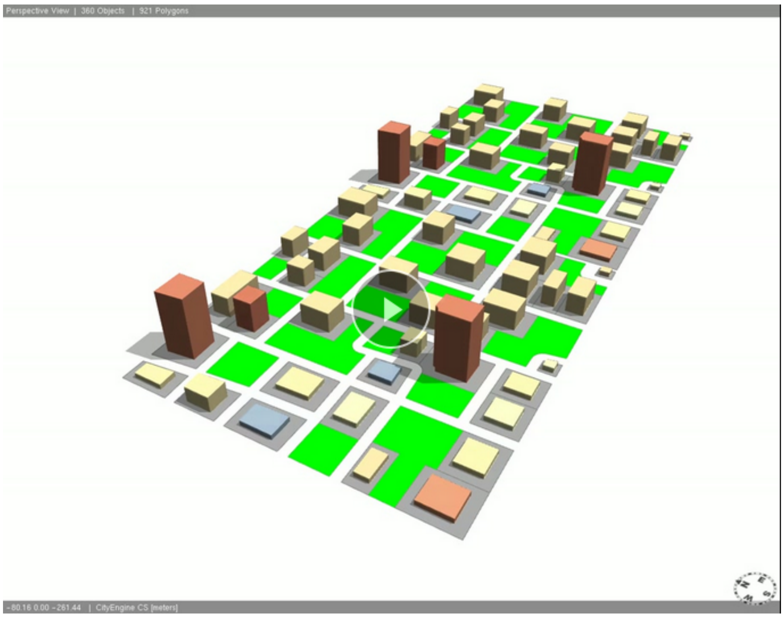

FIGURE 1.3: Animated demonstration of the log-scaled dynamic energy heating demand map
Click here to go to the animation link.

TABLE 1.1: Heating-Cooling Breakpoints in the Interface

heating		cooling	
kBtu	Ton	kBtu	Ton
5	60	2	24
22	264	7	84
50	600	15	180
91	1092	26	312
136	1632	56	672
213	2556	72	864

In the dynamic energy map interface design, the researcher applied a discrete color encoding with a seven-class bivariate choropleth representation (Table 1.1). The break points are calculated purely with the Quantile method in Section **??**. This allows for a quantitative legend that can depict more specific energy demand information. An animation with this discrete color scheme is also created and can be viewed and downloaded through this link.

Although the initial conditions of the map instances using the continuous and discrete encoding method are different: the animation with continuous color encoding depicts only one variable (gas heating) while the animation with discrete color encoding depicts two variables (space heating and cooling), the researcher observed that the continuous color encoding method seems to be better in demonstrating the general pattern of energy changing behavior. Further evaluations are needed to compare these two approaches and justify the design choice of a discrete or continuous color scheme.

1.3 Interactive Dynamic Map Interface

1.3.1 General Layout

The general layout of the dynamic map interface is displayed in Figure 1.4. It contains the following major sections :

(A) A snapshot of the dynamic energy map interface

(B) Dynamic Map Interface Layout

FIGURE 1.4: Dynamic Map Interface Layout

- A main map display on the upper left that shows the 2D or 3D version of the dynamic energy map with energy data encoded as the color of buildings.

- Four sliders that controls the linear and periodical navigation of the map image display and data plot.
- A "Building Initial Look-up Table" in the center bottom that defines the two-letter building type initials on the main map display window.
- A series of energy demand plots for four major building sectors (top right) and the whole community (lower right).
- A series of buttons and option menus on the lower right.

  The top row of the buttons performs forward (+) or backward (-) time navigation with time step of 24h or 1h. The bottom row of the buttons contains a "plot" button that plots the energy profile graphs of the 16 benchmark buildings (if "single" is chosen for option menu") or the plot of the aggregated community (if "community" or "group" is chosen), a "legend" button that shows the current legend, and a "clear" button that clears the selection in the 2D mode.

The following sections will provide more detailed explanation of the interface.

1.3.2 Main Display Window

As is mentioned in [2], the choice of 2D representation vs. 3D representation is one of the debated decisions in the world of cartographic data visualization [2]. 2D maps are 1) easier to navigate and 2) the operation of selecting an element or a region is easier to perform in a 2D map. Another important advantage of 2D map is that it has better theory support [7]: Jacques Bertin defined the seven visual variables in the graphic sign-system and their construction rules to effectively convey geographic information [**?**]. However the principles and variables of 3D or 4D maps (space-time map) are not thoroughly investigated [7]. This situation makes the design of 3D maps more difficult. However 2D maps "drastically simplify reality and thus do not give credit to the highly complex capabilities of human spatial cognition" [7]. Regarding this, a 3D map is rich in geometry representation and can provide realistic scenes. The realistic scene is important in conveying information to not only researchers but also the general public. The difference in surrounding building height and their exterior reflection properties could influence the exterior shading of a building, so the height and density distribution of a community could influence its total energy demand. The correlation of 3D community configurations with difference in building height, density and exterior surface properties on the total community level energy performance could also be identified with a 3D display. The current project does not use the whole-community energy simulation and this capability of the 3D display is not demonstrated. Integration with whole-community simulation thus could be one of the later development of the current project. However, the 3D display can both be an advantage or disadvantage based on the actual map usage. According to Tufte's data-ink ratio theory, the extra non-crucial richness of information should be eliminated to make the most important information stand out [9]. For an energy map, variables such as geothermal energy potential, biomass potential could be represented as 2D layers while solar energy potential might be more suitable to be visualized in 3D layers since its performance is influenced by exterior shading in an urban environment.

For the current dynamic energy map interface, both 2D and 3D representations are provided for the users to select. The main map display window on the top left is used for displaying the 2D / 3D dynamic map of the conceptual model. The lower left of the main map display window displays the current time for the image and data plots. By selecting the 2D / 3D option in the option menu on the lower right, the user can choose between 2D and 3D display. The 3D display provides a more realistic view of the community model. The building geometry is simplified in the current model in order to emphasize the color changing between frames without introducing distraction from complicated building geometry. Additional building details or features could be added to make the display more realistic. In the 2D display, the user can click on a single building or select a group of buildings to display their energy profile plots or the aggregated energy profile plots (Figure 1.5).

1.3.3 Bivariate Map Legend

1.3.3.1 Symbol Chosen

The major reason for choosing the 3D energy dynamic map display is to use it to provide a more realistic urban environment context. In the Dutch Heat map by Dobbelsteen et al. , the quantity of energy demand of each building or region is represented by extruding

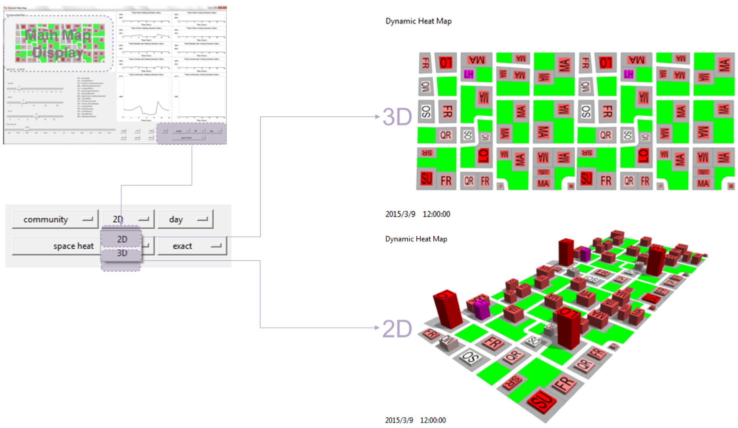

FIGURE 1.5: The current interface provides the choices of viewing 2D and 3D map display by toggling the 2D/3D option menu

the building or region by a corresponding height encoding its energy demand or supply [10]. This approach provides an easy way of aggregating energy demand and supply by adding up geometry height. For this project this approach was not chosen for the map design because it creates shape distortion and will impair the goal of providing a realistic urban environment vision.

In order to represent space heating demand and cooling demand on the same map, a common map design case is encountered in the current project: bivariate map design that visualizes the correlation of two variables on a same map. Elmer points out the challenge of the bivariate map design as a result of its increased information density. He presented eight possible types of representation for bivariate maps (Figure 1.6): "shaded cartographer, rectangle map, bar chart, value by alpha, choropleth with graduated symbol, bivariate choropleth, spoke glyph and shaded texture" [11]. In order to incorporate the bivariate map symbols to the current 3D model without introducing too much shape distortion, the researcher did not choose the representation with dimensional changes, i.e. the changing of building height, width, depth or the size changing of building centroid are not chosen in the map design of the current project. The choices are the ones that involves color or texture, i.e. "bivariate choropleth, value by alpha and shaded texture" (Figure 1.6). Among these three choices, bivariate choropleth representation has the highest accuracy rate [11], hence the researcher chose bivariate choropleth as the representation of the current map interface design.

In the current interface design, users can click on the "Legend" button and a legend used for encoding the 2D and 3D map will be displayed. To assist legend reading and

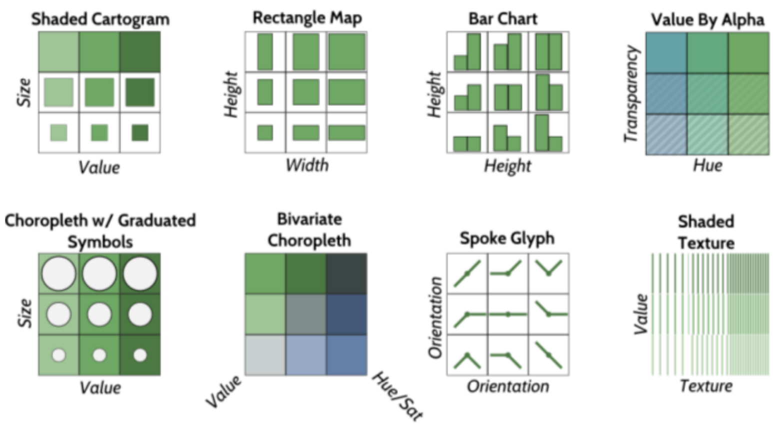

FIGURE 1.6: The eight bivariate map display approaches tested in Elmer's [11] study

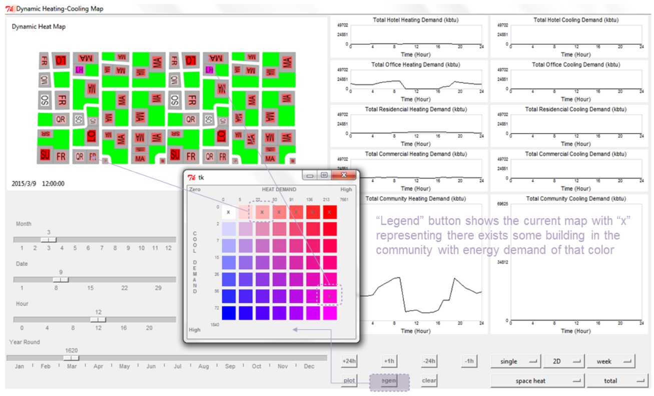

FIGURE 1.7: The seven-class-bivariate choropleth legend is used in the dynamic energy map interface design with red representing high space heating demand, blue representing high space cooling demand. "x" in the legend corresponds to colors appear in the current map display window

color comparison between the map and the legend, tick marks of "x" are added to the legend to indicate the color appeared in the map Figure 1.7. How to use the legend to identify the buildings or building groups that have large energy recovery potential is demonstrated in Section 1.4.1.

1.3.4 Time Sliders and Navigation Buttons

The lower left section contains a series of sliders for controlling interactive navigation of the image sequence and the corresponding data plot.

Harrower and Fabrikant classify time into two types: linear and cyclic [3]. The former represents the periodical changes and the latter represents the linear changes of spatial temporal variables. To address this, the design of the current interface includes both an overall time navigation utility and time navigation utilities that facilitate jumps with time steps corresponding to the natural period of energy data, such as month, day and hour. This design choice is anticipated to facilitate the representation of both linear changes and periodical changes of energy usage in the community.

There are three shorter "periodical" sliders on the lower left of the interface. One unit of position change in the "month" slider results in a forward or backward jump of one month in time. The total number of positions in the "month" slider equals the number of months in a year (which is the next level of time unit regarding month). The jump step for "date" slider is one day and the number of positions in the "date" slider is the number of days per month. Similarly for the "hour" slider, the jump step is one hour and the number of positions in the "hour" slider is the number of hours per day. Suppose the current time in display is 2015/1/1 12:00:00. By moving the month slider, viewers can see the energy demand in the form of map image and data plot for 2015/2/1 12:00:00, 2015/3/1 12:00:00, . . . , 2015/12/1 12:00:00. Similarly, if viewers pull the "date" slider, they can compare the different energy demand of this hour (12:00:00) throughout the whole month. With the hour slider, viewers can compare the energy demand between different hours of a day.

There is a longer "linear slider" on the bottom left of the interface. It has a time step of an hour and a navigation range of a year (8760 hours). It allows users to globally navigate through all 8760 hours of the year.

There are four buttons (+24h, +1h, -24h, -1h) on the bottom right of the interface. They provide a micro level adjustment of time.

1.3.5 Data Plot

1.3.5.1 Methods to Show Plot

There are three ways to view energy data plots in the dynamic energy map interface:

1) **By viewing the right hand side of the interface.**

   The dynamic data plots are directly shown on the right of the interface. They depict the energy demand of the four major building types (Hotel, Office, Residential and Commercial buildings) and the community.

Space heating and cooling energy demand is displayed in Figure 1.4. The interface can also display the electricity and heating demand for the CHP plant sizing application. These plots starts from the current time showing on the time sliders with a fixed plotting range of 24h.

2) **By clicking on the "plot" button on the lower right of the interface.**

If the "single" option in the option menu is chosen before one clicks the "plot" button, a data plot will be created for each building type (Figure 1.8). If the "community" option in the option menu is chosen before one clicks the "plot", a data plot for the community will be created (Figure 1.9).

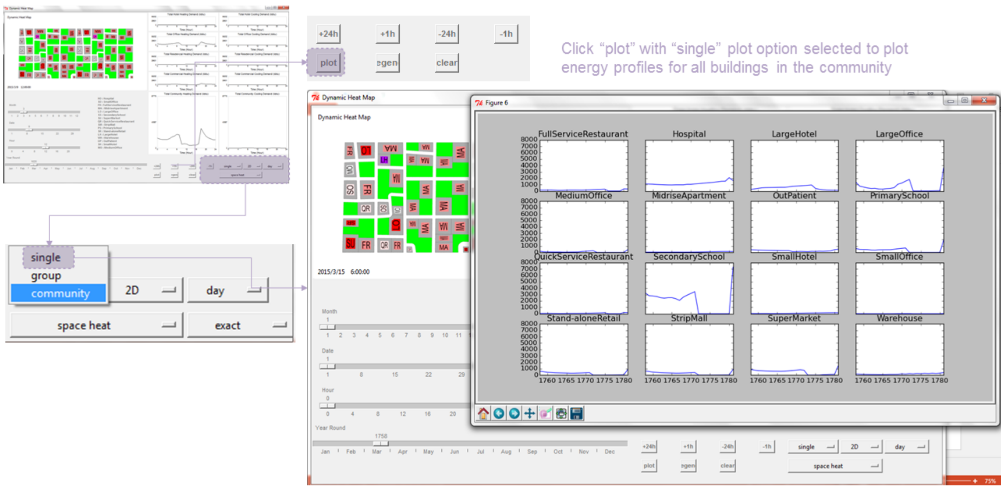

FIGURE 1.8: The plot shows the space heating energy demand plot of each of the 16 benchmark buildings

3) **By clicking on the building footprint in the 2D map display.**

A building is "selected" if the user clicks on its foot print. Each new click of a building footprint will add a new copy of that building to the selection set. The selection set can be cleared by pressing the "clear" button. If "single" option is chosen in the option menu before clicking on a building's foot print, a data plot will be created for the building the viewer just clicked on (Figure 1.10). If "group" is chosen in the option menu, each click of a building's footprint will create a data plot for the current selection set (Figure 1.11). This function is important in assessing the building group level energy recovery potential. Users can assess the energy recovery potential of the building or building group that rejects heat. They can also assess the space heating demand of the group of surrounding buildings of the reject heat

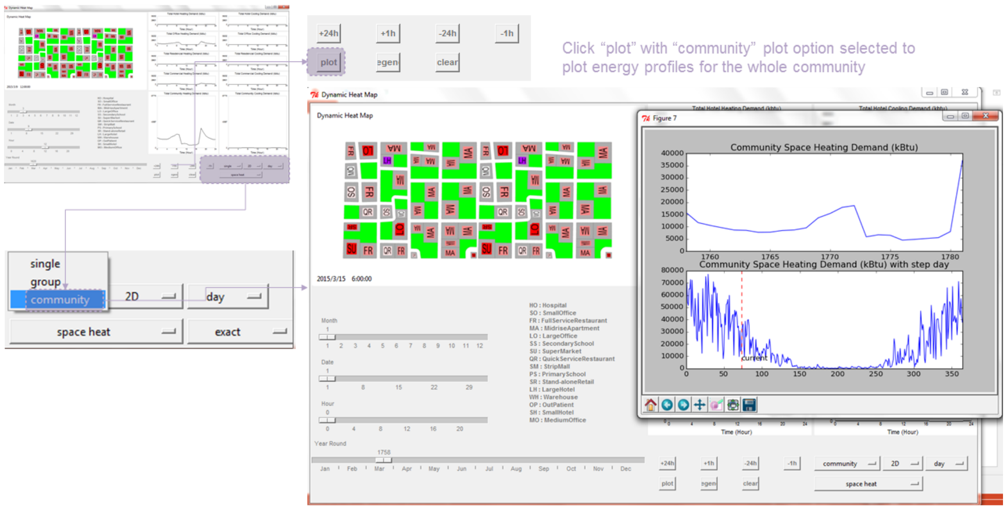

FIGURE 1.9: The plot shows the aggregated space heating energy demand for the whole community

producers. By comparing these two graphs the users can assess the effectiveness of energy recovery strategy in reducing the space heating demand of the group of buildings. A demonstration of presented in Section 1.4.1

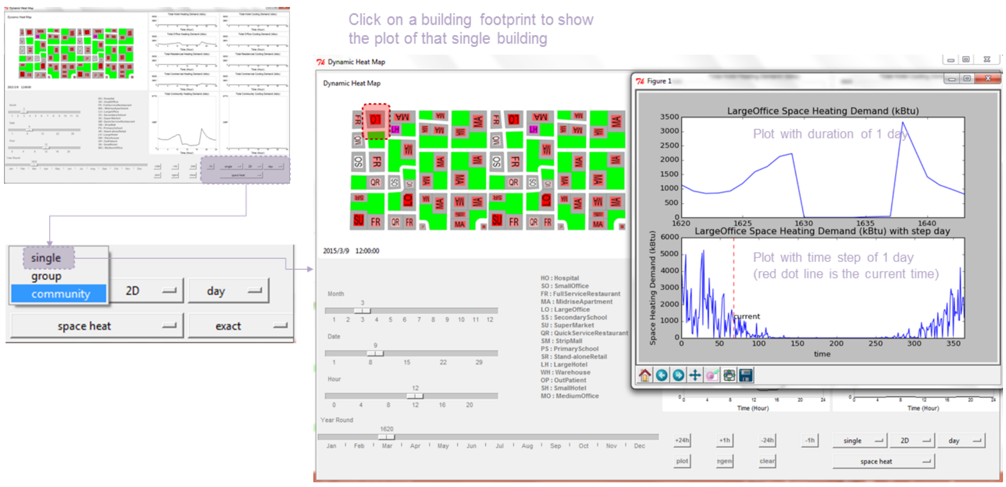

FIGURE 1.10: Click on a building foot print shows the energy plot of this building

1.3.5.2 Providing Temporal Context in Data Plot

Brownrigg suggested that it is necessary to provide a temporal context in a space-time map: “To comprehend how drastically or subtly something is changing, how fast or slow, in what direction, in relative to its environment, etc., demands some knowledge of the history of the change, an awareness of the objects’ properties before and after the change.” [2].

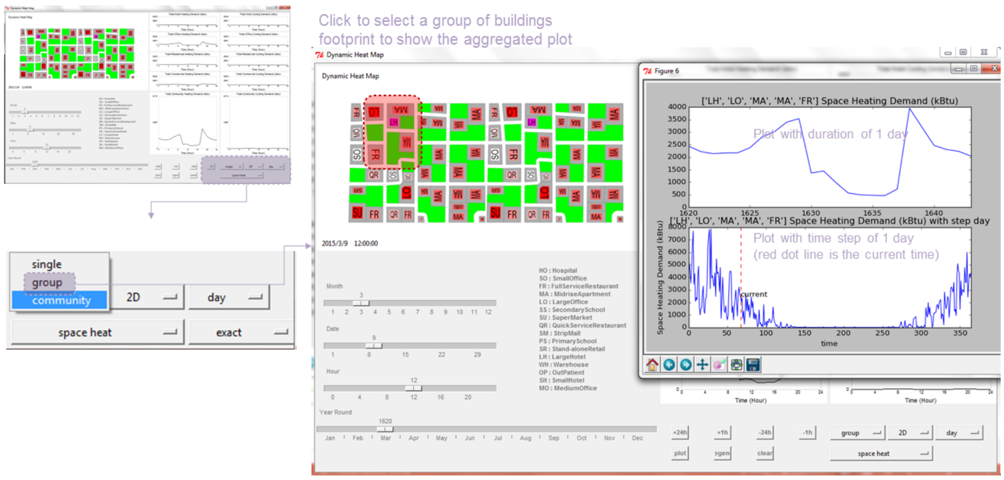

FIGURE 1.11: Click on a building footprint shows the energy plot for the selected building group

In the current map image display, the temporal context is created by providing three "periodical" slider bars that allows the user to jump with time steps of month, day and hour.

In data plots, the temporal context is created by providing a "longitude" and "latitude" comparison of energy demand. "Longitude" here refers to the comparison of adjacent time spots. It shows what the states of the direct future or past comparing to the state of the current time. "Latitude" here refers to the comparison of the current time spot with all similar time instances, for example, all 12:00:00 energy demand of the year. It shows how the current instance differ from similar instances.

For the current interface design, the top plot presents a longitude temporal context of the energy demand of the incoming 24h, week or month. Corresponding to the duration of time of the top plot (24h or one week or one month), the bottom plot presents the latitude demand context of the same hour with a step of one day, one week or one month. For example, in Figure 1.13, the top plot shows the Space Heating Demand for Large Office from 2015/3/9 12:00:00 to 2015/3/10 11:00:00, with a duration of 24h. The bottom plot shows the energy demand of the Large Office for all 12:00:00 of the 365 days of the year (the red dot line indicates the 12:00:00 of around the 70th day of the year, which is the date of Mar. 9th).

By providing the temporal context, the viewers are provided with a general understanding of whether the changing of energy demand behavior is drastic or subtle and whether

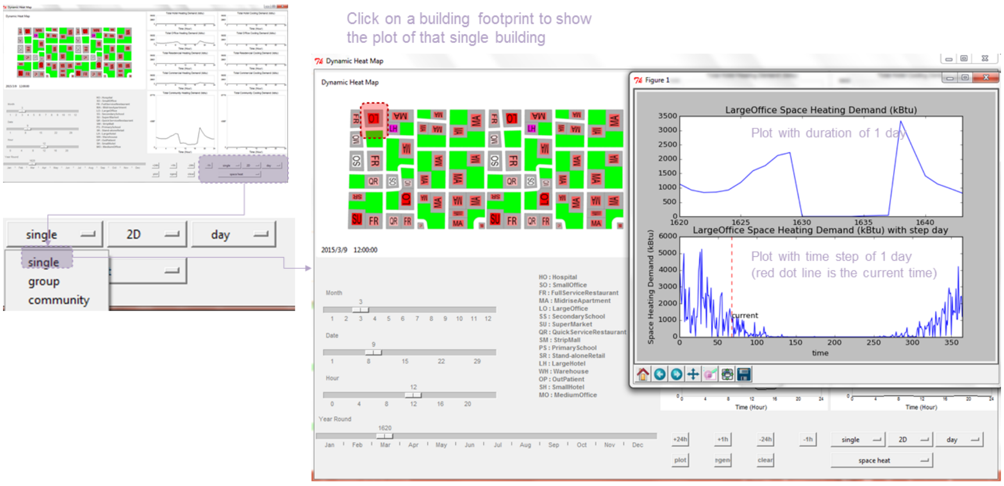

FIGURE 1.12: The data plot presents the longitude and latitude comparison of energy demand, the top plot presents a temporal context of the energy demand of the next 24h, the bottom plot presents the time context of the demand of the same hour throughout the 365 days of the year

FIGURE 1.13: By changing option in the option menu. User can choose to display a longitude latitude comparison with the time unit of day, week or month. The top plot shows the energy demand for the next day, week and month from left to right; the bottom plot shows the energy demand of this hour in the 365 days of year, 52 weeks of a year or the 12 months of a year from left to right

a drastic change is coming and whether the current demand is high, low or moderate comparing to the overall distribution over time and space.

1.4 Use Case Demonstrations

1.4.1 Use Case I: Identification of Energy Recovery Opportunity

In this section, the researcher present a general approach on how to use the dynamic energy map interface to identify the energy recovery opportunities. The process of space cooling will produce reject heat. As is explained in Section **??**, the ammount of cooling-induced reject heat is positively corelated to the cooling demand. Thus a building with high cooling demand will also have a large amount of reject heat. The reject heat from this building or group of buildings could possibly be recovered for use within the building such as pre-heating water or outside air or be transmitted to other buildings that have space heating demand so that the total space heating demand of the group of buildings could be reduced.

For the interface design in the current study, the researcher used a bivariate color ramp in space heating and cooling energy demand data representation that depicts the hourly space heating and space cooling demand on the same map (Figure 1.14) . Red represents high heating demand and blue represents high cooling demand. The closer the color cell is to the top, the lower cooling demand. The closer the color cell is to the left, the lower heating demand. The cells on the diagonal line (purple colored cells) represent buildings that have relatively similar heating and cooling demand. The cells to the upper right of the diagonal represents buildings that are heating dominated and the cells to the lower left of the diagonal represent buildings that are cooling dominated. The current breakpoints are decided through the "Quantile method" [12] for demonstration.

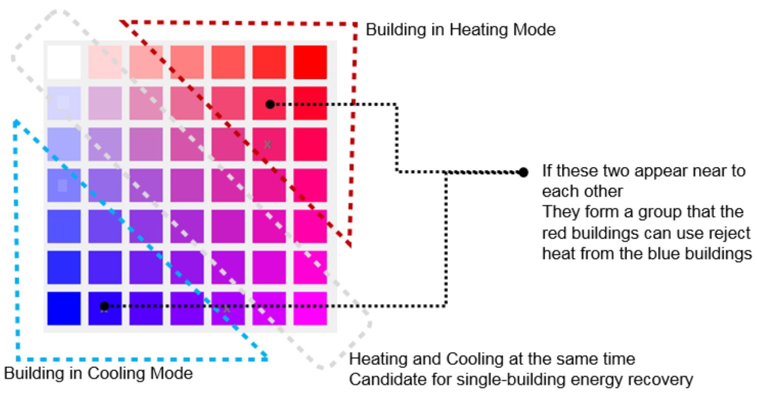

FIGURE 1.14: The bivariate color ramp displays two variables at the same time: space heating and space cooling. It better displays the co-relation between these two variables and thus helps users to identify energy recovery opportunities

With the dynamic energy map, the buildings colored in one of the colors in the bottom rows of the legend are buildings with high cooling demand. With the dynamic energy map, one can identify the potential reject heat suppliers and consumers over time (Figure 1.15).

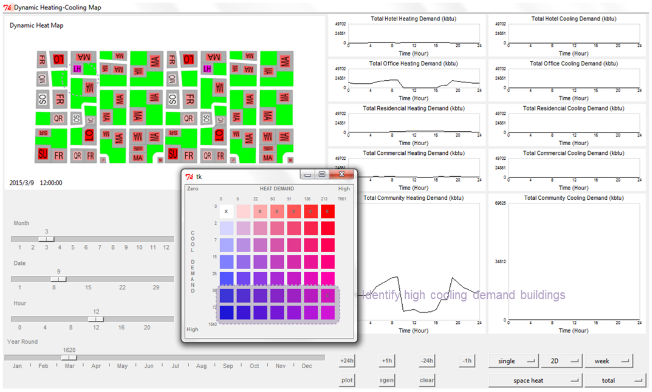

FIGURE 1.15: In the demonstration, buildings with a high cooling demand have colors on the bottom rows of the legend, thus Large Hotel and Large Office are identified as potential reject heat energy suppliers

Users can then calculate the "energy recovery potential" in the dynamic energy map with a specified time duration and step (Figure 1.16).

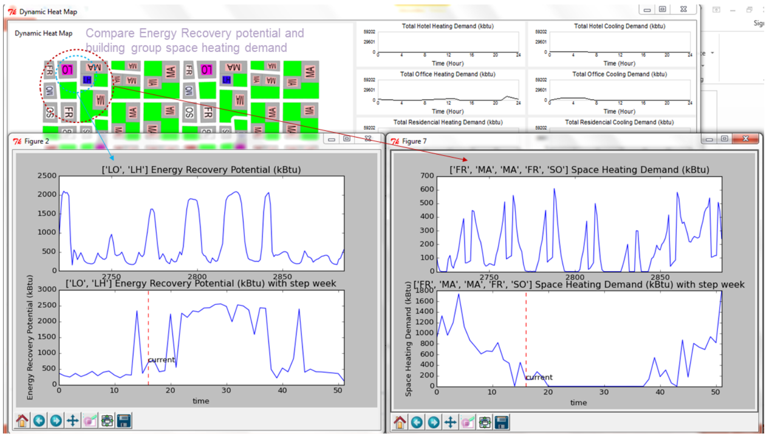

FIGURE 1.16: The users can calculate the energy recovery potential of the group of reject heat suppliers (Large Office and Large Hotel). They can also calculate the total heat demand of the surrounding buildings with space heating demand (two FirstService Resturant, two Midrise Apartment and one Small Office).

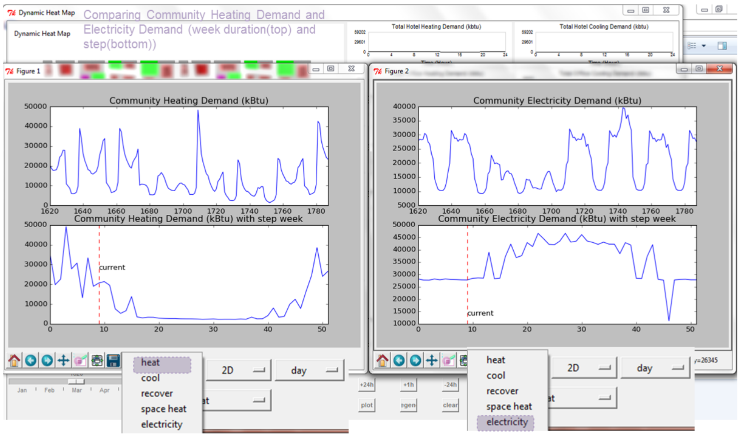

FIGURE 1.17: In this example, the users compare the week-wise heating and electricity demand

1.4.2 Use Case II: Sizing CHP Plant

1.5 Implementation tools and strategy

The software or platform involved in the project include EnergyPlus for building simulation, CityEngine for 3D modeling and image generation.

Imagemagic is used for converting and resizing images. For the creation of animated maps, “ffmepeg” was used for connect image sequences to animation.

Python 2.7 for interface design. The interface is written in Python2.7 with standard Tkinter graphic package including the data plot section. Pandas and numpy packages are used in data manipulation. Matplotlib and ggplot are used for creating data plots.

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